

## Research Article

# A Method of Selecting Project Delivery System with Pythagorean Fuzzy Set Group Decision-Making Method and Prospect Theory

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Received 10 May 2023; Revised 23 August 2023; Accepted 25 September 2023; Published 25 October 2023

Academic Editor: Morteza Bagherpour

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Project delivery system (PDS) is critical to the success of a construction project. There is a strong link between contract type and project delivery method. Therefore, this paper presents a framework for multiobjective decision-making that combines project delivery method and contract type for the first time. Furthermore, the Pythagorean fuzzy set (PFS) is introduced in the decision-making model. A new method for PDS group decision-making is constructed by combining PFS with the entropy weighting method and prospect theory. In order to improve the reliability of decision-making, this paper proposes a new method for determining decision-maker weights based on comprehensive weights, which is used to distinguish the professional level of decision-makers. Finally, the PDS decision-making process is elaborated through case analysis, the effectiveness of the decision-making model is verified, and the differences between the PDS decision-making method and the traditional method are discussed, which proves the reliability of the method. The method proposed in this paper can provide support for the owner's PDS selection.

## 1. Introduction

Investment in the Chinese construction industry has significantly increased in the past decades and is predicted to continue growing in the coming years [1]. Therefore, how to increase the investment efficiency of construction project is very important, which is concerned by the whole society [2]. Engineering project construction process is a series of transaction process. Practice has proved that project delivery system (PDS) is one of the critical factors affecting the success of a project [3–5]. How to design the most appropriate PDS for a project is crucial to the success of the project construction [3, 6].

Many scholars have done corresponding research on PDS selection. Chen et al. [3] and Qiang et al. [7] analyzed the indicators influencing PDS selection. Chen and Yang [8] categorized the indicators for PDS selection into four groups: project objectives, project characteristics, owner and contractor characteristics, and external environment. Feghaly et al.

[9] proposed 13 factors that affect the selection of PDS for water treatment plants. Raouf and Al-Ghamdi [10] analyzed the effectiveness of different PDSs in green buildings. Franz et al. [11] compared the performance of three traditional PDSs (design-bid-build (DBB), construction management at risk, and design-build (DB)) in the US construction industry by using the data of 212 projects. Mostafavi and Karamouz [12] constructed a PDS selection method based on a fuzzy set and the TOPSIS method. Chen et al. [3] developed a PDS selection support method based on the DEA-BND model to help owners to make a decision. Li et al. [13] proposed a PDS selection decision model using an unascertained measurement model and information entropy; An et al. [6] constructed a PDS selection decision support method using an interval-valued intuitionistic fuzzy set. Nguyen et al. [14] proposed an empirical inference system for highway project delivery selection methods using fuzzy pattern recognition. Zhong et al. [15] proposed a project delivery method selection framework using a design structure matrix.

The above decision method could provide support for the owner's PDS selection but leaves some issues unanswered. The above study only analyses the decision-making problem of project delivery method or identifies the factors influencing of PDS selection. In fact, the project delivery method is closely related to the contract type, especially in China. The contract type and the project delivery method are related and affect each other and thus cannot be considered independently when choosing a PDS. These factors make PDS composed of "project delivery method" + "contract type" (e.g., DBB + unit price contract and EPC + lump sum contract). Most existing studies on PDS selection fail to consider the contract type. In addition, in the decision-making process of PDS, it is common to invite experts to score and make decisions. However, existing research does not consider the professional fields of experts in building decision models, which leads to low reliability of decision results.

Based on the aforementioned research gap, this paper combines the project delivery method and contract type to establish a new PDS decision-making framework. Taking into account the ambiguity of the decision-making process and the psychological aversion to risk on the part of the decision-maker, this paper combines the Pythagorean fuzzy set (PFS) with prospect theory to construct a new PDS decision-making method. In order to improve the reliability of decision-making, this paper proposes a new method for determining decision-maker weights based on comprehensive weights, which is used to distinguish the professional level of decision-makers. The purpose of this paper is to propose a more effective PDS decision-making method to support the PDS selection of owners. The rest of this paper is arranged as follows: Section 2 is preliminaries. Section 3 describes the PDS selection decision-making model. Section 4 verifies the applicability of the decision-making method through a case study. Section 5 is the discussion section. Section 6 is the conclusions of this study.

## 2. Preliminaries

### 2.1. A Brief Introduction to Pythagorean Fuzzy Set

*Definition 1.* For the nonempty set  $X = \{x_1, x_2, \dots, x_n\}$ , an intuitionistic fuzzy set  $I$  on  $X$  is

$$I = \{ \langle x, I(u_I(x), v_I(x)) \mid x \in X \}, \quad (1)$$

where  $u_I(x) \in [0, 1]$  is called the membership degree;  $v_I(x) \in [0, 1]$  is called the nonmembership degree; and  $0 < u_I(x) + v_I(x) < 1$ . According to the degree of membership and nonmembership, the hesitation degree can be calculated as  $\pi_I(x) = 1 - u_I(x) - v_I(x)$  [16]. On the basis of intuitionistic fuzzy theory, Yager [17] proposed the Pythagorean fuzzy set (PFS). Compared with intuitionistic fuzzy set, PFS has more uncertainty and stronger applicability.

*Definition 2.* For the nonempty set  $X = \{x_1, x_2, \dots, x_n\}$ , a PFS  $P$  on  $X$  is

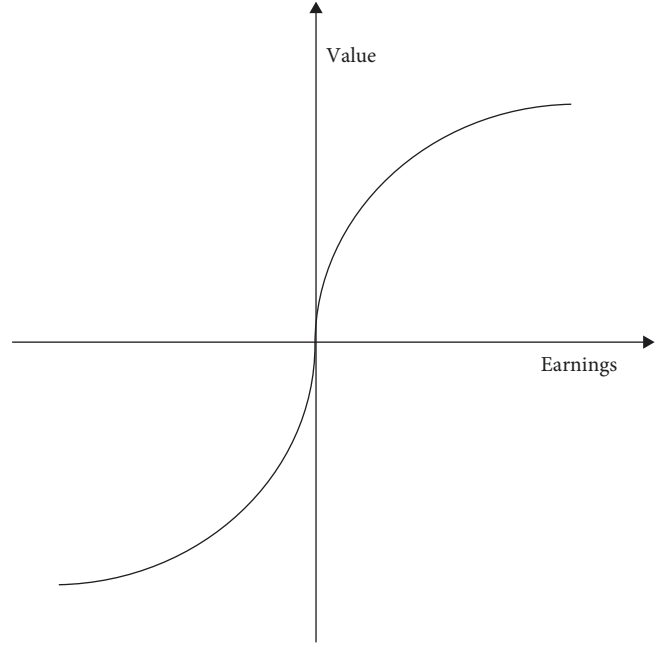


FIGURE 1: Value function.

$$P = \{ \langle x, P(u_p(x), v_p(x)) \mid x \in X \}, \quad (2)$$

where  $u_p(x) \in [0, 1]$  is also the membership degree;  $v_p(x) \in [0, 1]$  is the nonmembership degree; and  $(u_p(x))^2 + (v_p(x))^2 < 1$ . The hesitation degree can be calculated as  $\pi_p(x) = \sqrt{1 - (u_p(x))^2 - (v_p(x))^2}$  [18]. For brevity,  $(u_p(x), v_p(x))$  is called the Pythagorean fuzzy number, and  $P(u_p(x), v_p(x))$  for short [19].

*2.2. A Brief Introduction to Prospect Theory.* Zadeh and Kahneman [20] proposed the concept of prospect theory based on game theory and psychology. Prospect theory means that before the decision-making behavior occurs, the decision-maker will set a reference point according to the plan. When the decision-maker has benefits, and the benefits are close to the reference point, he will avoid risks. However, when the decision-maker is faced with a loss, and the loss is close to the reference point, the decision-maker is more inclined to take risks. Decision-makers are loss-averse. The value function and the probability weight function are the most important parts of prospect theory.

Value functions are usually constructed based on risk-return and risk loss. It contains two independent variables: one is the benchmark reference point, and the other is the amount of change relative to the benchmark reference point. The value function is S-shaped in terms of gain and loss. In addition, the slope of the value function in the gain part is smaller than the slope of the loss part. The value curve is shown in Figure 1 [21].

TABLE 1: Indicators that influence PDS selection.

Indicators of PDS selection	Project characteristics ( $B_1$ )	The economic attributes of the project ( $C_1$ )
		The scale of the project ( $C_2$ )
		The complexity of the project ( $C_3$ )
		The degree of interference of the subproject ( $C_4$ )
	Owner's needs and preferences ( $B_2$ )	Cost objective ( $C_5$ )
		Schedule objective ( $C_6$ )
		Quality objective ( $C_7$ )
		Risk attitude of the owner ( $C_8$ )
	Construction environment ( $B_2$ )	Management ability of the owner ( $C_9$ )
		The owner's preference ( $C_{10}$ )
		Resettlement ( $C_{11}$ )
		Construction site conditions of the project ( $C_{12}$ )
		Policy and regulation ( $C_{13}$ )
		Construction market development level ( $C_{14}$ )

The value function can be expressed as follows:

$$v(x) = \begin{cases} x^\alpha, & x \geq 0 \\ -\lambda(-x)^\beta, & x \leq 0, \end{cases} \quad (3)$$

where  $\alpha$  is the concave-convex degree of the gain region;  $\beta$  is the concave-convex degree of the loss region;  $\lambda$  is the degree of loss aversion;  $\alpha < 1$ ,  $\beta < 1$ , and  $\lambda > 1$ .

For the probability function  $\omega(p)$ , it is related to the objective probability  $p$  and represents the influence of the probability of an event on its foreground value. The probability weight function has the following characteristics [22]:

- (1)  $\omega(0) = 0, \omega(1) = 1$ ;
- (2) When  $r$  exists and satisfies  $0 < r < 1, \omega(rp) > rp$ ;
- (3) When the objective probability is large,  $\omega(p) < p$ ;  
when the objective probability is very small,  $\omega(p) > p$ ;
- (4) When  $0 < p < 1$ , there is  $\omega(p) + \omega(1-p) < 1$ ;
- (5) For any  $0 < p, q, r < 1$ , there is  $\frac{\omega(pq)}{\omega(p)} < \frac{\omega(pqr)}{\omega(pr)}$ ;
- (6)  $\omega(p) = \frac{p^\gamma}{[p^\gamma + (1-p)^\gamma]^{1/\gamma}}$ ,  $\gamma$  is fitting parameter.

### 3. PDS Selection Decision-Making Model

**3.1. Indicators Affecting PDS Selection.** PDS decision is very complex and influenced by many factors. PDS usually comprises "project delivery method" + "contract type." According to the results of the questionnaire survey [23], indicators affecting PDS selection are summarized, as shown in Table 1.

**3.2. The Process of PDS Selection.** For an engineering project PDS selection problem, assume that  $A = (A_1, A_2, \dots, A_m)$  represent the alternatives set,  $C = (C_1, C_2, \dots, C_n)$  represents the attributes set, and  $D = (D_1, D_2, \dots, D_l)$  represent the decision-makers set. The PDS selection process is as follows:

Step 1: Individual evaluation of decision-makers.

First, decision-makers are asked to evaluate each alternative over the indicators. The evaluation results are required to be given by utilizing PFSs. Let the decision evaluation results are  $Y^k = (Y^1, Y^2, \dots, Y^l)$ , among them

$$Y_k = \begin{matrix} & C_1 & C_2 & \dots & C_n \\ A_1 & \begin{bmatrix} Y_{11}^k & Y_{12}^k & \dots & Y_{1n}^k \end{bmatrix} \\ A_2 & \begin{bmatrix} Y_{21}^k & Y_{22}^k & \dots & Y_{2n}^k \end{bmatrix} \\ \vdots & \begin{bmatrix} \vdots & \vdots & \ddots & \vdots \end{bmatrix} \\ A_m & \begin{bmatrix} Y_{m1}^k & Y_{m2}^k & \dots & Y_{mn}^k \end{bmatrix} \end{matrix}, \quad (4)$$

where  $Y_{ij}^k = (u_{ij}^k, v_{ij}^k)$  represents the decision-maker  $D_k$ 's evaluation value of  $A_i$  for  $C_j$ .  $i = 1, 2, \dots, m, j = 1, 2, \dots, n$ , and  $k = 1, 2, \dots, l$ .

Step 2: Determination of the weights of decision-makers.

The determination of the weight of decision-makers is very important to the reliability of the decision [24]. In the PDS selection process, decision-makers are often invited from different fields. They are usually familiar with the attributes of their specialty, but usually not all of them [6]. Therefore, in order to solve this problem and improve the reliability of the decision, this study proposes a more refined decision-makers weight determination method based on the utility level of the evaluation information.

The level of information utility can be calculated by the distance measure formula. The average decision matrix  $\bar{R} = (\bar{r}_{ij})_{m \times n}$  can represent the average level of evaluation information given by the decision group, and the matrix can be calculated by the following:

$$\bar{r}_{ij} = (\bar{u}_{ij}, \bar{v}_{ij}) = \left( \sqrt{1 - \prod_{k=1}^l (1 - (u_{ij}^k)^2)^{1/l}}, \prod_{k=1}^l (v_{ij}^k)^{1/l} \right). \quad (5)$$

The Hamming distance [19] and Chebyshev distance [25] of  $R^k = (r_{ij}^k)_{n \times m}$  and  $\bar{R} = (\bar{r}_{ij})_{m \times n}$  can be calculated as follows [26]:

$$d_1(R^k, \bar{R}) = \frac{1}{4nm} \sum_{i=1}^n \sum_{j=1}^m (|(u_{ij}^k)^2 - (\bar{u}_{ij})^2| + |(v_{ij}^k)^2 - (\bar{v}_{ij})^2| + |(\pi_{ij}^k)^2 - (\bar{\pi}_{ij})^2|) \quad (k = 1, 2, \dots, l), \quad (6)$$

$$d_{+\infty}(R^k, \bar{R}) = \max_{\substack{1 \leq j \leq m \\ 1 \leq i \leq n}} \left\{ |(u_{ij}^k)^2 - (\bar{u}_{ij})^2|, |(v_{ij}^k)^2 - (\bar{v}_{ij})^2|, |(\pi_{ij}^k)^2 - (\bar{\pi}_{ij})^2| \right\} \quad (k = 1, 2, \dots, l). \quad (7)$$

According to Hamming distance and Chebyshev distance, the comprehensive distance can be calculated as follows:

$$d(R^k, \bar{R}) = \rho d_1(R^k, \bar{R}) + (1 - \rho) d_{+\infty}(R^k, \bar{R}), \quad (8)$$

where  $\rho \in [0, 1]$  is the control parameter.

According to the comprehensive distance, the weights of decision-makers can be defined as follows [26]:

$$\lambda_k = \frac{1 - d(R^k, \bar{R})}{\sum_{k=1}^l (d(R^k, \bar{R}))} \quad (k = 1, 2, \dots, l). \quad (9)$$

Step 3: Aggregate individual evaluation information.

After the decision-makers' weights are determined, we need to aggregate the individual evaluation information to get the collective evaluation matrix. The following equation can be used to compute the collective decision matrix  $R = (\tilde{r}_{ij})_{n \times m}$  [27]:

$$\tilde{r}_{ij} = (\tilde{u}_{ij}, \tilde{v}_{ij}) = \left( \sqrt{1 - \prod_{k=1}^l (1 - (u_{ij}^k)^2)^{\lambda_k}}, \prod_{k=1}^l (v_{ij}^k)^{\lambda_k} \right). \quad (10)$$

Step 4: Attributes weight determination.

Entropy can effectively describe the uncertainty and unpredictability of incomplete information. The entropy value of the PFS reflects the amount of information that can be extracted. The larger the entropy value, the less information about the PFS can be extracted, and the smaller the

weight of the PFS. Pythagorean fuzzy entropy can be calculated by utilizing the following equation [27]:

$$E_{A_j} = \frac{1}{n} \sum_{i=1}^n \left( 1 - \frac{|\tilde{u}_{ij}^k|^2 - \tilde{v}_{ij}^k|^2}{1 + \tilde{\pi}_{ij}^k|^2} \right). \quad (11)$$

Then, the following equation can be used to calculate the weights of the attributes.

$$w_j = \frac{1 - E_{A_j}}{\sum_{i=1}^n (1 - E_{A_j})}. \quad (12)$$

Step 5: Transformed Pythagorean fuzzy decision matrix into the interval number decision matrix.

The group Pythagorean fuzzy decision matrix  $R = (\tilde{r}_{ij})_{n \times m}$  can be transformed into interval number decision matrix  $X = (x_{ij}^u, x_{ij}^v)$  by utilizing the following equation [28]:

$$\begin{aligned} x_{ij}^u &= \tilde{u}_{ij} \\ x_{ij}^v &= \sqrt{1 - (\tilde{v}_{ij})^2}. \end{aligned} \quad (13)$$

Step 6: Normalization of interval number decision matrix.

The normalization process of interval number decision matrix is as follows:

$$r_{ij}^u = \frac{x_{ij}^u}{\sqrt{\sum_{i=1}^n (x_{ij}^v)^2}}, \quad r_{ij}^v = \frac{x_{ij}^v}{\sqrt{\sum_{i=1}^n (x_{ij}^v)^2}}, \quad (14)$$

$$r_{ij}^u = \frac{1/x_{ij}^v}{\sqrt{\sum_{i=1}^m (1/x_{ij}^u)^2}}, r_{ij}^v = \frac{1/x_{ij}^u}{\sqrt{\sum_{i=1}^m (1/x_{ij}^v)^2}}. \quad (15)$$

Step 7: Positive and negative ideal distance.

The selection of the reference points is very important to the application of prospect theory [29]. This study takes the positive and negative ideal reference points to judge the gains and losses. The positive and negative ideal reference points are as follows:

$$r_{ij}^+ = [r_{ij}^{u+}, r_{ij}^{v+}] = \left[ \max_{i=1}^n (r_{ij}^u), \max_i (r_{ij}^v) \right], \quad (16)$$

$$r_{ij}^- = [r_{ij}^{u-}, r_{ij}^{v-}] = \left[ \min_{i=1}^n (r_{ij}^u), \min_{i=1}^n (r_{ij}^v) \right]. \quad (17)$$

The gain and loss in the value function can be represented by the distance between the indicator and the corresponding positive and negative ideal reference point. Let the normalized distance between the  $j$  index of decision plan  $A_i$  and the positive and negative ideal reference points is  $d(r_{ij}, r_{ij}^+)$  and  $d(r_{ij}, r_{ij}^-)$ .  $d(r_{ij}, r_{ij}^+)$  and  $d(r_{ij}, r_{ij}^-)$  can be computed by utilizing the following equations [30]:

$$d(r_{ij}, r_{ij}^+) = \sqrt{(r_{ij}^u - r_{ij}^{u+})^2 + (r_{ij}^v - r_{ij}^{v+})^2}, \quad (18)$$

$$d(r_{ij}, r_{ij}^-) = \sqrt{(r_{ij}^u - r_{ij}^{u-})^2 + (r_{ij}^v - r_{ij}^{v-})^2}. \quad (19)$$

Step 8: Value analysis.

According to Equations (3) and (4), the value of the  $j$ th indicator of alternative  $A_i$  is calculated as follows:

$$v^+(d(r_{ij}, r_{ij}^-)) = d(r_{ij}, r_{ij}^-)^\beta, \quad (20)$$

$$v^-(d(r_{ij}, r_{ij}^+)) = -\theta d(r_{ij}, r_{ij}^+)^\alpha, \quad (21)$$

where  $\alpha$  and  $\beta$  ( $\alpha \geq 0, \beta \leq 1$ ) represent the degree of preference for gains and losses;  $\theta$  represents the risk avoidance coefficient.

Step 9: Probability weighting calculation for gain and loss.

Let the probability of indicators influencing decision-making is  $w = (w_1, w_2, \dots, w_m)$ , and the probability weight function of gains and losses is set as  $w^+(w_j)$  and  $w^-(w_j)$ .

$w^+(w_j)$  and  $w^-(w_j)$  can be computed by utilizing the following equations [31]:

$$w^+(w_j) = \frac{w_j^\gamma}{(w_j^\gamma + (1 - w_j)^\gamma)^{1/\gamma}}, \quad (22)$$

$$w^-(w_j) = \frac{w_j^\delta}{(w_j^\delta + (1 - w_j)^\delta)^{1/\delta}}, \quad (23)$$

where  $\gamma$  is risk-gain attitude coefficients, and  $0 < \gamma < 1$ ;  $\delta$  is the risk-loss attitude coefficient, and  $0 < \delta < 1$ .

Step 10: Decision-making.

The prospect value of alternative  $A_i$  ( $i = 1, 2, \dots, n$ ) can be obtained as follows [31]:

$$V_i = \sum_{j=1}^m v^+(d(r_{ij}, r_{ij}^-)) w^+(P_j) + \sum_{j=1}^m v^-(d(r_{ij}, r_{ij}^+)) w^-(P_j). \quad (24)$$

The higher the value of  $V_i$ , the better the corresponding alternative. Then, the alternatives can be ranked according to the prospect values. Finally, the optimal alternative can be obtained according to the ranking results.

## 4. Case Study

**4.1. Brief Introduction.** The A pumping station is located in Jiangsu Province, China, and the engineering project includes the design of the pumping station, the construction of the station's civil engineering works, and the installation of water pump units. This pumping station is relatively important and requires high standards in engineering design and construction. Before the construction of the project, the owner intends to select the most suitable PDS from the four PDSs: CM + TC ( $A_1$ ), DBB + UPC ( $A_2$ ), DB + TC ( $A_3$ ), and DB + UPC ( $A_4$ ). The owner invited four experts from related fields to assist in the selection of the PDS. Based on the constructed index system, experts use Pythagorean fuzzy numbers to evaluate and score the secondary indicators.

**4.2. PDS Selection.** For this case, the PDS decision-making process is as follows:

Step 1: Individual evaluation of decision-makers.

According to the actual situation of the project, the four experts were first invited to give the ratings of each alternative over the evaluation indicators. The evaluation results need to be presented using Pythagorean fuzzy numbers. The evaluation results are shown in Table 2.

Step 2: Determination of the weights of decision-makers.

According to the evaluation results of decision-makers in Table 2, using the formula for calculating the weight of decision-makers (Equations (5)–(9)), the weight of decision-





TABLE 3: Collective decision matrix.

Alternatives	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>	C <sub>5</sub>	C <sub>6</sub>	C <sub>7</sub>
A <sub>1</sub>	0.6633	0.2915	0.7523	0.3605	0.5469	0.3962	0.5498
A <sub>2</sub>	0.8553	0.2342	0.6932	0.3603	0.5896	0.4369	0.7132
A <sub>3</sub>	0.8692	0.2513	0.8353	0.3097	0.7201	0.3320	0.7537
A <sub>4</sub>	0.7478	0.3365	0.6485	0.3607	0.4992	0.3750	0.5297

Continued

Alternatives	C <sub>8</sub>	C <sub>9</sub>	C <sub>10</sub>	C <sub>11</sub>	C <sub>12</sub>	C <sub>13</sub>	C <sub>14</sub>
A <sub>1</sub>	0.5472	0.4141	0.7296	0.3429	0.8113	0.3147	0.6308
A <sub>2</sub>	0.6856	0.2835	0.6838	0.3128	0.7151	0.3590	0.6986
A <sub>3</sub>	0.6674	0.3296	0.7701	0.2831	0.7388	0.2938	0.6992
A <sub>4</sub>	0.6518	0.2475	0.5401	0.4112	0.6687	0.5022	0.5892

TABLE 4: Interval number decision matrix.

Alternatives	C <sub>1</sub>		C <sub>2</sub>		C <sub>3</sub>		C <sub>4</sub>		C <sub>5</sub>	
	$x_{ij}^u$	$x_{ij}^v$	$x_{ij}^u$	$x_{ij}^v$	$x_{ij}^u$	$x_{ij}^v$	$x_{ij}^u$	$x_{ij}^v$	$x_{ij}^u$	$x_{ij}^v$
A <sub>1</sub>	0.4399	0.9150	0.5660	0.8700	0.2991	0.8430	0.3023	0.8182	0.3198	0.7827
A <sub>2</sub>	0.7316	0.9451	0.4805	0.8702	0.3477	0.8091	0.5086	0.8961	0.4818	0.8427
A <sub>3</sub>	0.7555	0.9368	0.6978	0.9041	0.5186	0.8898	0.5681	0.9645	0.4198	0.8835
A <sub>4</sub>	0.0172	0.8868	0.4206	0.8699	0.2492	0.8594	0.2806	0.8859	0.3120	0.8602

Continued

Alternatives	C <sub>6</sub>		C <sub>7</sub>		C <sub>8</sub>		C <sub>9</sub>		C <sub>10</sub>	
	$x_{ij}^u$	$x_{ij}^v$	$x_{ij}^u$	$x_{ij}^v$	$x_{ij}^u$	$x_{ij}^v$	$x_{ij}^u$	$x_{ij}^v$	$x_{ij}^u$	$x_{ij}^v$
A <sub>1</sub>	0.4893	0.8397	0.2384	0.7350	0.2995	0.8286	0.5323	0.8824	0.6581	0.9010
A <sub>2</sub>	0.7112	0.8765	0.3626	0.7852	0.4700	0.9196	0.4676	0.9022	0.5114	0.8711
A <sub>3</sub>	0.5068	0.8579	0.1505	0.6741	0.4454	0.8913	0.5930	0.9198	0.5459	0.9137
A <sub>4</sub>	0.4034	0.8275	0.3913	0.8535	0.4248	0.9387	0.2917	0.8309	0.4472	0.7478

Continued

Alternatives	C <sub>11</sub>		C <sub>12</sub>		C <sub>13</sub>		C <sub>14</sub>		
	$x_{ij}^u$	$x_{ij}^v$	$x_{ij}^u$	$x_{ij}^v$	$x_{ij}^u$	$x_{ij}^v$	$x_{ij}^u$	$x_{ij}^v$	
A <sub>1</sub>		0.3979	0.9157	0.4024	0.8416	0.3488	0.9153	0.4335	0.7586
A <sub>2</sub>		0.4881	0.8920	0.4528	0.8629	0.5080	0.8698	0.6357	0.8695
A <sub>3</sub>		0.4889	0.9194	0.5257	0.8193	0.5636	0.8042	0.5390	0.9121
A <sub>4</sub>		0.3471	0.7663	0.3565	0.7831	0.3852	0.8438	0.3307	0.6870

makers can be calculated as  $\lambda = (0.2430, 0.2493, 0.2623, 0.2453)$ .

Step 3: Aggregate individual evaluation information.

After the weight of decision-makers is determined, the individual evaluation information can be aggregated by utilizing Equation (10). The evaluation matrix after aggregation is shown in Table 3.

Step 4: Attributes weight determination.

According to the evaluation information and the attributes weight calculation method provided in the preceding text (Equations (11) and (12)), the weights of attributes can be calculated as follows:

$w = (0.1312, 0.0981, 0.0431, 0.0653, 0.0478, 0.0900, 0.0308, 0.0630, 0.0787, 0.0928, 0.0643, 0.0578, 0.0686, 0.0684)$ .

Step 5: Transformed Pythagorean fuzzy decision matrix into interval number decision matrix.

According to the transformation formula (Equations (13)), the group Pythagorean fuzzy decision matrix can be transformed into an interval number decision matrix. The interval number decision matrixes after transformed are presented in Table 4.

Step 6: Normalization of interval number decision matrix.

TABLE 5: Normalized decision matrix.

Alternatives	C <sub>1</sub>		C <sub>2</sub>		C <sub>3</sub>		C <sub>4</sub>		C <sub>5</sub>	
	$r_{ij}^u$	$r_{ij}^v$	$r_{ij}^u$	$r_{ij}^v$	$r_{ij}^u$	$r_{ij}^v$	$r_{ij}^u$	$r_{ij}^v$	$r_{ij}^u$	$r_{ij}^v$
A <sub>1</sub>	0.2292	0.6562	0.3019	0.5913	0.1622	0.7088	0.1601	0.6352	0.1742	0.6320
A <sub>2</sub>	0.3812	0.6778	0.2563	0.5914	0.1885	0.6803	0.2694	0.6956	0.2625	0.6806
A <sub>3</sub>	0.3936	0.6719	0.3722	0.6144	0.2812	0.7481	0.3009	0.7487	0.2287	0.7135
A <sub>4</sub>	0.0090	0.6360	0.2244	0.5912	0.1351	0.7225	0.1486	0.6876	0.1700	0.6946
Continuated										
Alternatives	C <sub>6</sub>		C <sub>7</sub>		C <sub>8</sub>		C <sub>9</sub>		C <sub>10</sub>	
	$x_{ij}^u$	$x_{ij}^v$	$x_{ij}^u$	$x_{ij}^v$	$x_{ij}^u$	$x_{ij}^v$	$x_{ij}^u$	$x_{ij}^v$	$x_{ij}^u$	$x_{ij}^v$
A <sub>1</sub>	0.2653	0.5780	0.1366	0.6876	0.1583	0.6470	0.2831	0.6428	0.3552	0.6127
A <sub>2</sub>	0.3856	0.6033	0.2077	0.7346	0.2485	0.7181	0.2487	0.6572	0.2760	0.5924
A <sub>3</sub>	0.2748	0.5905	0.0862	0.6306	0.2355	0.6961	0.3154	0.6700	0.2946	0.6213
A <sub>4</sub>	0.2187	0.5696	0.2241	0.7984	0.2246	0.7331	0.1551	0.6053	0.2413	0.5085
Continuated										
Alternatives	C <sub>11</sub>		C <sub>12</sub>		C <sub>13</sub>		C <sub>14</sub>			
	$r_{ij}^u$	$r_{ij}^v$	$r_{ij}^u$	$r_{ij}^v$	$r_{ij}^u$	$r_{ij}^v$	$r_{ij}^u$	$r_{ij}^v$		
A <sub>1</sub>		0.2129	0.6978	0.2213	0.6385	0.1882	0.6812	0.2413	0.5448	
A <sub>2</sub>		0.2611	0.6797	0.2490	0.6546	0.2742	0.6473	0.3539	0.6244	
A <sub>3</sub>		0.2616	0.7007	0.2891	0.6215	0.3041	0.5985	0.3000	0.6550	
A <sub>4</sub>		0.1857	0.5839	0.1960	0.5941	0.2079	0.6280	0.1841	0.4934	

The group Pythagorean fuzzy decision matrix can be transformed into interval number decision matrix by utilizing Equations (14) and (15). The results are presented in Table 5.

Step 7: Calculate the positive and negative ideal distance.

According to interval number decision matrix and the ideal distance calculation method (Equations (16)–(19)), the positive and negative ideal distances  $d(r_{ij}, r_{ij}^+)$  and  $d(r_{ij}, r_{ij}^-)$  of the normalized decision matrix can be calculated. The calculation results are shown in Table 6.

Step 8: Value analysis.

According to Equations (20) and (21), the value function values of each alternative can be calculated, as shown in Table 7.

Step 9: Probability weighting calculation for gain and loss.

According to Equations (22) and (23), the probability weights of gain and loss for each indicator can be calculated. The result is shown in Table 8.

Step 10: Decision-making.

According to the probability weights of gain and loss for each indicator and the prospect value calculation method (Equation (24)), the prospect values of every available PDS can be calculated as follows:

$$V_1 = -0.3469, V_2 = 0.0792, V_3 = 0.1688, V_4 = -0.7640. \tag{25}$$

Then, the alternatives can be ranked according to the prospect value. The larger the prospect values, the more optimal the solution. For this project, the ranking order is  $V_3 > V_2 > V_1 > V_4$ . Obviously, DB + TC (A<sub>3</sub>) is the best PDS for this project.

### 5. Discussion

Based on the PFS, this paper introduces the weight of decision-makers considering the different degrees of expert expertise and combines it with the improved entropy weight method and prospect theory to construct a new PDS decision-making method. Through model calculation and analysis, it was ultimately determined that DB + TC(A<sub>3</sub>) was the optimal PDS for the case project. In order to validate the effectiveness of the PDS decision-making method constructed in this paper, the decision results are compared with the results of the TOPSIS model and the fuzzy comprehensive evaluation model. The results are shown in Table 9.

From Table 9, we can see that the calculation results of the three methods rank DBB + UPC(A<sub>2</sub>) and DB + TC(A<sub>3</sub>) in the top two positions. However, the difference is that the decision model proposed in this paper ranks the scheme DB + TC(A<sub>3</sub>) first, while the other two decision models rank the scheme DBB + UPC(A<sub>2</sub>) first. The difference is mainly due to the fact that the prospect theory focuses more on choosing a certain solution with a small return rather than a scheme



TABLE 6: Positive and negative ideal distances.

Alternatives	C <sub>1</sub>		C <sub>2</sub>		C <sub>3</sub>		C <sub>4</sub>		C <sub>5</sub>	
	d <sup>+</sup>	d <sup>-</sup>	d <sup>+</sup>	d <sup>-</sup>	d <sup>+</sup>	d <sup>-</sup>	d <sup>+</sup>	d <sup>-</sup>	d <sup>+</sup>	d <sup>-</sup>
A <sub>1</sub>	0.1658	0.2212	0.0740	0.0776	0.1253	0.0393	0.1808	0.0115	0.1201	0.0042
A <sub>2</sub>	0.0125	0.3745	0.1181	0.0320	0.1148	0.0534	0.0617	0.1351	0.0329	0.1044
A <sub>3</sub>	0.0059	0.3863	0.0000	0.1497	0.0000	0.1610	0.0000	0.1899	0.0338	0.1004
A <sub>4</sub>	0.3869	0.0000	0.1497	0.0000	0.1483	0.0423	0.1641	0.0525	0.0944	0.0626
Continuated										
Alternatives	C <sub>6</sub>		C <sub>7</sub>		C <sub>8</sub>		C <sub>9</sub>		C <sub>10</sub>	
	d <sup>+</sup>	d <sup>-</sup>	d <sup>+</sup>	d <sup>-</sup>	d <sup>+</sup>	d <sup>-</sup>	d <sup>+</sup>	d <sup>-</sup>	d <sup>+</sup>	d <sup>-</sup>
A <sub>1</sub>	0.1229	0.0473	0.1412	0.0761	0.1246	0.0000	0.0423	0.1333	0.0086	0.1543
A <sub>2</sub>	0.0000	0.1702	0.0659	0.1599	0.0149	0.1148	0.0679	0.1070	0.0843	0.0908
A <sub>3</sub>	0.1115	0.0599	0.2172	0.0000	0.0392	0.0914	0.0000	0.1729	0.0606	0.1247
A <sub>4</sub>	0.1702	0.0000	0.0000	0.2172	0.0239	0.1086	0.1729	0.0000	0.1603	0.0000
Continuated										
Alternatives	C <sub>11</sub>		C <sub>12</sub>		C <sub>13</sub>		C <sub>14</sub>			
	d <sup>+</sup>	d <sup>-</sup>	d <sup>+</sup>	d <sup>-</sup>	d <sup>+</sup>	d <sup>-</sup>	d <sup>+</sup>	d <sup>-</sup>		
A <sub>1</sub>	0.0488	0.1171	0.0697	0.0511	0.1159	0.0827	0.1575	0.0769		
A <sub>2</sub>	0.0209	0.1219	0.0401	0.0804	0.0452	0.0988	0.0306	0.2145		
A <sub>3</sub>	0.0000	0.1392	0.0331	0.0970	0.0827	0.1159	0.0539	0.1989		
A <sub>4</sub>	0.1392	0.0000	0.1110	0.0000	0.1100	0.0354	0.2344	0.0000		

TABLE 7: Value function.

Alternatives	C <sub>1</sub>		C <sub>2</sub>		C <sub>3</sub>		C <sub>4</sub>		C <sub>5</sub>	
	v <sup>+</sup>	v <sup>-</sup>	v <sup>+</sup>	v <sup>-</sup>	v <sup>+</sup>	v <sup>-</sup>	v <sup>+</sup>	v <sup>-</sup>	v <sup>+</sup>	v <sup>-</sup>
A <sub>1</sub>	0.2651	-0.4629	0.1054	-0.2276	0.0580	-0.3618	0.0197	-0.4996	0.0082	-0.3484
A <sub>2</sub>	0.4214	-0.0474	0.0483	-0.3435	0.0759	-0.3350	0.1718	-0.1939	0.1370	-0.1115
A <sub>3</sub>	0.4330	-0.0248	0.1880	0.0000	0.2005	0.0000	0.2318	0.0000	0.1323	-0.1142
A <sub>4</sub>	0.0000	-0.9756	0.0000	-0.4229	0.0618	-0.4195	0.0747	-0.4585	0.0873	-0.2819
Continuated										
Alternatives	C <sub>6</sub>		C <sub>7</sub>		C <sub>8</sub>		C <sub>9</sub>		C <sub>10</sub>	
	v <sup>+</sup>	v <sup>-</sup>	v <sup>+</sup>	v <sup>-</sup>	v <sup>+</sup>	v <sup>-</sup>	v <sup>+</sup>	v <sup>-</sup>	v <sup>+</sup>	v <sup>-</sup>
A <sub>1</sub>	0.0682	-0.3557	0.1036	-0.4018	0.0000	-0.3600	0.1698	-0.1391	0.1931	-0.0344
A <sub>2</sub>	0.2105	0.0000	0.1992	-0.2055	0.1489	-0.0557	0.1399	-0.2110	0.1210	-0.2553
A <sub>3</sub>	0.0839	-0.3265	0.0000	-0.5869	0.1218	-0.1301	0.2134	0.0000	0.1601	-0.1908
A <sub>4</sub>	0.0000	-0.4737	0.2609	0.0000	0.1417	-0.0841	0.0000	-0.4801	0.0000	-0.4492
Continuated										
Alternatives	C <sub>11</sub>		C <sub>12</sub>		C <sub>13</sub>		C <sub>14</sub>			
	v <sup>+</sup>	v <sup>-</sup>	v <sup>+</sup>	v <sup>-</sup>	v <sup>+</sup>	v <sup>-</sup>	v <sup>+</sup>	v <sup>-</sup>		
A <sub>1</sub>	0.1514	-0.1577	0.0730	-0.2158	0.1115	-0.3377	0.1047	-0.4425		
A <sub>2</sub>	0.1569	-0.0749	0.1089	-0.1327	0.1304	-0.1476	0.2580	-0.1046		
A <sub>3</sub>	0.1764	0.0000	0.1284	-0.1121	0.1501	-0.2509	0.2414	-0.1721		
A <sub>4</sub>	0.0000	-0.3969	0.0000	-0.3252	0.0529	-0.3225	0.0000	-0.6278		

TABLE 8: Probability weights of gain and loss.

Alternatives	C <sub>1</sub>		C <sub>2</sub>		C <sub>3</sub>		C <sub>4</sub>		C <sub>5</sub>	
	w <sup>+</sup>	w <sup>-</sup>	w <sup>+</sup>	w <sup>-</sup>	w <sup>+</sup>	w <sup>-</sup>	w <sup>+</sup>	w <sup>-</sup>	w <sup>+</sup>	w <sup>-</sup>
	0.2127	0.2002	0.1846	0.1682	0.1219	0.1015	0.1508	0.1314	0.1287	0.1084
Continuated										
Alternatives	C <sub>6</sub>		C <sub>7</sub>		C <sub>8</sub>		C <sub>9</sub>		C <sub>10</sub>	
	w <sup>+</sup>	w <sup>-</sup>	w <sup>+</sup>	w <sup>-</sup>	w <sup>+</sup>	w <sup>-</sup>	w <sup>+</sup>	w <sup>+</sup>	w <sup>-</sup>	w <sup>+</sup>
	0.1769	0.1597	0.1023	0.0823	0.1480	0.1285	0.1655	0.1472	0.1796	0.1627
Continuated										
Alternatives	C <sub>11</sub>		C <sub>12</sub>		C <sub>13</sub>		C <sub>14</sub>			
	w <sup>+</sup>	w <sup>-</sup>	w <sup>+</sup>	w <sup>-</sup>	w <sup>+</sup>	w <sup>-</sup>	w <sup>+</sup>	w <sup>-</sup>	w <sup>+</sup>	w <sup>-</sup>
		0.1496	0.1301	0.1418	0.1219	0.1546	0.1354	0.1543	0.1351	

TABLE 9: Results of different PDS decision-making methods.

Alternatives	The decision-making method proposed in this paper	TOPSIS	Fuzzy comprehensive evaluation model
EPC + TC(A <sub>1</sub> )	3	4	3
DBB + UPC(A <sub>2</sub> )	2	1	1
DB + TC(A <sub>3</sub> )	1	2	2
DB + UPC(A <sub>4</sub> )	4	3	4

with a large risk and a large return. The DB + TC (A<sub>3</sub>) scheme can reduce the risks associated with estimating the quantity of the project and project management for the owner, which is beneficial for them.

## 6. Conclusions

PDS selection has a great impact on the performance of the engineering project. Considering that in typical situations, the project delivery method and contract type have a high correlation, we have developed, for the first time, a new PDS decision framework based on a combination of “project delivery method” and “contract type.” In the PDS decision-making process, given the ambiguity of evaluation criteria and the differences in decision-makers’ expertise, we innovatively introduced the PFS and decision-maker weights to enhance the reliability of decision outcomes. Furthermore, we innovatively combined the above-mentioned methods with entropy weighting and prospect theory to construct a PDS decision model. Finally, the effectiveness of the proposed decision-making model was verified through case analysis and comparison analysis of the evaluation results of the TOPSIS model and the fuzzy comprehensive evaluation model. The PDS decision-making method proposed in this paper is more reliable and can provide support for the owner’s PDS selection.

Although this study has made every effort to consider all factors related to PDS selection, there are still some shortcomings. For example, we did not fully consider the impact of owner management mode on the selection of PDS. This will be a direction for future research. In addition, we are also

striving to promote the application of the PDS decision model in actual engineering and further optimize the proposed decision method based on its practical application effects.

## Data Availability

The data used to support the findings of this study are included within the article.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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