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Research Article

Time-Dependent Reliability Analysis of Fracture Failure of Corroded Cast Iron Water Pipes and Bayesian Updating for Lifetime Prediction

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Affected by the underground soil environment, buried cast iron pipelines are subject to corrosion during their long-term service, resulting in damage accumulation to the pipe wall and the eventual fracture failure of pipes. This paper aims to propose a probabilistic method to quantitively assess the time-dependent reliability of fracture failure of corroded cast iron pipes. A Gamma-based corrosion process of the pipe wall is derived according to the reported corrosion models. The first-order reliability method and the Monte Carlo simulation are used to cross-validate and conduct the time-dependent reliability analysis based on the fracture failure criterion. Furthermore, uncertain physical parameters of the pipes are updated by the Bayesian Markov Chain Monte Carlo (MCMC) algorithm based on the regional historical data of failed pipes, and lifetime predictions of buried cast iron pipelines are then obtained. A worked example is provided to illustrate the application of the proposed method. It is found that the Gamma process can well simulate the corrosion process hence can be employed to calculate the probability of pipe fracture failure. Sensitivity analysis reveals that the pipe internal water pressure, the fracture toughness, and the geometry of corrosion pits are the most influential parameters to the probability of fracture failure. It is also found that the predicted lifetime of corroded cast iron pipes in the worked example decreases from 110 to 85 years after the Bayesian updating.

1. Introduction

Shallow buried cast iron pipes have been applied in underground infrastructure for more than 100 years. At present, a large number of pipes have suffered different degrees of underground environmental corrosion, which leads to reduction of pipe mechanical properties and deterioration of pipes performance [1]. Buried cast iron pipes will eventually fracture when the corrosion-induced pipe wall damage exceeds the limit, which causes bursts of water main pipes consequently and high-pressure water continuously washing the surrounding soil, resulting in secondary disasters such as the ground subsidence and the instability of adjacent structures [2–4]. Considering the uncertainty in time-dependent corrosion process and influencing factors of pipe mechanical parameters, the traditional deterministic analysis may not provide objective assessment of pipe fracture failure behavior. Therefore, it is urgent to propose a probabilistic method that can more accurately evaluate the risk of fracture failure of corroded cast iron pipes, which can be further applied in taking engineering strategies for pipe maintenance.

Corrosion has been well known as the main reason for the performance deterioration of cast iron pipes in service [5, 6]. The corrosion process of the pipe wall is very complicated, which is basically influenced by factors such as pipe metal material and soil properties [7]. The literature research shows that many researchers have studied the growth of corrosion depth and several corrosion models have been proposed to represent the corrosion process. For example, Rossum [8] proposed a power law corrosion model that links soil properties and the corrosion process for the first time. Soil resistivity and pH were considered in this model. Kucera and Mattsson [9] introduced a power law model that was widely used in practical engineering, and the parameters of the model could be determined according to the actual burial conditions of the pipes. Rajani and Makar [10] presented a two-phase corrosion model of which the corrosion rate is large and changes exponentially at the initial service stage, while it becomes small and tends to be linear in the later stage.

Some other literature suggested that the deterioration process of corroded pipes can be considered as a Gamma process [11–13]. In recent years, only a few studies have made some applications in this context. Mahmoodian and Alani [14] used several groups of observational corrosion data of the cast iron pipes to estimate the parameters of the Gamma process and studied the growth of the corrosion depth. Li et al. [15] considered the temporal and spatial variability of the corrosion process of the cast iron pipes, the theory of random field, and the Gamma process were employed to simulate the corrosion growth. Yang et al. [16] developed a Gamma-based stochastic resistance degradation model by incorporating the spatial degradation into a nonstationary degradation process, and conducted reliability assessment considering the effect of time-dependent parameters in the Gamma process.

Without loss of generality, the deterioration caused by corrosion of the pipe wall probabilistically increases with the increase of service time, thus it is necessary to conduct time-dependent reliability analysis to accurately assess the risk of pipe failures at different service stages [17–21]. To name a few, Sadiq et al. [22] proposed a method to study the reliability of cast iron pipes by defining a safety factor between the residual strength of the pipes and the maximum external stress generated by various kinds of loads. Considering the uncertainty in some physical parameters, the Monte Carlo simulation (MCS) was used to propagate various uncertainties into the safety factor and eventually to calculate the probability of failure. Li and Mahmoodian [23] demonstrated a time-dependent reliability method to quantitatively assess the risk of cast iron pipes. The first passage probability theory was employed to calculate the probability of failure, and the remaining service life of the pipe was predicted. Miran et al. [24] assessed the time-dependent reliability of aging pipelines based on the developed power-low function of time model for predicting the maximum corrosion depth, and effects of newly generated corrosion pits on pipe failure could also be considered. Ji et al. [25] presented a time-to-failure assessment of cast iron pipes by studying the stress concentration caused by corrosion of the pipe wall. The probabilistic physical model was established to calculate the probability of failure and the truncated Weibull distribution was used to fit the failure curves as well as to predict the hazard rate of the cast iron pipes. Mulenga et al. [26] predicted failure probability of cast iron pipes with high aspect ratio corrosion pits based on physics of failure, the time-dependent upcrossing method was employed to quantify the failure probability.

The literature research shows that most previous studies consider the pipe failure criterion based more on the strength criterion than the fracture criterion. However, as cast iron is typically of brittle materials, a real inspection of the failed pipes reveals that most of the failure modes of buried cast iron pipes are fracture failures caused by the expansion of the initial crack [27]. There are only a few recent works combined with this context, so probabilistic analysis of pipe fracture failure will be the main topic in this study. Further literature review shows that the regional historical data of failed pipes was often hardly used [28]. It is well recognized that Bayesian theory is the fundamental theory in statistical inference to update our beliefs in events with the real data in hand, in the later sections of this paper, we will make some efforts on further analysis of the data to update important pipe physical parameters based on the Bayesian theory. After the Bayesian updating, it would be possible to more accurately assess the risk of pipe fracture failures [29].

In this study, a probabilistic method to systematically evaluate the risk of fracture failure of buried cast iron pipelines is presented. Gamma process is employed to simulate the corrosion process of cast iron pipe wall based on the reported corrosion model. Then time-dependent reliability analysis of pipes is conducted to calculate the probability of fracture failure at different service stages by the first-order reliability method (FORM) and the MCS. The parameter sensitivity analysis is also carried out to find the most influential parameters contributing to facture failure of pipes. Combined with the regional historical data of failed pipes in the worked example, the Bayesian Markov Chain Monte Carlo (MCMC) method is employed to update the physical parameters of the pipes and to improve the prediction of service lifetime of the corroded pipelines.

2. Corrosion Process

Generally speaking, both the internal and external of the pipe wall will suffer from corrosion. In this study, only external corrosion is considered since most water main pipelines are lined with polymeric or cement mortar liners. Besides, it should be stated here that the load coupled effect, such as the corrosion-fatigue-load coupled effect on the mechanism of the corrosion process is beyond the scope of this paper, since we mainly focus on the quantitative depth index of corrosion process which is substituted into the derived limit state function for time-dependent reliability analysis of pipe fracture failure in later sections.

Most studies have revealed that corrosion rate has a large value at the initial stage while decreases significantly as the pipes undergo ageing. In other words, the corrosion process shows the property of self-inhibition. Some empirical corrosion models have been proposed to predict the corrosion depth of the pipe wall. In this study, a two-phase corrosion model will be used to calculate the corrosion depth. The model can reflect the monotonic growth of the corrosion depth with the increase of service time, as well as the characteristics of a large corrosion rate in the initial stage and a small corrosion rate in the middle and late stages [10]:

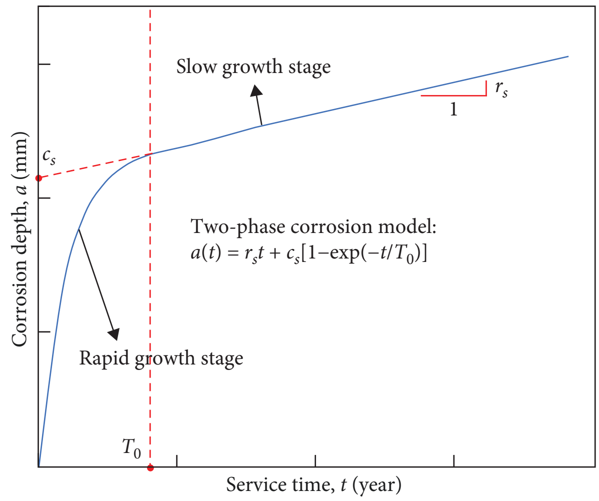

Figure 1: Schematic illustration of two-phase corrosion model [10].

$$a(t) = r_st + c_s[1 - \exp(-t/T_0)], \tag{1}$$

where a is the corrosion depth when the pipe age is t years; T_0 is the transit point of time from where the corrosion rate approaches a steady state in equilibrium with the surrounding environment after an initially high rate; and r_s and c_s are two corrosion parameters representing the interception and long-term corrosion rate, respectively, as shown in Figure 1.

The above two-phase corrosion model is actually an empirical description of the corrosion process, which may not be able to accurately reflect the development of corrosion over the long-lasting service lifetime. In addition, the model contains three random variables, which may lead to low efficiency while conducting the time-dependent reliability analysis of the pipes. Considering that the physical process of natural corrosion follows the Gamma process, this study will combine the preliminary knowledge of the corrosion depth calculated by the corrosion model with a systematic estimation of the parameters of a Gamma process associated to the corrosion model, as shown in Figure 2. Note that the Gamma process is capable of simulating the monotonic increase of the corrosion depth and the required mathematical calculation is relatively simple. In this way, we can directly capture the corrosion model using Gamma distributions of the corrosion depth at different service stages.

Mathematically, a Gamma process is composed of a series of Gamma distributions with a monotonically increasing shape parameter $\alpha(t)$ and an identical scale parameter λ. The corrosion depth of the pipe wall can be represented by a Gamma process as $\{a(t), t>0\}$ with the following properties: $a(0) = 0$ with probability of one; $a(t)$ has independent increments; and $[a(T) - a(t)]$ follows a Gamma distribution $\mathrm{Ga}(\alpha(T) - \alpha(t), \lambda)$, for all $T > t \geq 0$.The probability density function (PDF) of the corrosion depth $a(t)$ is given by Mahmoodian and Alani [14]

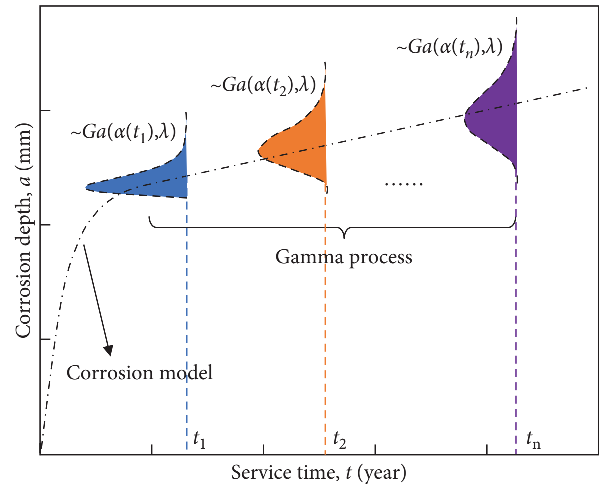

Figure 2: Schematic of corrosion process simulated by the Gamma process.

$$\mathrm{Ga}(a|\alpha(t), \lambda) = \frac{\lambda^{\alpha(t)}}{\Gamma[\alpha(t)]} a^{\alpha(t)-1} e^{-\lambda a} (a>0), \tag{2}$$

where $\Gamma(\cdot)$ is the Gamma function, and the shape parameter $\alpha(t)$ can be expressed as

$$\alpha(t) = \mu t^{\gamma}, \tag{3}$$

where μ and γ are two physical parameters, and γ can be taken as 1.0 for describing the corrosion of cast iron pipes [30]. The parameters μ and λ can be estimated by the method of maximum likelihood according to the corrosion data calculated by the corrosion model. Corrosion data are composed of pipe age t_i, $i = 0, 1, 2, \ldots, n$ where $0 = t_0 < t_1 < t_2 <, \cdots, < t_n$, the corresponding corrosion depth a_i, $i = 0, 1, 2, \ldots, n$ where $0 = a_0 \leq a_1 \leq a_2 \leq, \cdots, \leq a_n$, and the corrosion increments which can be expressed as $\delta_i = a_i - a_{i-1}$, $i = 1, 2, 3, \ldots, n$. According to the Gamma process, the likelihood function can be obtained as the product of the PDFs of the corrosion increments δ_i as follows:

$$\begin{aligned} L(\delta_1, \delta_2, \ldots, \delta_n|\mu, \lambda) &= \prod_{i=1}^{n} f_{a(t_i)-a(t_{i-1})}(\delta_i) \\ &= \prod_{i=1}^{n} \frac{\lambda^{\mu\left(t_i^{\gamma}-t_{i-1}^{\gamma}\right)}}{\Gamma[\mu(t_i^{\gamma} - t_{i-1}^{\gamma})]} \delta_i^{\mu\left(t_i^{\gamma}-t_{i-1}^{\gamma}\right)} e^{-\lambda\delta_i}. \end{aligned} \tag{4}$$

The maximizing of the likelihood function is equivalent to the maximizing of its logarithm, and it can be realized by setting their derivatives to zero. Thus, the estimation for the parameters μ and λ can be obtained by solving

$$\frac{\partial \log L(\mu, \lambda)}{\partial \mu} = 0, \tag{5}$$

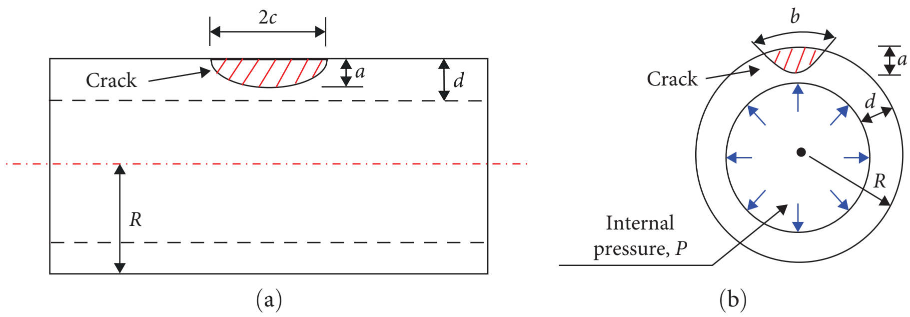

FIGURE 3: Details of a corroded pipe: (a) longitudinal view; and (b) circumferential view.

$$\frac{\partial \log L(\mu,\lambda)}{\partial \lambda} = 0. \tag{6}$$

The estimation for the parameter $\widehat{\lambda}$ can be determined by van Noortwijk [31]

$$\widehat{\lambda} = \frac{\widehat{\mu} t_n^{\gamma}}{a_n}. \tag{7}$$

To obtain the estimation of the parameter $\widehat{\mu}$, the following equation is to be solved. It should be noted that $\widehat{\mu}$ in Equation (8) has to be solved by iterative methods.

$$\sum_{i=1}^{n} \left(t_i^{\gamma} - t_{i-1}^{\gamma}\right)\left\{\psi\left[\widehat{\mu}\left(t_i^{\gamma} - t_{i-1}^{\gamma}\right)\right] - \log\delta_i\right\} = t_n^{\gamma} \log\left(\frac{\widehat{\mu} t_n^{\gamma}}{a_n}\right), \tag{8}$$

where $\psi(x)$ is the Digamma function that is the derivative of the logarithm of the Gamma function:

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)} = \frac{\partial \log(\Gamma(x))}{\partial x}. \tag{9}$$

3. Probability of Fracture Failure

3.1. Stress Intensity Factor. Cast iron pipelines usually suffer from various degrees of corrosion that generally occurs on the external surface of the pipe wall as discussed earlier, which may lead to the development of corrosion pits. Most of the literature suggests that the shape of corrosion pits are cuboids, spheres, and semiellipsoids [32, 33], while the sharp corrosion pit with a considerable corrosion depth is less considered [34]. However, corrosion pits with narrow openings and sharp shapes can be regarded as the initial cracks which cause the eventual fracture of the pipes.

In order to evaluate the breakage condition of such corroded cast iron pipes, the concept of the stress intensity factor is employed to quantitatively describe the strength of the stress field near the crack tip. In general, there are three modes of cracks in fracture mechanics, which are the opening mode (Mode I), the shear mode (Mode II), and the tear mode (Mode III). In this study, only the Mode I crack is considered since it has been commonly found to be the dominant cracking condition in observed fracture failure pipes. Considering the geometry of the corrosion pit and the internal water pressure, the stress intensity factor is determined by Raju and Newman [35]

TABLE 1: Constants in the influencing coefficient functions.

Scenario of the function	ξ_1	ξ_2	ξ_3
$f(a/c > 0.8)$	0.3143	0.0570	0.4568
$f(a/c \le 0.8)$	0.3296	0.1551	0.5538

$$K_{\mathrm{I}} = \frac{PR}{d}\sqrt{\pi a} f\left(\frac{a}{c}, \frac{a}{d}, \frac{d}{R}, \frac{b}{c}\right), \tag{10}$$

where P is the internal water pressure; R is the pipe radius; d is the pipe wall thickness; a is the corrosion depth; $f(\cdot)$ is the influencing coefficient function; c is the half length of the corrosion pit; and b is the width of the corrosion pit. Some details of a corroded pipe modeling are shown in Figure 3.

Furthermore, Wang et al. [36] carried out a series of 3D finite-element analyses to obtain the stress intensity factor of sharp corrosion pits with different geometry. The results showed that the influencing coefficient functions $f(\cdot)$ were expressed as follows:

$$f\left(\frac{a}{c}, \frac{a}{d}, \frac{d}{R}, \frac{b}{c}\right) = \xi_1 \frac{c}{\sqrt{ad}} e^{\frac{d}{2R} + \frac{a}{2d} - \frac{b}{2c}} + \xi_2 \sqrt{\frac{d}{a}} e^{-\frac{b}{2c}} + \xi_3 e^{-\frac{d}{2R}} \quad (a/c > 0.8), \tag{11}$$

$$f\left(\frac{a}{c}, \frac{a}{d}, \frac{d}{R}, \frac{b}{c}\right) = \xi_1 \sqrt{\frac{c}{a}} e^{\frac{d}{2R} + \frac{a}{d} - \frac{3b}{2c}} + \xi_2 \sqrt{\frac{d}{a}} e^{-\frac{2b}{R} - \frac{2b}{c}} + \xi_3 \frac{\sqrt{ab}}{c} e^{\frac{a}{2d} - \frac{b}{2c}} \quad (a/c \le 0.8), \tag{12}$$

where ξ_1, ξ_2, and ξ_3 are constants as summarized in Table 1.

3.2. Failure Criterion Definition. Based on the fracture mechanics, a pressurized cast iron pipe will fracture when the stress intensity factor at the crack tip exceeds the fracture toughness of the pipe material. The limit state function of the corroded pipes can be expressed according to the theory of structural reliability as

$$G(K_c, K_{\mathrm{I}}, t) = K_c - K_{\mathrm{I}}(\mathbf{x}, t), \quad (13)$$

where K_c is the fracture toughness, K_{I} is the stress intensity factor determined by Equation (10), $\mathbf{x}$ is the vector containing all random variables, and t denotes the service time. G is determined according to the given values of t and $\mathbf{x}$. In summary, the fracture failure of pipes is defined when $G < 0$, no failure when $G > 0$, and $G = 0$ denotes the limit state of pipe fracture.

3.3. Probability of Fracture Failure. The probability of fracture failure of the corroded pipes can be determined by

$$P_f = P(G \leq 0) = P[K_c \leq K_{\mathrm{I}}(\mathbf{x}, t)]. \quad (14)$$

The limit state function of the pipe fracture includes a number of physical parameters, all of which are subject to some degrees of uncertainty. Equation (13) is highly nonlinear, which cannot be determined analytically. Alternatively, the MCS and the FORM can be employed to numerically estimate the failure probability [37]. The MCS is usually regraded as a standard method in reliability problems, it calculates the probability of failure through a large number of samplings as follows:

$$P_f(t) = \frac{M[G(K_c, K_{\mathrm{I}}, t) \leq 0]}{N}, \quad (15)$$

where M is the number of samples for $G < 0$; and N is the total number of samples. It is well recognized that the accuracy of the MCS mainly depends on the value of N, which can be roughly determined according to the following formula [38]:

$$N \geq \frac{100}{P'_f}, \quad (16)$$

where N is the total number of simulations; and P'_f is the estimated failure probability.

Compared to the MCS, the basic concept of the FORM is that when the random variables are transformed from their original space (x-space) to the standard normal space (u-space), the reliability index β can be simply regarded as the minimum distance from the origin to the limit state surface in u-space and can be determined as follows [37]:

$$\beta = \min_{G=0} \sqrt{\mathbf{u}^T \mathbf{u}} = \sqrt{(\mathbf{u}^*)^T \mathbf{u}^*}, \quad (17)$$

where $\mathbf{u}$ are the vectors of random variables that are on the limit state surface in u-space, among which $\mathbf{u}^*$ is closest to the origin, called as the design point. The above calculation process for β can be efficiently realized by the Solver in the EXCEL [39]. The probability of pipe failure can be determined by

$$P_f = \Phi(-\beta), \quad (18)$$

where is the cumulative distribution function of a standard normal variable.

Different random variables show different effects on fracture failure of pipes. Due to the lack of full knowledge of random variables, it is of great importance to find the most influential variables so that further studies can focus on them. In this study, the concept of probability sensitivity index (PSI) is utilized to quantitatively evaluate the contribution of each random variable to the probability of failure. Note that the greater the absolute value of the PSI, the greater the influence of this parameter on the probability of failure. Positive PSIs indicate that increasing the values of the parameters will increase the probability of failure and vice versa. The PSI can be determined as follows [40]:

$$\alpha_i = \frac{-\frac{\partial G}{\partial u_i}\sigma(u_i)}{\sqrt{\sum_{j=1}^{n}\left[\frac{\partial G}{\partial u_j}\sigma(u_j)\right]^2}} \Big| \mathbf{u} = \mathbf{u}^*, \quad (19)$$

where α_i is the PSI of the random variable u_i ($i = 0, 1, 2, \ldots, n$); G is the limit state function as determined by Equation (13); $\sigma(u_i)$ is the standard deviation of u_i; n is the number of random variables; and $\mathbf{u}^*$ is the design point solved by the FORM.

4. Parameter Updating by Bayesian MCMC Algorithm

Obviously, time-dependent reliability analysis of fracture failure of corroded cast iron pipes should be based on the acquirement of accurate statistical information of pipe physical parameters. Unfortunately, the statistical information of these parameters is usually not fully available due to the complexity of underground space and the difficulty of pipelines inspection. The prior information based on engineering experience is often used for reliability analysis.

From probability theory, we can learn that Bayesian theory is one of the two most important branches in statistical inference, and its basic essence is to update our beliefs in events in our daily life. In this study, we consider that as we already have prior information of pipe parameters and recorded data of failed pipes in hand, which means that it actually meets the necessity of establishing the likelihood function in Bayesian formula, thus it is feasible for us to employ Bayesian method to update key pipe physical parameters. In this way, it would be possible to more accurately evaluate the risk of fracture failure of cast iron pipes with surface sharp corrosion pits, as more reliable statistical information for reliability analysis is obtained. The main process of updating parameters by the Bayesian method is briefly introduced below.

Note that a simplified physical model can be used to represent a natural time-dependent physical process. The predictions of the physical model at different time instants can be expressed as follows:

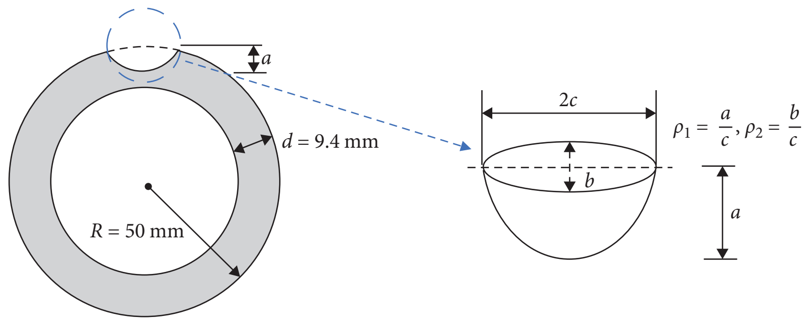

FIGURE 4: Schematic of corroded cast iron pipes in the worked example.

$$g(\widehat{\mathbf{x}}; t) = g(x_1, x_2, \ldots, x_n; t), \tag{20}$$

where $g(\cdot)$ is the prediction of the physical model, $\widehat{\mathbf{x}} = (x_1, x_2, x_3, \ldots, x_n)$ is the vector containing all input parameters; and t denotes the time.

Due to the simplification of the model and the uncertainty in the input parameters, the model predictions may not be very accurate and are always subject to some errors. If measurements are taken on the response quantity at some discrete points in time, we have the following relationship:

$$g_{\text{mes}}(\widehat{\mathbf{x}}; t) = g(\widehat{\mathbf{x}}; t) + e, \tag{21}$$

where $g_{\text{mes}}(\cdot)$ is the measurement value of the response quantity; and e is the error between the measurement value and the model prediction. A simple assumption is that the error e can be regarded as a random variable following a zero-mean normal distribution with variance ${\sigma_e}^2$. Therefore, if we have measurement values $g_{\text{mes},j}$ of the response quantity at discrete points of time t_j ($j = 1, 2, 3, \ldots, Y$), the likelihood function of $\widehat{\mathbf{x}}$ can be expressed as follows [41]:

$$\begin{aligned} &L\left[g_{\text{mes},1}, \ldots, g_{\text{mes},Y} | \widehat{\mathbf{x}}\right] \\ &= \prod_{j=1}^{Y} \frac{1}{\sigma_e \sqrt{2\pi}} \exp\left[-\frac{1}{2} \frac{\left[g_{\text{mes},j} - g\left(\widehat{\mathbf{x}}; t_j\right)\right]^2}{{\sigma_e}^2}\right], \end{aligned} \tag{22}$$

where $g\left(\widehat{\mathbf{x}}; t_j\right)$ is the model prediction at the time t_j. From Bayesian theory, the posterior PDF of $\widehat{\mathbf{x}}$, $f_{\widehat{\mathbf{X}}}(\widehat{\mathbf{x}})$ can be estimated using the likelihood function $L\left[g_{\text{mes},1}, \ldots, g_{\text{mes},Y} | \widehat{\mathbf{x}}\right]$ and the prior PDF of $\widehat{\mathbf{x}}$, $p_{\widehat{\mathbf{X}}}(\widehat{\mathbf{x}})$ as follows:

$$f_{\widehat{\mathbf{X}}}(\widehat{\mathbf{x}}) = f_{\widehat{\mathbf{X}}}\left[\widehat{\mathbf{x}} | g_{\text{mes},1}, \ldots, g_{\text{mes},Y}\right] = k p_{\widehat{\mathbf{X}}}(\widehat{\mathbf{x}}) L\left[g_{\text{mes},1}, \ldots, g_{\text{mes},Y} | \widehat{\mathbf{x}}\right], \tag{23}$$

where k is a normalization constant. Note that the above equation is referred to as indirect Bayesian updating since the likelihood function depends on the measurement values $g_{\text{mes}}(\cdot)$ and the predictions of the model $g(\cdot)$ instead of on the parameter $\widehat{\mathbf{x}}$ by itself.

It is difficult to solve Equation (23) since it is a multidimensional parameter updating problem, especially for the normalization constant k. This study uses the MCMC simulation algorithm to deal with the problem. For the complex target distribution which is hard to sample by common methods, MCMC can ensure the samples following the target distribution through its specific sampling algorithm, such as the Metropolis–Hastings algorithm, the Gibbs sampling, the Slice sampling, and so forth. The statistical inference can easily be made by calculating the moments of the obtained samples. In this study, the Metropolis–Hastings algorithm based on the Bayesian framework is employed to update those key physical parameters of cast iron pipelines. The prior means of the uncertain parameters are used as initial values to perform the sampling according to the Metropolis–Hastings algorithm. The acceptance rate of each step of sampling is α, which is calculated by [42]

$$\alpha = \min\left(1, \frac{L(D|x'|p(x')q(x)x')}{L(D|x|p(x)q(x')x)}\right), \tag{24}$$

where $L(D|x)$ is the likelihood function; D is the data (such as $g_{\text{mes},1}, \ldots, g_{\text{mes},Y}$ in Equation (23)); $p(x)$ is the prior information; and $q(x|x')$ is the proposal distribution. The MCMC algorithm does not need to evaluate the normalization constant k since it cancels out in Equation (24). After s steps of sampling, the samples are $x_0, x_1, x_2, \ldots, x_{i-2}, x_{i-1}, x_i, \ldots, x_s$ which have the property of Markov chain, as x_i is only dependent on x_{i-1} ($i = 1, 2, \ldots, s$) while is independent on $x_0, x_1, x_2, \ldots, x_{i-2}$. After deleting the first m samples as executing the burn-in process, the stationary set of the samples $\{x_m, x_{m+1}, x_{m+2}, \ldots, x_s\}$ is our target PDF.

5. Worked Example

To illustrate the proposed method for evaluating the probability of fracture failure of cast iron pipelines, a worked example of corroded cast iron pipe cohorts in Australia is provided in this section [43]. The pipes have an external diameter of 100 mm and the wall thickness of 9.4 mm. Considering the uncertainty in the geometry of corrosion pits, two parameters ρ_1 and ρ_2 are introduced here and both of them are considered to be random variables, which represent the ratio of the pit depth and width of the corrosion pit to the semilong axis of the corrosion pit, respectively, as shown in Figure 4. Corrosion parameters, fracture toughness, and internal pressure are all considered to be random variables. The prior statistical information of random variables for the

Table 2: Prior statistical information of random variables for the cast iron pipes.

Symbol	Mean	Coefficient of variation	Distribution
r_s	0.02 mm	0.2	Lognormal
c_s	6 mm/year	0.1	Lognormal
$1/T_0$	0.15 1/year	0.3	Lognormal
P	0.45 MPa	0.15	Normal
K_c	$14\ \mathrm{MPa}\cdot\sqrt{\mathrm{m}}$	0.05	Normal
ρ_1	1	0.1	Lognormal
ρ_2	0.5	0.1	Lognormal

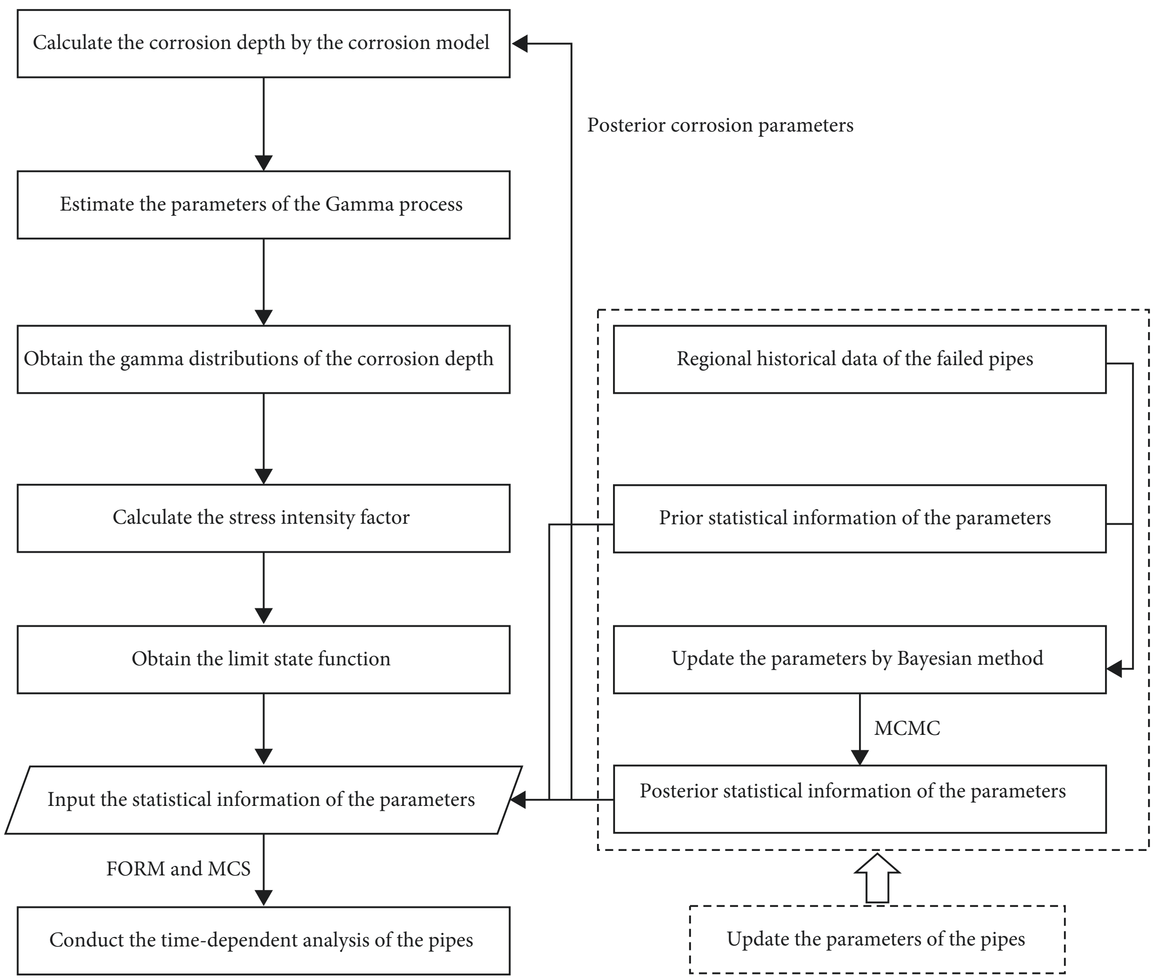

Figure 5: Flowchart of the time-dependent reliability analysis for the corroded cast iron pipes.

pipes is shown in Table 2. It should be noted that the bias factor of each random variable, which is the ratio of the mean value to the nominal value [44], is not considered in this study, as the prior statistical information is from other references and some of them are just determined by a rough estimation. Besides, we assume the pipes had suffered from corrosion since they were installed. The flowchart of the worked example is shown in Figure 5 and the main steps are summarized as follows:

Step 1: calculate the corrosion depth according to the two-phase corrosion model (Equation (1)), and use the result to estimate the parameters of the Gamma process (Equations (7) and (8)), then Gamma distributions of the corrosion depth at different service stages are obtained.

Step 2: calculate the stress intensity factor according to the corrosion depth, and substitute it into Equation (13) to obtain the limit state function. Input the prior information of the parameters, then calculate the probability of pipes fracture failure by the FORM (Equation (18)) and the MCS (Equation (15)).

Step 3: based on the prior information of the parameters, employ the Bayesian MCMC algorithm to solve Equation (23) to update the parameters at each discrete time point

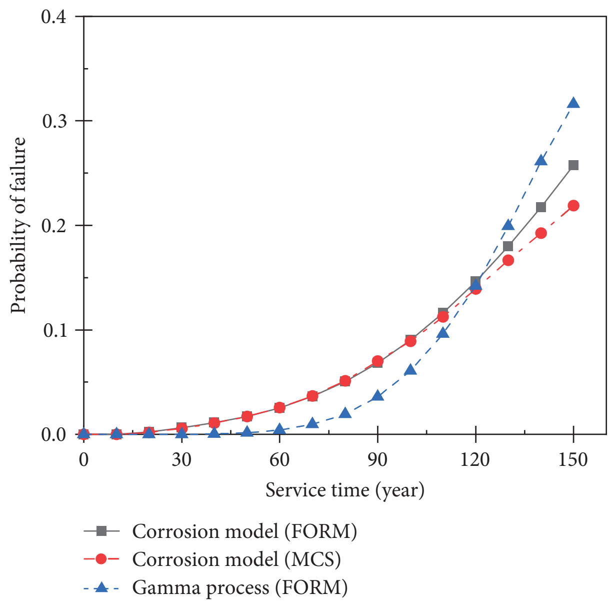

Figure 6: Probability of fracture failure of the corroded cast iron pipes.

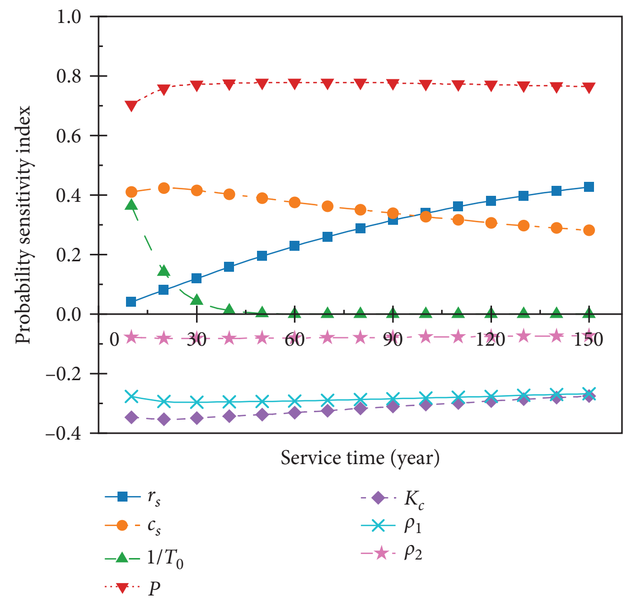

Figure 7: Change of probability sensitivity indices with time.

which is the service lifetime of failed pipes in historical data. Use the posterior information of the parameters to repeat Step 1 and Step 2 to obtain the results of posterior time-dependent analysis and lifetime prediction of the corroded pipes.

5.1. Results of Prior Time-Dependent Analysis. The probability of fracture failure of corroded cast iron pipes over the period of 0–150 years is first calculated using the FORM (Equation (18)) and the MCS (Equation (15), $N = 100{,}000$) based on the corrosion model, as shown in Figure 6. As shown in Figure 6, the probability of failure calculated by the FORM is in satisfactory agreement with the results from the MCS, which verifies the accuracy of the FORM in this problem. The corrosion depth obtained from the corrosion model is further used to estimate the parameters of the Gamma process, and the result shows that the shape parameter $\alpha(t) = 0.095t$, and scale parameter $\lambda = 1.578$. The average maximum corrosion depth predicted by the Gamma process is 9.0 mm at 150 years, which is of 2.2% difference with that determined by the two-phase corrosion model, hence it can be concluded that the Gamma process can well simulate the corrosion process of the pipe wall.

The Gamma process-based probability of fracture failure of the pipes is also calculated by the FORM according to Equation (18) and shown in Figure 6. It is found that the probability of failure calculated by the Gamma process is basically in consistency with the results from the corrosion model. The discrepancy between the Gamma process and the corrosion model can be explained: the parameters of the Gamma process estimated from the method of maximum likelihood will be constrained by the corrosion data that is determined by the corrosion model, while the corrosion model is not accurate enough to predict the corrosion depth especially at the initial service stage of the pipes. In this paper, we make some new attempts that more parameter analysis of pipe fracture failure can be conducted based on the Gamma process as it is more convenient than the traditional corrosion model, since the number of random variables based on the analysis of Gamma process are less. Therefore, it can be concluded that although the probability of failure calculated by the Gamma process may not be so accurate compared to the MCS, we believe it is still valuable and can be further improved. Considering the efficiency of the Gamma process in conducting the time-dependent reliability analysis, this study will focus on the Gamma process-based results in the following analysis.

The change of PSIs of random variables with time is calculated based on Equation (19) and shown in Figure 7. It is found that the internal pressure P, fracture toughness K_c, and the geometry parameter ρ_1 have the most significant influence on the probability of failure, which indicates that further study should focus on these parameters. Parameters with quite small PSIs, such as $1/T_0$, can be considered as a constant for pipes with a service life of more than 40 years. The probability of failure for pipe internal pressure with different coefficients of variation (COV) is further studied, as shown in Figure 8. It can be found that the larger the COV, the larger the probability of failure, and the discrepancy for different COVs decreases with the increase of service time.

The probability of fracture failure for different values of fracture toughness is shown in Figure 9. As shown in Figure 9, the greater the fracture toughness, the smaller the probability of failure, which is obviously self-evident. Further analysis indicates that the fracture toughness increases from 14 to 16 $\mathrm{MPa}\cdot\sqrt{\mathrm{m}}$, only increasing by about 14.3%, the probability of failure can be reduced by about 75.0% at the initial service stage and about an average of 60.0% in the whole service lifetime. This has certain guiding significance, for example, the optimal design considering both the cost of manufacture

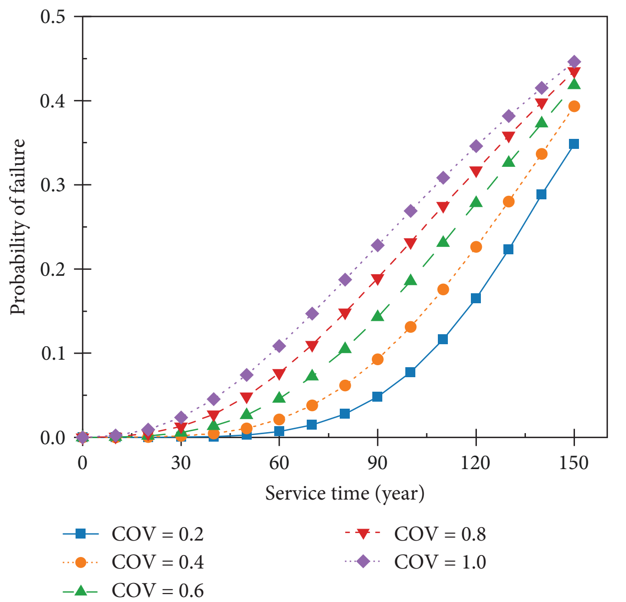

Figure 8: Probability of pipe fracture failure for internal pressure with different COVs.

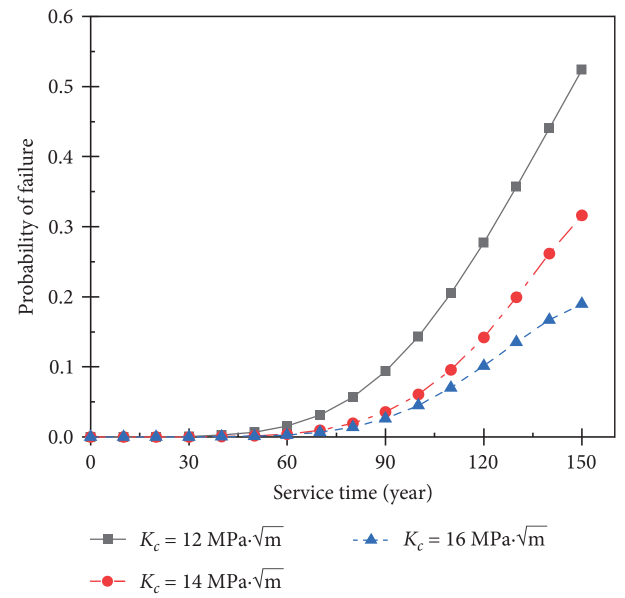

Figure 9: Probability of pipe fracture failure for different values of fracture toughness K_c.

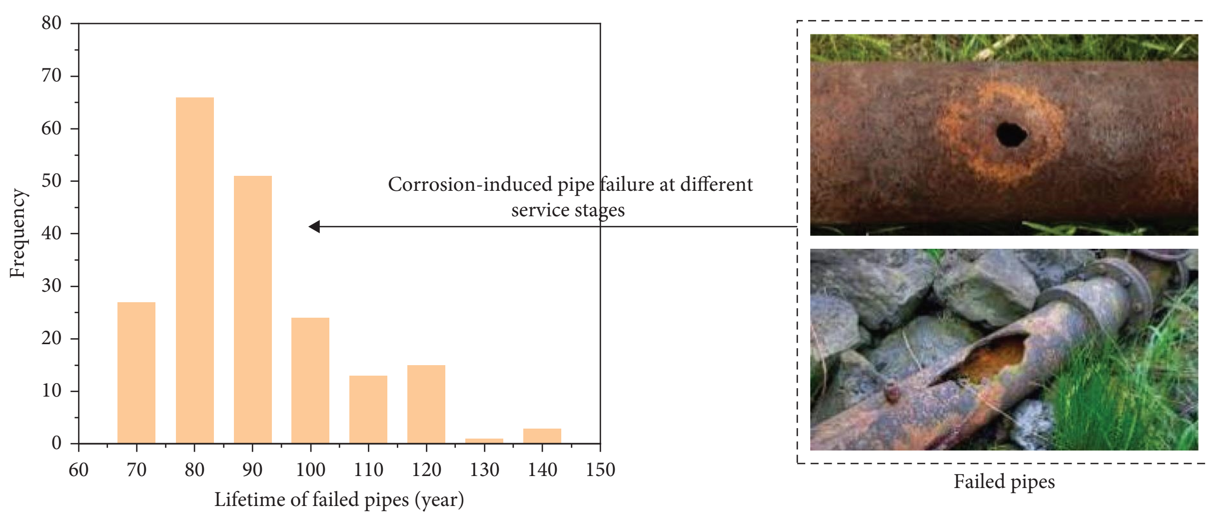

Figure 10: Failure data of service lifetime of failed pipes used for parameter updating.

and the risk of fracture failure should be conducted before the casting of pipes, and a reasonable value of fracture toughness should be determined to better balance the cost and the risk.

5.2. Results of Posterior Time-Dependent Analysis. According to the observational failure data of small diameter pipes provided by the Australian water utilities in the Advanced Condition Assessment and Pipe Failure Prediction project [45], the failure data used for the parameter updating in this study is shown in Figure 10, which is composed of service lifetime t_j ($j = 1, 2, 3, \ldots, 200$) of failed pipes. The MCMC simulations based on the Bayesian framework are conducted to update key physical parameters of the pipes (Equation (23)), and Figure 11 shows the trace plots of the parameters with 10,000 simulations in total. A burn-in of the first 5,000 simulations is executed and the posterior statistical information of random variables is shown in Table 3. It is found that the internal pressure P, the corrosion parameter c_s, and the fracture toughness K_c show different degrees of change after the Bayesian updating. The prior and posterior PDFs of these parameters are shown in Figure 12.The posterior corrosion parameters are used to estimate the parameters of the Gamma process, and the prior and posterior parameters of the Gamma process are shown in Table 4. The time-dependent analysis is conducted again according to the updated Gamma process in Table 4 and the parameters in Table 3. The comparison of Gamma process-based probability of failure before and after the updating of parameters is shown in Figure 13. It can be seen that the posterior probability of failure increases significantly, and at the same service

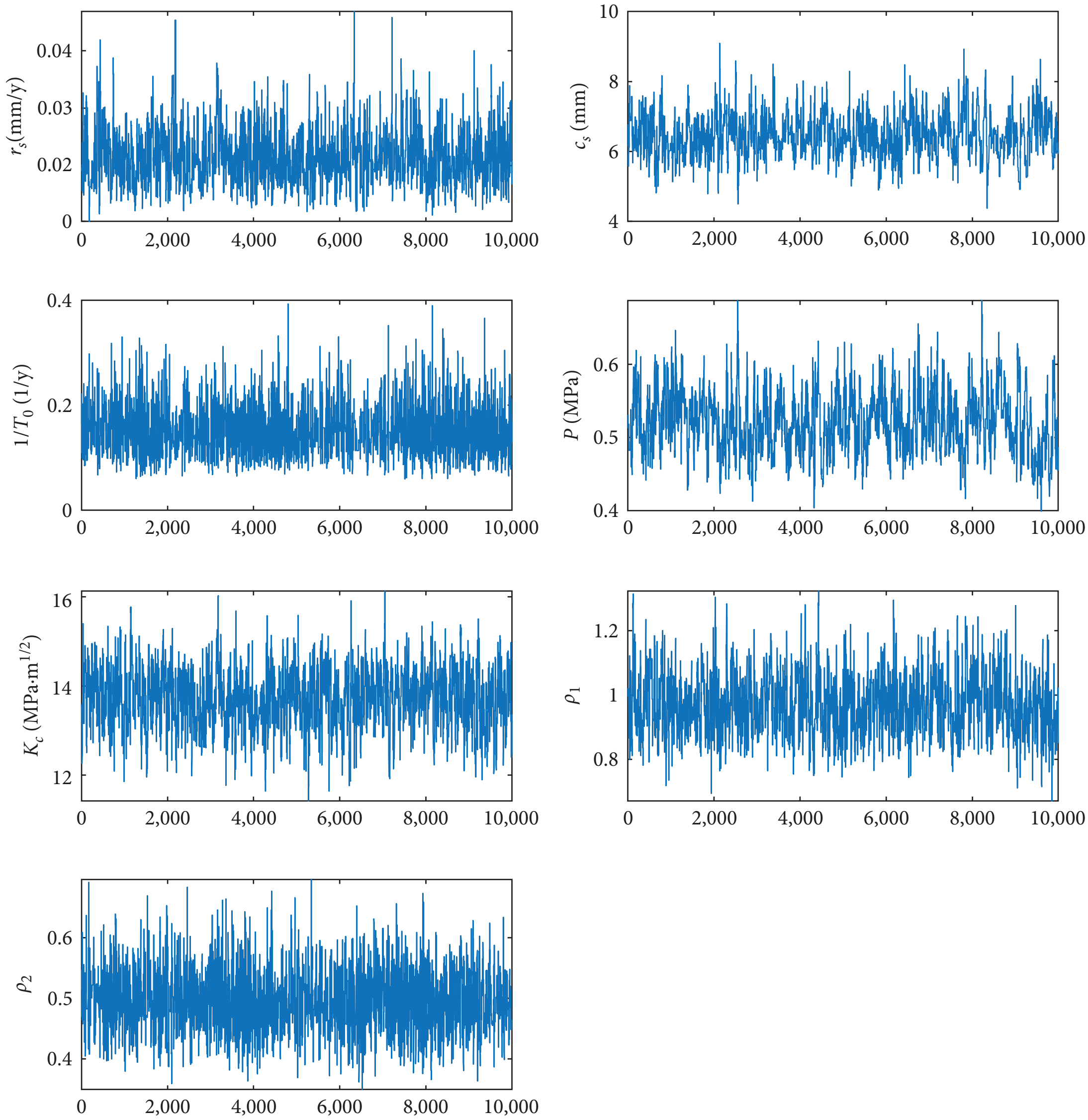

Figure 11: Trace plots of parameters by MCMC simulations.

Table 3: Posterior statistical information of random variables for the cast iron pipes.

Symbol	Mean	Standard deviation	Distribution
r_s	0.02 mm	0.004	Lognormal
c_s	6.47 mm/year	0.641	Lognormal
$1/T_0$	0.15 1/y	0.044	Lognormal
P	0.52 MPa	0.047	Normal
K_c	13.67 MPa$\cdot\sqrt{\text{m}}$	0.695	Normal
ρ_1	0.973	0.089	Lognormal
ρ_2	0.497	0.048	Lognormal

time it is about twice the prior probability of failure. This observation can be further explained as that we may underestimate the prior values of some physical parameters due to the lack of full knowledge, while more reliable statistical inference about these parameters can be made after the Bayesian updating since the observational failure data of the pipes are reasonably considered in the process of updating.

Setting the acceptable probability of failure as 0.1 as which has widely been used in other literature for representing the limit risk level of predicting service lifetime of corroded cast iron pipes [14, 23, 46], it can be shown in Figure 13 that the service life t_L for the pipe is 110 years before the updating of the parameters, while it reduces to 85 years with a decrease of 25 years after the Bayesian updating. Considering this

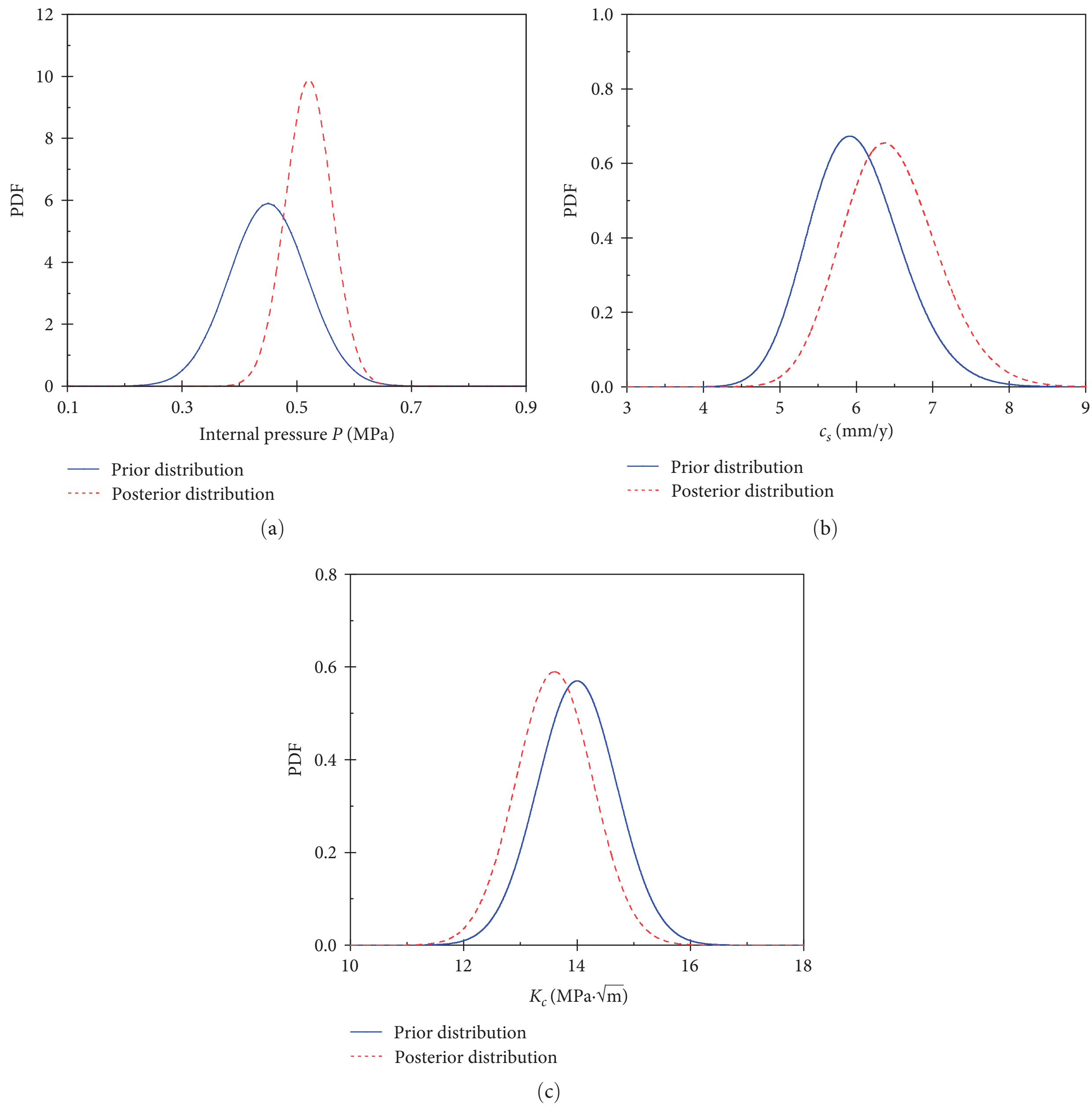

Figure 12: Prior and posterior PDFs of random variables: (a) internal pressure P; (b) corrosion parameter c_s; and (c) fracture toughness K_c.

Table 4: Prior and posterior parameters of Gamma process.

Parameters of Gamma process	Prior	Posterior
Shape parameter $\alpha(t)$	$0.096t$	$0.088t$
Scale parameter λ	1.602	1.454

difference, we want to emphasize in this paper that time-dependent reliability analysis of pipe fracture failure should be conducted based on more accurate information of pipe parameters, otherwise it is difficult for us to make reasonable predictions on corroded pipes. Besides, the posterior service life obtained here after the Bayesian updating is just an updated version of the prior one, it is more reliable but actually it can still be updated again if there are newly recorded data of pipes. Therefore, it can be concluded that it is of great significance to update physical parameters based on the historical data of failed pipes, as more accurate probabilistic analysis of fracture failure and lifetime prediction of the cast iron pipe can be carried out after the Bayesian updating of parameters.

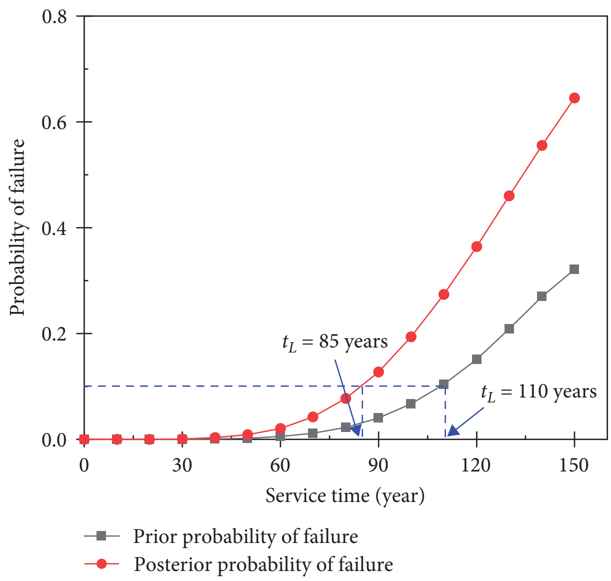

FIGURE 13: Gamma process-based probability of pipe fracture failure before and after the Bayesian updating of random variables.

6. Conclusions

In this study, a method to evaluate the risk of fracture failure of buried cast iron pipes was presented. In this method, the Gamma process was employed to simulate the corrosion process of cast iron pipes, and the Gamma-based time-dependent reliability analysis was carried out by the FORM and the MCS based on the failure criterion of pipe fracture. The updating of the reliability analysis and lifetime predictions of the pipes were obtained according to the Bayesian MCMC algorithm. Some conclusions can be drawn as follows:

(1) Gamma process could well simulate the corrosion process of the pipe wall hence can be used to efficiently conduct the time-dependent analysis of fracture failure of cast iron pipes. The probability of failure calculated by the Gamma process was in good agreement with that determined by the corrosion model.

(2) The parameter sensitivity analysis showed that the internal pressure, fracture toughness, and the geometry parameter of the corrosion pit were the most influential parameters on the probability of pipe fracture failure. The influence of COV of the internal pressure P is further studied and it was found that the larger the COV, the larger the probability of failure. Also, the greater the fracture toughness, the smaller the probability of failure.

(3) The Bayesian MCMC algorithm was employed to update key physical parameters of the pipe according to the regional historical data of failed pipes. Posterior reliability analysis results showed that the probability of fracture failure increased significantly after the parameter updating, which was almost twice the prior probability of failure. It was also found that when the acceptable probability of failure was 0.1, the predictive lifetime of the pipe was reduced to 85 years, which was 25 years less than the predicted lifetime before the Bayesian updating.

It can be concluded that the presented method in this study can calculate the probability of fracture failure and predict the lifetime of buried cast iron water main pipelines with reasonable accuracy. It should be noted that the parameters of the Gamma process in this study were estimated based on the corrosion depth calculated by a two-phase corrosion model representing the whole service lifetime. Future work will focus on the development of more accurate corrosion models and the estimation of parameters of the Gamma process with real corrosion data related to underground pipeline fracture failure.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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