Research Article

Experimental Study on the Size Effect on the Probability Distribution of Concrete Compressive Strength

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Understanding the size effect on the probability distribution of concrete compressive strength is critical to accurately determining its probability distribution and to correctly calculating reliability. Compressive strength tests were performed on concrete cubic specimens with side lengths of 70 mm, 100 mm, and 150 mm, respectively. The Kolmogorov-Smirnov test results revealed that concrete compressive strength of each specimen size followed a normal probability distribution; that is, a change in specimen size did not affect the type of probability distribution. The point and interval estimates of the mean and standard deviation of concrete compressive strength showed that both the mean and standard deviation decreased, and the latter decreased to a greater extent with an increase in specimen size. The calculation results show that the standard value of concrete compressive strength also has a significant size effect, but the standard value increases with an increase in specimen size, which is opposite to the change pattern of concrete compressive strength mean and standard deviation. Finally, a reliability analysis was carried out using the compressive strength of a cubic concrete specimen with a size of 150 mm as an example, and the probability distributions for different specimen sizes were found to have a great influence on the reliability of the calculation results. Therefore, it is important to account for the size effect on the probability distribution of concrete compressive strength in practical engineering.

1. Introduction

Since Mayer published a monograph on structural safety in 1926 [1] that, for the first time, systematically discussed the concept of using probability theory to study structural safety, probabilistic reliability theory has been generally valued in both academic and engineering communities and has become a popular topic in civil engineering research [2–7], with relevant findings adopted by several codes [8–10] and applied to many practical projects.

When probabilistic reliability analysis and design of engineering structures are carried out, the following two conditions should be fulfilled. First, a reasonable limit state equation should be available. Thus far, many studies have been conducted on establishing the corresponding limit state equations for different engineering structures, the discussion of which is not repeated herein. Second, the probability distributions of relevant random variables should be accurately obtained, which is the basic prerequisite for probabilistic reliability analysis; otherwise, no correct conclusion can be drawn even with reasonable calculation models. In particular, the compressive strength of concrete is one of the most widely used basic variables in the analysis of engineering structures, as it is not only critical in probabilistic reliability analysis, but also the basis for determining the values of important indicators such as standard and design values of concrete compressive strength. For example, reference [8] clearly stipulates that, in the case of sufficient test data, the standard value of concrete strength should be determined according to indexes such as the quantile value (generally taken as 0.05), expectation, and standard deviation of the probability distribution of the strength. Therefore, it is crucial to have accurate and applicable probability distributions of concrete compressive strength for the design of engineering structures.

Many researchers have conducted fruitful studies to obtain accurate probability distributions of concrete compressive strength. Bartlett and MacGregor [11] statistically
analyzed the mean and variance of C20 concrete. Xiao et al. [12] found that the statistical characteristics of the compressive strength of recycled aggregate concrete were comparable to those of ordinary concrete of the same strength. The statistical characteristics of compressive strength were not significantly different. Wang et al. [13] established a statistical relationship between the compressive strength and service life of concrete using 10,317 specimens. Kilinc et al. [14] drilled 264 concrete specimens with a diameter of 28 mm from existing buildings and statistically evaluated their compressive strength. Chen et al. [15] investigated the probability distribution of concrete strength through more than 200 tests. Sahoo et al. [16] experimentally studied silica fume concrete using 490 specimens and proposed a probabilistic model that can reflect the variability in the mechanical properties of silica fume concrete. In the above studies, the results were obtained based on conventional tests of standard-size (generally cubic or cylindrical specimens) or small-size specimens; these are currently the common test methods used to determine the probability distribution of material strength. That is, a standard-size cubic specimen (e.g., that with a side length of 150 mm) was first used to measure the probability distribution of concrete compressive strength for this specific size, and then, the probability distribution of concrete compressive strength of real-size structural members was assumed to be the same as that of the standard-size specimen regardless of the real size of the structural members in actual engineering projects, and on this basis, a probabilistic reliability analysis was performed. This assumption obviously lacks a scientific basis because there is no theoretical and experimental data to prove that the probability distribution of concrete compressive strength remains the same for different sizes. Therefore, the correctness and rationality of the above assumption are questionable. Whether the difference in size has an impact on the probability distributions of random variables (e.g., probability distribution type, mean, and variance) must be confirmed experimentally. That is, the question of whether there is a size effect on the probability distribution of concrete compressive strength should be investigated. The above assumption is reasonable and feasible only if it shows that the probability distribution of concrete compressive strength has no size effect; otherwise, the probability distribution obtained by the above approach will not be applicable for practical engineering; that is, the results of the reliability analysis and the calculated standard values of compressive strength obtained based on the above assumption are likely to be wrong.

Therefore, whether there is a size effect on the probability distribution of concrete compressive strength is a question that must be answered. The finding is expected to have important theoretical and practical significance, because the reliability theory has been adopted by relevant codes in many countries, and concrete is one of the most widely used building materials; hence, numerous structural design problems would arise without an accurate probability distribution of concrete compressive strength. However, targeted research on whether there is a size effect on the probability distribution of compressive strength of concrete is lacking, posing an urgent problem, which we attempt to solve in this study. In fact, many experimental studies have demonstrated a significant size effect on the mean compressive strength of concrete [17–23], with most of these studies showing that the mean compressive strength of concrete decreases with increasing size. These research results provide a good reference for studying the size effect on the probability distribution of concrete compressive strength. However, existing studies have only investigated the size effect on the mean compressive strength but have not explored the size effect on the probability distribution of compressive strength comprehensively, which includes not only the mean, but also the probability distribution type and standard deviation. Thus far, no literature has studied the influence of a change in specimen size on the probability distribution type and standard deviation of compressive strength, nor has it analyzed the influence of the size effect on the probability distribution of concrete compressive strength on the standard value of concrete compressive strength and the corresponding reliability analysis results. To fill this research gap, it is necessary to experimentally demonstrate whether the probability distribution of concrete compressive strength has a size effect.

In summary, whether the probability distribution of concrete compressive strength has a size effect is crucial to design calculations of concrete structures; however, there are no relevant research findings. Hence, it is necessary to conduct an in-depth experimental study on this topic. To this end, in this study, compressive strength tests on concrete cubic specimens of three different sizes were first conducted. On this basis, the size effect on the probability distribution of concrete compressive strength was investigated through Kolmogorov-Smirnov (K-S) tests of the probability distribution type, point and interval estimation of the numerical characteristics of the concrete compressive strength, and proof by contradiction. Finally, how the size effect on the probability distribution of concrete compressive strength influenced the standard value of concrete compressive strength and the results of reliability analysis were discussed. The experimental results showed that there was a significant size effect on the probability distribution of concrete compressive strength and that analysis results without consideration of the size effect likely differ considerably from the actual situation.

2. Test Overview

C30 concrete was tested in this experiment. It was made of the following materials: ordinary Portland cement with a strength of 42.5 MPa, S95 mineral powder with a specific surface area of 470 m\(^2\)/kg and a density \(\geq 2.8\) g/cm\(^3\), fly ash, with a specific surface area of 425 m\(^2\)/kg, medium sand with a fineness modulus of 2.6 as fine aggregate, continuously graded crushed stone with a particle size of 5–20 mm as coarse aggregate, and tap water. Three sizes of concrete specimens were prepared using the same raw materials, mix ratios, and curing conditions. Table 1 shows the main chemical compositions of cement, mineral powder, and fly ash. The mix ratios of the concrete specimens are shown in
Table 1: Chemical composition (mass fraction, %) of binder materials.

<table>
<thead>
<tr>
<th>Chemical composition</th>
<th>SiO₂</th>
<th>Al₂O₃</th>
<th>Fe₂O₃</th>
<th>CaO</th>
<th>MgO</th>
<th>SO₃</th>
<th>Na₂O</th>
<th>TiO₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cement</td>
<td>50.56</td>
<td>36.78</td>
<td>1.95</td>
<td>0.62</td>
<td>0.22</td>
<td>1.41</td>
<td>0.42</td>
<td>2.10</td>
</tr>
<tr>
<td>Mineral powder</td>
<td>34.80</td>
<td>13.70</td>
<td>1.96</td>
<td>35.85</td>
<td>8.90</td>
<td>2.61</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Fly ash</td>
<td>50.35</td>
<td>33.82</td>
<td>5.16</td>
<td>5.50</td>
<td>1.58</td>
<td>0.40</td>
<td>0.70</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 2. The specimens were demolded 24 h after molding and cured for 28 days in a curing room with a temperature controlled at (20 ± 1) °C and a relative humidity of 95%.

Several factors were considered in the selection of the specimen sizes. First, in terms of size difference, if there is indeed a size effect on concrete compressive strength, the greater the difference in specimen size, the greater the difference in the probability distribution of the concrete compressive strength. Therefore, to better reveal the size effect, the difference in the specimen size during the test should not be too small, and if the test conditions allow, the larger the difference between the various sizes, the better. Second, regarding the minimum and maximum sizes of specimens, on the one hand, the minimum size should not be too small; otherwise, the test results may be greatly influenced by other factors, which would mask the size effect on the probability distribution of the concrete compressive strength. On the other hand, the maximum size should not be too large to not exceed the maximum loading value of the test equipment. For this reason and based on reference [24], cubic specimens with side lengths of 70 mm, 100 mm, and 150 mm were used in this study. According to probability theory, the larger the number of specimens, the closer the test data to the true probability distribution of concrete compressive strength; conversely, too few specimens may lead to a large error between in the test data and the true probability distribution. In addition, specimen preparation is a complex process, and the corresponding test cost can be high. Therefore, it is necessary to determine a reasonable number of specimens so that the test data can reflect the true probability distribution as closely as possible, while the test cost is controlled. Jiang et al. [25] suggested that the number of specimens for each set of tests be more than 10 in order to obtain reasonable characteristic values. Hence, 30 specimens of each size (70 mm, 100 mm, and 150 mm) were prepared in this study. However, due to problems such as the prepared specimens not meeting the requirements and sudden malfunction of the test equipment during the test, the test results of a few specimens were invalid. Ultimately, 29, 24, and 30 valid test datasets were obtained for the 70 mm, 100 mm, and 150 mm concrete specimens, respectively. Water-saturated specimens were used in the tests to eliminate an influence from water content. In addition, to eliminate the influence of the loading rate, specimens of each of the three sizes were subjected to strain-controlled loading at a rate of 0.0001/s. The test was carried out on a compression testing machine with a maximum load of 2000 kN. Each specimen was numbered in the form of AA-BB, with AA representing the side length of the specimen and BB representing the BB-th specimen. For example, 70–01 represented the first cubic specimen with a side length of 70 mm, and 100–20 represented the 20th cubic specimen with a side length of 100 mm.

The concrete compressive strength of each specimen obtained through the test is shown in Table 3.

3. Size Effect on the Probability Distribution of the Concrete Compressive Strength

The probability distribution of concrete compressive strength is determined by the probability distribution type and numerical characteristics. Therefore, the influences of size changes on the probability distribution type and numerical characteristics, which mainly include the mean (i.e., mathematical expectation) and standard deviation, of concrete compressive strength are analyzed based on the test data in Table 3. Note that the mean of concrete compressive strength has been studied extensively in the literature, where it was consistently concluded that there is a size effect on the mean. However, no special experimental study has been performed about the size effect on the standard deviation of concrete compressive strength. Therefore, this study focused on the investigation of whether there is also a size effect on the standard deviation of compressive strength.

3.1. Size Effect on the Probability Distribution Type of the Concrete Compressive Strength

A distribution hypothesis test is generally used to determine the probability distribution type of a random variable. Currently, there are various commonly used distribution hypothesis tests [26], of which the K-S test does not require a large sample size and has the advantages of good robustness and full data utilization.

The basic idea of the K-S test method is described as follows: for a random variable $X$, assuming that its theoretical cumulative distribution function is $F_X(x)$ and that the empirical cumulative distribution function obtained from the sample observations is $F_n(x)$, where $n$ is the sample size, $F_n(x)$ can be expressed as follows according to the method for establishing the empirical distribution function:

$$F_n(x) = \begin{cases} 0, & x < x_1 \\ \frac{i}{n}, & x_i \leq x < x_{i+1} \\ 1, & x \geq x_n \end{cases},$$  

(1)

where $x_1, x_2, \ldots$, and $x_n$ are the arranged sample data.
The specific steps of the K-S test method can be found in the literature [27], and there are also directly callable functions in the scientific computing software MATLAB.

Existing studies generally assume that the compressive strength of concrete follows a normal distribution. Therefore, this study used the K-S test to test the probability distribution types of the concrete compressive strength of the three specimen sizes to analyze whether they all follow a normal distribution. The distribution hypothesis test was performed at the significance level \( \alpha = 0.05 \), i.e., at the 95% confidence level. The test results are shown in Table 4.

The analysis results in Table 4 show that the K-S value of the concrete compressive strength of each of the three specimen sizes is smaller than the critical value for accepting the hypothesis, thus indicating that the concrete compressive strength always follows the normal distribution; that is, the size change does not change its probability distribution type.

### 3.2. Size Effect on the Numerical Characteristics of the Concrete Compressive Strength

Mean and standard deviation are the commonly used numerical characteristics of random variables. In the following, the size effect on the mean and standard deviation is analyzed using the point and interval estimation of the mean and standard deviation of the concrete compressive strength as well as proof by contradiction.

First, the point estimation of the mean and standard deviation of the concrete compressive strength was conducted based on probability and statistics theory. The results are shown in Table 5.

As seen in Table 5, the point estimate of the mean concrete compressive strength decreases from 45.60 MPa for the size of 70 mm to 43.04 MPa for the size of 150 mm, a decrease of 5.61%. The point estimate of the standard deviation of the concrete compressive strength decreases from 4.55 MPa for the size of 70 mm to 1.69 MPa for the size of 150 mm, a decrease of 62.86%, which is 11.20 times the decrease percentage of the mean. Therefore, both mean and standard deviation of concrete compressive strength are subjected to a size effect, and both decrease with increasing size. The decreasing trends are shown in Figure 1, in which the mean and standard deviation are made dimensionless using equations (3) and (4), respectively.

### Table 2: Concrete mix ratio.

<table>
<thead>
<tr>
<th>Strength</th>
<th>Cement</th>
<th>Mineral powder</th>
<th>Fly ash</th>
<th>Fine aggregate</th>
<th>Coarse aggregate</th>
<th>Water</th>
</tr>
</thead>
<tbody>
<tr>
<td>C30</td>
<td>210 kg/m³</td>
<td>100 kg/m³</td>
<td>70 kg/m³</td>
<td>800 kg/m³</td>
<td>1050 kg/m³</td>
<td>175 kg/m³</td>
</tr>
</tbody>
</table>

### Table 3: Concrete compressive strengths of the tested cubic specimens.

<table>
<thead>
<tr>
<th>Specimen no.</th>
<th>f'cm (MPa)</th>
<th>f'cm (MPa)</th>
<th>f'cm (MPa)</th>
<th>f'cm (MPa)</th>
<th>f'cm (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>70–01</td>
<td>37.35</td>
<td>70–07</td>
<td>45.24</td>
<td>70–13</td>
<td>48.93</td>
</tr>
<tr>
<td>70–02</td>
<td>40.14</td>
<td>70–08</td>
<td>40.22</td>
<td>70–14</td>
<td>51.29</td>
</tr>
<tr>
<td>70–03</td>
<td>39.65</td>
<td>70–09</td>
<td>46.52</td>
<td>70–15</td>
<td>36.62</td>
</tr>
<tr>
<td>70–04</td>
<td>43.31</td>
<td>70–10</td>
<td>48.71</td>
<td>70–16</td>
<td>48.34</td>
</tr>
<tr>
<td>70–05</td>
<td>48.74</td>
<td>70–11</td>
<td>41.10</td>
<td>70–17</td>
<td>44.39</td>
</tr>
<tr>
<td>70–06</td>
<td>41.64</td>
<td>70–12</td>
<td>45.40</td>
<td>70–18</td>
<td>49.72</td>
</tr>
<tr>
<td>70–20</td>
<td>41.01</td>
<td>70–21</td>
<td>41.63</td>
<td>70–22</td>
<td>49.45</td>
</tr>
<tr>
<td>70–21</td>
<td>51.60</td>
<td>70–23</td>
<td>38.77</td>
<td>70–24</td>
<td>51.63</td>
</tr>
<tr>
<td>70–22</td>
<td>46.90</td>
<td>70–25</td>
<td>53.77</td>
<td>70–26</td>
<td>44.39</td>
</tr>
<tr>
<td>70–23</td>
<td>36.00</td>
<td>70–27</td>
<td>43.23</td>
<td>70–28</td>
<td>43.96</td>
</tr>
<tr>
<td>70–24</td>
<td>42.33</td>
<td>70–29</td>
<td>46.77</td>
<td>70–30</td>
<td>46.67</td>
</tr>
<tr>
<td>70–25</td>
<td>44.36</td>
<td>70–31</td>
<td>46.45</td>
<td>70–32</td>
<td>44.60</td>
</tr>
</tbody>
</table>

The K-S test method creates a table for \( D_n^\alpha \), which is available in most probability and statistics books and is not provided here due to space limitations.

For the hypothesis, thus indicating that the concrete compressive strength always follows the normal distribution; that is, the size change does not change its probability distribution type.

\[ F_X(x) \text{ and } F_n(x) \text{ belong to the same type of probability distribution if, in the whole range of random variable } X, \text{ the maximum difference } D_n \text{ between } F_X(x) \text{ and } F_n(x) \text{ satisfies the following formula:} \]

\[ D_n = \max_{-\infty < x < \infty} |F_X(x) - F_n(x)| < D_n^\alpha, \]  

\[ \text{where } D_n \text{ is a random variable with a distribution depending on } n, D_n^\alpha \text{ is the critical value at the significance level } \alpha; \text{ that is, the critical value at which the assumption that } F_X(x) \text{ and } F_n(x) \text{ belong to the same type of probability distribution type holds.} \]

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\[ D_n = \max_{-\infty < x < \infty} |F_X(x) - F_n(x)| < D_n^\alpha, \]  

\[ \text{where } D_n \text{ is a random variable with a distribution depending on } n, D_n^\alpha \text{ is the critical value at the significance level } \alpha; \text{ that is, the critical value at which the assumption that } F_X(x) \text{ and } F_n(x) \text{ belong to the same type of probability distribution type holds.} \]

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\[ \text{where } D_n \text{ is a random variable with a distribution depending on } n, D_n^\alpha \text{ is the critical value at the significance level } \alpha; \text{ that is, the critical value at which the assumption that } F_X(x) \text{ and } F_n(x) \text{ belong to the same type of probability distribution type holds.} \]
themean concrete compressive strength of specimens with a size of 70 mm.

\[ \sigma_i' = \frac{\sigma_i}{\sigma_{70}}, \]  

(4)

where \( \sigma_i' \) and \( \sigma_i \) are the dimensionless standard deviation and the standard deviation of the concrete compressive strength of specimens with a size of \( i \), respectively, and \( \sigma_{70} \) is the standard deviation of the concrete compressive strength of specimens with a size of 70 mm.

In summary, there is a size effect on both the mean and standard deviation of the concrete compressive strength, with the size effect on the latter being more significant than on the former. Therefore, the size effect on the standard deviation is expected to have a greater impact on the reliability analysis results and, accordingly, should receive more attention in engineering practice.

The above results from the point estimation of the standard deviation of the concrete compressive strength indicate that there is a size effect on the standard deviation. To further confirm the credibility of this conclusion, the interval estimation of the standard deviation of the concrete compressive strength was conducted with a confidence level of 95%. The interval estimates (i.e., confidence intervals) of the standard deviation of the concrete compressive strength are shown in Table 6.

Table 6 shows that the confidence (i.e., probability) that the standard deviation of the concrete compressive strength for the size of 70 mm falls within the confidence interval [3.61, 6.15] is 95% and that the confidence that the standard deviation of the concrete compressive strength for the size of 150 mm falls within the confidence interval [1.34, 2.27] is 95%. Hence, the confidence interval of the standard deviation for the size of 70 mm is completely different from that for the size of 150 mm, thus demonstrating that the standard deviation of the concrete compressive strength varies in size. Similarly, the confidence intervals of the standard deviations for sizes of 70 mm and 100 mm (or those for sizes of 100 mm and 150 mm) with a 95% confidence level are quite different. These results all indicate that there is a size effect on the standard deviation of the concrete compressive strength.

In addition, the existence of the size effect on the standard deviation of the concrete compressive strength is also shown by proof by contradiction using Monte Carlo simulation sampling with the following specific procedure.

(1) Assuming that there is no size effect on the standard deviation of the concrete compressive strength, that is, the test data for the different sizes given in Table 3 all follow the probability distribution of the same population, the 83 sets of test data in Table 3 are all the sample data of this population. The K-S test performed using these 83 sets of test data shows that the population follows normal distribution, with the point estimates of the mean and standard deviation being 44.21 MPa and 3.40 MPa, respectively.

(2) Under the above assumptions and based on probability statistics theory, the 29, 24, and 30 sample datasets for specimen sizes of 70 mm, 100 mm, and 150 mm, respectively, in Table 3 can be regarded as data with sample sizes of 29, 24, and 30, respectively, from the same population. Table 5 shows that the mean and standard deviation of the concrete compressive strength corresponding to the test data for each specimen size are different from those obtained from the above 83 sets of test data as a whole population, as

\[
\begin{align*}
\text{Size (mm)} & & \text{Assumed distribution type} & & \text{K-S value (} D_{n} \text{)} & & \text{Critical value for accepting the hypothesis (} D_{\alpha}^{n} \text{)} & & \text{Result} \\
70 & & \text{Normal distribution} & & 0.1052 & & 0.2457 & & \text{Accept} \\
100 & & \text{Normal distribution} & & 0.1403 & & 0.2693 & & \text{Accept} \\
150 & & \text{Normal distribution} & & 0.0953 & & 0.2417 & & \text{Accept} \\
\end{align*}
\]
shown in Table 7. If the assumption that there is no size effect on the standard deviation of concrete compressive strength holds, the explanation for the existence of the difference in Table 7 based on probability theory is that the mean and standard deviation estimated from the test data deviate from those of the population due to the limited test samples. According to probability and statistics theory, the larger the number of test data samples, the smaller the above deviation. Thus, as the number of test samples tends to infinity, the above deviation tends to zero. Therefore, the occurrence probability of the difference shown in Table 7 can be calculated based on the relationship between the number of samples and the deviation. If the occurrence probability of the difference in Table 7 is large, the assumption that there is no size effect on the standard deviation of the concrete compressive strength is acceptable; conversely, if the occurrence probability of the difference is small, the above assumption is unacceptable; that is, it is shown that there is a size effect on the standard deviation. Therefore, in the following, the occurrence probability of the differences shown in Table 7 is calculated by considering only the difference in the standard deviation and by considering the differences in both the mean and standard deviation, respectively.

(3) The occurrence probability considering only the difference in the standard deviation is calculated as follows. Since it is impossible to perform an unlimited number of tests in the actual project, it is assumed here that the point estimates of the mean and standard deviation of the 83 sets of test data in Table 3 are the mean and standard deviation of the distribution of the population. The calculations show that the corresponding error has little impact on the final analysis results, so the above treatment is reasonable and feasible. Therefore, the distribution of the population has a mean and standard deviation of 44.21 MPa and 3.40 MPa, respectively, denoted as \( \mu \sim N(44.21, 3.40^2) \). Through MATLAB programming, 29 samples are randomly selected each time for the case of the size of 70 mm (because there are 29 concrete specimens with a size of 70 mm in Table 3) from the population that follows the probability distribution of \( \sigma_3 \sim N(44.21, 3.40^2) \), and the standard deviation of the 29 samples as well as its difference from 3.40 MPa is calculated. This process is repeated 10 million times, and according to the differences in Table 7 and the statistical definition of the probability of occurrence [26], the number of times, \( N \), that the difference is greater than or equal to 1.15 is counted. Then, the occurrence probability of the difference being greater than or equal to 1.15 is

\[
P = \frac{N}{10000000}.
\] (5)

The meaning of the probability shown in equation (5) is the probability that the difference between the standard deviation based on 10 million simulated samples with a sample size of 29 and the standard deviation of the population distribution when there is no size effect on the standard deviation of the concrete compressive strength is greater than 1.15, which is the difference in the test. If the assumption of "no size effect on the standard deviation of the concrete compressive strength" is valid, the probability calculated by equation (5) should be large; otherwise, if the probability obtained from equation (5) is small, the above assumption is not valid; that is, there exists a size effect on the standard deviation. Therefore, the probability obtained from equation (5) is the probability that the assumption of "no size effect on the standard deviation of the concrete compressive strength" holds. The specific calculation results are shown in Table 8. Similarly, the probability that the difference between the standard deviation for the specimen size of 150 mm and the sample size of 30 and 3.40 MPa is less than or equal to –1.71 is calculated, as shown in Table 8.

(4) The occurrence probability considering the differences in both the mean and standard deviation is calculated as follows. The basic principle is the same as that in (3), except that the possibility that both the mean and standard deviation deviate from the respective true values of the population due to the limited test data is considered here. For example, for the case of the specimen size of 70 mm, 29 samples are randomly selected from the population each time, and the mean \( \mu \) and standard deviation \( \sigma_i \) of the 29 samples are calculated, followed by the calculation of the difference \( D_i \) between \( \mu_i \) and 44.21 and the difference \( d_i \) between \( \sigma_i \) and 3.40. This process is repeated 10 million times, and according to the differences in Table 7, the numbers of times that \( D_i \) is greater than or equal to 1.39 and that \( d_i \) is greater than or equal to 1.15 are counted, and the resulting probability is the probability that the assumption of "no size effect on the standard deviation of the concrete compressive strength" is valid. Similarly, the relevant results can be calculated for

<table>
<thead>
<tr>
<th>Size (mm)</th>
<th>Confidence level (%)</th>
<th>Confidence interval of the standard deviation of the compressive strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>95</td>
<td>[3.61, 6.15]</td>
</tr>
<tr>
<td>100</td>
<td>95</td>
<td>[2.22, 4.01]</td>
</tr>
<tr>
<td>150</td>
<td>95</td>
<td>[1.34, 2.27]</td>
</tr>
</tbody>
</table>

Table 6: Interval estimates of the standard deviation of the concrete compressive strength.
the case of the specimen size of 50 mm and the sample size of 30. The calculation results are shown in Table 9.

As seen from Table 8, when only the difference in standard deviation is considered, the probability of \( d_i \leq -1.71 \) is less than 1 in 1 million, and the probability of \( d_i \geq 1.15 \) is also only 0.62%, indicating that the probability that the assumption of “no size effect on the standard deviation of the concrete compressive strength” holds is very small; that is, the original assumption is not valid. Thus, it is shown that there is a size effect on the standard deviation of the concrete compressive strength. When the differences in both mean and standard deviation are considered, the same conclusion can be obtained based on the results in Table 9, and the probability of the assumption being valid is even lower.

Thus far, this study uses the K-S test to prove that the probability distribution type of the concrete compressive strength follows normal distribution and does not experience a size effect. Through point estimation, interval estimation, and the proof by contradiction, it is shown that there is a size effect on both the mean and standard deviation of the concrete compressive strength, with the size effect on the standard deviation being more significant than that on the mean. This finding is an important one in this study, and it also demonstrates that the existing methods for determining the probability distribution of concrete compressive strength are problematic, because these methods are based on a view that there is no size effect on the mean and standard deviation of concrete compressive strength, which is inconsistent with reality.

Existing research results generally agree that the size effect on the mean compressive strength of materials can mainly be attributed to their initial defects and inhomogeneity [28], which also explains the size effect on the standard deviation of the compressive strength of materials. Initial defects and inhomogeneity are common in brittle materials such as concrete. Therefore, although the concrete in the tests of this study was prepared using the mix ratio in Table 2, the conclusion that there is a size effect on the probability distribution of concrete compressive strength can be extended to other concrete.

### 4. Size Effect on the Standard Value of Concrete Compressive Strength

According to the Unified Standard for Reliability Design of Engineering Structures (GB 50153-2008) [8], the standard value of concrete compressive strength is taken as the 0.05 quantile of the probability distribution. Then, when concrete compressive strength is normally distributed, its standard value is as follows:

\[
f_k = \mu_l - 1.645\sigma_l,
\]

where \( \mu_l \) and \( \sigma_l \) are the expected value (mean) and standard deviation of the concrete compressive strength, respectively.

Existing studies have not specifically investigated the impact of the size effect on the probability distribution of concrete compressive strength on the standard value of concrete compressive strength, which is thus analyzed in this study. Table 5 shows that both the mean and standard deviation of the concrete compressive strength decrease with increasing size (from 70 mm through 100 mm to 150 mm). The existing studies of the size effect on concrete compressive strength have been limited to the mean of the compressive strength, while the size effect on the standard deviation has not been studied. Therefore, if only the size effect on the mean concrete compressive strength is considered, it is naturally concluded that the standard value of concrete compressive strength decreases with increasing size. Is this truly the case? Equation (6) shows that the standard value of the concrete compressive strength is related to both the mean and standard deviation of the compressive strength. For this reason, the mean and standard deviation of the concrete compressive strength for the sizes of 70 mm, 100 mm, and 150 mm in Table 5 are substituted into equation (6) to calculate the standard value of the compressive strength corresponding to each size. The detailed results are shown in Table 10 and Figure 2.

The results in Table 10 and Figure 2 show that, according to the principle that “the standard value of the concrete compressive strength is taken as the 0.05 quantile of the probability distribution of the compressive strength” stipulated in specification [8], the larger the specimen size, the higher the standard value of concrete compressive strength, which is opposite to the variation pattern of the mean compressive strength decreasing with increasing size shown in Figure 2. This is because although the mean compressive strength decreases with increasing size, the standard deviation of the compressive strength decreases more with increasing size (this has been analyzed in Section 3.2), resulting in the standard value of the compressive strength calculated by equation (6) increasing with increasing size. This
phenomenon has not been reported in the existing literature, and this finding has important implications for the determination of the standard value of the concrete compressive strength. Therefore, the impact of the size effect should be considered when determining the standard value of the concrete compressive strength; otherwise, the calculation results may not reflect the engineering reality.

5. Impact of the Size Effect on the Probability Distribution of the Compressive Strength on the Reliability Results

Section 3 shows that there is a size effect on both the mean and standard deviation of the concrete compressive strength. Its impact on the reliability calculation results should be further analyzed because this is the basis for whether the size effect needs to be considered in an actual project. The size effect on the mean and standard deviation of concrete compressive strength does not need to be considered if it has little impact on the reliability calculation results, and whether it should be considered is discussed next.

Reliability analysis of the compressive bearing capacity of a concrete cubic specimen with a size of 150 mm under a compression of 810 kN is performed as an example. The concrete specimen loses its compressive bearing capacity when it fails under compression, so its failure mode is failure under compression.

Then, based on this failure mode and the formula for verifying the compressive bearing capacity of a normal section in the literature [29], the limit state equation for the compressive bearing capacity of the 150 mm concrete specimen can be established as follows:

\[ g = \sigma_c \cdot A - 810000, \quad (7) \]

where \( \sigma_c \) is the concrete compressive strength (MPa), and \( A \) is the load-bearing area of the specimen (22500 mm²).

\( \sigma_c \) is a random variable in the probabilistic reliability analysis. Since the 150 mm concrete specimen is used as an example, the probability distribution of the compressive strength corresponding to the size of 150 mm in Table 5, \( \sigma_{c150} \sim N(43.04, 1.69^2) \), should be substituted into equation (7) to obtain the correct reliability calculation result. The calculated failure probability is shown in Table 11 and Figure 3; this is the true failure probability. Similarly, if the probability distributions obtained from 70 mm and 100 mm specimens are used for the reliability analysis of the compressive bearing capacity of the 150 mm specimen, \( \sigma_{c70} \sim N(45.60, 4.55^2) \) and \( \sigma_{c100} \sim N(43.98, 2.86^2) \) in Table 5 should be substituted into equation (7) for the calculation, and the corresponding failure probabilities are obtained as shown in Table 11 and Figure 3.

As seen from Table 11 and Figure 3, the true failure probability of the 150 mm concrete specimen is 0.00152%, while the failure probabilities calculated using the probability distributions for the sizes of 70 mm and 100 mm, i.e., \( \sigma_{c70} \sim N(45.60, 4.55^2) \) and \( \sigma_{c100} \sim N(43.98, 2.86^2) \), are 1.740% and 0.265%, which are 1144.74 and 174.34 times the true failure probability, respectively. Hence, the failure probabilities obtained from the probability distributions for the three sizes differ greatly, and the failure probability decreases greatly as the size increases. To further prove the generality of this conclusion, the reliability is recalculated by changing the compression on the 150 mm concrete specimen to 850 kN and 890 kN, and the corresponding failure probabilities are obtained, as shown in Tables 12 and 13, respectively. The calculation results show that the size effect on the probability distribution of concrete compressive strength has a significant impact on the reliability calculation results and should not be ignored in engineering practice.

In summary, the reliability calculation results obtained using the concrete compressive strength based on small-scale rather than full-size specimen tests may not meet the
reliability requirements, but the reliability requirements can be met in actual large-size projects. That is, the use of small-scale tests to determine the probability distribution of the concrete compressive strength may lead to misjudgment in reliability analysis, and the greater the difference between the test size and the real size, the greater the possibility of misjudgment.

Therefore, when the actual structural size differs greatly from the specimen size, it is not feasible to directly use the probability distribution obtained from small-size specimen tests for the reliability analysis of an actual structural project, so the size effect on the probability distribution of the concrete compressive strength should be considered.

Thus far, this study has demonstrated that there is a size effect on the probability distribution of concrete compressive strength, and the size effect has a significant influence on the standard value of the compressive strength and reliability results. Therefore, the size effect on the probability distribution of concrete compressive strength cannot be ignored. However, such problems are not recognized in existing research, leading to general deficiencies in the reliability calculation results and the standard values of concrete compressive strength when using existing methods.

6. Conclusions

Aiming at the question of whether there is a size effect on the probability distribution of concrete compressive strength, we conducted a special experimental study to investigate the influence of size changes on the probability distribution of concrete compressive strength. The following conclusions can be drawn:

(1) The K-S test shows that a size change has no influence on the probability distribution type of the concrete compressive strength; the probability distributions for each of the three specimen sizes are normal distributions.

(2) Point estimation, interval estimation, and proof by contradiction all show that there is a size effect on both the mean and standard deviation of the concrete compressive strength, that its value decreases with increasing size, and that the size effect on standard deviation is more significant.

(3) The definition of the standard value of concrete compressive strength (equation (6)) shows that it is related not only to the mean, but also to the standard deviation of concrete compressive strength. Compared with the mean, the standard deviation decreases more with an increase in the specimen size, resulting in an increase in the standard value of concrete compressive strength with an increase in specimen size. This result opposed the pattern seen for the mean value of concrete compressive strength, which decreases with an increase in specimen size.

(4) The reliability analysis results show that the probability distribution for different sizes has a great influence on the reliability calculation results, and the size differences of concrete structural members widely exist in practical projects. Therefore, when the sample size is significantly different from the actual size, one should pay attention to the influence of the size effect on the reliability analysis so as to avoid incorrect conclusions.

Data Availability

The data used to support the findings of this study are available from the author upon request.
Conflicts of Interest

The author declares no conflicts of interest.

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