Hindawi
Advances in Civil Engineering
Volume 2022, Article ID 9124208, 9 pages
https://doi.org/10.1155/2022/9124208

Research Article

A Statistic Damage Model of Rocks considering the Effect of Loading Rate

Kai Cui,[1,2] Bin Hu ,[1] and Jing Li [1]

[1]*School of Resources and Environmental Engineering, Wuhan University of Science and Technology, Wuhan, Hubei 430081, China*
[2]*School of Applied Engineering, Henan University of Science and Technology, Sanmenxia, Henan 472000, China*

Correspondence should be addressed to Bin Hu; hbin74@wust.edu.cn and Jing Li; lijing1994@wust.edu.cn

Received 14 December 2021; Revised 4 January 2022; Accepted 15 January 2022; Published 9 February 2022

Academic Editor: Bing Bai

This paper develops a new statistic damage model for rock to mainly study the effect of a loading rate on its mechanical behaviours. The proposed model adopts a new loading rate-dependent damage density function and is capable of describing the macroscopic damage accumulation process for rock samples subjected to external high-speed dynamic loadings. The proposed model can also account for the residual strength of rocks by introducing a modified equivalent strain principle, which considers the contribution of the friction force to the strength of rocks. The friction force is generated by the movements of the nearby microcracks. The predicted stress-strain curves by the proposed model agree with the measured data of salty rock under the conditions of various confining pressures and loading rates. It can be found that both the peak strength and the corresponding axial strain are increased at high-speed loading conditions. At the same time, a transition from ductile failure to brittle failure can be observed in rock samples.

1. Introduction

The mechanical behaviours such as strength and deformation of rock are affected by loading rates [1–4]. The microcracks distributed inside the rock samples will be propagated and result in macroscopic damage [5–9]. Mathematical description of the dynamic loading-induced damage accumulation process plays a crucial role in modelling the rock behaviours.

In terms of strength, [10] proposed dynamic strength criteria for rock-like materials based on theoretical derivation to describe the uniaxial strength characteristics from quasistatic to dynamic uniformly [11] explored the effect of loading rate on the dynamic yield strength of sandstone through experiments. In addition, [12,13] carried out a series of dynamic experiments on the Bukit Timah granite. Based on the experimental results, they modified the Mohr-Coulomb strength criterion of rock to account for the effect of loading rates, which can accurately describe the dynamic response of rocks. Similarly, [14] proposed another strength criterion for the uniaxial dynamic strength of rocks based on the work of [15] and established a statistical damage model which can reasonably simulate the dynamic deformation process of rock.

At the deformation aspect, [16–19] used the SHPB apparatus to conduct empirical research on the uniaxial dynamic behaviour of salt rock under different confining pressures and loading rates and obtained its steady and dynamic stress-strain curves. The experimental results showed that the salt peak strength of the rock corresponding axial strain would increase significantly with the increase of the loading rate. To simulate the dynamic deformation process of rock, [20] established a dynamic damage model of rock based on the viscoplastic theory by assuming that rock damage was only related to dynamic strain energy. However, [21] suggested that the stress level and the loading rate can also affect the damage law of rock.

The statistic damage models can account for the propagation process of microcracks and characterise the strength and deformation behaviours of rocks, which have been widely applied in geotechnical engineering. [22]developed a damage model to study the effect of freeze-thaw cycles and confining pressures on the mechanical properties of rocks. [23] applied an equivalent continuum damage model for modelling the impact of weakness planes in rock masses on the stability of tunnels. The temperature may also affect the mechanical properties, which can be described using the statistic damage theory [24–26].

Although the above statistic damage models are widely applied in practice, some of them cannot describe the residual strength of rocks. To overcome such limitation, [27] introduced a rock damage scale factor following the assumption that the damaged components of rocks can still resist external force-induced deformation. At the same time, [28] modified the Lemaitre strain equivalence principle to include the friction force between the nearby microcracks.

This paper aimed to develop a dynamic statistic damage model to study the effect of loading rates on rocks' strength and deformation behaviours. First, the evolution law of the damage factor was assumed to be affected by the loading rates to describe the macroscopic damage accumulation process when the rock samples were subjected to external dynamic loading. Second, the proposed model adopted the modified Lemaitre strain equivalence assumption to consider the residual strength of the rock. The proposed model was used to predict the experimental results of salt rock under different confining pressures and loading rate conditions.

2. Dynamic Stress-Strain Curve of Rock

To study the effect of loading rates on the mechanical behaviours of salty rocks, [16] conducted a series of triaxial tests using the SPHB apparatus under the conditions of various confining pressures and loading rates. The measured stress-strain curves are presented in Figure 1; it can be found that the tangent modulus of rocks samples is gradually decreased during the early loading process, which is mainly induced by the damage of microstructure and the accumulation of plastic deformation. Besides, the stress-strain curves exhibit the feature of strain-softening after the peak strength state as the shear strength decreases considerably against the axial strain. At a given confining pressure, increasing the loading rate will lead to a higher peak shear strength, together with a larger corresponding axial strain, indicating that the strength of salt rock is enhanced at a high-speed loading condition.

Figure 2 shows the dynamic yield strength of the salty rock samples which is not affected by the loading rate ranging from 0.1–1.0/s, and the relationship between the yield strength and the confining pressure under the quasi-static loading conditions can be described using the conventional Mohr-Coulomb strength criteria. However, further increasing the loading rate will significantly increase the dynamic yield strength of rock samples; the most apparent increment happens in the case of $\sigma_3 = 5\,\text{MPa}$, where the dynamic yield strength is approximately increased from 20 MPa to 62 MPa for the loading rates of 1.0 and 10^3/s, respectively. To address such a strength enhancement phenomenon, [14] suggested that the strength of rocks consists of two different components: one is the quasi-static strength that only depends on the confining pressure, whereas the other is the inertia force-induced strength. Increasing the loading rate will increase the inertia force-induced strength, which will play a primary role in forming the strength of rocks.

3. Statistical Damage Rock Model

3.1. Damage Factor. When rock is subjected to external forces, microcracks will gradually initiate and propagate to form random-distributed microcracks. The macroscopic damage accumulation process will decrease the tangent elastic modulus of rock samples and lead to a final failure.

Within the statistic damage theory, rock is assumed to consist of many microscope elements, and some of them will be damaged due to external forces. Assuming N is the number of all microscopic elements within rock and N_d is the number of damaged elements, the damage factor D is usually defined as the ratio between N_d and N as

$$D = \frac{N_d}{N}, \quad 0 \le D \le 1. \tag{1}$$

The Weibull distribution is used herein to describe the strength of microscopic elements, namely,

$$P(F) = \frac{m}{F_0}\left(\frac{F}{F_0}\right)^{m-1} \exp\left[-\left(\frac{F}{F_0}\right)^{m}\right], \tag{2}$$

where F is the element strength parameter corresponding to the failure criterion of rocks, which may be a function of the stress level or axial strain. m and F_0 are the shape and scale parameters, respectively.

We assume that the incremental number of damaged microscopic elements subjected to external forces is calculated as

$$dN_d = NP(F)dF\ . \tag{3}$$

Substituting equations (1) and (2) into (3), the damage factor D can be integrated as

$$\begin{aligned} D &= \int_0^F P(F)\mathrm{d}F \\ &= 1 - \exp\left[-\left(\frac{F}{F_0}\right)^{m}\right]. \end{aligned} \tag{4}$$

The damage factor D can quantify the macroscopic damage accumulation process. When D equals 1.0, rock samples will be completely damaged and lose the capability of resisting further deformation.

Figure 3 shows the effect of parameters m and F_0 on the evolution of D concerning F. On the one hand, it can be found that the evolution curve will conduct counterclockwise rotation by increasing the value of m. On the other hand, increasing the values of F_0 will significantly accelerate the macroscopic damage accumulation process.

3.2. Stress-Strain Relation. This study assumes that only a part of the rock sample will be damaged when subjected to

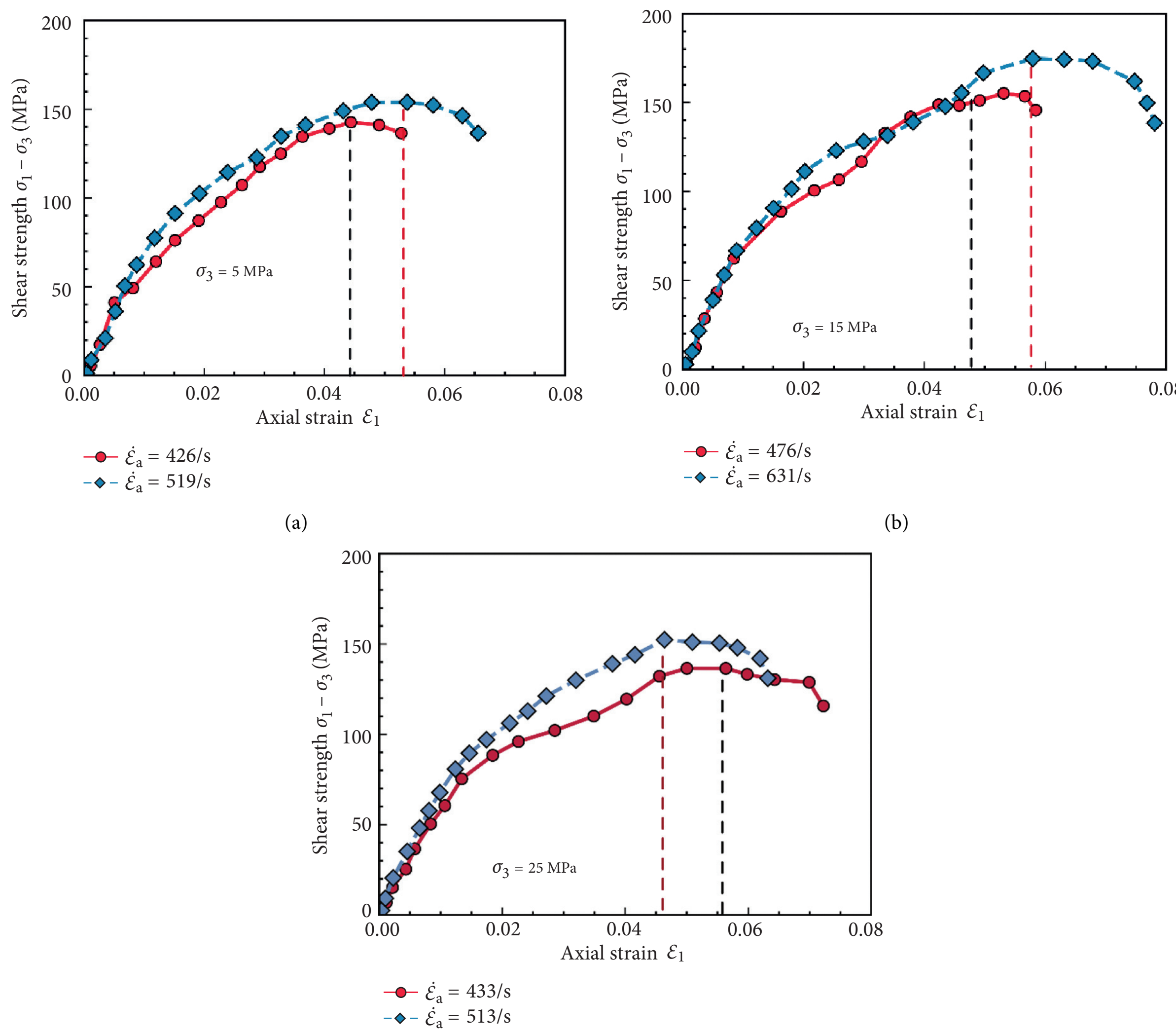


Figure 1: Dynamic stress-strain curves of salt rock [16] under different confining pressures and loading rates. (a) Confining pressure σ_3 = 5MPa. (b) Confining pressure σ_3 = 15MPa. (c) Confining pressure σ_3 = 25MPa.

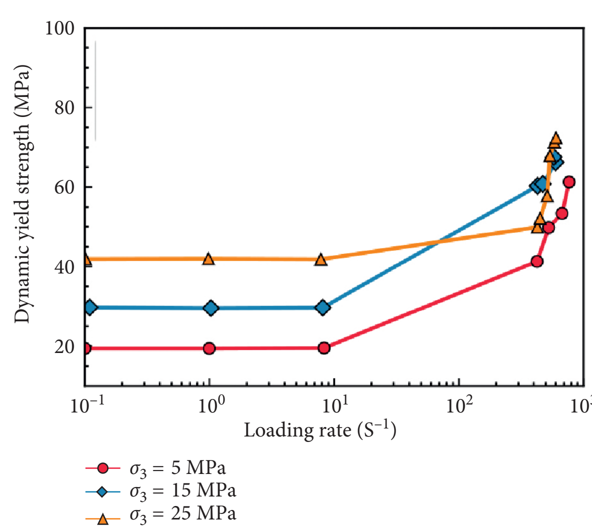


Figure 2: Dynamic yield strength of salt rock [16] corresponding to the loading rate at different confining pressures.

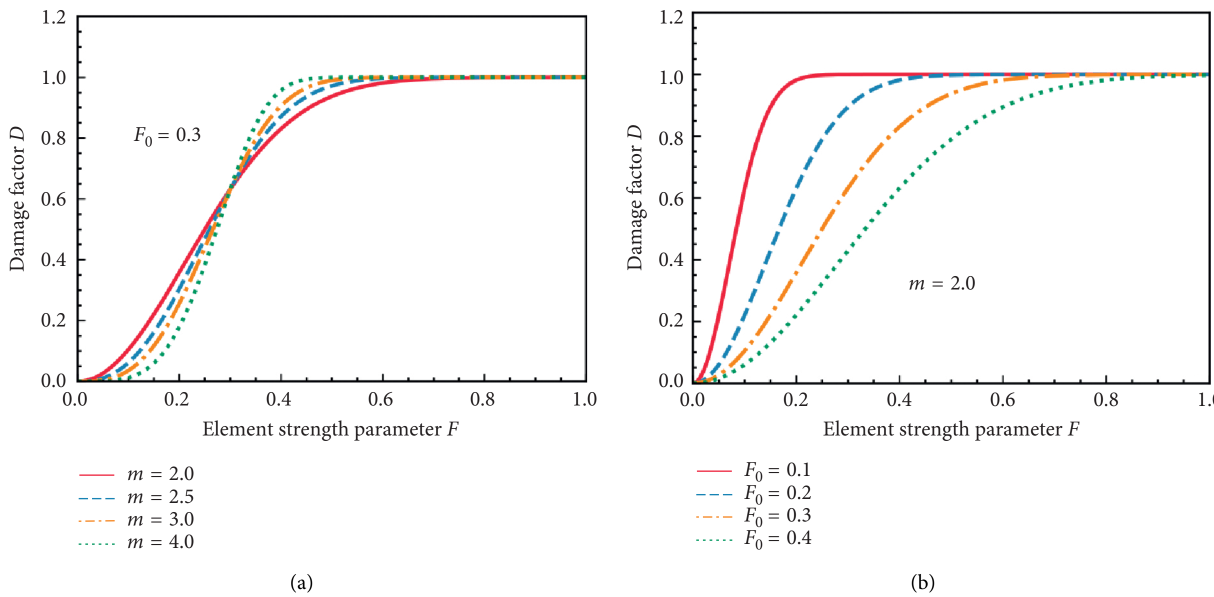


Figure 3: Effect of parameters m and F_0 on the evolution of D concerning F. (a) Parameter m. (b) Parameter F_0.

external forces. The intact component will perform elastic response obeying the generalized Hook's law:

$$\sigma_i' = E\varepsilon_i' + \mu\left(\sigma_j' + \sigma_k'\right), i, j, k = 1, 2, 3\ldots, \tag{5}$$

where $\sigma\prime$ and $\varepsilon\prime$ are the undamaged stress and strain, respectively. E is the elastic modulus, and μ is the Poisson's ratio.

According to the equivalent strain principle, it can be considered that the rock is composed of damaged and undamaged materials under external load, and the damaged part of the material does not have any bearing capacity. The equivalent stress σ is defined as

$$\sigma_i = \sigma_i'(1 - D), \quad i = 1, 2, 3. \tag{6}$$

Substituting equation (6) into (5), the stress-strain relation of rock considering the macroscopic damage accumulation is given by

$$\sigma_i = E\varepsilon_i(1 - D) + \mu\left(\sigma_j + \sigma_k\right). \tag{7}$$

We note that equation (7) cannot predict the residual strength of rock samples at different confining pressures because D will turn to be zero after the rock samples are entirely damaged. To overcome this limitation, [28] suggested that the friction force between the nearby microcracks can contribute to the residual strength of rocks. After that, the stress-strain relation is modified to account for the residual strength σ_{residual} as

$$\sigma_i = E\varepsilon_i(1 - D) + D\sigma_{\text{residual}} + \mu\left(\sigma_j + \sigma_k\right), \tag{8}$$

where σ_{residual} is the residual strength of the rock. When $\sigma_{\text{residual}} = 0$, (8) will degenerate to the original Lemaitre strain equivalence assumption.

To consider the effect of the loading rate on the mechanical behavior of rocks, the evolution law of the damage ratio D in (9) is modified as

$$D = 1 - \exp\left\{-\left[\frac{\varepsilon_1}{F_0}\left(1 + \alpha\ln\left(\frac{\dot{\varepsilon}_1}{\varepsilon_{\text{ref}}}\right)\right)\right]^m\right\}, \tag{9}$$

where α is a non-negative material constant, and ε_{ref} is the reference loading rate that is chosen as $\dot{\varepsilon}_{\text{ref}} = 10^{-5}$.

3.3. Model Parameter Calibration. The proposed statistic damage model for rocks includes six material parameters. The values of the elastic modulus E, Poisson's ratio μ, and the residual strength of σ_{residual} can be determined through the measured stress-strain curve of rock samples at a specific confining pressure. Parameter α should be calibrated by conducting another test with a different loading rate.

To calibrate the parameters m and F_0, the measured peak strength state of the rock samples should be used. For a conventional triaxial test ($\sigma_2 = \sigma_3$), the stress-strain relation represented in (8) is simplified as

$$\sigma_1 = E\varepsilon_1(1 - D) + D\sigma_{\text{residual}} + 2\mu\sigma_3. \tag{10}$$

Once the peak strength state is reached, the derivative of σ_1 with the corresponding ε_1 should be zero, namely,

$$\frac{\partial\sigma_1}{\partial\varepsilon_1}\Big|_{\varepsilon_1=\varepsilon_{\text{peak}},\sigma_1=\sigma_{\text{peak}}} = 0, \tag{11}$$

where σ_{peak} and $\varepsilon_{\text{peak}}$ are the peak strength and the related axial strain.

Substituting (10) into (9) and calculating the derivatives, we obtain

Table 1: Model parameters for the proposed model.

Material parameters	Value
Elastic modulus E (MPa)	800.0
Poisson's ratio μ	0.5
Residual strength σ_{residual} (MPa)	1.5
Peak strength σ_{peak} (MPa)	10.0
Axial strain at the peak strength state $\varepsilon_{\text{peak}}$	0.025
Rate-related material constant α	1.0

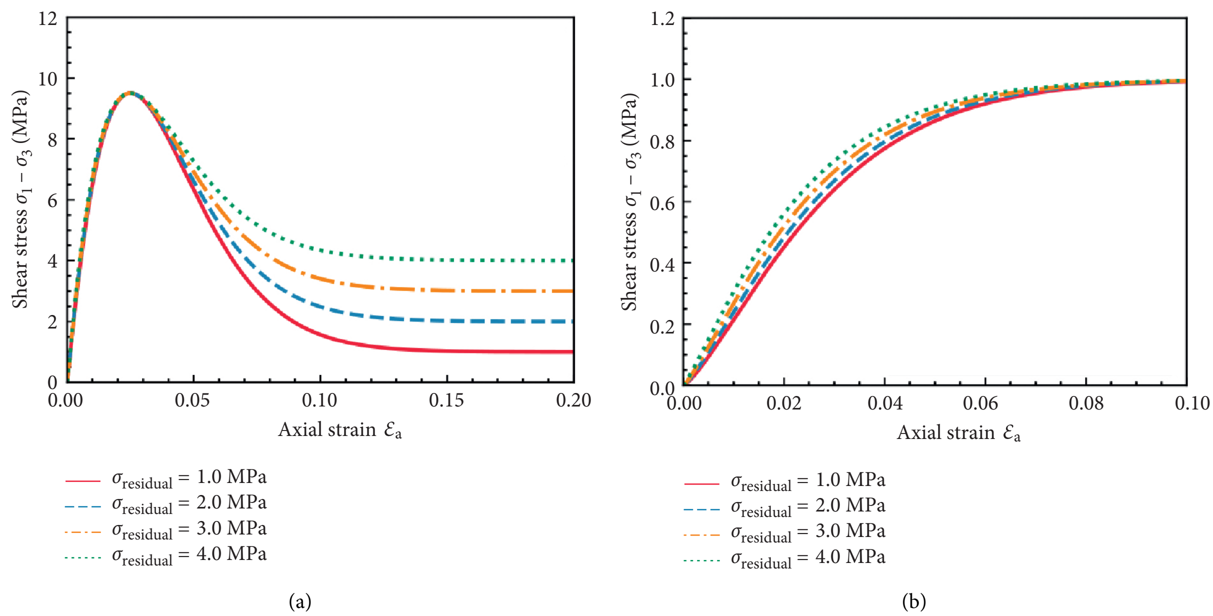


Figure 4: Effect of residual strength σ_{residual} on stress-strain curve and damage curve of rock. (a) Stress-strain curve. (b) Damage curve.

$$\left\{\frac{\varepsilon_1}{F_0}\left[1+\alpha\ln\left(\frac{\dot{\varepsilon}_1}{\varepsilon_{\text{ref}}}\right)\right]\right\}^{m-1}=\frac{EF_0}{mE\varepsilon_{\text{peak}}-m\sigma_{\text{residual}}}. \tag{12}$$

The stress-strain relation at the peak strength state can be expressed as

$$\sigma_{\text{peak}}=(1-D))\cdot E\varepsilon_{\text{peak}}+D\cdot\sigma_{\text{residual}}+2\nu\sigma_3. \tag{13}$$

Solving equations (12) and (13) will lead to the exact expression of parameters m and F_0 as

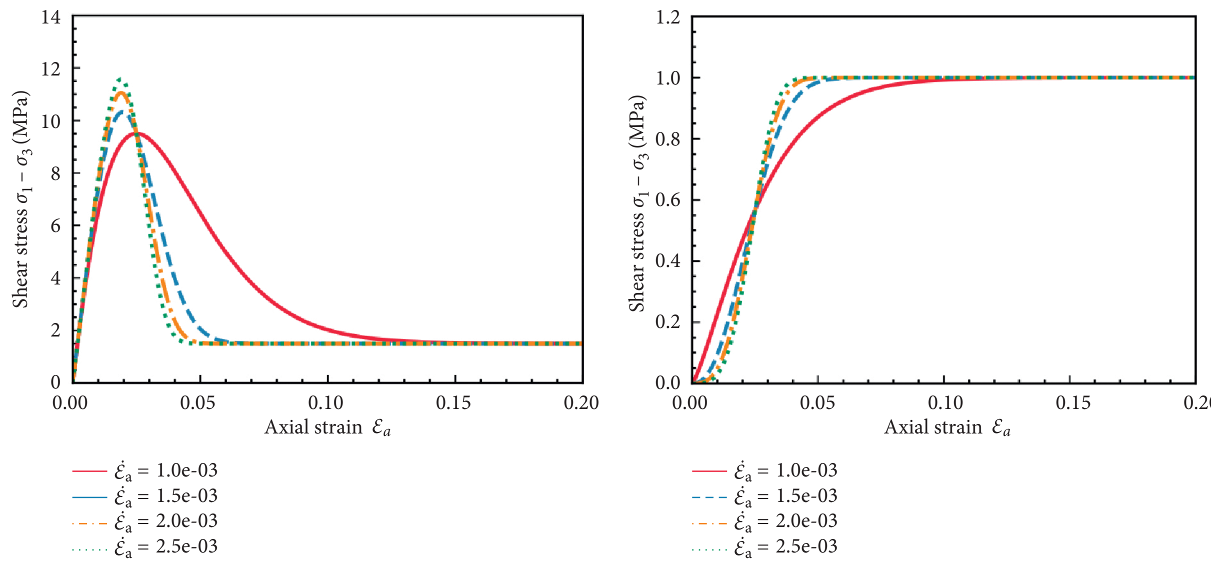
$$m=-\frac{E\varepsilon_{\text{peak}}\left[1+\alpha\ln\left(\dot{\varepsilon}_1/\varepsilon_{\text{ref}}\right)\right]}{\left(E\varepsilon_{\text{peak}}-\sigma_{\text{residual}}\right)\left[\ln\left(\sigma_{\text{peak}}^{-2\nu\sigma_3-\sigma_{\text{residual}}}/E\varepsilon_{\text{peak}}^{-\sigma}\text{ residual}\right)\right]},$$
$$F_0=\left\{\frac{mE\varepsilon_{\text{peak}}-m\sigma_{\text{residual}}}{E}\left[\varepsilon_{\text{peak}}\left(1+\alpha\ln\left(\frac{\dot{\varepsilon}_1}{\varepsilon_{ref}}\right)\right)\right]^{m-1}\right\}^{1/m}. \tag{14}$$

3.4. Parameter Experiment. To investigate the performance of the proposed statistical damage model in modelling the dynamic mechanical behaviours of rocks, a series of case studies were conducted here by letting the confining pressure be 0.5 MPa, and the values of the corresponding model parameters are given in Table 1.

First, we studied the effect of the residual strength on the stress-strain curves of rocks. Unlike the previous statistical damage models, the proposed model introduced a modified equivalent strain principle to account for the contribution of the friction force between the propagated nearby microcracks and the strength of rocks. In the cases of σ_{residual} were chosen to be 1.0, 2.0, 3.0, and 4.0 MPa, the predicted stress-strain curves shown in Figure 4(a) indicated that the strain-softening feature was weakened during the post-failure process, and the rock samples turn to be more ductile, which was confirmed by the evolution of the damage factor against the axial strain, as shown in Figure 4(b).

Second, this paper explored the influence of loading rate on rock dynamic stress-strain curve and damage evolution law, and the calculation results are shown in Figure 5. From Figure 5(a), it can be seen that the loading rate does not affect the residual strength of rock. However, the peak strength of rock σ_{peak} will gradually increase with a gradual increase of $\dot{\varepsilon}_1$. On the contrary, the peak strength of rock σ_{peak} will gradually decrease. The calculation results are consistent with the experimental results of rock under dynamic loading.

According to Figure 5(b), the damage factor D will gradually decrease with the increase of loading rate in a small

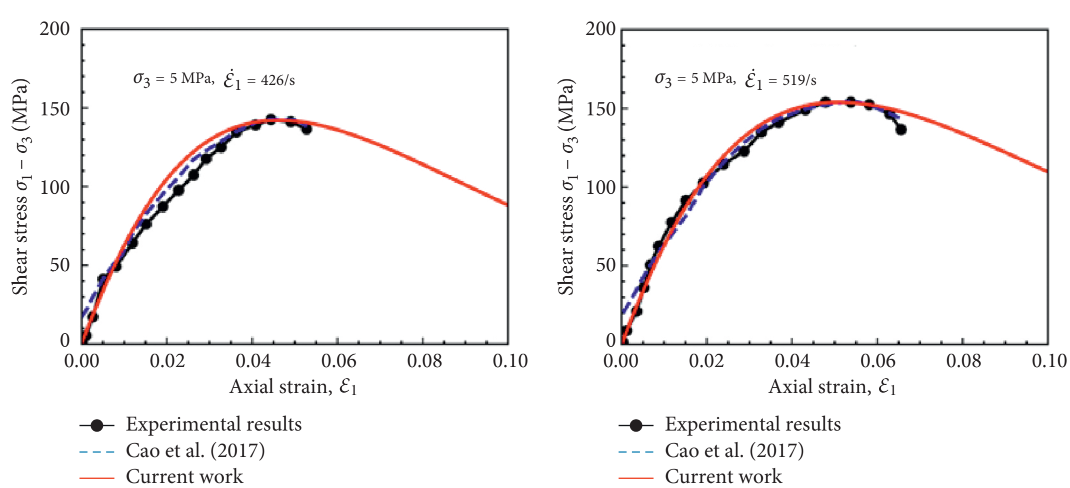


(a) (b)

FIGURE 5: Effect of loading rates on stress-strain curve and damage curve of rock $\dot{\varepsilon}_1$. (a) Stress-strain curve. (b) Damage curve.

TABLE 2: Model parameters of different kinds of rock.

σ_3/MPa	$\dot{\varepsilon}$/S^{-1}	E/GPA	μ	m	F_0
5	426	7.12	0.25	1.182	0.053
	519	7.21	0.25	1.155	0.057
15	476	7.13	0.25	1.119	0.055
	631	7.24	0.25	1.209	0.063
25	433	6.08	0.25	0.932	0.046
	513	6.95	0.25	1.094	0.052

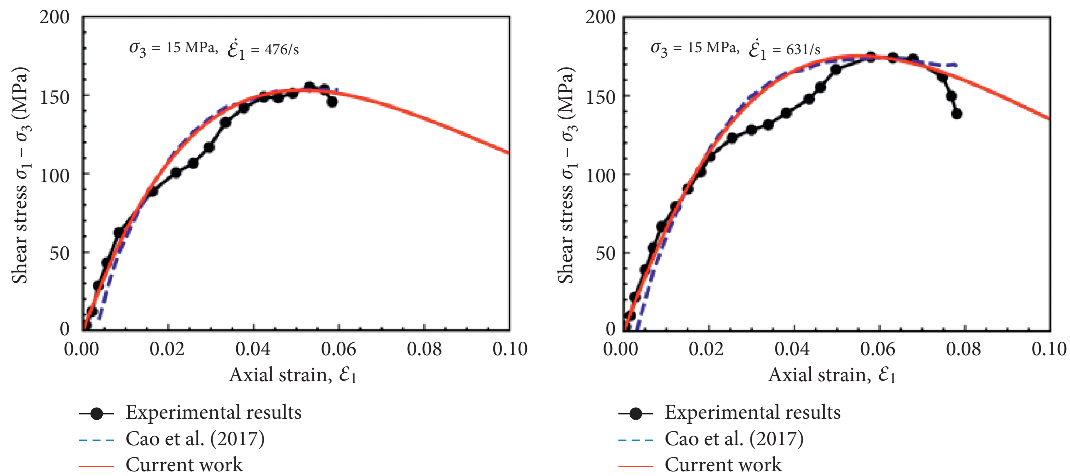


(a) (b)

FIGURE 6: Comparison between experimental and model results of dynamic stress-strain curves of salt rock [16] under confining pressure σ_3 = 5MPa. (a) $\dot{\varepsilon}_1 = 426/s$. (b) $\dot{\varepsilon}_1 = 519/s$.

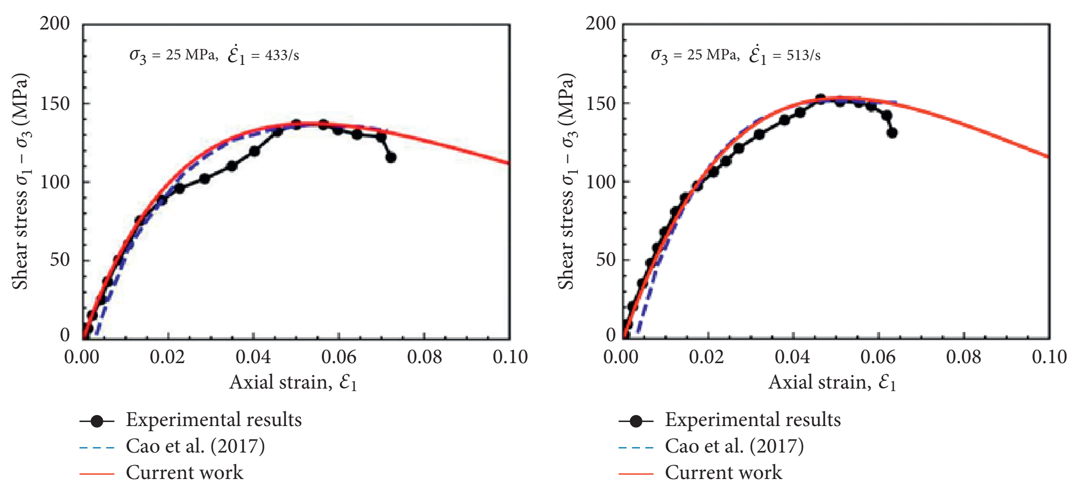
strain range ($\varepsilon_1 \leq 0.025$), and the tangent modulus of rock will also increase. However, with the further increase of axial strain, the growth rate is accelerated with the increase of loading rate, leading to brittle failures of rock more efficiently, and the stress-strain curve will have a noticeable "stress drop" phenomenon.

The experimental results show that the rock statistical damage model established in this paper can better describe



(a) (b)

Figure 7: Comparison between experimental and model results of dynamic stress-strain curves of salt rock [16] under confining pressure $\sigma_3 = 15$MPa. (a) $\dot{\varepsilon}_1 = 476/s$. (b) $\dot{\varepsilon}_1 = 631/s$.



(a) (b)

Figure 8: Comparison between experimental and model results of dynamic stress-strain curves of salt rock [16] under confining pressure $\sigma_3 = 25$MPa. (a) $\dot{\varepsilon}_1 = 433/s$. (b) $\dot{\varepsilon}_1 = 513/s$.

the effect of loading rate on the dynamic, progressive failure, and the residual strength of rock.

4. Model Verification

To further verify the performance of the proposed model, especially in describing the effect of loading rate on the dynamic yield strength of rocks, the experimental results of salty rocks are adopted here again. Table 2 gives the values of the model parameters with different confining pressures and loading rates.

The measured and predicted stress-strain curves by the proposed model are shown in Figures 6–8, together with the predictions by [14] as a comparison. Although the two statistic damage models predict similar results and can fit the measured dynamic yield strength quite well, the stress-strain curve predicted by [14] has an unreasonable shear strength before the external force is applied. Such limitation is overcome by the proposed model, which can better account for the mechanical response of rock samples during the early loading stage. It is noted that the proposed model can be conveniently extended based on further experimental investigation to consider the effect of loading rate on the residual strength of rocks.

5. Conclusion

This paper develops a new statistic damage model for rock to study its dynamic mechanical behaviours. The evolution law of the damage factor is assumed to depend on the loading ratio. Besides, the proposed model adopts a modified equivalent strain principle to improve its performance in modelling the residual strength of rocks. The main conclusions are summarized as follows:

(1) The microscopic damage accumulation process is affected by the loading rate, resulting in a higher peak strength of rocks. The evolution law of the damage factor is in terms of the axial strain and loading rate.

(2) The modified equivalent strain principle enables the proposed model to predict a reasonable residual strength of rocks mainly formed by the friction force generated by the movement of the nearby microcracks.

(3) The proposed model can well describe the stress-strain curves of salty rocks, providing a promising tool to study the dynamic behaviours of rocks.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China under Grant nos. U1802243 and 41672317, in part by the Hubei Province Technical Innovation Special (major projects) Project under Grant no. 2017ACA184, and in part by the Major Science and Technology Projects of WUST Cultivate Innovation Teams under Grant no. 2018TDX01.

References

[1] S. J. Xie, H. Lin, Y. F. Chen, Y. Wang, R. Cao, and J. Li, "Statistical damage shear constitutive model of rock joints under seepage pressure," *Frontiers of Earth Science*, vol. 8, no. 3, p. 232, Article ID 00232, 2020.

[2] S. Xie, H. Lin, Y. Wang et al., "A statistical damage constitutive model considering whole joint shear deformation," *International Journal of Damage Mechanics*, vol. 29, no. 6, pp. 988–1008, 2020.

[3] W. J. Liu, Z. W. Dan, Y. J. Jia, and X. H. Zhu, "On the statistical damage constitutive model and damage evolution of hard rock at high-temperature," *Geotechnical & Geological Engineering*, vol. 38, no. 6, pp. 4307–4318, 2020.

[4] H. B. Jiang, K. N. Li, and X. B. Hou, "Statistical damage model of rocks reflecting strain softening considering the influences of both damage threshold and residual strength," *Arabian Journal of Geosciences*, vol. 13, no. 7, pp. 1–8, 2020.

[5] Y. Pan, Z. M. Zhao, L. He, and G. Wu, "A nonlinear statistical damage constitutive model for porous rocks," *Advances in Civil Engineering*, vol. 2020, no. B3, 12 pages, Article ID 8851914, 2020.

[6] C. T. Zhou, K. Zhang, H. B. Wang, and Y. X. Xu, "A plastic strain based statistical damage model for brittle to ductile behaviour of rocks," *Geomechanics and Engineering*, vol. 21, no. 4, pp. 349–356, 2020.

[7] K. Chen, "Constitutive model of rock triaxial damage based on the rock strength statistics," *International Journal of Damage Mechanics*, vol. 29, no. 10, pp. 1487–1511, 2020.

[8] W. Liu, S. Zhang, and B. Sun, "Energy evolution of rock under different stress paths and establishment of A statistical damage model," *KSCE Journal of Civil Engineering*, vol. 23, no. 10, pp. 4274–4287, 2019.

[9] J. Lemaitre, *A Course on Damage Mechanics*, Springer, Berlin, Germany, 1992.

[10] J. Hu, Y. P. Yao, X. D. Zhang, and Y. Wei, "Dynamic strength criterion for rock-like materials," *Chinese Journal of Geotechnical Engineering*, vol. 42, no. 3, pp. 97–104, 2020.

[11] F. Q. Gong, D. H. Lu, X. B. Li, and Q. H. Rao, "Experimental research of sandstone dynamic strength criterion under different strain rates," *Rock and Soil Mechanics*, vol. 34, no. 9, pp. 2433–2441, 2013.

[12] H. B. Li, J. Zhao, and J. R. Li, "Experimental study on dynamic mechanical properties of granite under triaxial condition," *Explosion and Shock Waves*, vol. 24, no. 5, pp. 470–474, 2004.

[13] J. Zhao and H. B. Li, "Estimating the dynamic strength of rock using mohr-coulomb and hoek-brown criteria," *Chinese Journal of Rock Mechanics and Engineering*, vol. 22, no. 2, pp. 171–176, 2003.

[14] W. G. Cao, X. T. Lin, C. Zhang, and S. Yang, "A statistical damage simulation method of dynamic deformation process for rocks based on nonlinear dynamic strength criterion," *Chinese Journal of Rock Mechanics and Engineering*, vol. 36, no. 4, pp. 794–802, 2017.

[15] Q. H. Qian and C. Z. Qi, "Dynamic strength and dynamic fracture criteria of rock and rock mass," *Journal of Tong ji University(Natural Science)*, vol. 36, no. 12, pp. 1599–1605, 2008.

[16] Q. Fang, Z. Ruan, C. C. Zhai, X. Jiang, L. Chen, and W. Fang, "Split Hopkinson pressure bar test and numerical analysis of salt rock under confining pressure and temperature," *Chinese Journal of Rock Mechanics and Engineering*, vol. 31, no. 9, pp. 1756–1765, 2012.

[17] Q. Z. Wang, W. Li, and X. L. Song, "A method for testing dynamic tensile strength and elastic modulus of rock materials using SHPB," *Pure and Applied Geophysics*, vol. 163, no. 5-6, pp. 1091–1100, 2006.

[18] Q. Z. Wang, W. Li, and H. P. Xie, "Dynamic split tensile test of Flattened Brazilian Disc of rock with SHPB setup," *Mechanics of Materials*, vol. 41, no. 3, pp. 252–260, 2009.

[19] X. Li, Y. Zou, and Z. Zhou, "Numerical simulation of the rock SHPB test with a special shape striker based on the discrete element method," *Rock Mechanics and Rock Engineering*, vol. 47, no. 5, pp. 1693–1709, 2014.

[20] L. X. Xie, G. M. Zhao, and X. R. Meng, "Research on excess stress constitutive model of rock under impact load," *Chinese Journal of Rock Mechanics and Engineering*, vol. 32pp. 2772–2781, z1, 2013.

[21] W. G. Cao, H. Zhao, L. Zhang, and Y. J. Zhang, "Simulation method of dynamic triaxial deformation process for rock under invariable strain rate," *Chinese Journal of Geotechnical Engineering*, vol. 32, no. 11, pp. 1658–1664, 2010.

[22] H. M. Zhang, X. Z. Meng, and G. S. Yang, "A study on mechanical properties and damage model of rock subjected to freeze-thaw cycles and confining pressure," *Cold Regions Science and Technology*, vol. 174, Article ID 103056, 2020.

[23] G. Xu, M. Gutierrez, C. He, and S. Wang, "Modeling of the effects of weakness planes in rock masses on the stability of tunnels using an equivalent continuum and damage model," *Acta Geotechnica*, vol. 16, no. 7, pp. 2143–2164, 2021.

[24] B. Bai, R. Zhou, G. Q. Cai, W. Hu, and G. C. Yang, "Coupled thermo-hydro-mechanical mechanism in view of the soil particle rearrangement of granular thermodynamics," *Computers and Geotechnics*, vol. 137, no. 8, Article ID 104272, 2021.
[25] B. Bai, G. C. Yang, T. Li, and G. S. Yang, "A thermodynamic constitutive model with temperature effect based on particle rearrangement for geomaterials," *Mechanics of Materials*, vol. 139, Article ID 103180, 2019.
[26] W. L. Feng, C. S. Qiao, T. Wang, M.-y Yu, S.-J. Niu, and Z.-Q. Jia, "Strain-softening composite damage model of rock under thermal environment," *Bulletin of Engineering Geology and the Environment*, vol. 79, no. 11, pp. 4321–4333, 2020.
[27] S. Q. Yang, W. Y. Xu, L. D. Wei, and C. D. Su, "Statistical constitutive model for rock damage under uniaxial compression and its experimental study," *Journal of Hohai University(Natural Sciences)*, vol. 32, no. 2, pp. 200–203, 2004.
[28] H. C. Li and S. Zhang, "A constitutive damage model of rock based on the assumption of modified Lemaitre strain equivalence hypothesis," *Rock and Soil Mechanics*, vol. 38, no. 5, pp. 1321–1326+1334, 2017.