Effect of Size on the Stability of Narrow Foundation Pits

1. Introduction

Currently, the trend of urban construction in China towards underground space is becoming more and more obvious; the importance of tube-tunnel and tube-network engineering is increasing daily. In such projects, the foundation pit is typically a long trench, and the width of which is small in comparison to its depth; supporting structures on both sides of the foundation pit are close to each other.

In practice, this kind of foundation pit is called the foundation trench, and the dependence of foundation pit deformation and stability on geometrical factors such as width, length, and depth is called the “size effect” [1]. By comparing practice with theory, Zeng and Yang [2] found that foundation pit width has a significant impact on uplift stability. Investigating antiuplift stability, Huang and Qin [3] found that the safety factor calculated by the foundation-bearing capacity model without considering the width term is too large. By comparing software simulation results, Wang and Zhang [4] found that the opposite retaining wall of a narrow foundation pit has a restrictive effect on the pit’s antiuplift stability. Therefore, the trapezoidal failure mode is proposed, which well considered the influence of the width of foundation pit on the antiuplift stability of narrow and long foundation pit. Qi [5] considered the influence of foundation pit width on the antiuplift safety factor K at the bottom of the foundation pit. As the pit width decreases, K increases at an increasingly fast rate. Qin et al. [6] found that increasing the depth-to-width ratio reduces the antiuplift stability coefficient. Wang et al. [7] proposed a deformation mechanism for narrow foundation pits based on the Terzaghi mechanism using the deformation increment of the Ricken distribution function and calculated K considering the geometric size of the foundation pit. Parameter analysis and case verification showed that the size effect is more obvious at smaller width-to-depth ratios and that the antiuplift safety coefficient is also larger. Xiao et al. [8] analysed and processed field-monitoring data from 92 foundation-pit excavations with different widths in soft soil areas of China, finding that narrow foundation pits showed an obvious size effect due to the influence of the opposite retaining wall. Ge [9] studied support structure deformation after the excavation of foundation pits with different length-to-width and width-to-depth ratios by
numerical simulation. Narrow underground railway founda-
tion pits exhibited a spatial effect: the displacement of the 
long side changed significantly. Previous studies have shown 
that the shape and size of the foundation pit have a signif-
ificant influence on deformation and stability. Wang [10, 11] 
proposed an improved algorithm considering these factors. 
In practical engineering, it is well established that narrow 
foundation pits are more stable. For example, as long as 
a transverse plate support is adopted, the retaining structure 
of a general shallow trench can maintain the stability of the 
foundation pit without insertion of soil below the pit bottom 
[12]. Research suggests that this is because the retaining 
structures on both sides of the trench restrain the pit 
bottom soil.

When the foundation pit cannot meet stability specifi-
cations, the embedded depth of supporting piles is usually 
increased. However, for a narrow foundation pit, utilising the 
size effect should be given priority, because the existing 
specification algorithms assume that the foundation pit is very 
wide and only take a one-sided retaining structure and the soil 
in the passive zone as the research objects. As a result, the 
influence of foundation pit width is not reflected in standard 
stability analysis by normative methods [12], resulting in great 
design waste. Some scholars have carried out fruitful research 
work, combined with the finite element numerical model, and 
analysed the narrow instability characteristics of the foun-
dation pit. Using Coulomb–Wedge analysis, Ying et al. [13] 
modelled the passive earth pressure of the translational rigid 
retaining wall of a narrow foundation pit and deduced an-
alytical formulas for the sliding crack surface inclination and 
passive earth pressure coefcient in a passive limit state. 
Zheng et al. [14] analysed the influence of the length and 
flexural strength of the retaining structure on the failure mode 
and safety factor; they proposed that the sliding surface 
touching the enclosure structure should be used to distinguish 
the wide and narrow foundation pits. Compared with a wide 
foundation pit, a narrow foundation pit has more advantages 
in stability because of its smaller size [15].

When applying a circular sliding mode, the sliding 
surface of a narrow foundation pit requires spatial effect 
corrections to improve the safety factor. Wang [16] sim-
plifed the uplift deformation of foundation pit as the plastic 
zone expansion problem caused by the load applied to the 
bottom of the pit on the elastic half space.444 At the same 
time, substantial progress has been made in developing an 
antioverturning stability algorithm [17, 18] that incorporates 
the foundation pit width into an antiuplift stability formula 
based on the circular arc-sliding mode [12, 19–22] and the 
foundation-bearing capacity mode [23]. In addition, Zhang 
et al. [24] and Liu et al. [21] have studied the uplift failure 
mechanism of long and narrow foundation pits via cen-
trifugal model tests. Such studies have rarely used numerical 
analysis to verify the size effect and classify foundation pits 
under realistic conditions. Hoszeinzadeh and Joosse [25–28] 
studied the inluence of soil coupling in the passive area of 
foundation pits on the stability of foundation pits, calculated 
the coefcient of passive earth pressure and active earth 
pressure, and introduced the strengthening factor of passive 
earth pressure to optimise the embedding depth of the 
envelope structure.

In the present study, fnite element simulations are 
conducted using standard algorithms that relect actual 
working conditions. A semi-infinite-medium elastic model 
not considering the size effect and an infinite-medium elastic 
model considering the size effect are created to simulate the 
mechanical characteristics of foundation pits under various 
working conditions. Combining the numerical results with 
results in the literature, this study reveals the essential 
mechanism of the foundation pit size effect and shows the 
thoretical reasons for the design waste in existing algo-
rithms. Finally, foundation pits are classifed according to 
their critical width, which can be used to optimise sup-
porting structures.

2. Analysis of Mechanical Deformation 
Characteristics of a Narrow Foundation Pit

In the model, semi-infinite and infinite elastic spaces are 
used to simulate unilateral structures and analyse the overall 
situation. Given the excavation depth $H$, the embedded 
depth $D$ of the supporting pile, and the variable foundation 
pit width $B$, the main shape variables and stress values may 
be extracted from the simulation results. The critical width of 
the foundation pit is determined by a single parameter 
depending on $B$: when the parameter is greater than the 
critical width, a semi-infinite elastic medium is used for the 
analysis; when it is less, an innite one is used, conforming to 
the stress characteristics of the narrow foundation pit. When 
the parameter is equal to the critical width, the result is the 
same for both cases.

The usual support method for a foundation trench was 
assumed: a steel sheet pile (type IV Larsen) and one support 
(a seamless steel pipe with a diameter of 609 mm and a wall 
 thickness of 12 mm). The construction process simulated an 
actual-layered excavation. The $U$-shaped steel sheet pile was 
converted into rectangular sections with equivalent mo-
mants of inertia. The Mohr–Coulomb plastic model was 
used for soil; the supporting structure was assumed to be 
elastic. For simplicity, only a single soil layer is considered 
and silty clay representative in Jingzhou area is selected. Its 
physical and mechanical parameters were selected on the 
basis of a geological exploration report. The analysis steps 
during modelling were set according to the actual con-
struction process: the steel sheet pile was applied, and the 
steel pipe support was arranged after the corresponding soil 
layer was excavated. In the process of balancing in situ stress, 
the supporting structural unit was “killed”; it was then ac-
tivated for the analysis that ensured the functioning of the 
subsequent support [29]. The contact between the steel sheet 
pile and soil was realized by binding constraints. The hori-
zontal displacement of the left and right boundaries of the 
model was limited, and the bottom surface was constrained 
by fixed supports. The boundary constraints of the model are 
shown in Figure 1. The material parameters in the modelling 
process are shown in Table 1.

The analysis was performed using ABAQUS fnite ele-
ment software. The grid was unevenly divided to improve
the accuracy and efficiency of the plane-strain calculations. The division of the grid should follow the principles of regularity, orderliness, and appropriate density, but to reduce the computational workload and facilitate convergence, it is permissible to increase the mesh density for more significant parts [30]. The size of this model was \(100 \times 50\) m. To meet the requirements of the two-dimensional model analysis area, the excavation boundary of the foundation pit is extended outward by more than twice the excavation depth and the bottom of the foundation pit is more than twice the excavation depth [31]. A total of 1536 units and 1617 nodes were divided, and the type of grid unit was CPE4R. When arranging seeds, the global seeds were arranged as 3, and the seeds of key parts were coded as 1. The mesh division diagram of the model is shown in Figure 2.

Three common length specifications (9, 12, and 15 m) were selected for steel sheet piles. Based on the actual project, three groups of numerical simulation tests were set (Table 2); the working conditions were numbered in the order of \(D\) from small to large. The simulation results for the equivalent plastic strain of a narrow foundation pit in a semi-infinite and infinite elastic medium are shown in Figure 3. Considering the small size of foundation pit in engineering practice, in order to display the failure mechanism clearly and intuitively, we took the working conditions as follows: \(H = 10\) m, \(B = 14\) m, and \(D = 15\) m. The simulation results are shown in Figures 4 and 5.

<table>
<thead>
<tr>
<th>Material</th>
<th>(\rho) (kg·m(^{-3}))</th>
<th>(E) (kPa)</th>
<th>(\mu)</th>
<th>(\varphi) (°)</th>
<th>(c) (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silty clay</td>
<td>1800</td>
<td>(1 \times 10^4)</td>
<td>0.3</td>
<td>20</td>
<td>19</td>
</tr>
<tr>
<td>Steel pipe support</td>
<td>7800</td>
<td>(2 \times 10^8)</td>
<td>0.3</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Sheet pile</td>
<td>7800</td>
<td>(2 \times 10^8)</td>
<td>0.3</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

It can be seen from Figure 3(b) that for a narrow foundation pit, the strain distribution of soil in the passive zone is similar under both conditions. The soil close to the base near the supporting piles on both sides produces the maximum equivalent plastic strain; this weak area is prone to stress under deformation during construction. However, the stress change in the case of the infinite space is small. For observational convenience, a working condition with large size was selected. At this time, the foundation pit’s width was close to critical, so the difference between the two cases was not significant. Ground overload and excavation cause a large pressure difference between the interior and exterior of the pit. When the shear strength of soil is exceeded, uplift instability characterized by stress strength failure will occur. As shown in Figure 5(a), the pressure difference forces the active zone on both sides to squeeze the soil in the passive zone and the support pile effectively limits the displacement of the soil outside the pit. The soil below the pit bottom undergoes uniform and small deformation that is significantly less than a maximum deformation of 50 mm at 2 m above the pit bottom (Figure 4), which is mainly due to the lack of internal support near the pit bottom, low stiffness of the support structure, and relative lack of deformation resistance. In Figure 5(b), because of excavation and unloading, the soil in the passive zone can only rise upward after the displacement on both sides is limited. The uplift is largest in the middle of the pit. Under the joint action of the pressure difference and support structure constraints, the soil on both sides has local stress concentrations and maximum equivalent plastic strain. It can be seen from Figure 4 that the top of the supporting pile has a displacement trend outside the foundation pit, whereas the direction of the substructure is opposite. Thus, the foundation pit shows certain “skirting” failure characteristics.

In total, eight quantities, including the axial force of the steel pipe support and the maximum stress and displacement...
in the horizontal direction of the steel sheet pile, were extracted from the simulation results. The curve intersections of different parameters under the same working conditions were located. Under the same working condition, the $B$ value at different intersections changed little, fluctuating slightly around a single value. This confirms the existence of critical foundation pit width. The results on each side of the critical value, however, were obviously different. Above the critical width, the two curves were very close and almost coincided, and the existing algorithm was fully applicable. Below it, however, the numerical difference was large; this was increasingly obvious as the pit became narrower. The current method of unilateral structural analysis is unduly conservative, resulting in great waste; the foundation pit size effect should be fully utilised.

### Table 2: Actual working conditions in simulations.

<table>
<thead>
<tr>
<th>Excavation depth $H$ (m)</th>
<th>Embedded retaining pile depth $D$ (m)</th>
<th>Foundation pit width $B$ (m)</th>
<th>Bottom support to pit bottom distance $d$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>5, 8</td>
<td>2, 4, 6, 8, 10, 12</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>3, 6, 9</td>
<td>3, 5, 7, 9, 11, 13</td>
<td>5</td>
</tr>
<tr>
<td>8</td>
<td>4, 7</td>
<td>4, 6, 8, 10, 12</td>
<td>7</td>
</tr>
</tbody>
</table>

### Figure 3: Equivalent plastic strain of a narrow foundation pit for (a) semi-infinite and (b) infinite elastic medium.

### Figure 4: The horizontal displacement of the supporting pile in the case of an infinite elastic space (m).

When calculating the antioverturning safety factor, it is assumed that, in the event of instability and failure of the foundation pit, the lowest support point is the rotation centre, and the wall bears the active earth pressure on one side of the active zone and the passive earth pressure on one side of the passive zone [32]. According to classical earth pressure theory, when the foundation pit overturns, the soil in the active area will form a slip-surface angle to the vertical direction of $(45\degree - \phi/2)$, where $\phi$ is the internal friction angle of the soil; in the passive area, the angle will be $(45\degree + \phi/2)$.

### 3. Study of the Influence of Width on Foundation Pit Stability

When calculating the antioverturning safety factor, it is assumed that, in the event of instability and failure of the foundation pit, the lowest support point is the rotation centre, and the wall bears the active earth pressure on one side of the active zone and the passive earth pressure on one side of the passive zone [32]. According to classical earth pressure theory, when the foundation pit overturns, the soil in the active area will form a slip-surface angle to the vertical direction of $(45\degree - \phi/2)$, where $\phi$ is the internal friction angle of the soil; in the passive area, the angle will be $(45\degree + \phi/2)$. Wang [17] discussed in detail the relationship between the formation of the sliding surface in overturning failure and the width of the foundation pit. If $\alpha$, the relative width of the pit, is defined as the ratio of the excavation width $B$ to the embedded depth $D$ of the supporting pile, it is not difficult to show that, when $\alpha$ satisfies $B/D < \tan (45\degree + \phi/2)$, the sliding surface of overturning failure obtained from Coulomb earth pressure theory will pass through the opposite retaining wall. Thus, the sliding surface cannot be formed as expected because of the restraint, so the
The antioverturning stability of the foundation pit is improved. On the other hand, slip-surface formation will not be affected by the opposite retaining wall. Therefore, in practice, when \( B/D < \tan(45° + (\phi/2)) \), the restraint due to retaining structures on both sides of the soil at the bottom of the pit should be considered. We can see this more intuitively in the schematic diagram, as shown in Figure 6.

The present standard method of calculating the circular slip stability safety factor of retaining structures assumes that the centre of the slip arc is located at the lowest support elevation. Wang [12] constructed an antiuplift safety factor algorithm considering the width of the foundation pit by making geometric assumptions and conducting physical analysis of the circular slip surface and the position of the centre of the circle. If \( d \) is the distance between the lowest support and the pit bottom, then (according to Wang’s assumption) the lowest support point is the centre of the circle and the distance \( d + D \) from it to the bottom end of the support structure is the radius of circular arc-sliding failure. Obviously, from the geometric relationship, when the width of the foundation pit satisfies \( B < \sqrt{(D + d)^2 - d^2} \), the circular slip surface is restrained by the opposite retaining wall. Then, the soil wedge in the passive zone is wedged tightly and cannot slide. Because the coexistence of the two forms of instability, when the foundation pit is damaged, affects the simulation results, the simulation value should be compared with the average theoretical value, and the antisliding stability of the foundation pit is improved. On the contrary, the sliding surface can be formed smoothly. Therefore, when \( B < \sqrt{(D + d)^2 - d^2} \), \( D \) should be appropriately reduced in the design. We can see this more intuitively in the schematic diagram, as shown in Figure 7.

The above analysis leads to different critical widths. Antioverturning theory involves three variables: \( B, D, \) and \( \phi \). Once the construction site is determined, \( \phi \) is fixed. Therefore, the critical value of \( B \) depends only on \( D \), and the two are related linearly. The graph of \( B \) against \( D \) is a straight
line passing through the origin in the plane coordinate system.

Circular slip theory, on the other hand, involves three variables: $B, D,$ and $d$; the critical value of $B$ depends on both $D$ and $d$. The graph of such a bivariate first-order functional relationship is a surface in a three-dimensional space. Therefore, it is impossible to consider the $B-D$ relationship a single curve. We considered $B$ the dependent variable and the actual working conditions $D, \phi$, and $d$ the independent variable, focusing mainly on the relationship between theoretical and simulated values under various working conditions.

We averaged the intersection positions of different result parameter curves under the same working conditions to eliminate errors and obtain more accurate critical values (see Table 3 and Figure 8) for the critical widths of foundation pits under different working conditions.

As shown in Figure 8, under different working conditions, the jagged lines representing the average simulated and theoretical values were of similar form, and the fluctuations around the average were also very similar. The simulated values were generally less than the average theoretical values. Except in a few cases, the difference between them increased with $D$. Thus, the finite element model was basically verified, showing that our theoretical analysis of the influence of width on stability is correct; the two expressions for the critical width of foundation pits are reasonable, and a narrow foundation pit is more stable.

To obtain the theoretical critical width from the geometry, it was necessary to make certain assumptions that the lowest support point was the rotation centre of the foundation pit. Whether these assumptions are consistent with the actual failure form of foundation pits remains to be discussed. In addition, in the simulation process, the diversity of contact modes between soil and supporting structure also affects the calculation results, and a variety of factors lead to slight differences between the two.

4. Critical Width and Classification of Foundation Pits

Existing foundation pit specifications fail to consider the restraint effect of the retaining structures on both sides of the soil at the bottom of the pit. When the pit is narrower than the critical width, this effect is significant. Therefore, the determination of the critical width is very important for the design and optimisation of narrow foundation pits. Wang [12, 17] obtained different critical widths from overturning and circular-slip failure theory; however, these values are given only in principle and do not take into account engineering practice. In engineering practice, both types of failure effects coexist when the foundation pit is unstable. In this way, the determination of the critical width of the foundation pit is not only more scientific and reasonable but is also more convenient for engineering application.

The feasible idea is to calculate both theoretical values under actual working conditions and adopt a practical value based on the smaller of the two so that the narrow foundation pit is not damaged. Let $B_\text{QC}$ and $B_{\text{HCmin}}$ respectively, represent the theoretical values of the antioverturning and the minimum circular slip. The smaller value of the two will be denoted $B_{\text{SC}}$, and the value to be used in practice will be denoted $B_{\text{GC}}$. When $D$ is constant, $B_{\text{SC}}$ is only associated with $\phi$; there is no minimum value. The circular slip $B_{\text{HC}}$ is only affected by $d$: the closer the support is to the base, the smaller is the theoretical value of $B_{\text{HC}}$. It is specified in the standard for foundation pits (JGJ120-2012) that the lower support distance should not be less than 3 m; therefore, $B_{\text{HCmin}}$ should be taken at $d = 3$ m. Thus, $\phi$ alone determines the relative magnitudes of $B_{\text{QC}}$ and $B_{\text{HCmin}}$.

Typical silty clay in the Jingzhou area will be used as an example when calculating $B_{\text{QC}}$. The internal friction angle of most soil in the area is concentrated at about 20°, so we assume $\phi = 20°$. The critical width of foundation pits under actual working conditions is shown in Table 4 and Figure 9 [33].

As shown in Figure 9, the theoretical value of the minimum circular slip under the actual working condition approximately meets the linear relationship $B_{\text{HCmin}} = 1.068 \times D + 2.04$. When $\phi > 30°$, $B_{\text{QC}} > B_{\text{HCmin}}$ for all $D \in [3, 9]$, so $B_{\text{GC}} = B_{\text{SC}} = B_{\text{HCmin}}$. When $14.5° \leq \phi \leq 30°$, the point where $B_{\text{QC}} = B_{\text{HCmin}}$ moves up the straight line

$$B_{\text{HCmin}} = 1.068 \times D + 2.04$$

from the point $(3, 5.20)$ to $(9, 11.62)$. Taking $\phi = 20°$ as an example, $B_{\text{QC}}$ and $B_{\text{HCmin}}$ intersect at $D = 5.68$ m. When $D \in [3, 5.68]$, Figure 9 shows that $B_{\text{QC}} \leq B_{\text{HCmin}}$ and that $B_{\text{SC}} = B_{\text{QC}} = D \times \tan (45° + 20°/2)$. However, when $D \in [5.68, 9]$, $B_{\text{HCmin}} > B_{\text{QC}}$ and therefore, $B_{\text{SC}} = B_{\text{HCmin}} = 1.068 \times D + 2.04$. In short, when $\phi = 20°$, the function expression of the real value is shown as follows:

$$B_{\text{SC}} = \min \{B_{\text{QC}}, B_{\text{HCmin}}\},$$

$$= \begin{cases} 
D \times \tan \left(45° + 20° \over 2\right), & 3 \leq D \leq 5.68, \\
1.068 \times D + 2.04, & 5.68 < D \leq 9.
\end{cases}$$

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$$= \begin{cases} 
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1.068 \times D + 2.04, & 5.68 < D \leq 9.
\end{cases}$$
Table 3: Critical width of foundation pit under different working conditions (m).

<table>
<thead>
<tr>
<th>Working conditions (H, D)</th>
<th>Range of B</th>
<th>Critical width (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Anti-overturning (theory)</td>
</tr>
<tr>
<td>1: (H = 6, D = 3)</td>
<td>3–7</td>
<td>4.28</td>
</tr>
<tr>
<td>2: (H = 8, D = 4)</td>
<td>4–8</td>
<td>5.71</td>
</tr>
<tr>
<td>3: (H = 4, D = 5)</td>
<td>2–8</td>
<td>7.14</td>
</tr>
<tr>
<td>4: (H = 6, D = 6)</td>
<td>3–9</td>
<td>8.57</td>
</tr>
<tr>
<td>5: (H = 8, D = 7)</td>
<td>4–12</td>
<td>10.00</td>
</tr>
<tr>
<td>6: (H = 4, D = 8)</td>
<td>2–12</td>
<td>11.43</td>
</tr>
<tr>
<td>7: (H = 6, D = 9)</td>
<td>3–13</td>
<td>12.85</td>
</tr>
</tbody>
</table>

![Figure 8: Critical width of foundation pits under different working conditions.](image)

Table 4: Critical width of foundation pits when supporting piles are embedded in different depths (m).

<table>
<thead>
<tr>
<th>The corresponding width of the two theories</th>
<th>Critical width of foundation pits when supporting piles are embedded in different depths (m)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>BQC (φ = 20°)</td>
<td>3.87 5.17 6.46 7.75 9.04 10.33 11.62</td>
<td></td>
</tr>
<tr>
<td>BQC (φ = 30°)</td>
<td>4.28 5.71 7.14 8.57 10.00 11.42 12.85</td>
<td></td>
</tr>
<tr>
<td>BHCmin</td>
<td>5.20 6.93 8.66 10.39 12.12 13.86 15.59</td>
<td></td>
</tr>
<tr>
<td>BSC (φ = 20°)</td>
<td>4.28 5.71 7.14 8.49 9.54 10.58 11.62</td>
<td></td>
</tr>
<tr>
<td>BGC (φ = 20°)</td>
<td>4.28 5.51 6.73 7.95 9.17 10.40 11.62</td>
<td></td>
</tr>
</tbody>
</table>

Such a piecewise function is inconvenient for applications. Instead, we construct the purple line shown in the figure, which connects the point (3, 3tan (45° + 20°/2)) with the point (9, 11.6), for the engineering application value of linear equations and keep a safe reserve. The equation of the line for the case φ = 20° is \( B_{GC} = 1.223 \times D + 0.62 \). Generally, for any φ in the interval \([14.5°, 30°]\), the position of (3, 3\tan (45° + \phi/2)) changes, and therefore, the equation of \( B_{GC} \) becomes \( B_{GC} = [1.94 - 0.5 \tan (45° + \phi/2)] \times D + 4.5 \tan (45° + \phi/2) \). When \( 0° < \phi < 14.5° \), Figure 9 shows that \( B_{QC} = B_{HCmin} \) for all \( D \in [3, 9] \) so that \( B_{GC} = B_{SC} = B_{QC} = D \tan (45° + \phi/2) \).

By considering the two forms of instability, overturning failure and circular slip failure, under actual working conditions and taking the smaller value, the practical critical width of the foundation pit has been obtained. This can be the basis of a practical classification of foundation pits, depending on the width \( B \), pile depth \( D \), and internal friction angle of soil at the bottom of the pit \( \phi \). We define a “narrow” pit as the one satisfying Equation (2). When Equation (2) holds, the restraint effect of retaining structures on both sides of the soil at the bottom of the pit should be considered. Otherwise, the foundation pit is classified as “wide,” and the standard algorithm is fully applicable.

\[
B < \begin{cases} 
1.068 \times D + 2.04, & 30° < \phi, D \in [3, 9], \\
1.94 - 0.5 \tan \left( \frac{\pi}{4} \cdot \frac{\phi}{2} \right) - 5.81, & 14.5° \leq 30°, D \in [3, 9], \\
D \times \tan \left( \frac{\pi}{4} \cdot \frac{\phi}{2} \right), & 0° < \phi < 14.5°, D \in [3, 9]. 
\end{cases}
\]

(2)

When determining the critical width of the foundation pit, not only the provisions on the insertion ratio of the retaining structure in the standard must be considered but also the principles of feasibility and economy. For example, when \( H = 4 \) m, one can theoretically make \( D = 4 \) m, but this would be wasteful and impractical. Therefore, the value range \( 3 \text{m} \leq D \leq 9 \text{m} \) can meet almost all the needs of practical engineering. The critical width given is only related to \( D \) and \( \phi \); the type of supporting pile has no influence on it, and the excavation depth \( H \) is not involved. \( B_{HCmin} \) is always determined with \( d = 3 \) m. In this way, when \( D \) is constant and \( H \) changes, there is no consequent change in the critical width, effectively avoiding the problem that the critical width is not uniquely defined for all \( H \). At the same time, it ensures that the foundation pit classification is widely applicable to different combinations of \( H \) and \( D \). In fact, a larger \( H \) implies a larger \( D \) in varying degrees, and \( B_{HC} \) cannot take the minimum value, which may increase the
critical width slightly. This shows that the critical width obtained by comparing the size of \( B_{\text{QC}} \) and \( B_{\text{HC}}\) is relatively conservative for ensuring safety; the greater the excavation depth, the more the influence of foundation pit width on stability. Although the foundation pit classification in this study was based on the geological conditions of the Jingzhou area, it is also applicable to other areas. It has certain reference significance for the support design and construction of narrow foundation pits.

5. Conclusions

(1) For a given excavation depth, the foundation pit shows obvious size effects with the change in width. A foundation pit should be classified as “narrow” when the width of the foundation pit is less than the critical width and “wide” when it is greater.

(2) The main factors affecting the stability of a narrow foundation pit are its width, the embedded depth of the supporting pile, and the internal friction angle of the soil, and width is the primary factor. When stability requirements are not met, the embedded depth of the supporting pile should not be unthinkingly increased; instead, the influence of the width on the foundation pit should be considered. In addition, for a foundation pit with poor soil quality, increasing the internal friction angle is also very effective for strengthening the soil in the passive zone and improving the shear strength.

(3) For the support of a narrow foundation pit, the restraint effect of retaining structures on the soil at the bottom of the pit should be fully used to optimise the design and reduce costs.

Data Availability

The data supporting the results of this study are available on request from the corresponding author.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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