

# Research Article

# A Novel Surface Subsidence Prediction Model Based on Stochastic Medium Theory for Inclined Coal Seam Mining

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Most coal resources are deposited in the form of inclined coal seams, and surface subsidence basin morphology induced by the mining of inclined coal seams is frequently skewed. The probability integral method with symmetrical distribution characteristics is widely used at present in surface subsidence prediction in coal mining in China. However, this method performs poorly when the inclined coal seam mining subsidence is predicted, and prediction accuracy decreases considerably with an increase in coal seam inclination. To solve this problem, this study first establishes three coordinate systems: a working surface rectangular coordinate system, a working face body-following coordinate system, and a surface rectangular coordinate system. Then, a random medium theory is applied to realize the superposition integral operation of the subsidence influence of unit mining in the working faces body-following coordinate system. Subsequently, the subsidence effect of a certain point on the surface is converted into the surface rectangular coordinate system. Finally, the inclined coal seam mining subsidence caused by the mining of the inclined coal seam units conforms to the Weibull polar distribution law, and the effectiveness of the prediction model is verified through examples. The relative mean squared error of the prediction is less than 10%. The results of the study can provide theoretical and technical support for the subsidence prediction of similar mining areas.

## 1. Introduction

China is the largest coal producer in the world. Coal resources play a key role in the primary energy structure, accounting for 60% of the energy produced. Large-scale mining of coal resources has resulted in large-scale overburden movement and surface disturbance [1–3]. In accordance with relevant statistics, the coal subsidence area in China is approximately  $1.4-2.0 \times 10^6$  hm<sup>2</sup>, and it is increasing at a rate of  $2.0 \times 10^4$  hm<sup>2</sup>/year. Surface disturbance has damaged the original topography, landform, and natural landscape of mines, causing considerable damage to land, water, and vegetation resources (Figure 1) and seriously damaging the living and ecological environments of mining areas [4–6].

According to the mining subsidence theory, when the dip angle of a coal seam is between  $0^{\circ}$  and  $8^{\circ}$ , it can be regarded as a horizontal coal seam. When the dip angle of the coal seam is greater than  $8^{\circ}$ , it is an inclined coal seam [7–9].

Available prediction methods for mining subsidence include the profile function, influence function, and typical curve methods [10, 11]. Among them, the influence function method is the most widely used at present. The most extensively used probability integration method in China is a type of influence function method [12, 13].

The probability integral method is a theoretical model based on random medium theory. It exhibits the normal distribution characteristics of the left–right symmetric distribution, which is highly suitable for predicting horizontal coal seam mining subsidence [14, 15].

However, most of the coal resources in China are located in inclined coal seams [16]. The mining subsidence of inclined coal seams demonstrates different geometric and mechanical



FIGURE 1: Surface damage caused by mining.

distribution characteristics from horizontal coal seams [7, 17–19]. The analysis of a large amount of data measured by surface movement observation stations shows that surface movement basins exhibit a certain skewness after the mining of inclined coal seams [20]. When solving the problem of subsidence prediction induced by inclined coal seam mining, a probability integral method with symmetrical distribution characteristics is frequently applied, producing unsatisfactory results. The greater the dip angle of a coal seam, the more serious the skewness of a moving basin and the higher the prediction error [21–23].

At present, research on the skewed distribution of surface subsidence focuses on two aspects: (1) the mechanism of the skewed distribution of surface movements from the aspects of geological structures, such as joints [24–26], and (2) the analysis of the skewness distribution law of the basin combined with the measured data of surface movements [27–30]. However, no research on surface subsidence prediction based on skewed distribution has been published. In the current study, a new prediction model suitable for the mining subsidence prediction of inclined coal seam is established by combining coordinate transformation and random medium theory.

## 2. Traditional Methods for Mining Subsidence Prediction in Inclined Coal Seams

In China, existing methods for the mining subsidence prediction of inclined coal seams are mostly based on the probability integral method, which is a subsidence prediction model based on the mining of a horizontal coal seam.

The probability integration method is based on the following four assumptions: (1) the rock mass is an isotropic heterogeneous medium; (2) the principle of linear superposition is followed; (3) the rock mass in the sagging zone only deforms without volume change; and (4) when time tends to infinity, the surface subsidence volume after stable movement is equal to the mined volume. Based on these assumptions, our



FIGURE 2: Principle of establishing probability integral method.

findings show that the surface subsidence during horizontal coal mining conforms to a normal distribution in space (Figure 2). However, the surface subsidence in inclined coal seam mining has lost its normal distribution characteristics in space. Therefore, when the probability integration method is used for inclined coal seam mining, corresponding treatment should be taken.

At present, the primary treatment method is the equivalent mining of an inclined coal seam with the mining of two horizontal coal seams. The key idea is to equate the mining effect caused by the mining of an inclined coal seam (CD) with the difference in the mining effect caused by the mining of two horizontal coal seams (CF and DE), as shown in Figure 3.

This equal influence principle considers the equivalence of mining influence in the horizontal direction, but it does not consider the equivalence of mining influence in the



FIGURE 3: Calculation principle diagram of inclined coal seam subsidence using probability integral method.

vertical direction. When the probability integral method is used to predict the mining subsidence of an inclined coal seam, a smaller dip angle of the coal seam results in a lower prediction error. However, the prediction error becomes increasingly high with the gradual increase of the dip angle.

Establish a coordinate system along the surface, with the *x*-axis pointing horizontally upward along the surface, the origin at point *O*, and the w(x)-axis plumb downward. When calculating the surface subsidence caused by semi-infinite mining of CF, the mining depth is  $H_1$ , which is the actual downward direction of the inclined coal seam, and the inflection point is at point *O* with x=0. The calculation formula for surface subsidence caused by semi-infinite mining of CF is as follows:

$$w(x) = \frac{w_o}{2} \left[ \operatorname{erf}\left(\frac{\sqrt{\pi}}{r}x\right) + 1 \right], \tag{1}$$

where  $w_o$  is the maximum subsidence value of the surface, erf() is the Gaussian error function, r is the main influence radius, m is the mining thickness, q is the subsidence coefficient, and  $\alpha$  is the dip angle of the coal seam.

When calculating the surface subsidence caused by semiinfinite mining DE, the mining depth is the actual mining boundary depth  $H_2$  in the uphill direction of the inclined coal seam, and the inflection point is at point  $x = l_1$ . The calculation formula for surface subsidence caused by semi-infinite mining DE is as follows:

$$w(x-l_1) = \frac{w_o}{2} \left[ \operatorname{erf}\left(\frac{\sqrt{\pi}}{r}(x-l_1)\right) + 1 \right], \quad (2)$$

where  $l_1 = (D_1 - s_1 - s_2) \frac{\sin(\theta_o + \alpha)}{\sin \theta_o}$ ,  $D_1$  is the actual mining length,  $s_1$  is the offset distance of the downhill inflection point,  $s_2$  is the offset distance of the uphill inflection point, and  $\theta_o$  is the propagation angle of mining influence.

The calculation formula for mining the inclined coal seam is as follows:

$$w^{o}(x) = w(x) - w(x - l_{1}).$$
 (3)

## 3. Construction of a New Subsidence Prediction Model for Inclined Coal Seams

3.1. Specific Ideas for Model Construction. According to the theory of random media, the surface movement caused by mining inclined coal seams is regarded as a random event. The specific steps are as follows:

- (1) A probability integral is used to represent the predicted formula for surface movement.
- (2) The superposition principle is used to calculate the influence of mining on each unit, and the surface movement caused by limited coal mining is obtained.

For unit mining, it can be considered as ball rolling. When mining horizontal coal seams, the probability of small balls *B* and *C* entering space *A* after the extraction of small balls *A* is 1/2 (Figure 4(a)). However, when mining inclined coal seams, the probability of small balls *B* and *C* entering space *A* is different after small ball *A* is extracted.

The random medium movement model for inclined seam mining is established, as shown in Figure 4(b). If the small ball in grid A is moved away and vacancies are available, then the small balls B and C will move toward the vacancies in grid A under its own weight. However, the probability of moving toward the two vacancies is different. The vertical line that passes through the center of the bottom of grids B and C divides grid A into two parts. The area proportion of the two parts represents the probability that grids B and C will move to ball in grid A.

The total area of grid *A* is as follows:

$$S = S_1 + S_2 = ab, \tag{4}$$

where *S* is the total area of grid *A*,  $S_1$  is the left area of grid *A*, and  $S_2$  is the right area of grid *A*.  $S_1$  and  $S_2$  can be calculated using the following formulas:



FIGURE 4: Movement model of stochastic medium: (a) horizontal coal seam; (b) inclined coal seam.



FIGURE 5: Coordinate systems of prediction model.

$$S_{1} = \frac{1}{2}ab + \frac{1}{2}b^{2}\tan\alpha,$$

$$S_{2} = \frac{1}{2}ab - \frac{1}{2}b^{2}\tan\alpha,$$
(5)

Then, the probabilities that grids *B* and *C* will move to the balls in grid *A* are as follows:

$$P_{S_1} = \frac{S_1}{S} = \frac{\frac{1}{2}ab + \frac{1}{2}b^2 \tan \alpha}{ab} = \frac{1}{2} + \frac{b}{2a} \tan \alpha, \quad (6)$$
$$P_{S_2} = \frac{S_2}{S} = \frac{\frac{1}{2}ab - \frac{1}{2}b^2 \tan \alpha}{ab} = \frac{1}{2} - \frac{b}{2a} \tan \alpha,$$

where  $P_{S_1}$  and  $P_{S_2}$  represent the probabilities that grids *B* and *C* will move to the ball in grid *A*.

To ensure the convenience of formula derivation, we need to establish relevant coordinate systems.

*3.2. Construction of Coordinate Systems.* Three sets of coordinate systems were established along the main section of the working face, namely, the working face rectangular coordinate system *soz*, the working face body following coordinate system *uov*, and the surface rectangular coordinate system *xOw*, as shown in Figure 5. The origin and orientation of the

three coordinate systems are provided in Table 1. The dip angle of the coal seam is  $\alpha$ .

In accordance with the relevant information of the coordinate systems shown in Figure 3, the conversion relationship between the working face body-following coordinate system and the working face rectangular coordinate system was obtained as follows:

$$\begin{cases} s = u \cos \alpha - v \sin \alpha \\ z = u \sin \alpha - v \cos \alpha \end{cases} \text{or} \begin{cases} u = s \cos \alpha + z \sin \alpha \\ v = -s \sin \alpha + z \cos \alpha \end{cases}.$$
(7)

The conversion relationship between the working face rectangular coordinate system and the surface rectangular coordinate system was obtained as follows:

$$\begin{cases} s = x\\ z = -w + H_1 \end{cases},\tag{8}$$

where  $H_1$  refers to the mining depth at the lower boundary of the working face.

The conversion relationship between the working face body following the coordinate system and the surface rectangular coordinate system was obtained by combining

TABLE 1: Information of coordination system
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Coordinate system	Origin of coordinate	Direction of coordinate system The <i>s</i> -axis is in the horizontal direction and the <i>z</i> -axis is in the vertical direction	
Working face rectangular coordinate system ( <i>soz</i> )	Point <i>o</i> at the bottom plate of the downhill boundary of the mining area		
Working face body following coordinate system ( <i>uov</i> )	Point <i>o</i> at the bottom plate of the downhill boundary of the mining area	The <i>u</i> -axis points from the downhill direction to the uphill direction along the coal seam, and the <i>v</i> -axis points upward along the normal direction of the coal seam	
Surface rectangular coordinate system ( <i>xOw</i> )	Corresponding point <i>O</i> of point o on the surface	The <i>x</i> -axis points to the horizontal direction of the surface, and the <i>w</i> -axis points to the vertical direction downward	

Equations (7) and (8) as follows:

$$\begin{cases} x = u \cos \alpha - v \sin \alpha \\ w = H_1 - u \sin \alpha + v \cos \alpha \end{cases}$$
(9)

*3.3. Surface Subsidence Prediction Model for Unit Mining.* According to Equation (6), the following equation is deduced in accordance with the probability superposition:

$$p(u, v + b) = P_{S_1} \times p\left(u - \frac{a}{2}, v\right) + P_{S_2} \times p\left(u + \frac{a}{2}, v\right).$$
(10)

The Taylor formula is used to expand the items in the above formula at point (u, v), and the approximate expression after expansion is as follows:

$$p(u, v + b) = p(u, v) + \frac{\alpha p(u, v)}{\alpha v} b$$

$$p\left(u - \frac{a}{2}, v\right) = p(u, v) - \frac{a}{2} \frac{\alpha p(u, v)}{\alpha u} + \frac{a^2}{8} \frac{\alpha p^2(u, v)}{\alpha u^2} .$$

$$p\left(u + \frac{a}{2}, v\right) = p(u, v) + \frac{a}{2} \frac{\alpha p(u, v)}{\alpha u} + \frac{a^2}{8} \frac{\alpha p^2(u, v)}{\alpha u^2}$$
(11)

By introducing Equation (11) into Equation (10), we can derive the following:

$$p(u,v) + \frac{\alpha p(u,v)}{\alpha v} b = P_{S_2}$$

$$\times \left[ p(u,v) - \frac{a}{2} \frac{\alpha p(u,v)}{\alpha u} + \frac{a^2}{8} \frac{\alpha p^2(u,v)}{\alpha u^2} \right] . \quad (12)$$

$$+ P_{S_1} \times \left[ p(u,v) + \frac{a}{2} \frac{\alpha p(u,v)}{\alpha u} + \frac{a^2}{8} \frac{\alpha p^2(u,v)}{\alpha u^2} \right]$$

By sorting out the preceding formula, we can obtain the following:

$$\frac{\alpha p(u,v)}{\alpha v} = \frac{a^2}{8b} \frac{\alpha p^2(u,v)}{\alpha u^2} - \frac{\tan \alpha}{2} \frac{\alpha p(u,v)}{\alpha u}.$$
 (13)

Let  $A = \frac{a^2}{8b^2}$ , and  $B = \frac{\tan \alpha}{2}$ . Then, Equation (13) can be simplified as follows:

$$\frac{\alpha p(u,v)}{\alpha v} = A \frac{\alpha p^2(u,v)}{\alpha u^2} - B \frac{\alpha p(u,v)}{\alpha u}.$$
 (14)

The boundary conditions of the model are as follows:

$$P(u,0) = \delta(u). \tag{15}$$

In accordance with the literature [8], the special solution of Equation (14) is the Weibull distribution function, as shown in the following equation:

$$P(u,v) = \frac{p}{d} \left(\frac{c-u}{d}\right)^{p-1} e^{-\left(\frac{c-u}{d}\right)^p},$$
(16)

where p is the morphological parameter, d is the scale parameter, and c is the curve translation distance.

Then, the subsidence value of surface points caused by the unit mining of inclined coal seams is as follows:

$$P(u,v) = \frac{p}{d} \left(\frac{c-u}{d}\right)^{p-1} e^{-\left(\frac{c-u}{d}\right)^p}.$$
 (17)

For the surface, the expression of the subsidence basin caused by unit mining is as follows:

$$W_e(u) = \frac{p}{d} \left(\frac{c-u}{d}\right)^{p-1} e^{-\left(\frac{c-u}{d}\right)^p}.$$
 (18)

The distribution of surface subsidence caused by the unit mining of inclined coal seams conforms to the Weibull skewness distribution law. 3.4. Surface Subsidence Prediction Model for Limited Mining of Inclined Coal Seams

#### (1) Subsidence

If the mining thickness is *m*, then the maximum subsidence value is as follows:

 $w_o = mq \cos \alpha, \tag{19}$ 

where  $w_o$  is the maximum subsidence value, *m* is the mining thickness, *q* is the subsidence coefficient, and  $\alpha$  is the coal seam dip angle.

Then, the subsidence formula of the abscissa surface point caused by the mining length and thickness of the coal seam is as follows:

$$W_{u}(u) = W_{O} \int_{0}^{l} \frac{p}{d} \left(\frac{c - (u - s)}{d}\right)^{p-1} e^{-\left(\frac{c - (u - s)}{d}\right)^{p}} ds = W_{O} \left[e^{-\left(\frac{c - (u + l)}{d}\right)^{p}} - e^{-\left(\frac{c - u}{d}\right)^{p}}\right].$$
 (20)

According to the mutual conversion relationship among the three coordinate systems, we can derive the following:

$$u = x \cos \alpha + (H_1 - w) \sin \alpha. \tag{21}$$

In the surface rectangular coordinate system, it can be regarded as w = 0. Substituting it into Equation (21), we can

obtain the following:

$$u = x \cos \alpha + H_1 \sin \alpha. \tag{22}$$

If this relationship is introduced into Equation (20), then we can derive the following:

$$W(x) = W_O \left[ e^{-\left(\frac{c - x\cos\alpha - H_1\sin\alpha + l}{d}\right)^p} - e^{-\left(\frac{c - x\cos\alpha - H_1\sin\alpha}{d}\right)^p} \right].$$
 (23)

The preceding formula is for calculating the subsidence of surface points with the abscissa caused by inclined coal seams with mining length l, thickness m, and dip angle  $\alpha$ .

Surface tilt is caused by uneven subsidence. The firstorder partial derivative of the subsidence formula in the *x*direction shows that the surface tilt formula is as follows:

(2) Horizontal movement

$$i(x) = \frac{W_0 \cos \alpha p}{d} \begin{bmatrix} \left(\frac{c - x \cos \alpha - H_1 \sin \alpha + l}{d}\right)^{p-1} e^{-\left(\frac{c - x \cos \alpha - H_1 \sin \alpha + l}{d}\right)^p} \\ -\left(\frac{c - x \cos \alpha - H_1 \sin \alpha}{d}\right)^{p-1} e^{-\left(\frac{c - x \cos \alpha - H_1 \sin \alpha}{d}\right)^p} \end{bmatrix}.$$
(24)

In accordance with the literature [8], and the horizontal movement of the surface is proportional to surface tilt, the horizontal deformation of the surface is proportional to

surface curvature. The calculation formula of the horizontal surface movement is as follows:

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FIGURE 6: Influence of shape parameter on curve: (a) shape parameter; (b) scale parameter; (c) curve translation distance.

$$U(x) = B \frac{W_0 \sin \alpha p}{d} \begin{bmatrix} \left(\frac{c - x \cos \alpha - H_1 \sin \alpha + l}{d}\right)^{p-1} e^{-\left(\frac{c - x \cos \alpha - H_1 \sin \alpha + l}{d}\right)^p} \\ -\left(\frac{c - x \cos \alpha - H_1 \sin \alpha}{d}\right)^{p-1} e^{-\left(\frac{c - x \cos \alpha - H_1 \sin \alpha}{d}\right)^p} \end{bmatrix},$$
(25)

where B is the scale coefficient.

## 4. Result and Discussion

4.1. Parameter Analysis. In accordance with the prediction model established above, the model has three parameters: shape parameter p, scale parameter d, and curve translation distance c. The three parameters are analyzed separately, as shown in Figure 6.

## (1) Shape parameter *p*

As shown in Figure 6(a), when the scale parameter is the same, the curves under different shape parameters p significantly vary. The larger the p-value, the narrower the curve shape.

(2) Scale parameter d

The value of the scale parameter is greater than 0. As shown in Figure 6(b), the scale parameter plays a role in reducing and enlarging the abscissa scale. However, it does not affect the morphology of the curve.

#### (3) Curve translation distance *c*

As shown in Figure 6(c), the curve translation distance determines the starting position of the curve and does not affect the shape of the curve.

4.2. Field Application. The measured data of the ground movement observation station was used as an example to verify the effectiveness of the model. The working face 2,051 of Liangbei Mine is located in Xuchang City, Henan Province. The main mining area of the working face 2,051 is the no. 21 coal seam, which adopts a backward comprehensive mechanized mining method, and the roof is managed by the full collapse method.

The direct roof of the coal seam is mainly sandstone, which is stable within the coal mine range, with a thickness of 0.49–36.92 m. The lithology is medium-coarse-grained sandstone, which is hard and has a compressive strength of 20.7–121 Mpa, a tensile strength of 1.2–4.3 Mpa, and a softening coefficient of 0.43–0.75. The stability is good. The coal seam floor is mostly composed of mudstone, sandy mudstone, carbonaceous mudstone, or fine sandstone mudstone interbedding. The tensile strength is 0.5–4.31 Mpa, and the rock mass stability is poor. Under the action of high-pressure water head, floor heave is prone to occur, causing water inrush.

The surface above the 2,051 working faces belongs to a slightly inclined piedmont plain, with a flat terrain and densely populated villages. The ground elevation is between +105 and +120 m.

The designed strike length of the working face 2,051 is about 770 m, with a dip length of 224 m. The actual strike mining is 664 m, with a dip mining of 224 m. The azimuth angle of the working face strike is about 85.5°. The average coal thickness of the working face is 5.0 m, and the coal seam structure is simple. The mining depth of the working face is 580 m, with a coal average seam inclination of about 18°. The thickness of the mining coal seam is 5.0 m. The stability of the coal seam is relatively stable. The mining of the working face started in October 2020 and ended in November 2022.

The ground movement observation line was set to the inclination half. We take the coordinates at the mining boundary as the origin and consider the positive value within the goaf area and the negative value outside the goaf area. A total of 25 monitoring points have been set up on the surface, as shown in Figure 7. The subsidence and horizontal movement values obtained from each monitoring point are shown in Table 2. Due to the destruction of points B24 and B25, the final monitoring data was not obtained.

According to the existing data and existing observation data in the mining area, the expected parameters of the probability integral method are as follows. Subsidence factor q = 0.715, displacement factor b = 0.38, offset distance of the



FIGURE 7: Layout of surface monitoring points.

TABLE 2: Measured data.

Point number	Subsidence	Horizontal movement
	value (mm)	value (mm)
B1	-2	5
B2	-4	-11
B3	-18	4
B4	-55	-1
B5	8	0
B6	-22	5
B7	0	4
B8	-26	5
B9	33	8
B10	-9	10
B11	37	22
B12	23	33
B13	93	55
B14	109	83
B15	201	94
B16	216	105
B17	376	286
B18	521	354
B19	757	437
B20	1,070	488
B21	1,508	412
B22	1,777	163
B23	1,793	-194

inflection point s = 90 m, tangent of major influent angle  $\tan \beta = 2.1$ , and influence transference angle  $\theta_0 = 87^0$ .

Based on the measured data and curve fitting method, the parameters of the Weibull prediction model obtained by the least square criterion are as follows: q = 0.72, p = 6.3, d = 287, c = 300, and B = 463.

The measured subsidence and horizontal movement values and the fitting curves are shown in Figure 8.

According to relevant literature, the reliability of prediction models is mainly measured by relative mean square error. When the relative mean square error of subsidence



FIGURE 8: Comparison between measured data and predicted results: (a) subsidence prediction value using Weibull distribution prediction model; (b) subsidence prediction value using traditional method; (c) horizontal movement prediction value using Weibull distribution prediction model; (d) horizontal movement prediction value using traditional method.

TABLE 3: Prediction precision.

	Traditional method		Weibull distribution prediction model	
	Mean squared error (mm)	Relative mean squared error (%)	Mean squared error (mm)	Relative mean squared error (%)
Subsidence	65	3.6	49	2.7
Horizontal movement	80	16.4	46	9.4

is less than 10%, and the relative mean square error of the horizontal movement is less than 20%, the model has high reliability and can meet the requirements for engineering prediction accuracy. The mean squared error and relative mean squared error obtained from Equations (26) and (27) are presented in Table 2.

$$m_x = \pm \sqrt{\frac{[\Delta \Delta]}{n}},\tag{26}$$

$$m_d = \pm \frac{m_x}{v_m},\tag{27}$$

where  $m_x$  is the mean squared error,  $m_d$  is the relative mean squared error,  $\Delta$  is the difference between the measured and predicted values, n is the number of points, and  $v_m$  is the maximum movement value.

As indicated in Table 3, the subsidence relative mean squared error of the Weibull distribution prediction model is less than 5%, and the horizontal movement relative mean squared error of the Weibull distribution prediction model is less than 10%, which can meet engineering prediction accuracy requirements. In addition, the relative mean square error of the Weibull distribution prediction model is smaller than that of the probability integration method, indicating that the Weibull distribution prediction model has higher prediction

accuracy in predicting subsidence in inclined coal seam mining compared to the probability integration method.

4.3. Application Scope. The model established in this study is a mathematical statistical model. Thus, its application scope is limited. (1) The study area has no large geological structures, such as faults. (2) No major geological historical events, such as earthquakes, occur in the study area. However, when the study area has large geological structures or geological historical events, the prediction accuracy of the model seriously declines.

## 5. Conclusions

- (1) Based on the random medium theory, the rock layer was considered each discrete small ball, and the corresponding area above the small ball was considered the probability that the upper small ball would fall into the grid, and the surface movement of the inclined coal seam unit mining conformed to the Weibull distribution law.
- (2) In accordance with the Weibull distribution theory, a new surface subsidence prediction model of inclined coal seams was established by using the method of mutual coordinate transformation of multiple sets of coordinate systems.
- (3) Through case analysis, the mean squared error of subsidence was determined to be 2.7%, and the relative mean squared error of horizontal movement was 9.4%. The Weibull distribution prediction model fulfilled the engineering requirements when predicting the mining subsidence of inclined coal seams.

## **Data Availability**

The data used to support the findings of this study are available from the corresponding author upon request.

## **Conflicts of Interest**

The authors declare they have no conflicts of interest.

# **Authors' Contributions**

Conceptualization was done by W.Y.; methodology was done by W.Y.; software-related work was done by W.Y.; validation was done by W.Y.; formal analysis was done by J.G.; investigation was done by S.Y.; resources were provided by S.Y.; data curation was done by W.T.; writing—original draft preparation was done by W.Y.; writing—review and editing was done by S.Y.; visualization was done by J.G.; supervision was done by W.Y.; project administration was done by Y.Y.; funding acquisition was done by J.G. All authors have read and agreed to the published version of the manuscript.

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