

Research Article

Research on Mechanical Model of Canal Lining Plates under the Effect of Frost Heaving Force

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Frost heaving damage of canal lining in cold and dry areas is one of the important causes of canal leakage. In this paper, based on the bending theory of thin plate, a mechanical model of canal lining under the action of frost heaving force is established and solved. Through parametric and engineering case analysis, the following conclusions are drawn: under the action of frost heaving force, the bending moment, shear force, and internal force of the slope plate show a nonuniform distribution, and the maximum values of bending moment M_x and normal stress σ_x are close to the bottom third of the slope, which is consistent with the existing research and engineering practice. Compared with the theory of beam, the results of the theory of thin plates show that the internal forces and stresses increase at the free boundary (longitudinal expansion joint). The bending moment M_{xy} and stress τ_{xy} are maximum at the four corners of the plate: although the numerical magnitude is lower than that of M_x , it may cause stress concentration to damage the lining plate and thus cause break. The shear force at the longitudinal expansion joint may lead to fracture of the joint material, which needs to be considered in the design process. Due to the uniform distribution of the frost heaving force on the bottom plate lining, its deflection, internal forces, and stresses also show a uniform distribution. The research results can provide scientific reference for the design and operation and maintenance of water transmission canal lining in cold areas.

1. Introduction

China is a large irrigated agricultural production country, and agricultural water consumption accounts for about 63.2% of the total water consumption in the country; meanwhile, more than 50% of China's regions are in arid and semiarid areas, and agricultural water consumption in Gansu, Ningxia, Xinjiang, and Inner Mongolia accounts for more than 75% of the total local water consumption due to factors such as geographical location and climatic conditions [1]. In order to solve the problem of uneven spatiotemporal distribution of water resources, China has constructed a large number of water transmission projects, and more than 800,000 km of trunk and branch canals have been built nationwide, but due to problems such as canal leakage, the average water utilization coefficient of the canal system is

only about 0.5, resulting in a large amount of water waste [2]. Canal leakage prevention projects have largely improved the efficiency of water utilization and promoted the development of water conservation projects. China is vast, and the distribution area of multiyear permafrost zone and seasonal permafrost zone accounts for 21.5% and 53.5% of the national land area, respectively [3, 4], and the latitude span of these areas is large, and the climate is complex and diverse, which makes the service life of canal projects vary greatly from place to place. Especially in the vast northern cold regions such as northeast, northwest, and north China, the winter climate is cold and the low temperature lasts for a long time; for example, the winter temperature in Xinjiang is generally $-40\sim-10^\circ\text{C}$, the annual cumulative average daily negative temperature is $-1,000\sim-1,500^\circ\text{C}$, and the annual duration of negative temperature is about 130 days [5, 6]. At

the same time, the natural freezing depth in these areas is large, coupled with the repeated freezing and thawing action, resulting in serious frost damage problems in canal projects in general. Canal freezing and thawing disasters not only directly affect the use of the canal, wasting valuable water resources and making the land along the canal have secondary saline-alkalization, but also increase the number of engineering maintenance and operating costs, seriously limiting the efficiency of the project. Therefore, how to prevent and control the freezing and thawing damage of the cold area canal project has become a key problem that settle the development of agricultural production in the national economy of irrigation areas.

The canal lining damage in cold areas is mainly due to the uneven frost heaving of subsoil which makes the lining plate bending moment increase, coupled with the deterioration of lining plate and joint material caused by freezing and thawing action, which causes the fracture and damage to the lining plate. Wang [7] established a mechanical model of trapezoidal canal lining frost heaving damage and analyzed the internal force and dangerous cross section of lining under the action of frost heaving force based on the beam theory. Set et al. [8–11] used similar methods to theoretically analyze the frost heaving damage of lining in different structural forms, such as trapezoidal and U-shaped, and determined the location of the dangerous cross section of lining. Considering that the process of soil frost heaving deformation and the process of canal lining structure deformation in the process of canal frost heaving have an interactive relationship, the frost heaving damage characteristics of lining were analyzed based on the elastic foundation beam theory, which can better reflect the influence of canal subsoil characteristics on lining deformation [12–15].

Based on the beam theory and elastic foundation beam theory to analyze the frost heaving damage on the lining, it is better to predict the internal forces of the canal cross section under different structural forms or conditions, so as to establish the frost heaving damage judgment criterion. However, as a linear project, the size of the lining along the canal line inevitably affects the distribution characteristics of its internal forces and deformations. For small canals, the lining is usually constructed as cast-in-place concrete plates, and for large canals, although precast concrete plates are laid on the surface, a layer of cast-in-place concrete plate is still constructed at the base [16]. Therefore, the use of thin plate theory can better respond to the deformation and internal force characteristics of canal lining plates under the action of frost heaving forces. To this end, this paper takes water transmission canal lining in alpine regions as the research object, based on thin rectangular plate theory, considering the effect of frost heaving force and the boundary conditions of canal lining, combined with typical engineering cases, analyzing the deformation, internal force, and stress of the lining plate. The research results can provide reference for the prevention and control of frost heaving damage of canal lining in alpine regions.

2. Establishment and Solution of the Lining Mechanics Model

2.1. Basic Assumptions and Conventions. Uneven frost heaving of the canal foundation soil makes the liner plate swell, bulge, and fracture, which is the most serious and main form of damage to the liner plate by freezing [7, 16]. The most used precast plates and cast-in-place concrete lining canals are rigid lining, which are susceptible to frost damage for two reasons: on the one hand, because the canal is a groove structure, coupled with the inevitable leakage, water migration during the long-term freeze-thaw cycle process, and the groundwater embedment depth of different parts in canal subsoil, soil moisture content, surface temperature, and different freezing start time, frost heaving deformations of different sizes and directions are bound to be produced. On the other hand, the lining structure is usually an open thin plate structure with small elasticity of concrete material and low tensile strength, and its non-symmetrical structure makes it less resistant to deformation and prone to have bending damage.

Take the open system trapezoidal concrete lining canal in engineering as an example, as shown in Figure 1. Under certain geological, meteorological, and moisture conditions, moisture recharge is the main factor affecting soil frost heaving. For canal projects, groundwater is the main source of moisture recharge after winter shutdown [9, 12]. However, there are various patterns of frost heaving distribution along the depth direction under different conditions, such as large top with small bottom attenuation type, large middle with small ends type, and small top with large bottom type [3]. Among them, when the soil quality is homogeneous, the soil before freezing is closer to the water table, and the water content of the soil before freezing is higher; thus, the frost heaving is manifested as the large top with small bottom attenuation type. This frost heaving distribution characteristic is also used extensively in the analysis of canal lining frost heaving damage [7, 8, 10, 11]. Both the slope plate and the bottom plate are subjected to frost heaving forces; the top of the slope plate is generally set with a certain width of edge protection in the horizontal direction, having no frost heaving constraint or minor frost heaving constraint, while the bottom of the slope plate and the bottom plate are mutually constrained. Both edges of the bottom plate are constrained by the bottom of the slope plate. According to the characteristics of frost heaving distribution in the depth direction of the canal and the constraint situation of the lining, there is no frost heaving constraint or slight frost heaving constraint at the top of plate, so assuming that the normal frost heaving force on the slope plate is 0 at the top of the slope and maximum at the bottom of the slope, which is linearly distributed [12]. The bottom plate is subjected to a uniformly distributed normal frost heaving force. The canal lining is usually laid at an inclination, and the normal and tangential frost heaving forces are existing between the liner and the soil, and the eccentric press bending of the liner is

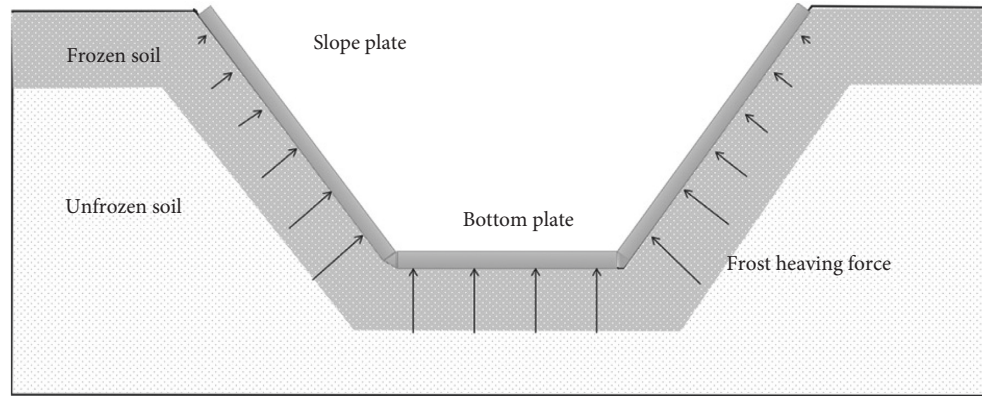


FIGURE 1: Schematic diagram of canal lining.

caused by the tangential frost heaving force. Corresponding to different engineering requirements, the thickness of the liner plate is usually 10~30 cm. Due to the small force arm, the bending moment caused by tangential frost heave force is smaller compared to that caused by normal frost heave force; therefore, only the effect of normal frost heave force is considered in the paper, which affects the accuracy of stress analysis to some extent, but considering the safety factor in the structural design process, the research results still have important reference value for engineering design. Combining the existing research results and practical engineering experience, the following assumptions and conventions are added to the establishment of the model [7, 13, 14]:

- (1) The canal lining mechanics model is simplified to a thin rectangular plate structure.
- (2) Due to the slow freezing process in winter, the lining deformation process is regarded as a quasistatic process. The deformation of frozen soil and lining are always coordinated in the process of frost heaving, and the lining is in ultimate equilibrium when the structure is damaged.
- (3) The canal lining deformation is in the range of linear elasticity, and only the small deformation of the lining is considered, the rotational effect of the microelements is ignored.
- (4) Frost heaving calculation only considers the deformation of frozen soil within the range of freezing depth and does not consider the solidification deformation of frozen soil outside the freezing depth.
- (5) After the completion of the canal, the lining self-weight and the foundation reaction force balance each other; the force analysis does not consider the influence of the lining self-weight and only considers the influence of the normal frost heaving force on the internal force of the lining plate.

Figure 2(a) shows the model of slope plate lining structure. The width along the canal line direction is b_1 , the height along the canal depth direction is a_1 , and the thickness of the liner plate is δ . The coordinate system is shown in the figure, the y -direction is the direction of the

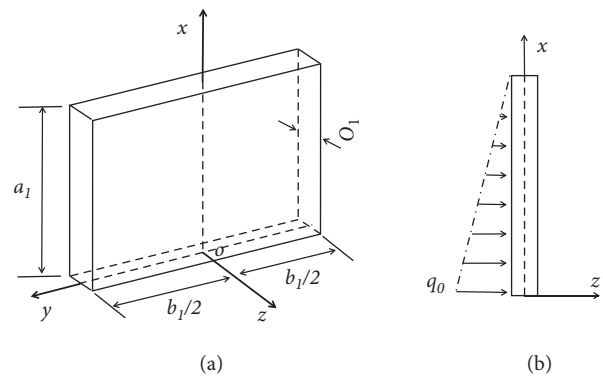


FIGURE 2: Diagram of force acting on the rectangular slope under frost heaving force.

canal line, the z -direction points to the canal inner slot (that is, the negative z -direction points to the soil body), and the x -direction points from the bottom to the top of the slope. Figure 2(b) shows the schematic diagram of the normal frost heaving force on the slope plate, and because the distance from the water table is different at different locations of the slope plate, the relationship between the amount of frost heaving at different locations and groundwater depth is a power function, which can be approximately regarded as a linear distribution [12]. It is assumed that the frost heaving force is linearly distributed, which is 0 at the top of the slope and maximum q_0 at the bottom of the slope [7].

Figure 3(a) shows the model of the bottom plate lining structure. The width along the canal line direction is b_2 , the height along the canal depth direction is a_2 , and the thickness of the liner plate remains δ . The coordinate system direction is the same as the slope plate definition. Figure 3(b) shows the schematic diagram of the frost heaving force on the bottom plate lining, and the whole bottom plate is subject to uniform frost heaving force.

When using beam theory to study the force deformation of the lining plate, the two ends of the beam are usually simplified to simply supported constraints (top of slope and bottom of slope) [7, 12, 15–17], so the two ends of the plate are also simplified to simply supported constraints in this paper, where the top of slope ($x = a_1$) and bottom of slope

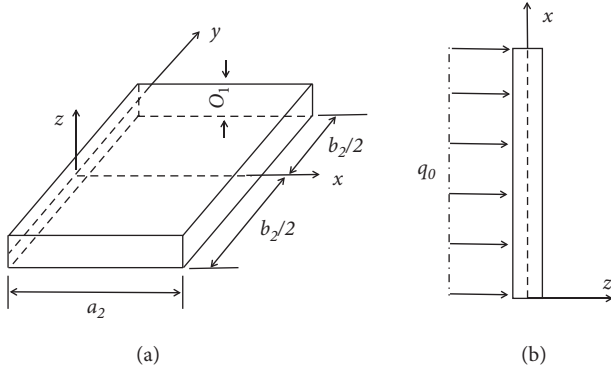


FIGURE 3: Diagram of force acting on the rectangular bottom plate under frost heaving force.

($x=0$ or a_2) are regarded as simply supported constraints. Considering that the canal lining will be set up with expansion joints at certain intervals along the longitudinal direction, the material stiffness of the expansion joints is much smaller compared with the concrete lining plate and the deformation allowed to occur is also large, so the boundary is simplified to a free boundary at $y=\pm b_1/2$ and $y=\pm b_2/2$. According to the above simplification, both the slope plate and the bottom plate boundary conditions are thin rectangular plates with two opposite edges simply supported and the remaining edges free. From Figures 2 and 3, the geometric structures and coordinates of the slope plate and the bottom plate are the same; therefore, for the convenience of deriving the equations, a_1 and a_2 are unified as a , and b_1 and b_2 are unified as b in the latter part. Finally, the distinction is made when the results are specifically calculated.

2.2. Model Building and Solution. The differential equation for the elastic deflection surface of the thin rectangular plate is [18]

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D}, \quad (1)$$

where D is the flexural stiffness of the lining plate, $D = E\delta^3/12(1 - \mu^2)$; E is the elastic modulus of the lining plate; μ is the Poisson's ratio of the lining plate.

The single trigonometric series solution method has been widely used for the solution of thin plate structures, which is suitable for both four-sided simply supported rectangular plates and rectangular plates with one pair of simply supported opposite edges, and the solution function has a simple form and fast convergence [19, 20]. From the model established in the previous section, the model has a boundary case with one pair of opposite edges simply supported and the other two edges free. The corresponding boundary conditions are as follows.

For slope plate and bottom plate lining, when $x=0$ and $x=a$,

$$w = 0, \quad \frac{\partial^2 w}{\partial x^2} = 0. \quad (2)$$

For slope plate and bottom plate lining, when $y = \pm b/2$,

$$\begin{aligned} -D \left(\frac{\partial^2 w}{\partial y^2} + \mu \frac{\partial^2 w}{\partial x^2} \right) &= 0, \\ -D \left[\frac{\partial^3 w}{\partial y^3} + (2 - \mu) \frac{\partial^3 w}{\partial y \partial x^2} \right] &= 0. \end{aligned} \quad (3)$$

Assuming that the solution of the control equation (1) is a single trigonometric series, the deflection surface function of a rectangular thin plate subjected to frost heaving forces with two simply supported opposite edges and two free opposite edges can be constructed:

$$w = \sum_{m=1}^{\infty} Y_m(y) \sin \frac{m\pi x}{a}, \quad (4)$$

where m is any positive integer and $Y_m(y)$ is independent of the independent variable x . Obviously, the above equation satisfies the simply supported boundary conditions, and the problem is solved if the function $Y_m(y)$, which satisfies the other two boundary conditions, is proved to satisfy the control equation at the same time.

The trigonometric series (4) is substituted into equation (1) to get

$$\begin{aligned} \sum_{m=1}^{\infty} \left[\frac{d^4 Y_m(y)}{dy^4} - 2 \left(\frac{m\pi}{a} \right)^2 \frac{d^2 Y_m(y)}{dy^2} + \left(\frac{m\pi}{a} \right)^4 Y_m(y) \right] \\ \cdot \sin \frac{m\pi x}{a} = \frac{q(x, y)}{D}. \end{aligned} \quad (5)$$

In the above equation, the left end of the equal sign is a trigonometric series with respect to the sine function, and to make the right end correspond to it, the right end of the equal sign is also expanded to a trigonometric series,

$$\frac{q(x, y)}{D} = \sum_{m=1}^{\infty} F_m(y) \sin \frac{m\pi x}{a}, \quad (6)$$

where $F_m(y)$ can be calculated by the following equation [19]:

$$F_m(y) = \frac{2}{Da} \int_0^a q(x, y) \sin \frac{m\pi x}{a} dx, \quad (7)$$

and substituting (6) into (5) yields

$$\begin{aligned} \sum_{m=1}^{\infty} \left[\frac{d^4 Y_m}{dy^4} - 2 \left(\frac{m\pi}{a} \right)^2 \frac{d^2 Y_m}{dy^2} + \left(\frac{m\pi}{a} \right)^4 Y_m \right] \\ \cdot \sin \frac{m\pi x}{a} = \sum_{m=1}^{\infty} F_m(y) \sin \frac{m\pi x}{a}. \end{aligned} \quad (8)$$

Since $\sin m\pi x/a$ is impossible to be constant equal to 0, it follows from the above equation that

$$\frac{d^4 Y_m}{dy^4} - 2\left(\frac{m\pi}{a}\right)^2 \frac{d^2 Y_m}{dy^2} + \left(\frac{m\pi}{a}\right)^4 Y_m = F_m(y), \quad (9)$$

the above equation is a 4th order ordinary differential equation with constant coefficients and the solution can be expressed as

$$Y_m = Y_m^{(0)} + Y_m^{(*)}, \quad (10)$$

where $Y_m^{(0)}$ is a homogeneous solution and $Y_m^{(*)}$ is a particular solution. The homogeneous solution is

$$Y_m^{(0)} = A_m \cos ha_m y + B_m \sin ha_m y + C_m a_m y \cos ha_m y + D_m a_m y \sin ha_m y, \quad (11)$$

where $a_m = m\pi/a$, A_m , B_m , C_m , and D_m are undetermined coefficients.

Thus, the general solution of (9) can be expressed as

$$Y_m = A_m \cos ha_m y + B_m \sin ha_m y + C_m a_m y \cos ha_m y + D_m a_m y \sin ha_m y + Y_m^{(*)}. \quad (12)$$

The particular solution $Y_m^{(*)}$ can be calculated by the form of $F_m(y)$, the undetermined coefficients A_m , B_m , C_m , and D_m can be calculated by the free boundary conditions at both ends.

Substituting (12) into (4) to obtain the expression for the deflection of the lining plate,

$$w = \sum_{m=1}^{\infty} (A_m \cos ha_m y + B_m \sin ha_m y + C_m a_m y \cos ha_m y + D_m a_m y \sin ha_m y + Y_m^{(*)}) \sin a_m x. \quad (13)$$

The particular solution of the deflection equation of the slope plate $Y_m^{(*)}$ is solved as follows, where the load on the slope plate is

$$q = q_0 \left(1 - \frac{x}{a_1}\right). \quad (14)$$

Substituting this into (8) yields

$$F_m(y) = \frac{2}{Da_1} \int_0^{a_1} q \sin \frac{m\pi x}{a_1} dx = \frac{2q_0}{Dm^2 \pi^2} (m\pi - \sin m\pi). \quad (15)$$

Assuming that the form of the particular solution is $Y_m^{(*)} = C_0$, substituting it into (9) could yield $C_0 = 2q_0 a_1^4 / Dm^6 \pi^6 (m\pi - \sin m\pi)$; in turn, the particular solution of the slope plate deflection equation is obtained:

$$Y_m^{(*)} = \frac{2q_0 a_1^4}{Dm^6 \pi^6} (m\pi - \sin m\pi). \quad (16)$$

Substitute the previous equation into equation (13) to obtain the slope plate general solution,

$$w = \sum_{m=1}^{\infty} \left[(A_m + C_m a_m y) \cos ha_m y + (B_m + D_m a_m y) \sin ha_m y + \frac{2q_0 a_1^4}{Dm^6 \pi^6} (m\pi - \sin m\pi) \right] \sin a_m x, \quad (17)$$

where the particular solution of the bottom plate deflection equation $Y_m^{(*)}$ is solved such that the load on the bottom plate is $q = q_0$. Substituting it into (8) yields

$$F_m(y) = \frac{2}{Da_2} \int_0^{a_2} q \sin \frac{m\pi x}{a_2} dx = \frac{2q_0}{Dm\pi} (1 - \cos m\pi). \quad (18)$$

Assuming that the particular solution is of the form $Y_m^{(*)} = C_1$, substituting it into (10) yields that $C_1 = 2q_0 a_2^4 / Dm^5 \pi^5 (1 - \cos m\pi)$; in turn, the particular solution of the deflection equation of the bottom plate is obtained:

$$Y_m^{(*)} = \frac{2q_0 a_2^4}{Dm^5 \pi^5} (1 - \cos m\pi). \quad (19)$$

Substitute the previous equation into equation (13) to obtain the general solution of the bottom plate:

$$w = \sum_{m=1}^{\infty} \left[(A_m + C_m a_m y) \cos ha_m y + (B_m + D_m a_m y) \sin ha_m y + \frac{2q_0 a_2^4}{Dm^5 \pi^5} (1 - \cos m\pi) \right] \sin a_m x. \quad (20)$$

Since the load, plate geometry, and boundary conditions are symmetric with the x -axis, the deflection w must be an even function of y , which is $w(x, y) = w(x, -y)$. Therefore, the solution to equations (17) and (20) is $B_m = C_m = 0$.

The general solutions for the slope plate and the bottom plate are simplified as

$$w = \sum_{m=1}^{\infty} \left[A_m \cos ha_m y + D_m a_m y \sin ha_m y + \frac{2q_0 a_1^4}{Dm^6 \pi^6} (m\pi - \sin m\pi) \right] \sin a_m x, \quad (21)$$

$$w = \sum_{m=1}^{\infty} \left[A_m \cos ha_m y + D_m a_m y \sin ha_m y + \frac{2q_0 a_2^4}{Dm^5 \pi^5} (1 - \cos m\pi) \right] \sin a_m x,$$

where A_m and D_m can be calculated by the boundary conditions when $y = b/2$.

Due to the good convergence of the single triangular series, taking the first 5 terms will obtain good accuracy results. The internal force in the plate can be calculated from the following:

$$\begin{aligned}
M_x &= -D \left(\frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2} \right), \\
M_y &= -D \left(\frac{\partial^2 w}{\partial y^2} + \mu \frac{\partial^2 w}{\partial x^2} \right), \\
M_{xy} &= -D(1 - \mu) \frac{\partial^2 w}{\partial x \partial y}, \\
Q_x &= -D \frac{\partial}{\partial x} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right), \\
Q_y &= -D \frac{\partial}{\partial y} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right),
\end{aligned} \tag{22}$$

where M_x is the cross-sectional moment, M_y is the longitudinal-section moment, M_{xy} is the torque, Q_x is the transverse shear force within the cross section, and Q_y is the transverse shear force within the longitudinal section.

3. Effect of Frost Heaving Forces on the Mechanical Properties of Lining

Figure 4 shows the comparison of the results of slope plate deflection calculated by using beam theory and plate theory, respectively. In Figure 4, the calculation method and parameters (slab thickness 0.2 m, modulus of elasticity 2.2×10^4 Mpa, ultimate tensile strain 0.5×10^{-4}) of the paper by Wang [7] are used for the beam theory calculation method, and the method of this paper is used for the rectangular plate theory calculation. It can be seen that the results obtained from the two calculation methods are in good agreement, and the calculation process in this paper can be considered accurate.

3.1. Mechanical Analysis of the Slope Plate. The canal lining plate is set up with expansion joints at a certain interval in the longitudinal direction to release the deformation caused by temperature and other factors, which is often taken as 3~6 m. The frost heaving force acting on the structure during the freezing of the soil is related to a variety of factors such as the nature of the soil, climatic conditions, and constraints, which makes the frost heaving force take a wide range of values; for example, the maximum frost heaving force of the selected slope plate is as 8 kPa in [7], the simulated value in [15] was 380~560 kPa, the measured frost heaving force in the field was 0.8~32 kPa in [21], and in [22], the frost heaving force in the calculation model was 1.22 kPa. Therefore, the frost heaving forces of 8 kPa, 20 kPa, 50 kPa, and 100 kPa were selected as the maximum values applied to the slope plate in [7], and the width of the slope plate was taken as 5 m and the height as 3 m. Then the variation law of each mechanical variable could be obtained. It can be seen from Figure 5 that the change of deflection of the lining plate under the effect of frost heaving force is basically uniform, and the value of deflection increases and then decreases from the bottom to the top of the plate and reaches the maximum near the midline of the plate. The deflection equivalent curve

is symmetrically distributed up and down with respect to the middle line of the plate, which is the same as the lining deflection variation law in the existing studies [12, 13]. With the gradual increase of the frost heaving force, the deflection value increases significantly, and the deflection at the midpoint of both edges of the plate (expansion joints) is greater than that at the center of the plate. From Figure 6(e), it can be observed that the maximum value of deflection is 0.3 mm when the maximum frost heaving force is 8 kPa; the maximum value of deflection is 0.7 mm when the maximum frost heaving force is 20 kPa; the maximum value of deflection is 1.9 mm when the maximum frost heaving force is 50 kPa; and the maximum value of deflection is 3.7 mm when the maximum frost heaving force is 100 kPa. The change in the size of the frost heaving force has a large effect on the deflection value.

The cross-sectional bending moment is the main internal force causing the damage of the lining plate. Figure 7 shows the effect of different frost heaving forces on the cross-sectional bending moment. It can be seen that the bending moment M_x has a similar variation trend as deflection when the lining plate is under the effect of frost heaving force, where the value of bending moment increases and then decreases from the bottom to the top of the plate. Unlike the deflection distribution, the bending moment is asymmetrically distributed along the height direction of the plate, which is caused by the nonuniformly distributed frost heaving force. The bending moment is the second order derivative of the deflection, which can further reflect the concavity of the deflection. The deflection at the midpoint of both edges of the plate (expansion joints) is greater than that at the center of the plate, which is the danger point. From Figure 7(e), it can be seen that when the value of bending moment M_x reaches the maximum at the height of 1.2 m of the plate and the maximum frost heaving force increases from 8 kPa to 20 kPa, the maximum value of bending moment M_x increases by 1.5 times; when the maximum frost heaving force increases to 50 kPa, the maximum value of bending moment M_x increases by 5.24 times; when the maximum frost heaving force increases to 100 kPa, the maximum value of bending moment M_x increases by 11.49 times. It can be seen that the change in the magnitude of the frost heaving force has a large effect on the value of bending moment M_x .

From Figure 8, it can be seen that the lining plate has a circular distribution of bending moment M_y under the effect of frost heaving force, and the value of bending moment M_y is higher as it goes to the center and reaches the maximum value at the height of 1.2 m. With the gradual increase of frost heaving force, the value of bending moment M_y also increases gradually, and the equivalent curves of bending moment M_y are arranged more and more closely. From Figure 8(e), when the maximum frost heaving force is 8 kPa, the maximum value of bending moment is 0.7 kN m; when the maximum frost heaving force increases from 8 kPa to 20 kPa, the maximum value of bending moment M_y increases 1.3 times; when the maximum frost heaving force increases to 50 kPa, the maximum value of bending moment M_y increases 5.3 times; and when the maximum frost

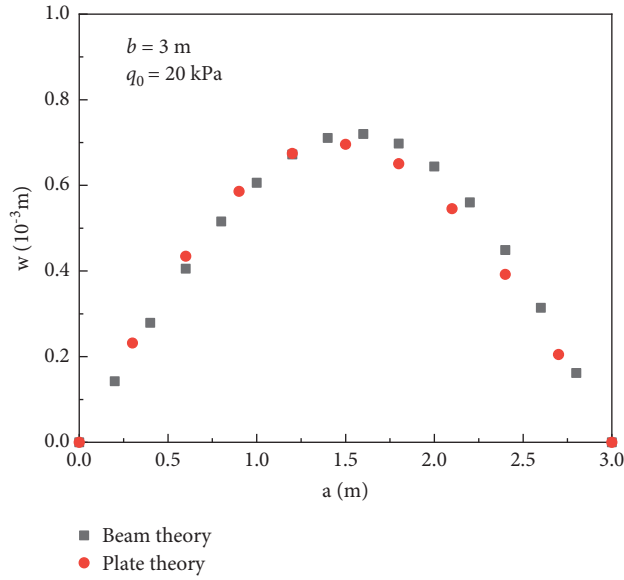


FIGURE 4: Deflection comparison between beam theory and plate theory.

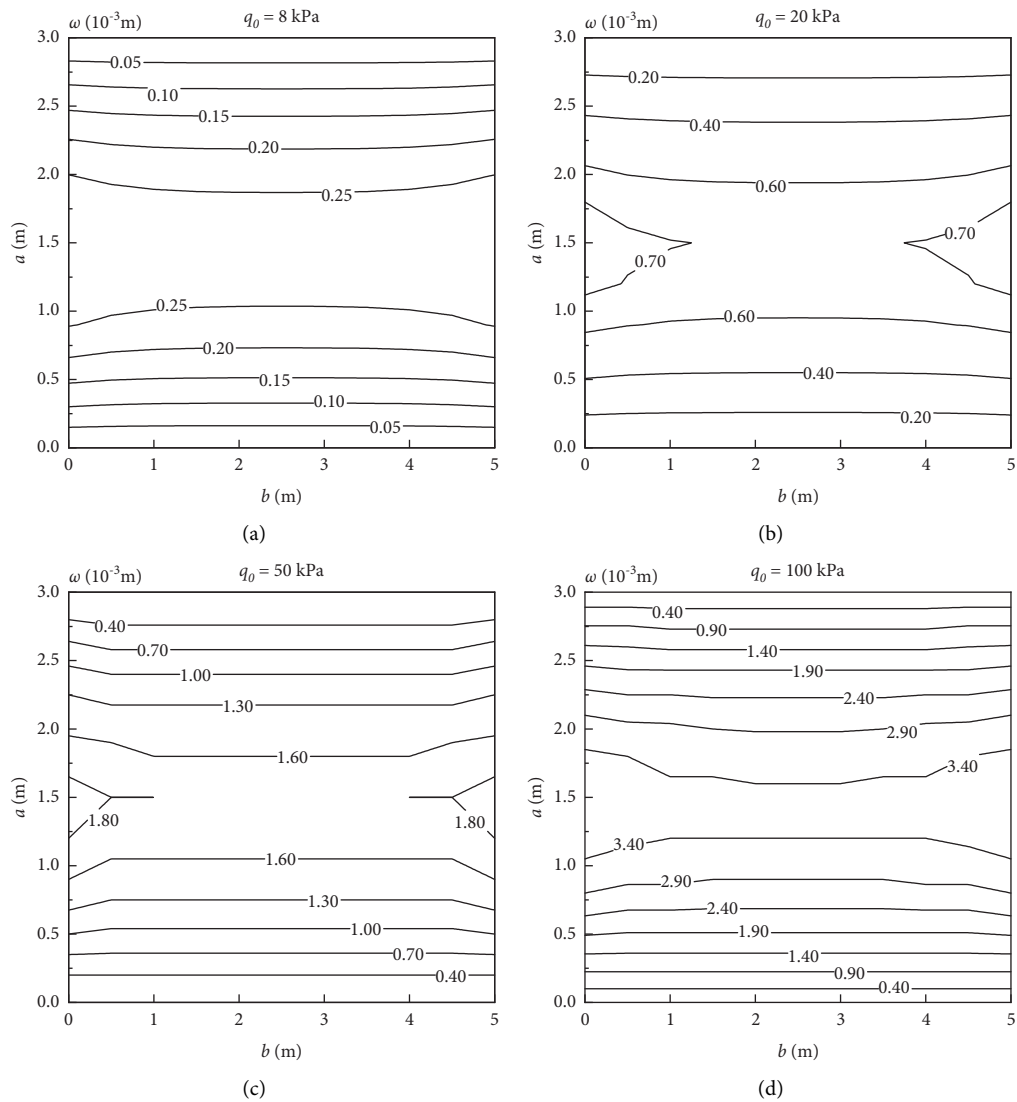


FIGURE 5: Continued.

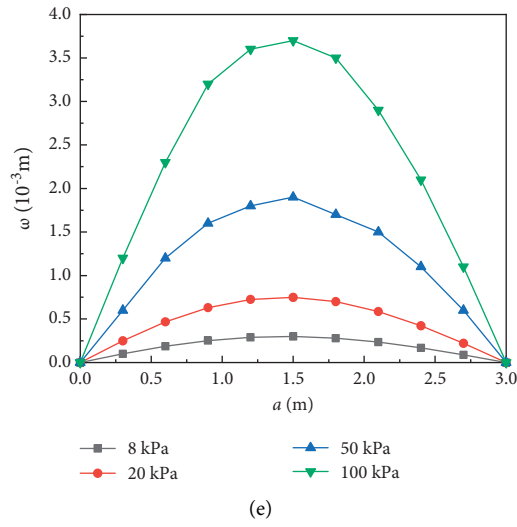


FIGURE 5: Effect of frost heaving force on deflection (w).

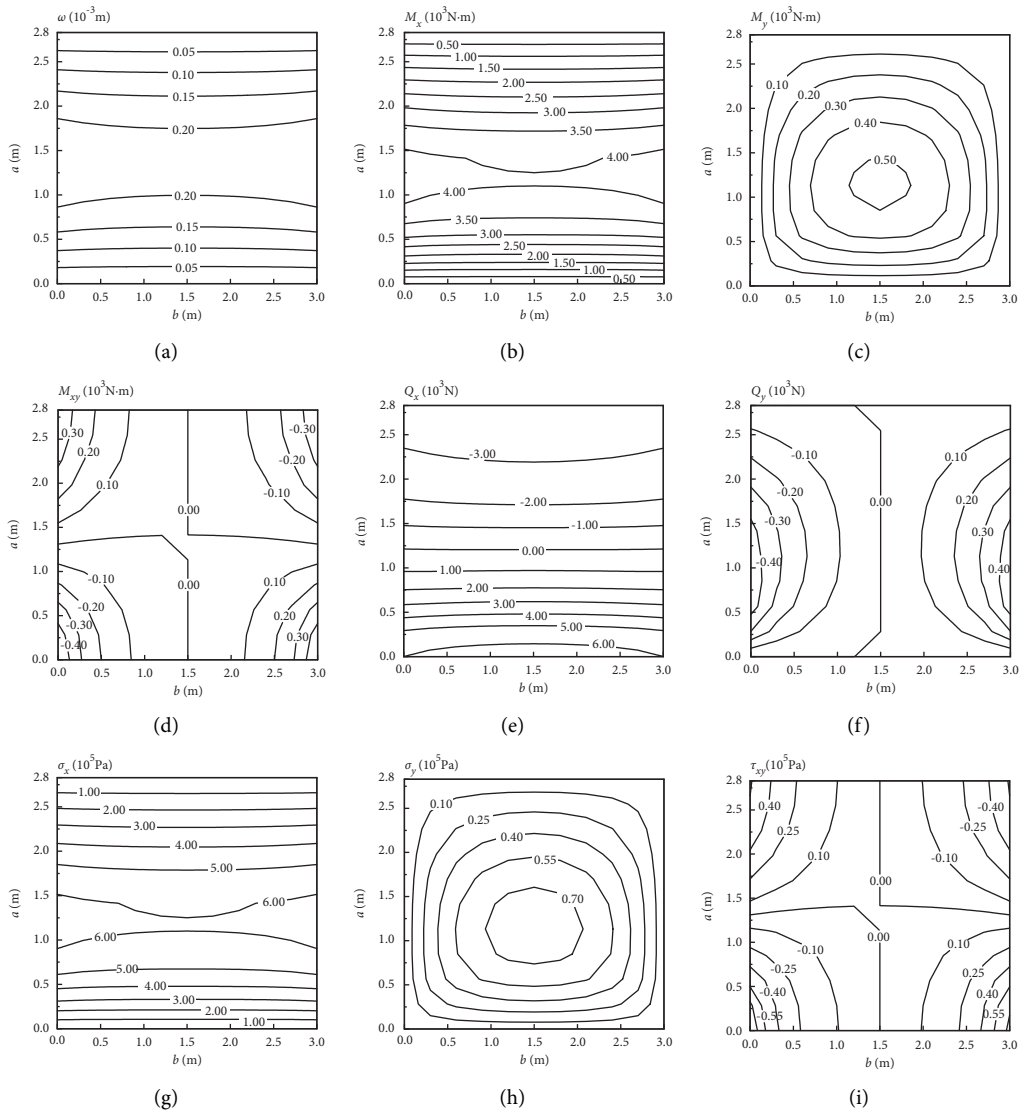


FIGURE 6: Distribution of deflection, internal force, and stress of slope plate.

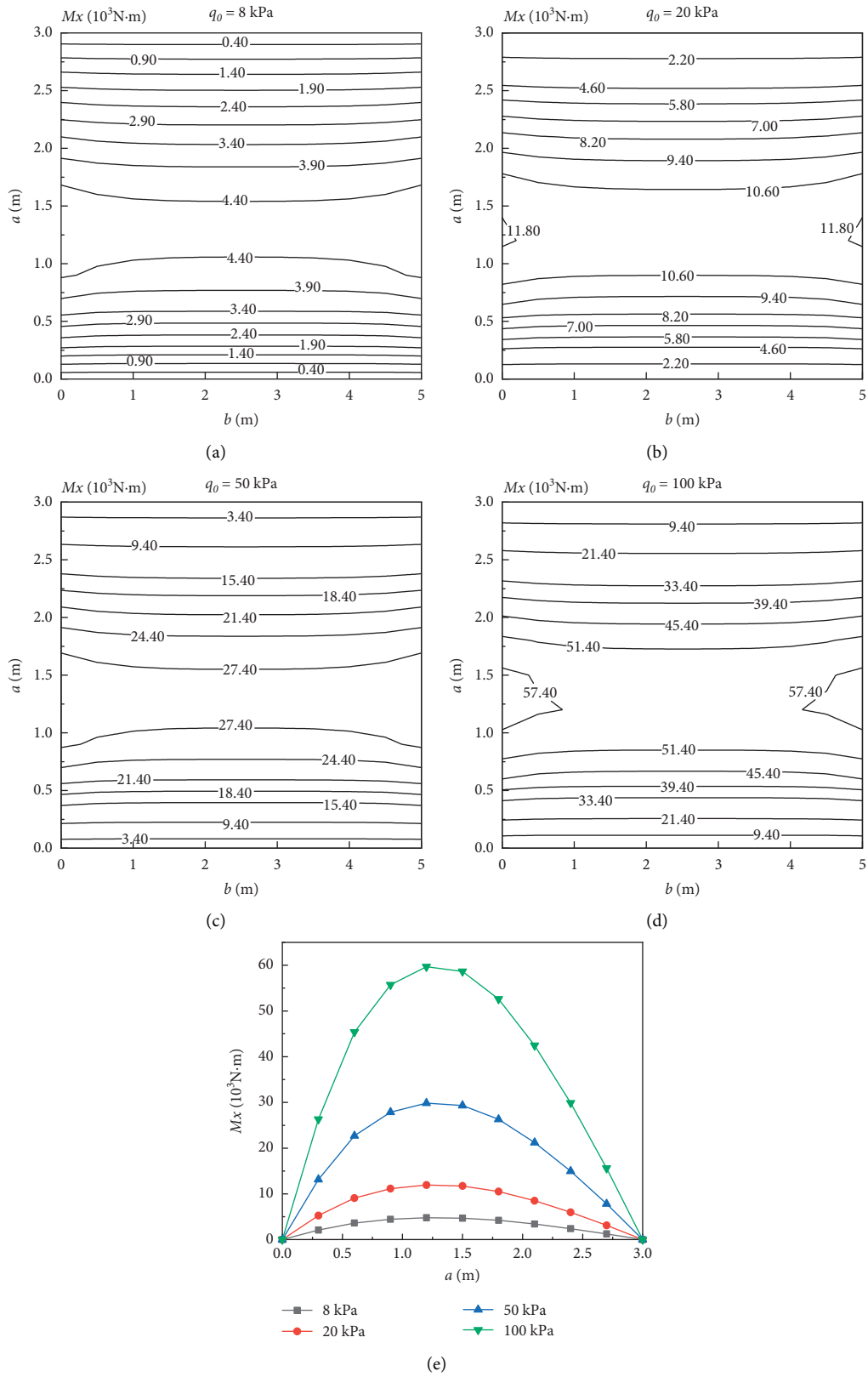


FIGURE 7: Effect of frost heaving force on bending moment M_x .

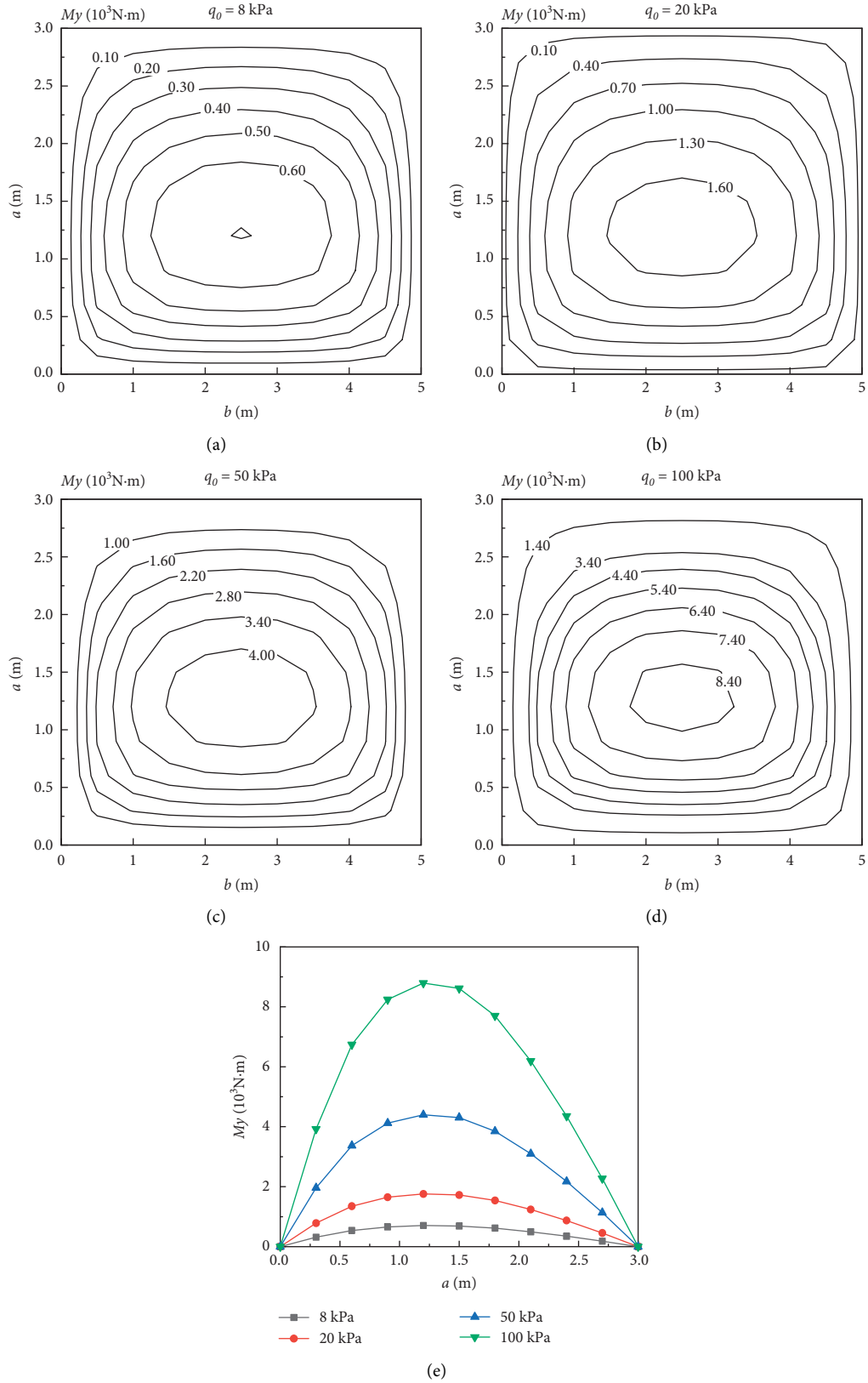


FIGURE 8: Effect of frost heaving force on bending moment M_y .

heaving force increases to 100 kPa, the maximum value of bending moment M_y increases 11.3 times. The increase multiplier is comparable to the bending moment M_y , which shows that the change in the magnitude of the frost heaving force also has a large effect on the value of the bending moment M_y .

From Figure 9, it can be seen that the equivalent curve of torque M_{xy} is symmetrically distributed with respect to the center of the lining plate, and the maximum values are at the four corners of the plate. If the horizontal axis is shifted upward, the first and third quadrants have negative torque, and the second and fourth quadrants have positive torque. Their maximum values are distributed at the corners of the lining plate, and the overlapping of torque on two adjacent sides is likely to cause stress concentration at the corners. From Figure 9(e), it can be seen that the increase in frost heaving force increases the value of torque M_{xy} .

Figure 10 shows the variation law of cross-sectional shear force. The figure shows that the shear force Q_x is negative on the upper side of the lining plate and positive on the lower side, reaching its maximum value at the position of the upper and lower edges of the plate, and is nonuniformly distributed up and down. The shear force at the upper and lower edges (simply supported restraints) is slightly larger in the middle than at the sides, and the shear force at the top of the slope is smaller than that at the bottom. From Figure 10(e), it can be seen that similar to the torque M_{xy} , the slope of the curve increases significantly by increasing the magnitude of the frost heaving force, indicating that increasing the frost heaving force will increase the value of the shear force Q_x .

As can be seen from Figure 11, the equivalent curve of the longitudinal interface shear force Q_y is symmetrically distributed with respect to the midline of the thin plate on the left and right, and the shear value decreases and then increases from the left to the right side of the plate. Although the shear forces at the left and right edges (joints) are not as large as the cross-sectional shear force, it may lead to fracture of the joint material and needs to be considered in the design process. The shear force Q_y is still nonuniformly distributed along the top to bottom of the plate. From Figure 11(e), it can be seen that the shear force Q_y reaches the maximum at the height of 0.9 m. The shear force Q_y gradually increases as the frost heaving force gradually increases. The maximum value of shear force increases from 0.5 kN to 6.2 kN when the frost heaving force increases from 8 kPa to 100 kPa.

Since the stress change of the lining plate corresponds to its internal force change, only the graph of the stress change in the plate at different frost heaving forces is given. From Figure 12(a), it can be seen that the value of stress x increases and then decreases from the bottom to the top of the plate and reaches the maximum at the height of 1.2 m of the plate. When the frost heaving force is 8 kPa, the maximum value of stress x is 7.2×10^5 Pa; when the frost heaving force is 100 kPa, the maximum value of stress x is 89.5×10^5 Pa, indicating that the change of the frost heaving force has a large effect on the value of stress x . From Figure 12(b), it can be seen that when the frost heaving force increases from 8 kPa to 100 kPa, the maximum value of stress y increases

from 1.1×10^5 Pa to 13.2×10^5 Pa. From Figure 12(c), it can be seen that the value of stress τ_{xy} increases significantly with the increase of the frost heaving force.

3.2. Mechanical Analysis of the Bottom Plate. The bottom plate can also be regarded as a thin rectangular plate subjected to the uniform frost heaving force. Take the width of the bottom plate as 5 m and the height as 3 m, applying the frost heaving forces of 8 kPa, 20 kPa, 50 kPa, and 100 kPa to the bottom plate in turn. Due to space limitations, only the data of the internal force and stress along the height direction of the plate are given. The bottom plate is subjected to uniform frost heaving forces, so the internal forces and stresses are also uniformly distributed along the height direction of the plate, but slightly larger at the free boundary than at other locations. From Figure 13(a), it can be seen that when the bottom plate lining is subjected to the action of frost heaving forces, the deflection value increases and then decreases from the bottom to the top of the plate and reaches the maximum at the midline position of the plate. The maximum value of deflection increases from 0.6 mm to 7.5 mm when the frost heaving force increases from 8 kPa to 100 kPa. It can be seen from Figure 12(b) that the variation trend of the bending moment M_x is similar to that of the deflection, and the value of bending moment increases first and then decreases from the bottom to the top of the plate. The maximum value of the bending moment M_x increases 11.48 times when the frost heaving force increased from 8 kPa to 100 kPa. From Figure 12(c), it can be seen that the maximum value of bending moment M_y increases by 11.3 times when the frost heaving force increases from 8 kPa to 100 kPa. From Figures 12(d)–12(f), it can be seen that increasing the magnitude of the frost heaving force will increase the values of bending moment M_{xy} , shear force Q_x , and shear force Q_y , and they are symmetrically distributed along the height of the plate. Figures 12(g)–12(i) show that when the frost heaving force increases from 8 kPa to 100 kPa, the stress σ_x increases by 12.6 times, the stress σ_y increases by 12.3 times, and the stress τ_{xy} increases by 12.3 times.

4. Engineering Case Analysis

Wang [7] calculated the frost heaving deformation and damage location of a plain concrete lined trapezoidal canal, and the following calculation was carried out using the parameters of the engineering case used by Wang, and the dimensional loading parameters of the canal are as follows. The height of bottom plate is 2 m, and the width is 3 m; the height of slope plate is 2.83 m, and the width is 3 m; the thickness of plates is 0.2 m, the frost heaving force of bottom plate is 7.3 kPa, and the maximum frost heaving force of slope plate is 8 kPa. The elastic modulus of the concrete is 2.2×10^4 MPa and Poisson's ratio is 0.16.

Figure 13 shows the deflection, internal force, and stress distribution of the slope plate. Figure 13(a) shows that the deflection is basically symmetrical along the height of the plate, and the maximum deflection value is 0.22 mm, which meets the deformation requirements of the lining plate.

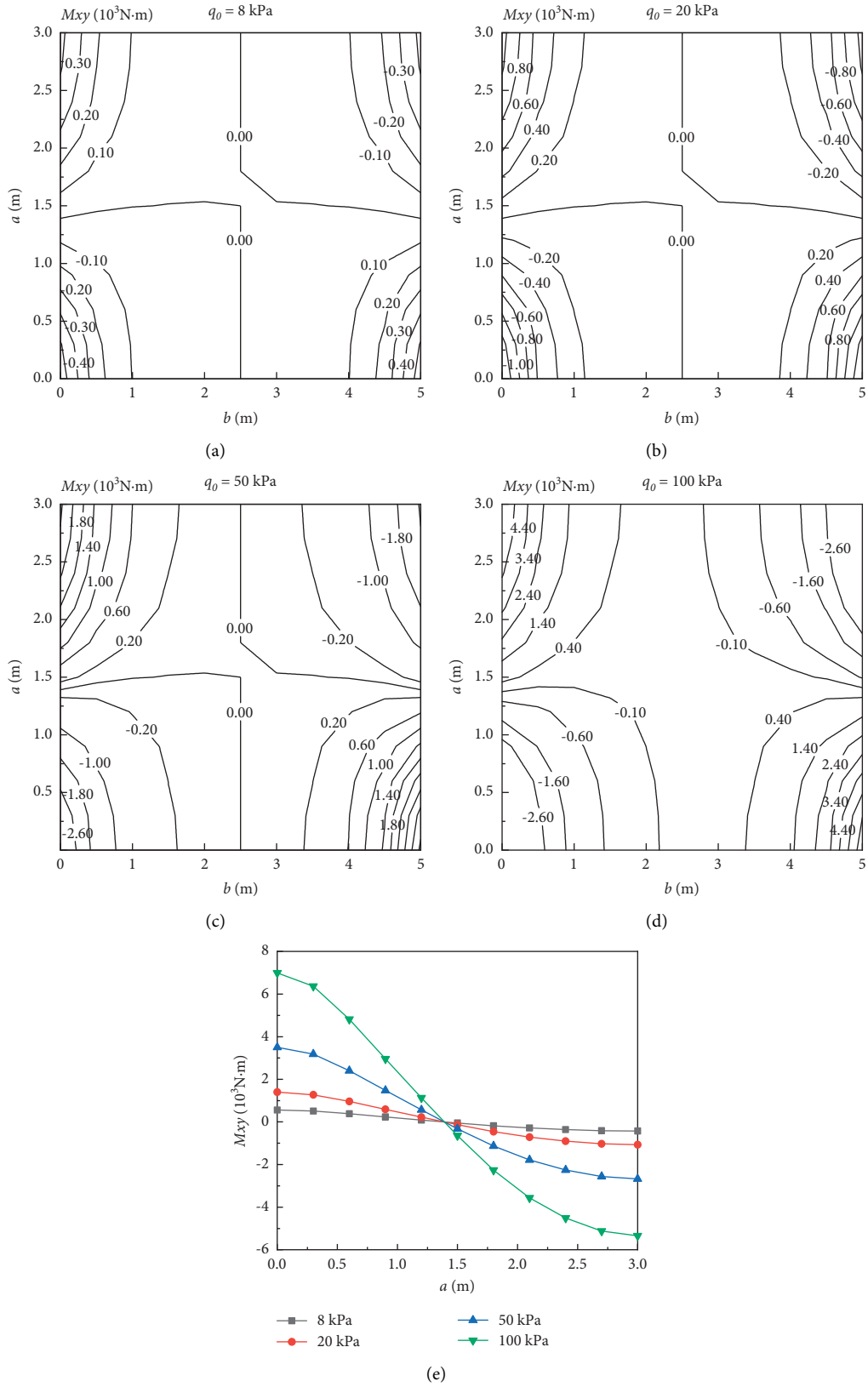


FIGURE 9: Effect of frost heaving force on bending moment M_{xy} .

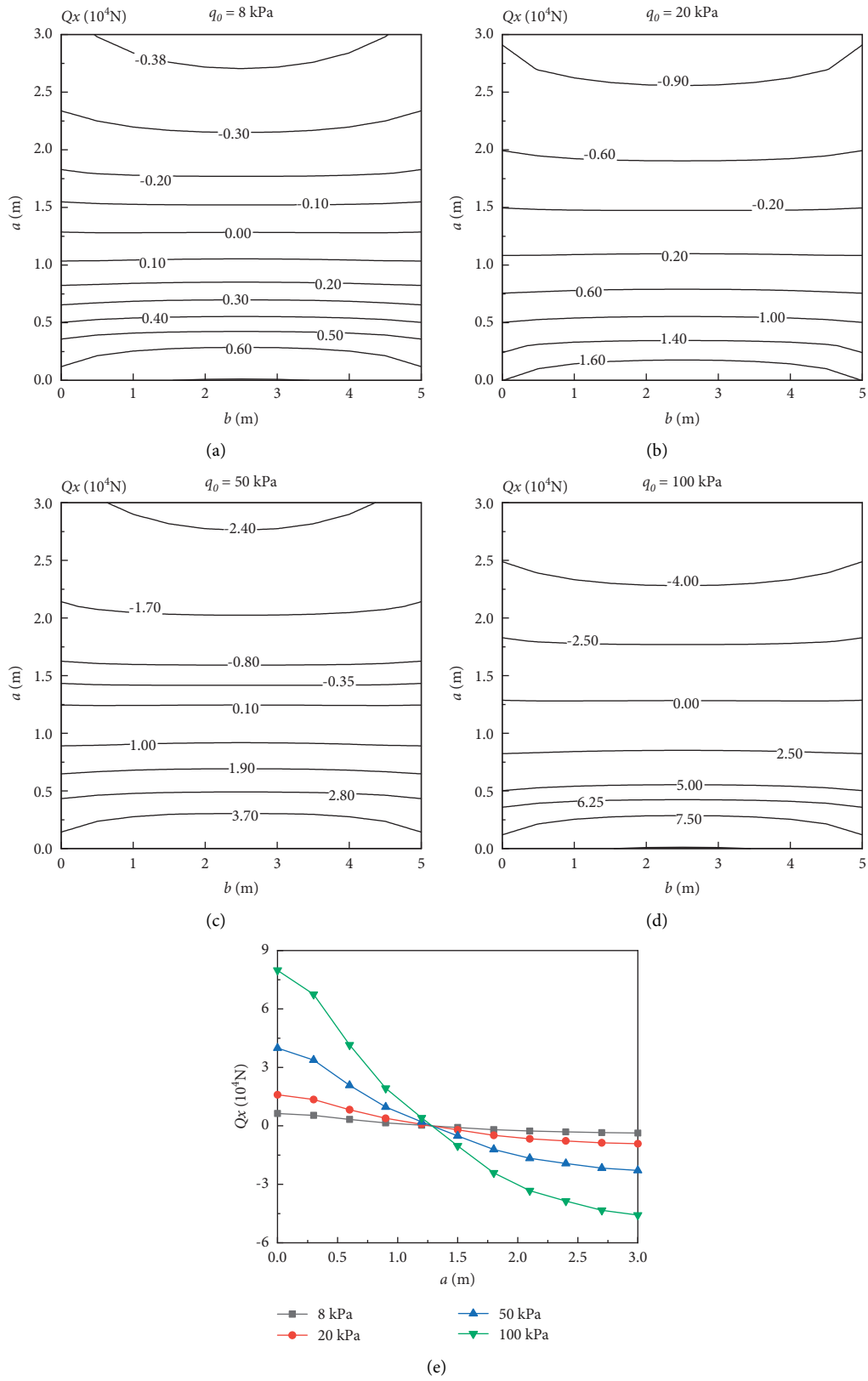


FIGURE 10: Effect of frost heaving force on shear force Q_x .

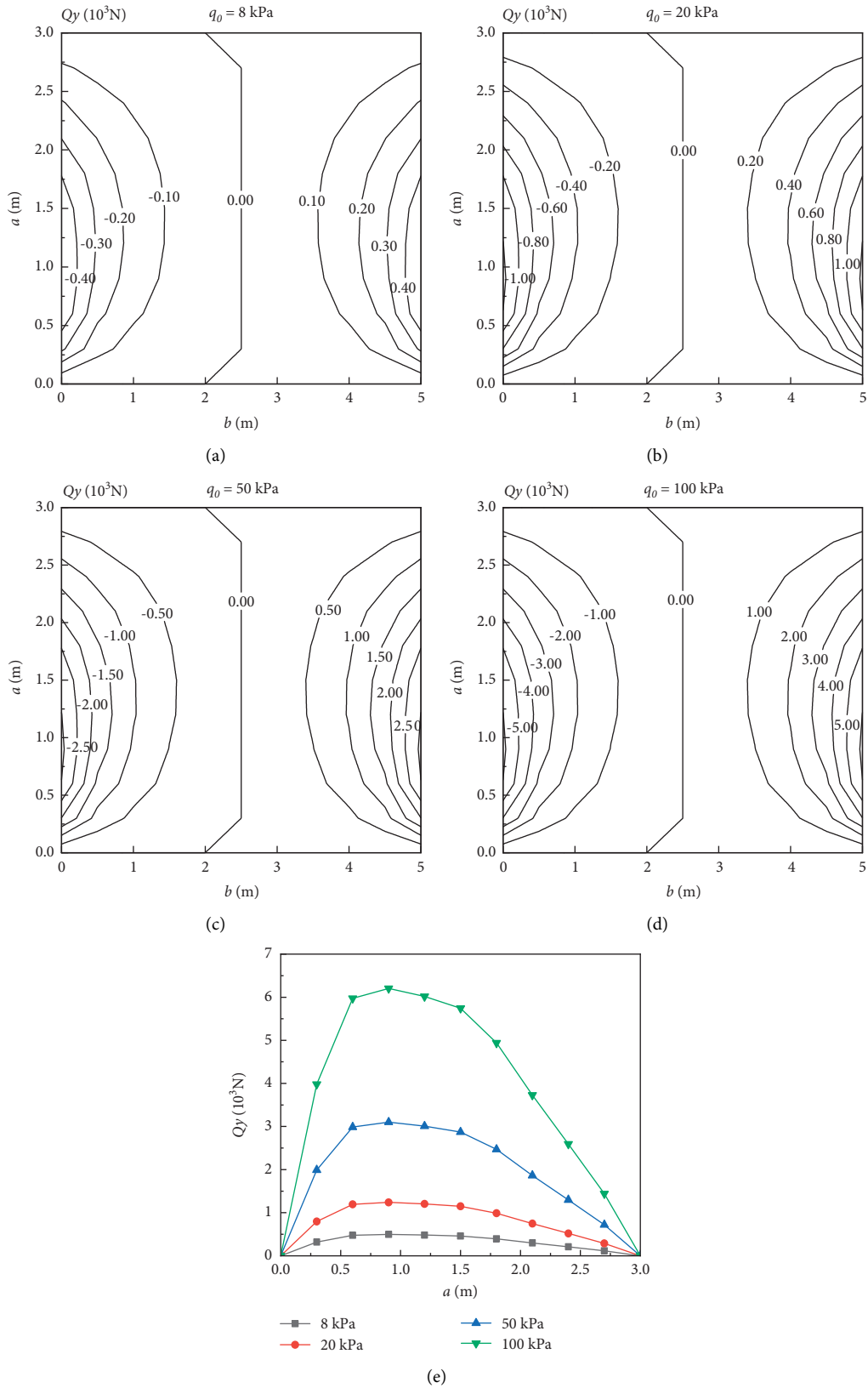


FIGURE 11: Effect of frost heaving force on shear force Q_y .

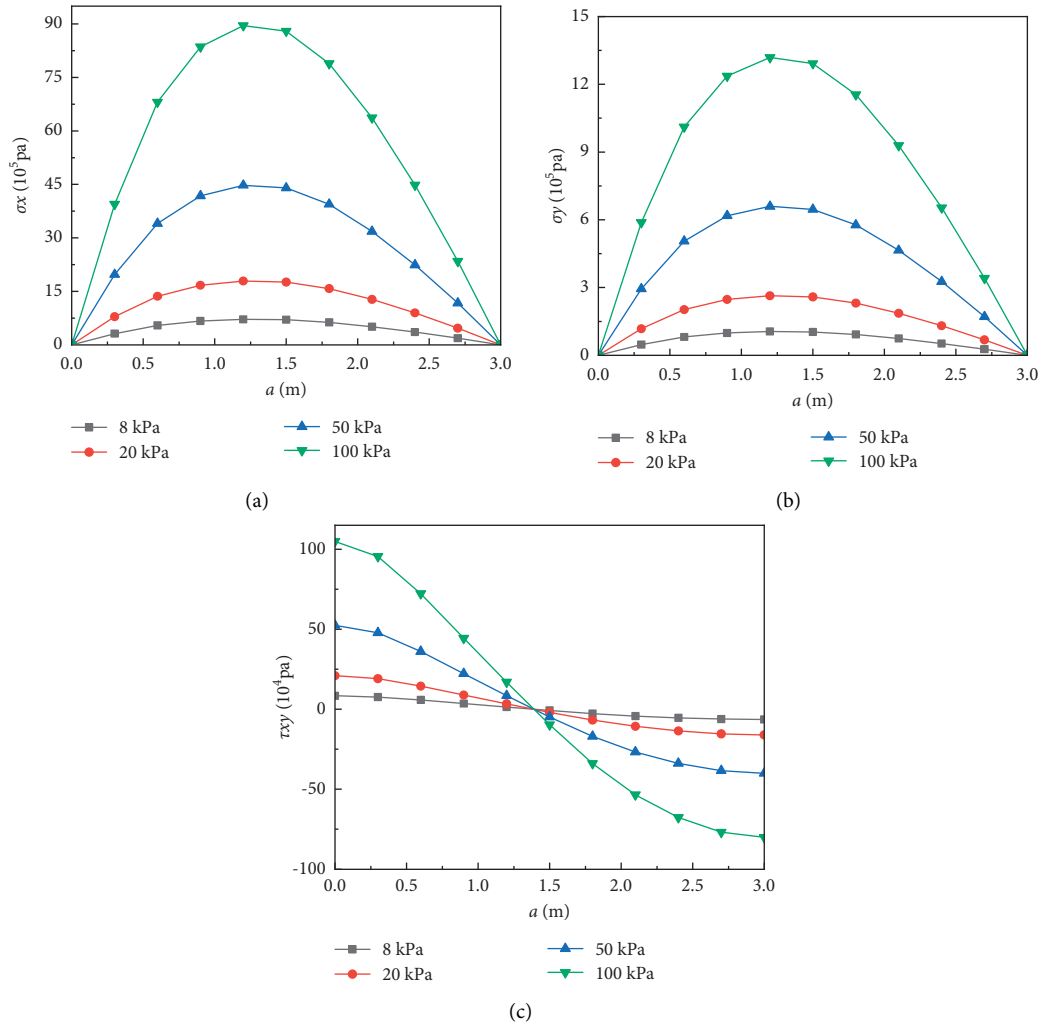


FIGURE 12: Effect of frost heaving force on stress.

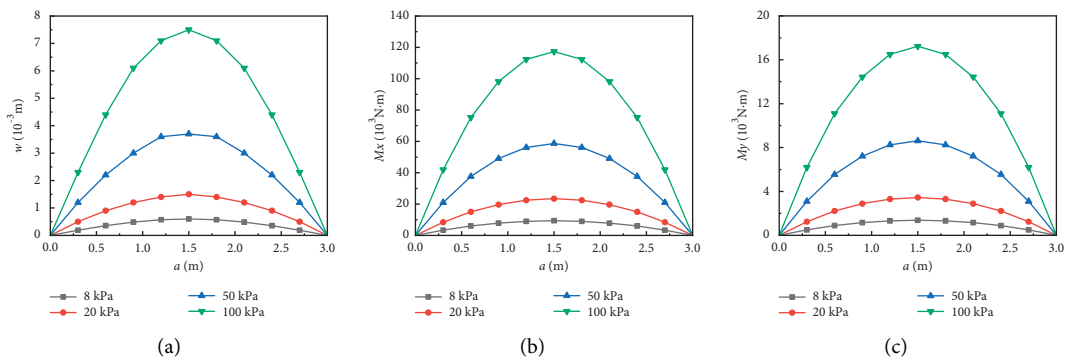


FIGURE 13: Continued.

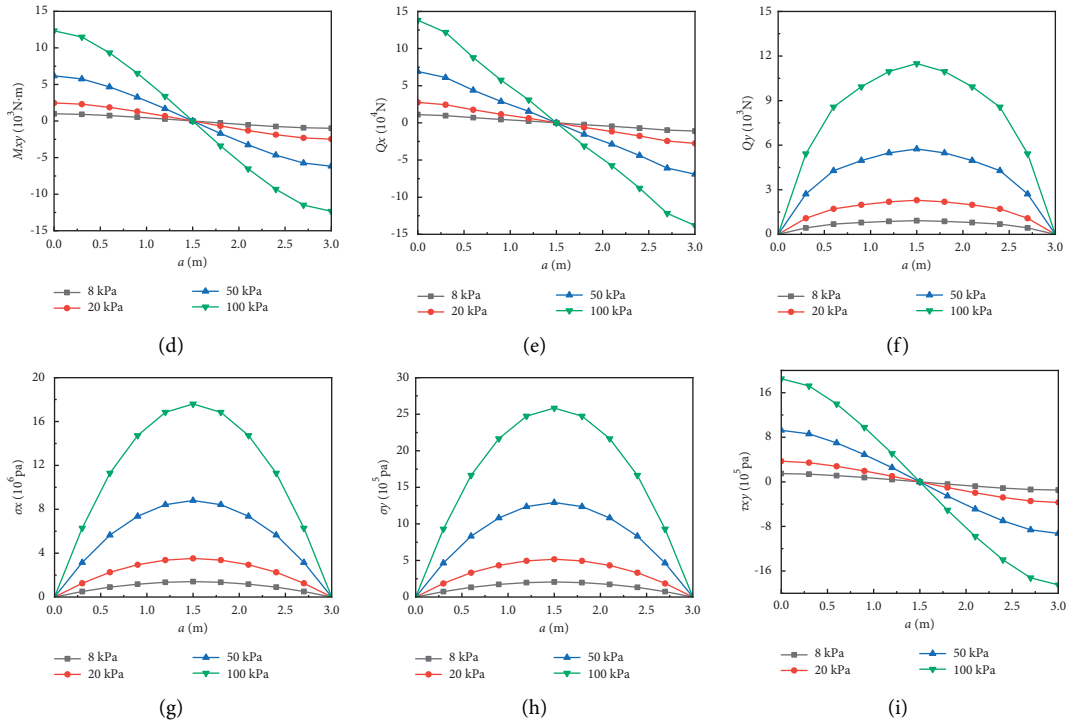


FIGURE 13: Effect of frost heaving force on deflection, internal force, and stress of the bottom plate.

Figure 13(b) shows that the bending moment M_x is asymmetrically distributed along the height direction of the plate. The bending moment M_x is the main internal force causing the longitudinal crack, and the maximum bending moment is near the location of the slope bottom, which is consistent with the references and the damage location in engineering practice [12, 23, 24]. The bending moment M_y (Figure 13(c)) is small at the edge and large at the center of the plate. The torque M_{xy} (Figure 13(d)) is maximum at the four corners of the plate and coincides with the geometric inflection points of the plate; although its numerical magnitude is lower than the value of the bending moment M_x , it may also cause stress concentration to damage the lining plate and thus cause leakage. The maximum values of shear forces Q_x and Q_y (Figures 13(e), (f)) are at the upper and lower edges and left and right edges, respectively, and are asymmetrically distributed along the height direction of the slope plate. The distribution laws of stresses σ_x , σ_y , τ_{xy} (Figures 13(g), (h), (i)) are the same as their corresponding internal force distribution laws.

5. Conclusion

(1) A mechanical model for deformation analysis of canal lining plates under the action of frost heaving force is established based on the theory of rectangular thin plate, the boundaries of rectangular thin plate are simplified to two pairs of simply supported opposite edges and the other two pairs of opposite edges are free (longitudinal expansion joints), and the solution process of the model is given.

- (2) Under the action of nonuniformly distributed frost heaving force, the slope plate has no obvious non-uniform distribution of deflection along the depth of the canal; the bending moment, shear force, and internal force show nonuniform distribution; and the maximum values of bending moment M_x and positive stress σ_x are biased to the bottom plate position. Compared with the beam theory, the calculation results of the plate theory show that the internal forces and stresses increase at the free boundary.
- (3) The bending moment M_{xy} and stress τ_{xy} are maximum at the four corners of the plate and geometrically coincide with the corners of the plate, although their numerical magnitudes are lower than that of the bending moment M_x , and they may also cause stress concentration to damage the lining plate and thus cause leakage. Although the shear forces at the left and right joint locations are not as large as the cross-sectional shear force, they may lead to fracture of the joint material, which needs to be considered in the design process.
- (4) Due to the uniform distribution of the frost heaving force on the bottom plate lining, its deflection, internal force, and stress also show uniform distribution.

Data Availability

Data can be obtained from the article and corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

Yangtao Liang was responsible for project administration. Fuping Zhang carried out formal analysis and wrote the original draft. Mingming Jing did data curation and formal analysis. Pengfei He carried out writing (original draft preparation) and formal analysis.

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