

Research Article

Improved Response Surface Method Based on Linear Gradient Iterative Criterion

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Aiming at the problem of big calculation error in solving reliability index by the traditional response surface method, an improved response surface method based on the linear gradient iteration criterion is proposed. First, the linear gradient iteration criterion is proposed to reduce the iteration step size with the increase of iteration times. It will improve the fitting accuracy of response surface and lead to a better convergence while approaching the limit state surface. Then, the reduction coefficient of the linear gradient iterative criterion is studied. The optimal value of coefficient is 0.2. The improved response surface method will get a more accurate reliability index quickly. Examples show that the proposed method has obvious advantages of high accuracy and efficiency. The application of this method can also be expanded in other similar engineering structure.

1. Introduction

With the application of probability theory in engineering, the reliability analysis is more and more widely used in practical engineering [1–4]. The first-order second-moment method [5], second-order second-moment method [6, 7], response surface method [8], and Monte Carlo method [9] are often used to calculate structure reliability index. In engineering application, it is often encountered that the limit state function is not existing or existing but with a high nonlinearity. It is generally difficult to calculate the reliability index in such cases. The response surface method has the advantage of fitting the limit state function to deal with these problems [10]. The traditional response surface method has low fitting errors and an acceptable calculation accuracy when the nonlinearity of the limit state function is low. When the nonlinearity is higher, the fitting error increases much more and so does the calculation error. It is a challenging problem for the traditional response surface method.

Many scholars have tried solving these problems. For example, Bucher and Bourgund [8] proposed a new adaptive interpolation scheme, which can characterize the system behavior quickly and accurately by response surface. This

response surface approach utilizes elementary statistical information on the basic variables (mean values and standard deviations) to increase efficiency and accuracy. Kang et al. [11] proposed an efficient response surface method using moving least squares (MLS) approximation to replace the traditional least squares approximation commonly used in response surface methods. MLS approximation gives higher weight to the test points near the most probable failure point (MPFP), which brings the response surface function closer to the limit state function at MPFP, so as to improve computational efficiency. Li et al. [12] proposed a stochastic response surface method for reliability analysis involving correlated non-normal random variables. The proposed method will substantially extend the application of the stochastic response surface method for reliability problems. Allaix and Carbone [13] proposed an iterative strategy to fit the limit state function near the checking point. This method uses an adaptive process combined with the FORM method to build successive response surfaces until it converges to the checking point. Gaspar et al. [14] proposed an efficient RSM with the Kriging interpolation models instead of the quadratic polynomial without cross terms. Goswami et al. [15] proposed an improved RSM based on

the moving least squares method to reduce the number of iterations. Wei et al. [16] proposed a modified iterative response surface method (called NDIRSM) to improve the accuracy and efficiency of structural reliability analysis. Based on the idea of double weight factor and vector projection method, Xia and Wang [17] proposed a new improved response surface method. However, these methods improve the accuracy while increasing the amount of calculation, and some improve the computational efficiency but affect the accuracy. Therefore, how to find a balance between computational efficiency, accuracy, and stability is a key issue.

As known, when the iteration step is too large, it will get a poor fitting response surface. Then, it will not converge to the true checking point and fall into a local convergence solution. When the iteration step is too small, it will get a better fitting response surface but with low efficiency. For large nonlinear limit state equations, if the iterative step size is too small from the beginning of the iterative process, it will even not converge. Therefore, a better fitting response surface function should be gradually reduced from large to small, to the range near the checking point.

In this paper, an improved response surface method based on the linear gradient iteration criterion is proposed. It improves the iteration of the traditional response surface method. Firstly, a linear gradient iteration criterion is proposed to reduce the iteration step size with the increase of iteration time, making the fitting of response surface function closer to the true response surface. Then, the coefficient of the linear gradient iterative criterion is studied and it is found that 0.2 is the most suitable value for the coefficient. The improved method improves the calculation accuracy and efficiency compared with the traditional response surface method and provides a new and effective method for reliability analysis of complex structures.

The outline of this paper is organized as follows: Section 2 briefly introduces the basic principle of the traditional response surface method. Section 3 introduces the definition of linear gradient iterative criterion. In Section 4, five different numerical examples are used to verify the proposed method, and the optimal coefficient values are determined according to the analysis of calculation results. The conclusion is summarized in Section 5.

2. Traditional Response Surface Method

The basic idea of the response surface method [18, 19] is to replace the implicit or hard-to-determine limit state function with a relatively easy-to-handle function (called response surface function). The key is the fitting of response

surface function, especially for the location near checking points. The form of response surface function should be as simple as possible. The most commonly used response surface function is the polynomial form. For a structure with basic random variable X , the response surface function can be set as follows:

$$g(X) \approx \hat{g}(X) = a + \sum_{i=1}^n b_i X_i + \sum_{i=1}^n c_i X_i^2 + \sum_{1 \leq i < j \leq n} d_{ij} X_i X_j, \quad (1)$$

where a, b_i, c_i, d_{ij} is the undetermined coefficient.

For an implicit limit state equation $g(X) = 0$, the traditional response surface method often uses incomplete quadratic polynomial $\hat{g}(X)$ without cross term:

$$\hat{g}(X) = a + \sum_{i=1}^n b_i X_i + \sum_{i=1}^n c_i X_i^2, \quad (2)$$

where a, b_i, c_i is the $2n+1$ unknown undetermined coefficient.

Response surface function needs to design a series of test points x_i and calculates the corresponding function values $z_i (i = 1, 2, \dots, n)$. It is general that the central composite design method [20] will be used to design test points. Figure 1 is the typical central composite design method. If the number of test points is more than the undetermined coefficient, the least square method will be used to calculate the undetermined coefficient. Usually, test points are selected near the mean point μ_X .

To fit the best response function, the test point must be close to or located at the true failure surface, which is defined by the limit state function $g(X) = 0$. According to the previous steps, the point x to be expanded can be obtained according to the previous steps. Then, we calculate the estimated value of checking point x^* according to the obtained function value. The new expansion point x is calculated by the following interpolation formula:

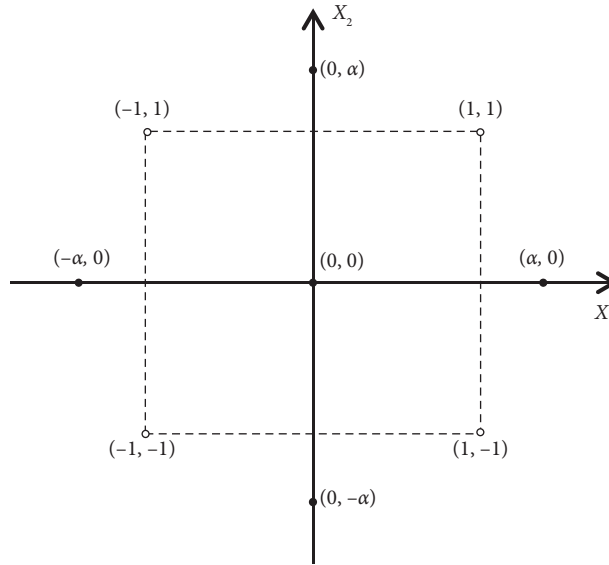
$$x = \mu_X + \frac{g(\mu_X)}{g(\mu_X) - g(x^*)} (x^* - \mu_X). \quad (3)$$

Based on the formula mentioned above, the linear equations can be formed as follows:

$$A\lambda = \hat{g}, \quad (4)$$

where $\lambda = (a, b_1, b_2, \dots, b_n, c_1, c_2, \dots, c_n)^T$, $\hat{g} = (\hat{g}_1, \hat{g}_2, \dots, \hat{g}_{2n+1})^T$, A is a coefficient matrix and has the following form, whose definition is explained in detail by Zhang [7]. Matrix A has $2n+1$ rows and $2n+1$ columns.

$$A = \begin{bmatrix} 1 & [x_i]_{1 \times n} & [x_i^2]_{1 \times n} \\ 1 & [x_i]_{1 \times n} & [x_i^2]_{1 \times n} \\ \vdots & \vdots & \vdots \\ 1 & [x_i]_{1 \times n} & [x_i^2]_{1 \times n} \end{bmatrix}_{2n+1 \times 2n+1} + \begin{bmatrix} 0 & [0]_{1 \times n} & [0]_{1 \times n} \\ [0]_{n \times 1} & -f \text{diag}[\sigma_{X_i}]_{n \times n} & \text{diag}[f^2 \sigma_{X_i}^2 - 2f x_i \sigma_{X_i}]_{n \times n} \\ [0]_{n \times 1} & f \text{diag}[\sigma_{X_i}]_{n \times n} & \text{diag}[f^2 \sigma_{X_i}^2 - 2f x_i \sigma_{X_i}]_{n \times n} \end{bmatrix}_{2n+1 \times 2n+1}. \quad (5)$$

FIGURE 1: Central composite design method ($\alpha = \sqrt{2}$).

3. Improved Response Surface Method Based on Linear Gradient Iterative Criterion

3.1. Linear Gradient Iteration Criterion. In order to accurately solve a structural reliability index, the response surface method often requires multiple iterations to make the response surface function highly approximate to the true function. The iterative process for calculating reliability by using the traditional response surface method is shown in Figure 2. In Figure 2(a), when the iteration step is too large, the checking point cannot be accurately converged to the limit state surface, and even the optimal failure point will be missed. It is clear that the fitting is poor. In Figure 2(b), when the iteration step is too small, it cannot approach the limit state surface quickly and hovers away from the limit state surface, it even leads to nonconvergence.

This article focuses on the problems mentioned above and improves the traditional response surface method. With increase of the iteration numbers, the proposed method in this paper makes iterative step size decrease gradually. The iterative process is shown in Figure 3. The suggested gradient iteration can be defined as follows:

$$f^{(k+1)} = \eta f^{(k)}, \quad (6)$$

where η is the reduction coefficient.

3.2. The Calculation Steps of the Improved Response Surface Method Based on the Linear Gradient Iteration Criterion

- (1) select the initial iteration point x , which usually is the mean value of μ_X .
- (2) Determine the initial value of f , initial $f = 3$. The follow-up iterative process is $f^{(k+1)} = \eta f^{(k)}$.
- (3) Using the least square method, the function response value \hat{g}_i at each point is calculated to form the coefficient matrix A

- (4) solve the undetermined coefficient a, b_i, c_i and solve equation (4)
- (5) calculate the reliability index and checking point x^*
- (6) calculate functional response values at x^* , generating a new x by using linear interpolation iteration of formula (3)
- (7) repeat steps (3)–(6) until $|\beta^{k+1} - \beta^k|/\beta^{k+1} < \varepsilon$, and ε is the accuracy requirement.

4. Examples

4.1. Example 1. The structure limit state equation is shown as follows:

$$g(X) = 2.2257 - 0.025\sqrt{2}(X_1 + X_2 - 20)^3/27 + 33 \times (X_1 - X_2)/140, \text{ Where } X \text{ obeys normal distribution, } X_1, X_2 \sim N(10, 3).$$

This example is calculated by the Monte Carlo method, traditional response surface method, and the proposed method in this study. The calculation results are shown in Table 1. The iteration process is shown in Figure 4. Based on 10^6 times calculation with the Monte Carlo method, the reliability index is 2.0751. It can be seen from Figure 4 that the nonlinearity of this function is high. Using the proposed method, the iteration step size can be gradually reduced with the increase in iteration time. Then, it quickly and accurately converges to the limit state surface. The reliability index is calculated as 2.2393. Comparing it with that of the Monte Carlo method, the relative error of the traditional response surface method is 20.38%. It is almost three times that of the relative error of the proposed method. The error is greatly reduced compared with the traditional response surface method. It can be seen from Table 1 that the number of iterations using the traditional response surface method is 28 times. The iteration number of the proposed method is only 8 times. It greatly reduces the number of iterations in the calculation process. The proposed method is an efficient and accurate method for the reliability calculation. In addition, it

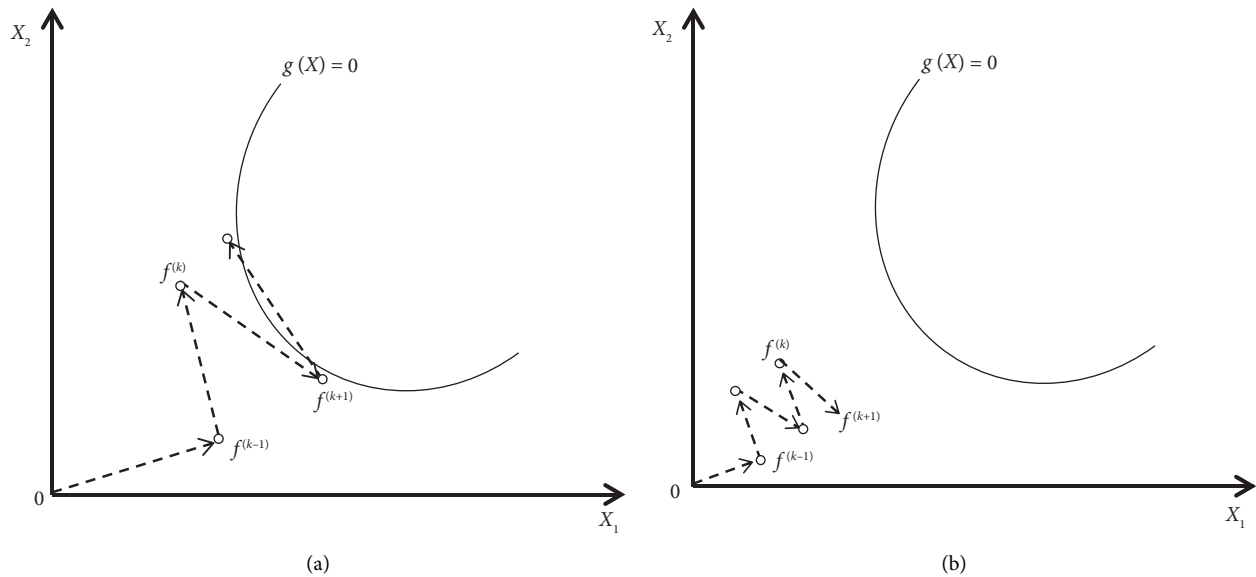


FIGURE 2: Traditional iterative process diagram. (a) Large step size. (b) Small step size.

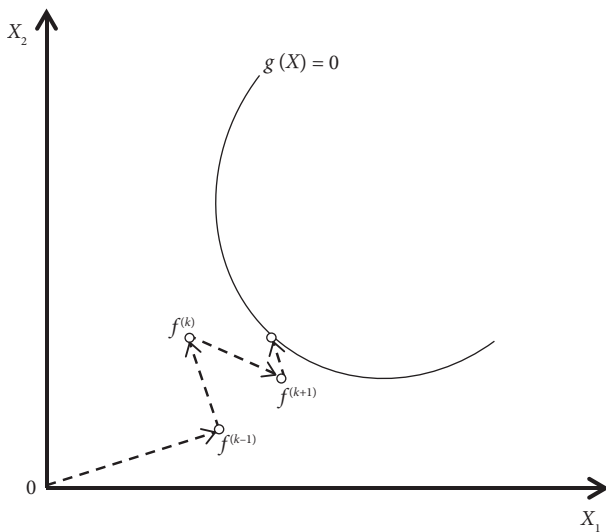


FIGURE 3: Iteration process diagram of gradient iteration criterion.

can be found that the number of iterations of the proposed method varies with the value of the reduction coefficient. If the decrease coefficient is too big or too small, it will lead to the increasing number of iterations. Additionally, when the reduction coefficient is 0.2, the calculation efficiency is the best. This example shows that the proposed method greatly reduces the number of iterations in the calculation process and the calculation error.

4.2. Example 2. The structure limit state equation is shown as follows:

$g(X) = 2 - X_2 - 0.1X_1^2 + 0.06X_1^3$, where X obeys normal distribution, $X_1, X_2 \sim N(0, 1)$.

This example is calculated also by the Monte Carlo method, traditional response surface method, and the

proposed method in this study. The calculation results are shown in Table 2. The iterative process is shown in Figure 5. Based on 10^6 times calculation with the Monte Carlo method, the reliability index is 1.8143. It can be treated as the exact solution of the question. When the traditional response surface method is used to calculate the reliability, there is still a certain distance between the checking point and the true checking point, resulting in big calculation error. By using the proposed method, the reliability index is 2.0000. From Table 2, it will be found that the relative error of the traditional response surface method is 22.33%, and the relative error of the proposed method is 10.24%. The error with the method given in this paper is only half of that of the traditional response surface method. The traditional response surface method has 9 iterations. However, the iteration number of the method in this paper is only 4 times. It indicates that the method in this paper not only has high calculation efficiency but also has relatively high accuracy. It can also be found that if the reduction coefficient is too big or too small, it will lead to the increasing number of iterations. When the reduction coefficient is 0.2, the calculation efficiency is the best.

4.3. Example 3. The structure limit state equation is shown as follows:

$g(X) = X_1 + 2X_2 + 2X_3 + X_4 - 5X_5 - 5X_6$, where X obeys lognormal distribution. Its distribution is shown in Table 3.

This example is also calculated by Monte Carlo method, traditional response surface method, and the proposed method in this paper. The calculation results are shown in Table 4. Based on 10^6 times calculation with the Monte Carlo method, the reliability index is 2.2516. It can be treated as the exact solution to the question. Both the traditional response surface method and the proposed method can converge to the true checking point. The accuracy of the proposed

TABLE 1: Calculation results of example 1.

Used method	Reduction coefficient	Iteration times	Failure probability	Reliability index	Relative error (%)
Monte Carlo method	—	—	19.0×10^{-3}	2.0751	—
Traditional response surface method	—	28	6.2×10^{-3}	2.4981	20.38
The proposed method	0.01	—	—	—	—
	0.05	9	12.6×10^{-3}	2.2393	7.91
	0.2	8	12.6×10^{-3}	2.2393	7.91
	0.4	10	12.6×10^{-3}	2.2393	7.91
	0.6	14	12.6×10^{-3}	2.2393	7.91
	0.8	28	12.6×10^{-3}	2.2393	7.91

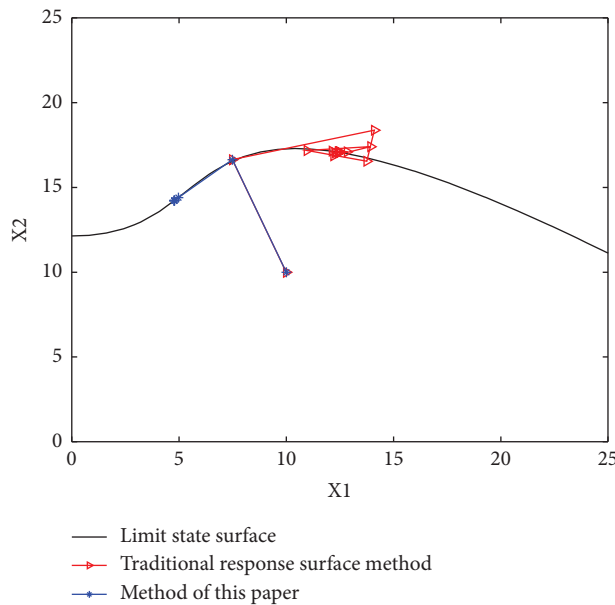


FIGURE 4: Iterative process of example.

TABLE 2: Calculation results of example 2.

Used method	Reduction coefficient	Iteration times	Failure probability	Reliability index	Relative error (%)
Monte Carlo method	—	—	3.48×10^{-2}	1.8143	—
Traditional response surface method	—	9	1.32×10^{-2}	2.2194	22.33
The proposed method	0.01	6	2.28×10^{-2}	2.0000	10.24
	0.05	5	2.28×10^{-2}	2.0000	10.24
	0.2	5	2.28×10^{-2}	2.0000	10.24
	0.4	7	2.28×10^{-2}	2.0000	10.24
	0.6	9	2.28×10^{-2}	2.0000	10.24
	0.8	16	2.27×10^{-2}	2.0009	10.28

method in this paper is slightly higher than that of the traditional response surface method. Additionally, the number of iterations is significantly less than that of the traditional response surface method. The iteration of traditional response surface method is more than 4 times that of the proposed method. In addition, the number of iterations varies with the reduction coefficient. It can also be found that if the reduction coefficient is too big or too small, it will lead

to the increasing number of iterations. When the reduction coefficient is 0.2, the calculation efficiency is the best.

4.4. Example 4. The plane frame structure with plastic hinges which is shown in Figure 6 is a three-branch series system [21]. This example considers the possibility of failure of the plane frame presented in Figure 6 by means of plastic hinge

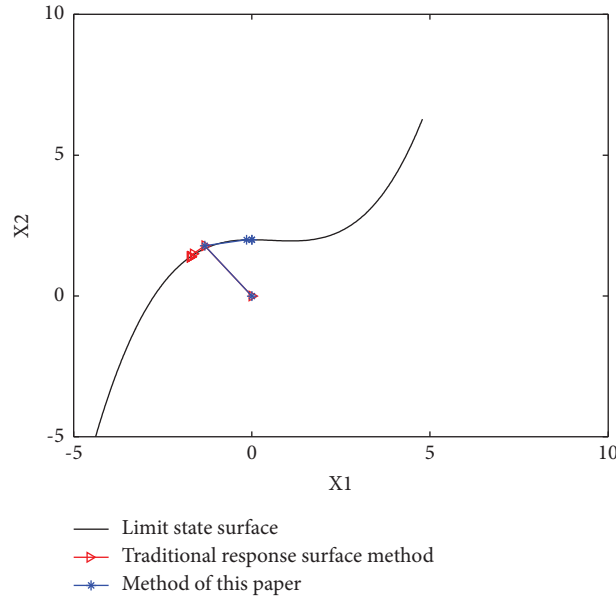


FIGURE 5: Iterative process of example 2.

TABLE 3: Random variable distribution of example 3.

Random variable	Dispersion pattern	Mean value	Standard deviation
X_1	Lognormal distribution	120	12
X_2	Lognormal distribution	120	12
X_3	Lognormal distribution	120	12
X_4	Lognormal distribution	120	12
X_5	Lognormal distribution	50	15
X_6	Lognormal distribution	40	12

TABLE 4: Calculation results of example 3.

Used method	Reduction coefficient	Iteration times	Failure probability	Reliability index	Relative error (%)
Monte Carlo method	—	—	1.22×10^{-2}	2.2516	—
Traditional response surface method	—	13	0.82×10^{-2}	2.4015	6.66
The proposed method	0.01	6	0.94×10^{-2}	2.3482	4.29
	0.05	4	0.94×10^{-2}	2.3482	4.29
	0.2	3	0.94×10^{-2}	2.3482	4.29
	0.4	3	0.94×10^{-2}	2.3482	4.29
	0.6	6	0.94×10^{-2}	2.3482	4.29
	0.8	9	0.94×10^{-2}	2.3482	4.29

mechanisms as investigated by Madsen et al. [22]. The failure function for this structure can be written as

$$g(x) = \min \begin{cases} x_1 + x_2 + x_4 + x_5 - 5x_6, \\ x_1 + 2x_3 + 2x_4 + x_5 - 5x_6 - 5x_7, \\ x_1 + 2x_3 + x_4 - 5x_7. \end{cases} \quad (7)$$

It represents a series system of three failure mechanisms given by the limit state function. The equation presents three design points, each one corresponding to a limit state function presented in equation. By analyzing the reliability

of each limit state function $g(x)$, the design point which is most likely to fail is obtained. The distribution of random variables is shown in Table 5.

Since the structure is a three-branch series system, one of the three branches failing means the failure of the whole structure. Therefore, the reliability index should take the smallest branch of the three branches. Based on 10^6 times calculation with the Monte Carlo method, the reliability index is 2.5911. It can be treated as the exact solution to the question. The calculation result is shown in Table 6. It is obvious that the accuracy of the proposed method is higher.

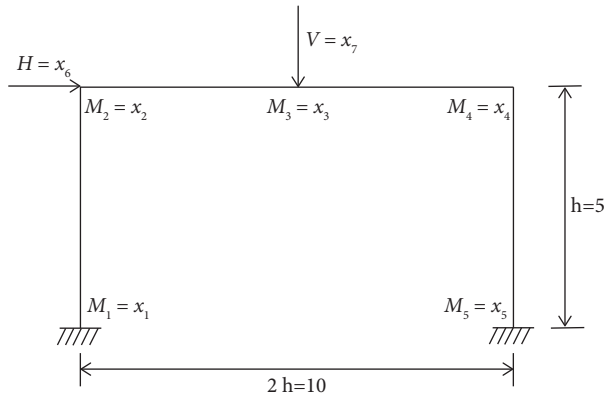


FIGURE 6: Plane frame work.

TABLE 5: Random variable distribution of example 4.

Random variable	Dispersion pattern	Mean value	Standard deviation
X_1	Lognormal distribution	134.9	13.49
X_2	Lognormal distribution	134.9	13.49
X_3	Lognormal distribution	134.9	13.49
X_4	Lognormal distribution	134.9	13.49
X_5	Lognormal distribution	134.9	13.49
X_6	Lognormal distribution	50	15
X_7	Lognormal distribution	40	12

TABLE 6: Calculation results of example 4.

Used method	Reduction coefficient	Iteration times	Failure probability	Reliability index	Relative error (%)
Monte Carlo method	—	—	0.0048	2.5911	—
Traditional response surface method	—	10	0.0031	2.7403	5.76
The proposed method	0.01	5	0.0033	2.7118	4.66
	0.05	3	0.0033	2.7118	4.66
	0.2	3	0.0033	2.7118	4.66
	0.4	4	0.0033	2.7118	4.66
	0.6	5	0.0033	2.7203	4.99
	0.8	5	0.0032	2.7221	5.06

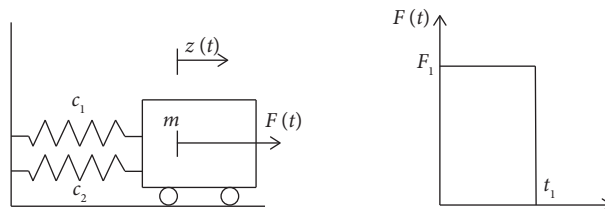


FIGURE 7: A nonlinear undamped single-degree-of-freedom structure.

TABLE 7: Random variable distribution of example 5.

Random variable	Variables to description	Dispersion pattern	Mean value	Coefficient of variation
x_1	m	Dispersion pattern	1	0.05
x_2	c_1	Dispersion pattern	1	0.10
x_3	c_2	Dispersion pattern	0.1	0.01
x_4	r	Dispersion pattern	0.5	0.05
x_5	F_1	Dispersion pattern	1	0.20
x_6	t_1	Dispersion pattern	1	0.20

TABLE 8: Calculation results of example 5.

Used method	Reduction coefficient	Iteration times	Failure probability	Reliability index	Relative error (%)
Monte Carlo method	—	—	0.0286	1.9012	—
Traditional response surface method	—	60	0.0258	1.9457	2.34
The proposed method	0.01	—	—	—	—
	0.05	—	—	—	—
	0.2	8	0.0259	1.9455	2.33
	0.4	8	0.0259	1.9455	2.33
	0.6	10	0.0259	1.9455	2.33
	0.8	16	0.0259	1.9455	2.33

The iteration number of the traditional response surface method is reduced from 10 to 3 compared with that of the proposed method. The proposed method uses less than one-third iterations of the traditional response surface method to reach a more precise result. It proves the efficiency and accuracy of the method in this paper. It can also be found that if the reduction coefficient is too big or too small, it will lead to the increasing number of iterations. When the reduction coefficient is 0.2, the calculation efficiency is the best and the calculation result is the most accurate.

4.5. Example 5. A nonlinear undamped single-degree-of-freedom structure [23] is shown in Figure 7. The distributions of random variables are shown in Table 7. The limit state function can be expressed as follows:

$$g(x) = 3x_4 - \frac{2x_5}{x_2 + x_3} \sin\left(\frac{x_6}{2} \sqrt{\frac{x_2 + x_3}{x_1}}\right). \quad (8)$$

It is a nonlinear undamped single-degree-of-freedom structure system with high dimensional random variables in this example. It also uses the Monte Carlo method, traditional response surface method, and the proposed method to calculate the reliability index. The results are shown in Table 8. The traditional response surface method needs 60 times iterations to converge. The proposed method only needs 8 times iterations. The new method greatly improves computational efficiency. Additionally, the calculation accuracy is slightly improved. It can also be found that if the reduction coefficient is too big or too small, it will lead to the increasing number of iterations. When the reduction coefficient is 0.2, the calculation efficiency is the best.

The advantages of the improved response method can be illustrated in previous examples. The new method improves the computational efficiency and accuracy. In addition, through the study of the reduction coefficient, it is proved that the calculation efficiency is the best when the reduction coefficient is generally 0.2.

5. Conclusion

This article presents an improved response surface method based on the linear gradient iterative criterion. The improved response surface method solves the problems of low efficiency and big error of the traditional response surface method. Finally, five examples are used to prove the

efficiency and accuracy of the proposed method. The main conclusions are shown as follows:

- (1) An improved response surface method based on the linear gradient iteration criterion is proposed. The method makes the iteration step size decrease with the increase of iteration times and improves fitting the accuracy of response surface. It can approach the limit state surface quickly and converge to the true checking point. Thus, the computational efficiency and accuracy of the traditional response surface method are improved.
- (2) The reduction coefficient of the linear gradient iterative criterion is also studied by examples. Examples show that the value of reduction coefficient has considerable influence on the number of iterations. A too big or too small decrease coefficient will lead to the increasing number of iterations. When the reduction coefficient is 0.2, the calculation efficiency is the best.

Data Availability

All data and models generated or used during the study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

YuXia conceptualized the study, developed methodology, performed formal analysis, reviewed and edited the article, and participated in funding acquisition. Wenzheng Kong developed methodology, wrote the original draft, and reviewed and edited the article. Yingye Yu and Yiying Hu collected resources and reviewed and edited the article. Jingyou Li collected resources, reviewed and edited the article, and participated in funding acquisition.

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