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Research Article

Parameter Determination Method in Analysis of Stratum Deformation Caused by Tunnel Excavation

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Based on the analytical method of formation deformation caused by tunnel excavation proposed by Verruijt and Sagaseta, combined with the basic principles of soil mechanics and elastic mechanics, a method for determining the pending parameters required in the analysis process is proposed. This method does not require the actual measurement of deformation values in the field, and the required parameters can be determined from geological data, so as to obtain the deformation values of any points of the whole stratum. According to engineering examples, when the pending parameters are obtained by this method, the difference between the resolved value of maximum surface settlement and the measured value can be controlled within 5.0%, and the analytical results can splendidly reflect the formation deformation law caused by tunnel excavation. Further combined with the analysis of the numerical simulation results, the proposed approximation method has certain shortcomings and deficiencies, mainly in the lack of lateral constraints on the displacement. When the measured data in the field are insufficient, this method can be used to estimate the deformation, and the analysis results are slightly larger than the measured values, which is safer.

1. Introduction

Tunnel excavation will destroy the original equilibrium state of the stratum and make the soil mass around the excavation space establish a new equilibrium gradually. This process will inevitably lead to the deformation of the soil mass near the excavation space. When the deformation exceeds a certain degree, the soil mass and surrounding structures will be damaged, leading to the occurrence of engineering accidents. Therefore, stratum deformation and ground settlement caused by tunnel excavation become one of the most important data in tunnel construction. At present, Peck's empirical formula, theoretical analysis method, and numerical simulation are the main analytical methods used to predict excavation deformation of engineering industry.

Peck [1] proposed the empirical method under a series of assumptions based on a large number of engineering measured data. Subsequently, Attewell and Hurrell [2] and O'Reilly and New [3] obtained the empirical formula for calculating the width and inflection point of settlement trough through the regression analysis of engineering measured data. Lin et al. [4], Lu et al. [5], and Chapman et al. [6] proposed empirical formulas for the variation of the inflection point and maximum value of the deformation curve with depth based on the plane strain principle and a large amount of engineering experience, which extended the surface deformation law to the whole stratum. In Peck's empirical method, the undetermined parameters were determined mainly by the field measured deformation and empirical formula. Although this method lacks strict theoretical basis, it is simple to operate and meet the requirements of practical engineering in terms of accuracy, so it has been applied in many practical engineering.

The analytical methods mainly include complex function analysis methods and approximate solution methods, which are based on fluid dynamics; in addition, there are analysis methods using elastic mechanics theory. Among them, the complex function analytic method was first obtained by Verruijt [7, 8] in the complex formula of plane elasticity. This method transforms the conventional semi-infinite space into a ring space by complex function, avoiding the problem of body force asymmetry. This solution method can get accurate conclusions, but the solution process requires complex function knowledge. Kong et al. [9] and Pieride et al. [10] combined the theory related to the complex variable function, and then derived an analytical solution for the ground deformation caused by tunnel excavation under inclined ground surface.

The approximate solution was first proposed by Sagaseta [11] when he studied the shrinkage deformation problem of shallow buried tunnels under the assumption of incompressible medium ($\nu = 0.5$). The formation loss will inevitably occurs in the deformation process caused by tunnel excavation, which is similar to the problem in fluid dynamics. Subsequently, Verruijt and Booker [12], based on Sagaseta's solution idea, extended the method to compressible medium ($\nu \neq 0.5$), and since then it has been formally applied to the analysis of stratum deformation caused by tunnel excavation. On this basis, Loganathan and Poulos [13] proposed the classical Loganathan formula for estimating the ground deformation due to ground loss in tunnel construction in soft soil areas by using the concept of gap parameter and combining the theories of Sagaseta and Verruijt with the theoretical analysis and fitting corrections to the measured data. Pieride et al. [14] combined Fourier analysis and Navier equation to derive the analytical solution of soil deformation around cylindrical bore in the drainage conditions.

In the application of complex variable function solutions and approximate solutions, parameter determination mainly relies on the graph search method, which is based on field measured deformation values given by Pinto [15] and Pinto and Whittle [16]. Although this method is clear in theory and accurate in results, it requires field measured deformation at specific locations as support, and is not practical when measured data are insufficient.

The analytical method of elastoplasticity should take the problem of gravity asymmetry into account, which is generally applicable to $r/h < 0.05$ ultra-deep-buried tunnels. The analysis of conventional tunnels will produce great errors. Li et al. [17] applied the theory of elasticity to solve the deformation distribution of the composite lining of deep-buried water conveyances in polar coordinates. Aghchai et al. [18] and others have described the various ideas in detail for solving this problem. This method is less used in conventional traffic tunnels because of its special conditions. Based on the three-dimensional strength theory, Chen et al. [19] presented an anisotropic elastic–plastic solution to the problem of ground deformation due to circular tunnel excavation. The elastoplasticity solution theory is mature. When gravity is ignored, the parameters can be calculated according to the conventional mechanical parameters. The numerical simulation method is also developed based on the foundation of elastoplastic mechanics. Wang et al. [20] analyzed the drainage and expansion response of cylindrical cavities under biaxial in situ stress by establishing a numerical model. Cao et al. [21] carried out a ground deformation analysis caused by tunnel excavation using the discrete element method. In addition, Feng et al. [22] predicted the ground deformation during pit excavation in composite strata based on an artificial swarm-backpropagation model; Gong et al. [23] studied the leakage problem of local joints of tunnel lining based on the theory of three-dimensional coupled hydrodynamics, which promotes the development of tunneling technology.

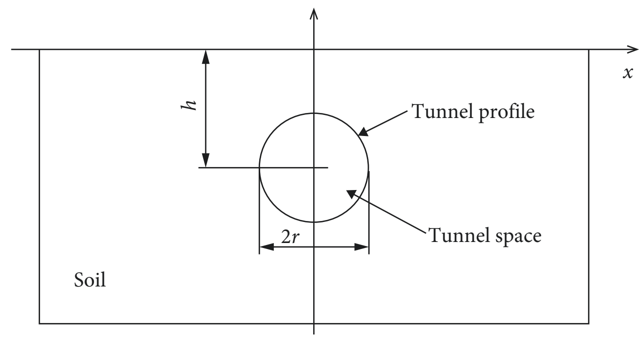

Figure 1: Circular tunnel in elastic semi-infinite space.

In this context, Verruijt complex function analytic method and Sagaseta's solution idea based on fluid dynamics approximation method are briefly introduced in this paper, which are also the basis of a series of analytical methods. A drawback of the parameter determination method proposed by Federico P. is its inability to be applied when there is insufficient field measured data. In order to address this issue, a approximate method for determining the unknown parameters is proposed. Then, taking the shield tunnel in an urban rail transit project as an example, the proposed approximate method is used to analyze the formation deformation caused by the shield tunneling, and the results are compared with the measured values in the field. At the same time, combined with numerical simulation, the shortcomings and deficiencies of the proposed approximate method are analyzed. Finally, according to the analysis results, the problems that should be paid attention to in the practical application of the analytical method are explained.

2. Solving Ideas of Analytical Method

Verruijt analytical method of complex function and Sagaseta's approximate solution method based on fluid dynamics provide two kinds of ideas to analyze the problem of stratum deformation caused by tunnel excavation. This paper mainly studies the two kinds of solutions, which are collectively referred to as analytic solutions. First, the two kinds of solution ideas are briefly introduced here.

2.1. Problem Description. In order to meet the requirements of plane strain problem, the tunnel is assumed to be infinitely long on the axis and the soil is an elastomer in a semi-infinite space, namely, a circular tunnel in a semi-infinite space, as shown in Figure 1, where the tunnel radius is r and the tunnel burial depth is h.

2.2. Solution of Complex Function. Verruijt's solution of complex function is to transform this problem from the conventional semi-infinite space (Figure 1) into a ring space (Figure 2) through the theory of complex function. The point in the transformed space forms a one-to-one correspondence with the point in the circular semi-infinite space.

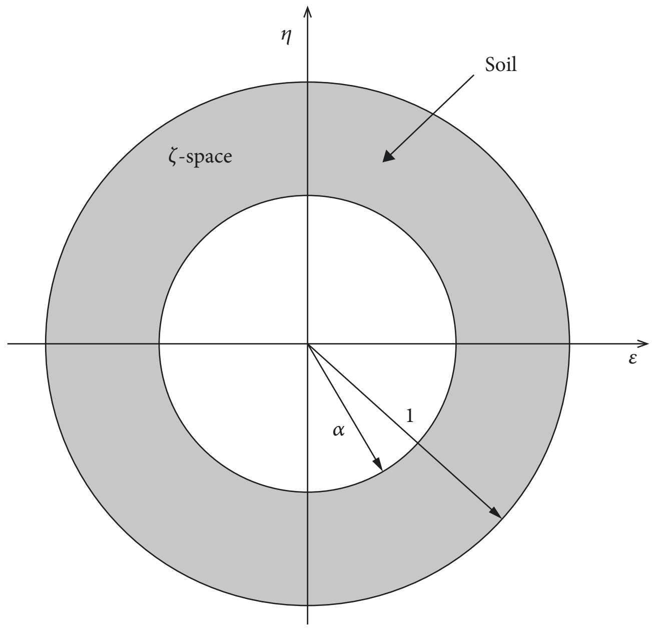

Figure 2: Annular space after conformal transformation.

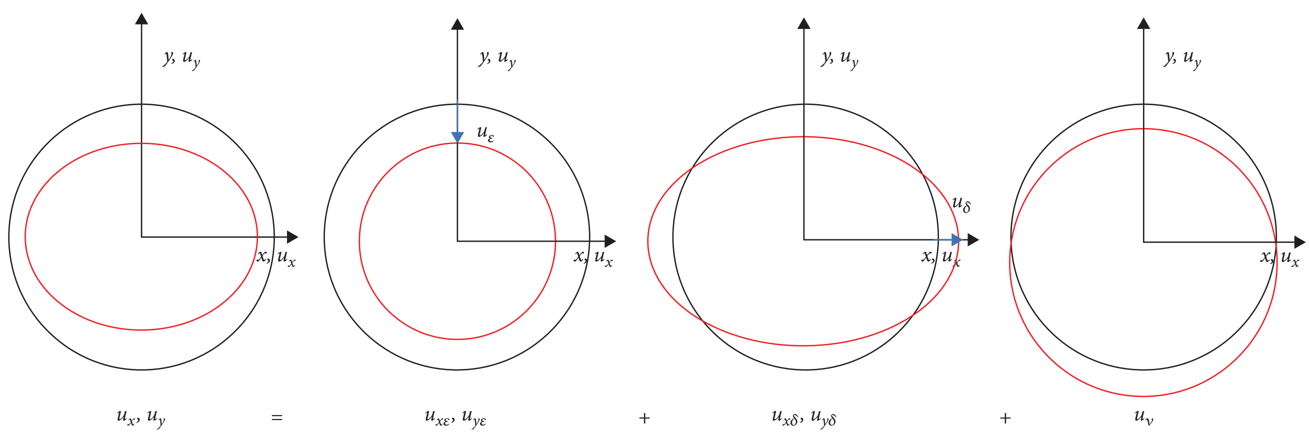

Figure 3: Approximate solution deformation decomposition diagram.

The final solution is expressed in terms of two functions $f(z)$ and $g(z)$, which are obtained by applying boundary conditions. The relation between displacement and these two functions is shown in Equation (1):

$$2Gu_z(z) = (3-4v)f(z) - z\frac{d\overline{f(z)}}{\overline{dz}} - \overline{g(z)}, \quad (1)$$

where G is shear modulus, v is Poisson's ratio, $\overline{dz}$ represents the derivative of complex conjugate, i is the imaginary number unit, $z = x + i \times y$, and $u_z = u_x + i \times u_y$.

In the new ring space, the Laurent series is used to expand the two functions, and the series expression of the two functions is obtained, as shown in Equations (2) and (3):

$$f(\xi) = a_0 + \sum a_k \times \xi^k + \sum b_k \times \xi^{-k}, \quad (2)$$

$$g(\xi) = c_0 + \sum c_k \times \xi^k + \sum d_k \times \xi^{-k}. \quad (3)$$

Then, the displacement function of substitution can be obtained by applying boundary conditions to solve a_k, b_k, c_k, and d_k. Undetermined parameters are introduced when boundary conditions are considered, as described detailedly in Section 2.4. The result obtained by this method is accurate in theory, so it is hereinafter referred to as "exact solution."

2.3. Approximate Solution Method Based on Fluid Dynamics. The Sagaseta approximate solution method was generalized by Verruijt from the hypothesis of incompressible media. In the approximate solution, the displacement is divided into three parts, namely, one is the uniform contraction part, the second is the pure deformation, the third is the vertical displacement component, as shown in Figure 3. The final displacement is the superposition of the three parts. When the error influence caused by vertical displacement is ignored, the displacement is shown in Equations (4) and (5).

$$u_x(x,y) = u_{x\varepsilon}(x,y,u_\varepsilon,\nu) + u_{x\delta}(x,y,u_\delta,\nu), \quad (4)$$

$$u_y(x,y) = u_{y\varepsilon}(x,y,u_\varepsilon,\nu) + u_{y\delta}(x,y,u_\delta,\nu), \quad (5)$$

where u_ε is uniform shrinkage deformation parameter and u_δ is pure deformation parameter.

Then, the parameters can be determined by boundary conditions, and the whole stratum deformation and displacement field can be obtained. The solution of this method is based on the extension of incompressible media, and the obtained solution is approximate, so it is hereinafter referred to as "approximate solution."

2.4. Boundary Conditions and Undetermined Parameters. In this problem, the boundary conditions include three:

(1) The displacement and stress at infinity are 0.
(2) The surface stress is 0.
(3) The amount of deformation of tunnel inner wall u_ε and u_δ is known.

For the boundary conditions (1) and (2), the complex function solution is applied by repeated recursion of the series, while the approximate solution is modified by surface shear stress. The boundary condition (3) is calculated as a known quantity.

For a practical tunnel project, the radius of the tunnel r and the depth of the tunnel h are known, and the mechanical parameters of each layer of soil are determined after the local layer conditions are determined. The application of analytical methods to analysis involves the transformation of the actual stratigraphic model to the simplified mechanical model (Figure 1). The mechanical parameters after transformation will not reflect the mechanical properties of a certain layer of soil, but reflect the deformation properties of the whole stratum.

3. Method of Parameter Determination in Analytic Analysis

As mentioned above, in addition to the known tunnel depth h and tunnel radius r, there are three undetermined parameters in the application of analytical method. The key to the application of analytical method is how to obtain these three parameters. As long as these three parameters are determined, the formation deformation and displacement field can be given by the formula.

3.1. Universal Method. According to the idea of function, Pinto [15] proposed a method. There is some functional relationship between the three parameters obtained by substituting and the formation deformation and displacement field. Therefore, three independent equations can be obtained through the functional relationship only by measuring the formation deformation values of the three positions that are independent of each other, so as to solve the three undetermined parameters.

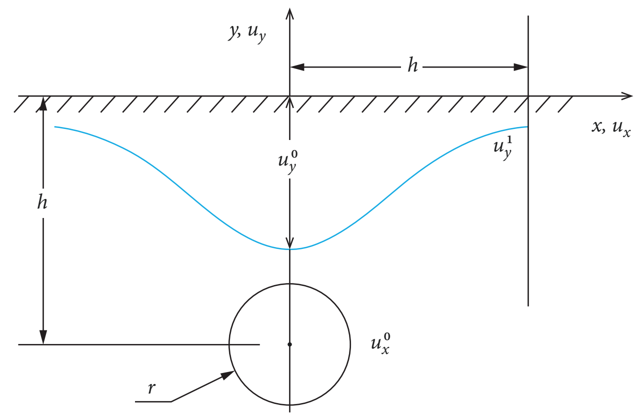

Figure 4: Measured deformation parameters at three locations.

However, due to the relatively complex functional relationship, it is relatively difficult to accurately solve it. In practice, the undetermined parameters can be obtained by drawing the influence line and then using the field measured data to look up the graph. For the convenience of measurement, the three measured values are, respectively, taken as the ground settlement directly above the tunnel u_y^0, the convergence of the horizontal clearance of the tunnel u_x^0, and the vertical settlement at the depth of h of the tunnel that is twice the deviation from the vertical axis of the tunnel u_y^1, as shown in Figure 4.

This method requires field measured parameters as input, that is, the whole formation deformation and displacement field can be obtained through finite field measured deformation values. However, at the beginning of the engineering design, the field measurement conditions are not available, and the field measurement data are insufficient, so it can not be applied, so it has certain limitations.

3.2. Approximation Method. Before the excavation of the actual project, there is no field measured deformation data, and there will be insufficient measured parameters on the site of the project. Based on the above analysis and the theory of soil mechanics and elastic mechanics, an approximate method for obtaining undetermined parameters is proposed in this paper. Under certain assumptions, approximate uniform shrinkage deformation parameter u_ε and pure deformation parameter u_δ are obtained. Poisson's ratio ν can be estimated according to geological data.

When the influence of lining stiffness and gravity is ignored, the problem of uniform shrinkage deformation parameters u_ε and pure deformation parameters u_δ can be regarded as the circular plate hole problem in elasticity. As shown in Figure 5, the uniform shrinkage deformation parameter u_ε represents the shrinkage amount of the tunnel contour under the external force component of uniform compression, while the pure deformation parameter u_δ represents the maximum deformation amount of the tunnel contour under the force of pure deformation.

According to Equations (6) to (8) and Equations (11) to (13) of the stress solution of the circular hole problem in a flat plate in elasticity (Figure 6), the strain component was obtained by substituting the physical equation in the form

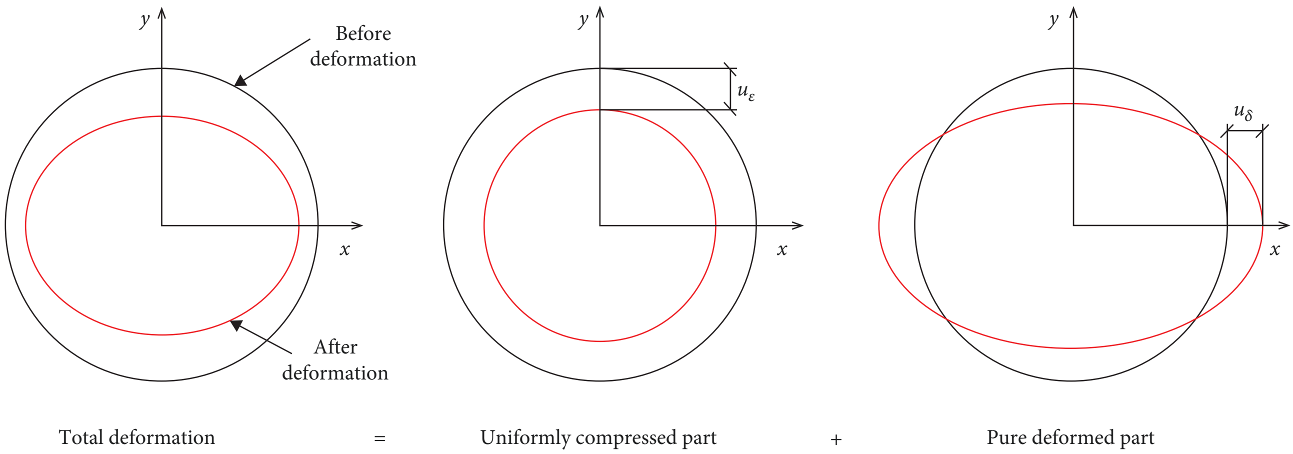

Figure 5: Physical meaning of deformation parameters.

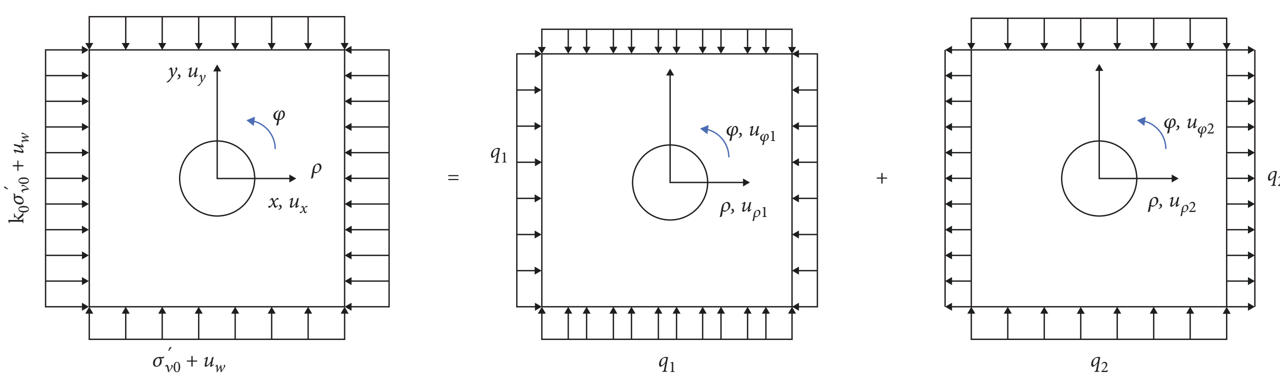

Figure 6: Calculation diagram of water and soil pressure.

of polar coordinates, and then the expression of the displacement component was obtained by integrating the geometric equation. Then, the boundary conditions of rigid body translation and rotation were taken into account. The explicit expressions of displacement components can be determined by the following Equations (9), (10), (14), and (15).

(1) Uniformly compressed part

Stress component:

$$\sigma_{\rho 1} = q_1\left(1 - \frac{r^2}{\rho^2}\right), \tag{6}$$

$$\sigma_{\rho 1} = -q_1\left(1 + \frac{r^2}{\rho^2}\right), \tag{7}$$

$$\tau_{\rho\varphi 1} = 0. \tag{8}$$

Displacement component:

$$u_{\rho 1} = \frac{q_1}{E}\left[(1-\nu)\rho + r^2\rho^{-1}(1+\nu)\right], \tag{9}$$

$$u_{\varphi 1} = 0. \tag{10}$$

(2) Pure deformed part

Stress component:

$$\sigma_{\rho 2} = q_2 \cos 2\varphi\left(1 - \frac{r^2}{\rho^2}\right)\left(1 - 3\frac{r^2}{\rho^2}\right), \tag{11}$$

$$\sigma_{\rho 2} = -q_2 \cos 2\varphi\left(1 + 3\frac{r^4}{\rho^4}\right), \tag{12}$$

$$\tau_{\rho\varphi 2} = -q_2 \sin 2\varphi\left(1 - \frac{r^2}{\rho^2}\right)\left(1 + 3\frac{r^2}{\rho^2}\right). \tag{13}$$

Displacement component:

$$u_{\rho 2} = \frac{q_2 \cos 2\varphi}{E}\left[(1+\nu)\rho + 4r^2\rho^{-1} - r^4\rho^{-3}(1+\nu)\right], \tag{14}$$

$$u_{\varphi 2} = \frac{-q_2 \sin 2\varphi}{E} \left[\begin{array}{c} (1+\nu)\rho \\ +2r^2\rho^{-1}(1-\nu) + r^4\rho^{-3}(1+\nu) \end{array} \right]. \tag{15}$$

According to the physical meaning of uniform shrinkage deformation parameter u_ε and pure deformation parameter u_δ, the values of the two parameters can be directly determined without considering the lining stiffness and gravity action, as shown in Equations (16) and (17):

$$u_\varepsilon = \frac{2q_1 r}{E}, \tag{16}$$

$$u_\delta = \frac{4q_2 r}{E}, \tag{17}$$

where r is the radius of tunnel, E is the elastic modulus of soil, and ν is Poisson's ratio. According to relevant theories of elasticity:

$$q_1 = \frac{1}{2}\left[\sigma'_{\nu 0}(k_0+1) + 2u_w\right], \tag{18}$$

$$q_2 = \frac{1}{2}\left[\sigma'_{\nu 0}(1-k_0)\right]. \tag{19}$$

Since the buried depth of tunnel, the radius of tunnel, and the Poisson's ratio of soil have great influence on the soil deformation, and the modulus of soil, the lining modulus, and the Poisson's ratio of lining have little influence on the calculation results of the soil deformation, two deformation parameters can be obtained by directly calculating the elastic modulus approximation when the soil layer is simple.

However, it is difficult to calculate the approximate value of the elastic modulus directly for the complex soil layer and the lining stiffness, so it is necessary to estimate the approximate value of the deformation value. It can be found that the ratio of the two has nothing to do with stiffness, but has something to do with Poisson's ratio, which is also the reason why Poisson's ratio should be selected for mechanical parameters after calculation model simplification. Therefore, relative deformation ρ_0 is defined as the negative number of the ratio of uniform shrinkage deformation parameter u_ε to pure deformation parameter u_δ, as shown in Equation (20). When the influence of lining stiffness is not taken into account, according to the theory of elasticity and the results of Equations (16) and (17), the relative deformation can be resolved into Equation (21):

$$\rho_0 = -\frac{u_\delta}{u_\varepsilon}, \tag{20}$$

$$\rho_0 = \frac{2(1-k_0)}{1+k_0+2r_u}, \tag{21}$$

$$r_u = \frac{u_w}{\sigma'_{\nu 0}}, \tag{22}$$

where k_0 is the soil pressure coefficient, obtained from geological exploration data, r_u is the pore pressure ratio, calculated by Equation (22), u_w is the water pressure at the calculated depth, and $\sigma'_{\nu 0}$ is the horizontal effective earth pressure at the calculated depth, which can be calculated according to the knowledge of soil mechanics.

In order to simplify the process, the calculation can be carried out according to the location of the center of the tunnel contour, that is, the calculation depth is the buried depth h of the tunnel, as shown in Figure 6. In practical application, it is necessary to calculate the relative deformation ρ_0 according to the geological data, and then estimate the uniform shrinkage deformation parameter u_ε according to the engineering situation, so that the pure deformation parameter u_δ can be obtained according to the relative deformation.

Especially for the shield tunnel, due to the particularity of construction process and structure, it can be directly assumed that part of shield tail gap shrinkage is uniform shrinkage deformation parameter. However, it should be noted that this method has certain defects. The estimation of uniform shrinkage deformation parameter u_ε directly determines the correctness of the final results, so it needs to be adjusted repeatedly in combination with field measured data and calculated results during application.

4. A Case Study

4.1. Engineering Background. The urban rail transit project in a certain city, the tunnel section is excavated by a shield machine with a diameter of 16.0 m, the thickness of the shield lining is 0.265 m, the tunnel space is 15.4 m, and the gap of the shield tail is $\Delta = 70$ mm. The soil layer parameters are shown in Table 1. The tunnel should be considered as a plane strain problem, and the values of elastic modulus and Poisson's ratio should be revised according to Equations (23) and (24). The data in the table are all revised data. Some sections are selected as research objects, and the field measured surface settlement data are shown in Table 2.

$$E' = \frac{E}{1-\nu^2}, \tag{23}$$

$$\nu' = \frac{\nu}{1-\nu}. \tag{24}$$

4.2. Analysis of Stratum Deformation Caused by Shield Excavation. Due to the lack of necessary field measured data, it is necessary to use approximate method to determine the undetermined parameters. For shield tunnel, it can be assumed that the partial shrinkage of shield tail gap is

Table 1: Soil layer parameter.

Serial number	Type of soil layer	Thickness (m)	Density (kg/m^3)	Compression modulus (MPa)	Elasticity modulus (MPa)	Cohesion (kPa)	Internal friction angle (°)	Poisson's ratio
1	Artificial soil	4.80	190 0	6.0	3.74	10.0	15	0.35
2	Fine medium sand	23.90	202 0	35.0	31.50	0	30	0.20
3	Powdery clay	2.30	194 0	15.0	11.45	35.0	12	0.29
4	Fine medium sand	4.30	206 0	60.0	54.00	0	34	0.20
5	Powdery clay	3.80	202 0	20.0	15.26	50.0	18	0.29
6	Fine medium sand	8.75	208 0	75.0	67.50	0	35	0.20
7	Clay powder soil	3.25	201 0	25.0	19.08	45.0	18	0.29
8	Fine medium sand	3.50	208 0	85.0	76.50	0	36	0.20
9	Powdery clay	4.30	201 0	21.0	15.60	51.0	12.9	0.30
10	Fine medium sand	22.75	210 0	95.0	85.50	0	36	0.20
11	Fine medium sand	–	210 0	115.0	103.50	0	38	0.20

Table 2: Field measured ground settlement data.

Section	Tunnel depth (m)	$x = -40$ m (mm)	$x = -20$ m (mm)	$x = -10$ m (mm)	$x = 0$ m (mm)	$x = 10$ m (mm)	$x = 20$ m (mm)	$x = 40$ m (mm)
DB1	37.50	0.09	1.70	3.11	5.41	4.50	1.53	0.01
DB2	37.70	0.10	2.11	4.43	5.53	3.49	1.64	0.03
DB3	37.90	0.10	1.98	4.26	5.53	3.56	1.59	0.02
DB4	38.50	0.12	2.34	4.88	5.50	4.02	1.86	0.13
DB5	38.70	0.09	2.10	4.14	5.48	4.49	1.61	0.07

uniform shrinkage deformation parameter. Considering the factors such as site geological conditions and shield grouting reinforcement, it is assumed that the 10% shrinkage of shield tail gap is the uniform shrinkage deformation parameter, $u_\varepsilon = 10\% \times \Delta = 7.0\,\text{mm}$.

According to soil layer, the Poisson's ratio is approximately taken as the weighted average of soil thickness $\nu = 0.3$. Then, according to the geological data and tunnel depth, the undetermined parameters were calculated according to the approximate method. The parameter calculation results are shown in Table 3.

Table 3: Analysis method pending parameter list.

Section	Tunnel depth (m)	u_ε (mm)	u_δ (mm)
DB1	37.50	7.0	−2.366
DB2	37.70	7.0	−2.366
DB3	37.90	7.0	−2.366
DB4	38.50	7.0	−2.359
DB5	38.70	7.0	−2.352

After the undetermined parameters are obtained, the formation deformation and displacement field can be obtained according to the analytical theory. For the exact solution and approximate solution of DB2 section, the formation deformation cloud map and ground deformation curve diagram are shown in Figure 7.

The analysis results of ground settlement at some points of each section and the errors between them and the measured values are shown in Table 4. Figure 8 shows the interpolation comparison curve between the analytical settlement value and the measured ground settlement value.

Taking DB2 section as an example, the measured maximum surface deformation is 5.53 mm, and the exact solution and approximate solution are 5.554 and 5.696 mm, respectively, the error is within 3.0%. The difference between the analysis results of the maximum surface subsidence of other sections and the measured value is less than 5.0%. This shows that, to a certain extent, the analytical method can reflect the deformation law of strata.

However, it can be found from the comparison of ground deformation curves that there is a large difference between measured settlement and analytical settlement at $x = \pm 10$ in DB2 section, reaching 50%, and the analysis results of other sections are basically similar, which indicate that there is a certain difference between the curve of the ground settlement trough of the analytical method and the curve fitting of the measured value, mainly in the degree of concavity of the curve.

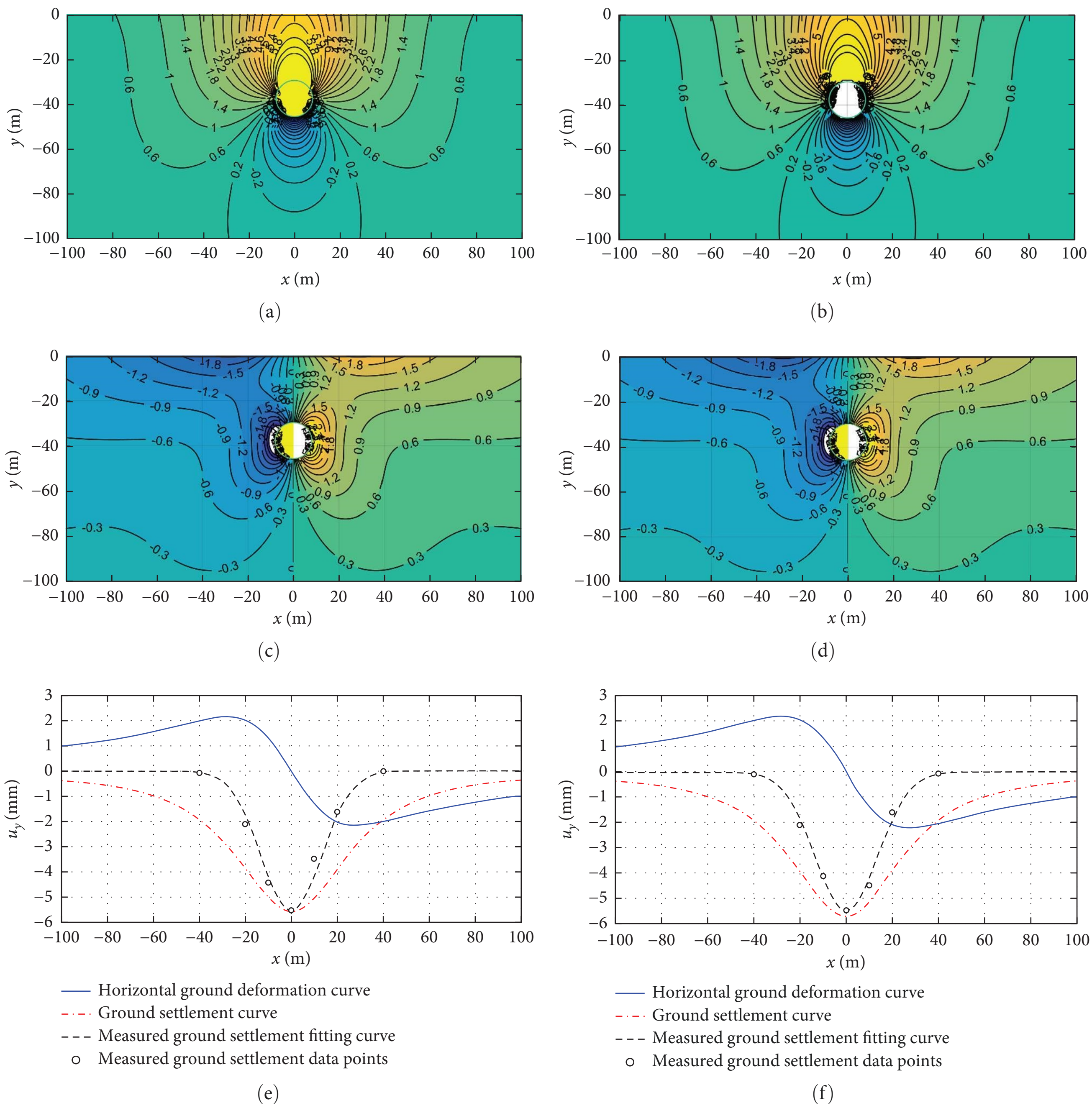

FIGURE 7: Analytical method stratigraphic deformation map of DB2 section: (a) exact solution of vertical displacement clouds; (b) approximate solution of vertical displacement clouds; (c) exact solution of horizontal displacement clouds; (d) approximate solution of horizontal displacement clouds; (e) exact solution of ground deformation curve; (f) approximate solution of ground deformation curve.

This is mainly because the approximation method is adopted in the acquisition of undetermined parameters. This method assumes that part of the contraction of shield tail gap is uniform contraction deformation parameter. Such approximation has an obvious shortcoming, that is, it cannot reflect the transverse constraint property of settlement curve. If the undetermined parameters are taken as inputs and the deformation and displacement field as outputs, the approximate method has no parameters that can reflect the inflection point of the ground settlement curve in the input, so it has certain errors.

However, this method can obtain the deformation and displacement field of the whole formation through simple calculation, and does not require the field measured deformation value. Compared with the Peck's empirical method, the displacement can be more comprehensive, and the theoretical basis is clear. Meanwhile, the settlement result obtained by the analytical method is generally slightly larger than the measured

Table 4: Analytical results and errors of the maximum subsidence of the surface.

Section	Tunnel depth (m)	x (m)	Measured deformation (mm)	Exact solution of settlement (mm)	Error with measured value (mm)	Approximate solution of settlement (mm)	Error with measured value (mm)
DB1	37.50	−10	3.11	5.030	2.392	5.158	2.048
DB1	37.50	0	5.41	5.582	0.118	5.726	0.316
DB1	37.50	10	4.50	5.030	1.002	5.158	0.658
DB2	37.70	−10	4.43	5.007	0.577	5.135	0.705
DB2	37.70	0	5.53	5.554	0.024	5.696	0.166
DB2	37.70	10	3.49	5.007	1.517	5.135	1.645
DB3	37.90	−10	4.26	5.010	0.75	5.113	0.853
DB3	37.90	0	5.53	5.526	−0.004	5.666	0.136
DB3	37.90	10	3.56	5.010	1.45	5.113	1.553
DB4	38.50	−10	4.88	4.930	0.05	5.046	0.166
DB4	38.50	0	6.07	5.441	−0.629	5.574	−0.496
DB4	38.50	10	4.02	4.930	0.91	5.046	1.026
DB5	38.70	−10	4.14	4.905	0.765	5.021	0.881
DB5	38.70	0	5.48	5.410	−0.07	5.541	0.061
DB5	38.70	10	4.49	4.905	0.415	5.021	0.531

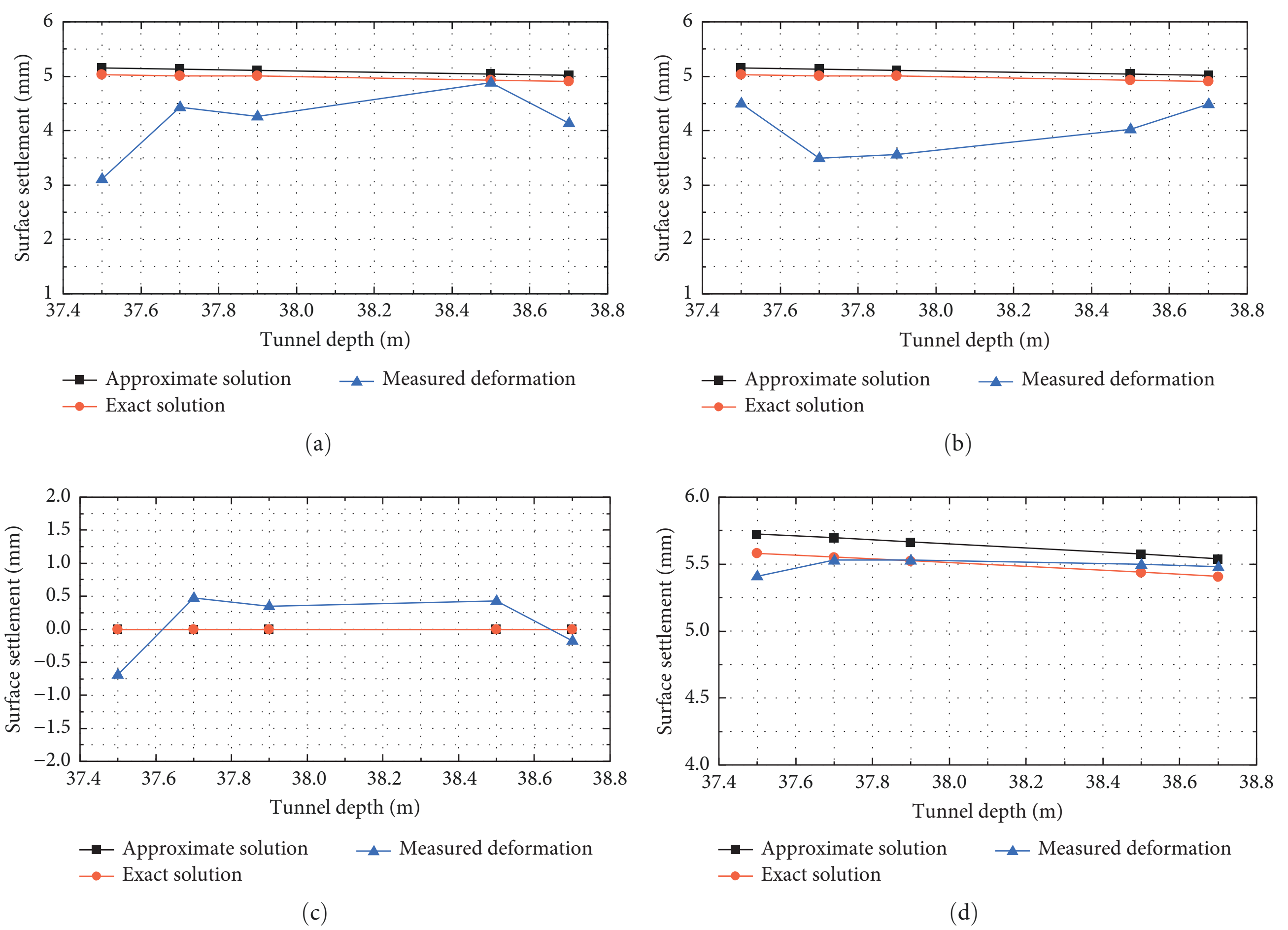

Figure 8: Analytical deformation and measured deformation comparison interpolation curve: (a) surface settlement comparison curve at $x = -10$; (b) surface settlement comparison curve at $x = 10$; (c) average surface settlement curve at $x = -10$ and $x = 10$; (d) surface settlement comparison curve at $x = 0$.

Table 5: Numerical simulation parameter.

	Thickness (m)	Density (kg/m^3)	Elasticity modulus (MPa)	Poisson's ratio
Concrete lining	0.265	289 0	350 00	0.16
Equivalency layer	0.140	210 0	–	0.20

Figure 9: Numerical simulation model diagram of DB2 section.

deformation value, it is more secure, so it is of certain practical value.

At the same time, according to the comparison between curve and cloud image, under the condition of the same section, the conclusion of the exact solution is slightly smaller than that of the approximate solution, and the depth of the burial is inversely proportional to the difference between the two solutions. The buried depth of the section selected in this paper satisfied $r/h < 0.21$, and the difference between the two solutions is less than 5.0%, which is similar to that of Federico P. These two solutions have the same conclusion that the error is small under deep buried condition.

Considering the requirement of symmetry, the ground settlement value should be symmetrical about the vertical axis of the tunnel, as shown in the fourth curve in Figure 8, which is well reflected by the analytical method, but there are some errors in the measured value. The measured results of some sections show large oscillations. On one hand, because the actual stratum is complex and changeable, there must be local stratum discontinuity, which causes the oscillation of the measured settlement. The other is due to measurement errors.

5. Simulation of Stratum Deformation Caused by Shield Excavation

In order to further demonstrate the applicability of the approximate method proposed in this paper, the Abaqus numerical simulation is used for analysis. In practical engineering, the dimensions of the shield lining, which can generally be regarded as known parameters, should be specifically designed and calculated according to the buried depth, span, and soil layer parameters of the tunnel. However, the material and strength of grouting reinforced layer should be determined according to the actual engineering. There are also factors such as stress release in the interval between excavation and support and grouting reinforcement. Considering the influence of a series of factors, the geostress release method adopts the equivalency layer method. The equivalency layer thickness is 140 mm, and the model size is consistent with the analytical method, which is 200×100 m. The values of each parameter are shown in Table 5.

Section DB2 is also taken as an example here. The modeling is shown in Figure 9, and the formation deformation cloud map of the numerical simulation method is obtained, as shown in Figures 10 and 11.

It has been explained above that there is little difference between the approximate solution and the exact solution. Here, the approximate solution is taken as an example to draw the ground deformation curve obtained by three methods: analysis, field measurement, and numerical simulation, as shown in Figure 12. It can be found from the curve that the fitting error of the three methods to the ground settlement curve is within a reasonable range, but the ground settlement value of the analytical results is a little larger, which indicates that the analytical method is safe. At the same time, by combining the curve with the cloud map of horizontal deformation obtained by the analytical method and the numerical simulation method, it can be found that there are some differences in horizontal deformation between the two methods, which is also caused by the approximation method to obtain the undetermined parameters of the analytical method.

The approximate method assumes that the uniform shrinkage deformation parameter is a constant. Based on the theory of elasticity, without considering a series of assumptions such as lining stiffness, gravity, and plasticity of soil, the pure deformation parameter that causing tunnel distortion is calculated by relative deformation and then inversely calculated. Meanwhile, using the theory of soil mechanics, the coefficient of earth pressure k_0, effective earth pressure, and water pressure are also introduced in the calculation. On one hand, such calculation differs from the basic assumption of the circular tunnel in the semi-infinite elastic space at the beginning of the analytical method. On the other hand, the method of calculating deformation parameters ρ_0 through relative deformation lacks lateral constraint on displacement. At the same time, the selection of calculation depth will also cause large changes in parameters, so there will be large errors. This method can be used to estimate the formation deformation caused by excavation.

The analysis results of the maximum surface settlement by the numerical simulation method of each section are shown in Table 6. Taking DB2 section as an example, the maximum surface settlement of numerical simulation is 5.52 mm, which has a small difference from the measured settlement of 5.53 mm. Meanwhile, the difference from the maximum surface settlement obtained by analytical method is also within 3.0%, which has a good fitting effect.

The maximum surface settlement results obtained from numerical simulation, analysis, and field measurement are drawn in a coordinate system after linear interpolation, as

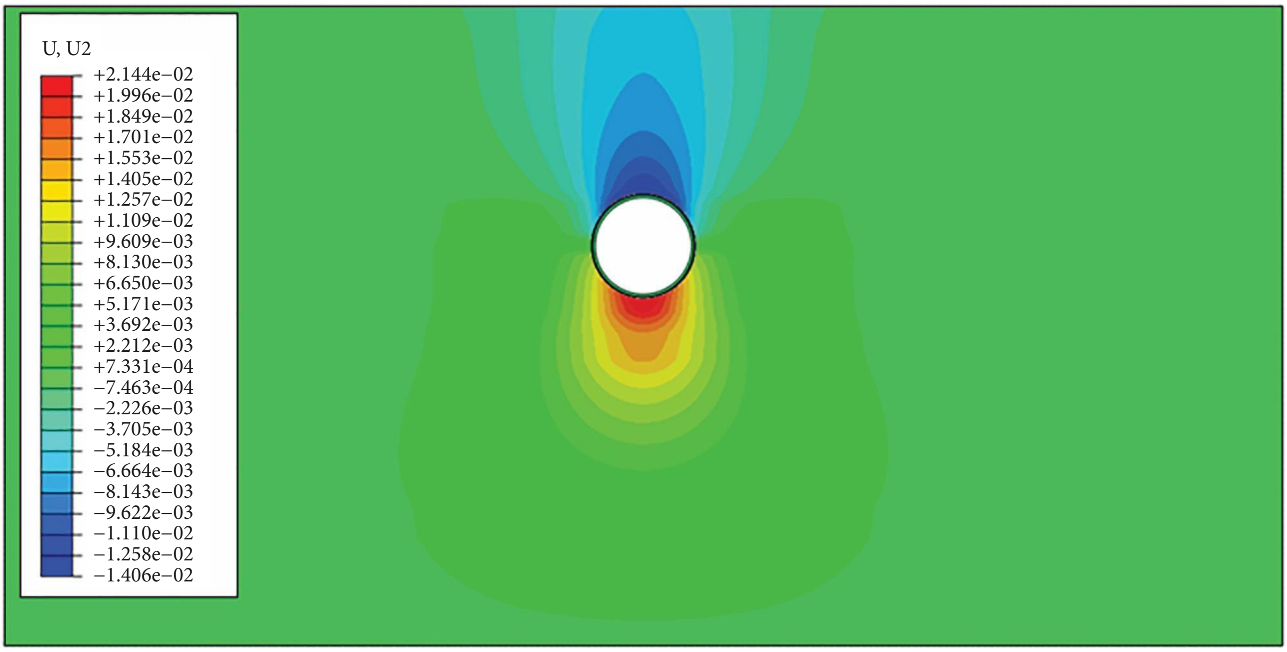

FIGURE 10: Numerical simulation of vertical displacement cloud map of DB2 section.

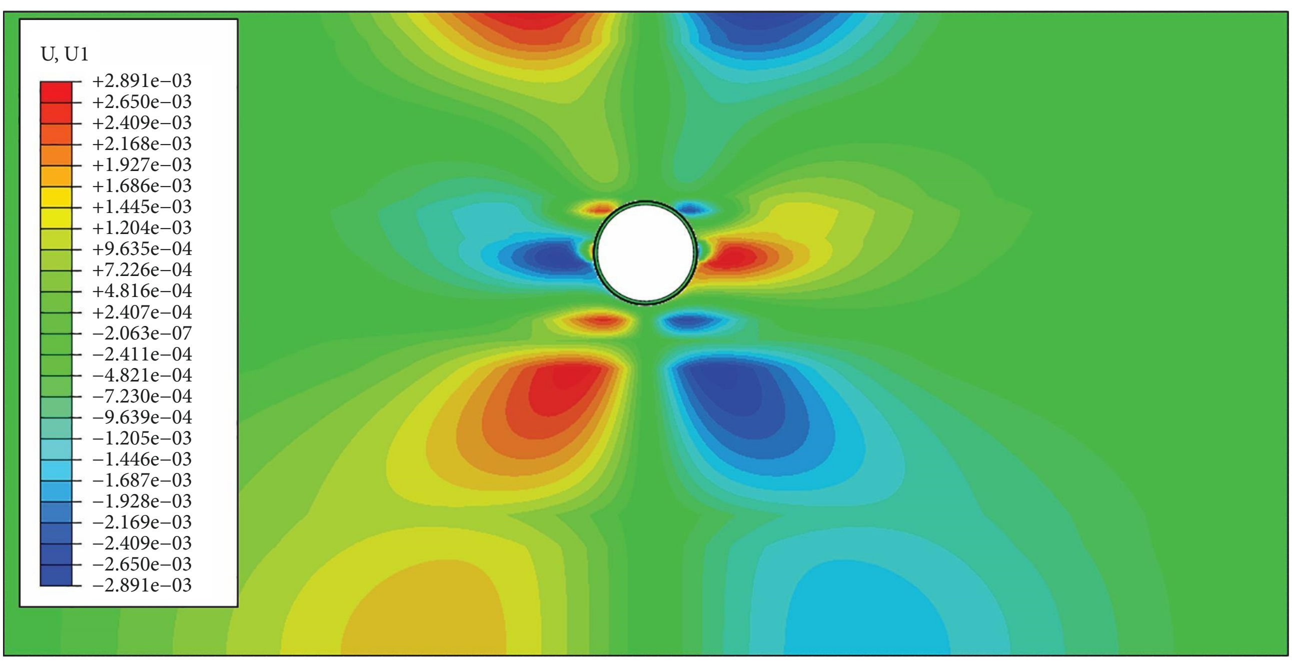

FIGURE 11: Numerical simulation horizontal displacement cloud map of DB2 section.

shown in Figure 13. Observing the curve, it is not difficult to find that the rules obtained by the four methods are similar, that is, the maximum surface settlement value decreases gradually with the increase of tunnel burial depth.

However, there are still some differences between the measured value and the theoretical value, for two reasons. First, the assumption of theoretical calculation model is insufficient; for example, the analytical method assumes that the whole stratum is of the same elastic material, and the parameters are obtained by using approximate method. The second is due to the uncertainties in the field, such as the accuracy of the measurement, local changes in the actual stratum, etc.

In the actual use of analytical method, considering the influence of these factors, in order to get a more accurate conclusion, on one hand, undetermined parameters should be obtained according to the general method, on the other hand, certain requirements should be made on the accuracy of measurement; if necessary, the calculation results of multiple sections should be compared.

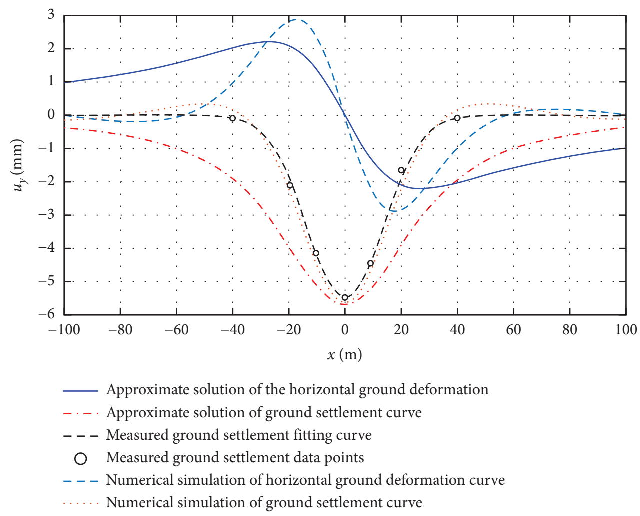

FIGURE 12: Ground deformation curve comparison chart.

TABLE 6: Maximum surface subsidence results of the four methods.

Section	Tunnel depth (m)	Numerical simulation of settlement (mm)	Approximate solution of settlement (mm)	Exact solution of settlement (mm)	Measured deformation (mm)
DB1	37.50	5.52	5.726	5.582	5.41
DB2	37.70	5.52	5.696	5.554	5.53
DB3	37.90	5.52	5.666	5.526	5.53
DB4	38.50	5.45	5.574	5.441	6.07
DB5	38.70	5.44	5.541	5.410	5.48

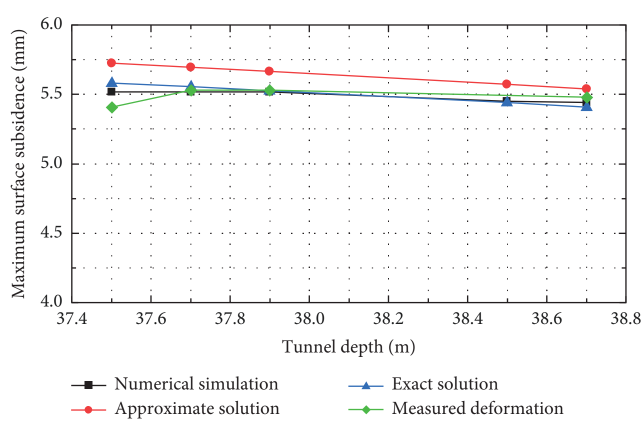

FIGURE 13: Interpolation curve of the maximum surface subsidence.

6. Conclusions

Based on the solution process and application ideas of Verruijt's complex variable function solution and Sagaseta's approximate solution, this paper proposes an approximate method for solving undetermined parameters. Then, taking a city shield tunnel project as an example, the formation deformation caused by tunnel penetration is analyzed by analytical method and numerical simulation method and compared with the actual monitoring data on site. The following conclusions are reached:

(1) When the field measured data are insufficient, the undetermined parameters caused by tunnel excavation in the analytical analysis of stratum deformation can be estimated by using the approximate method proposed in this paper combined with the theory of elasticity and soil mechanics. Elastic modulus and Poisson's ratio can be estimated when soil layer conditions are simple, while uniform shrinkage deformation parameters and Poisson's ratio can be estimated when soil layer conditions are complex. The latter is

especially suitable for shield tunnel, but the two are essentially the same.

(2) The approximate method can obtain the undetermined parameters caused by tunnel excavation in the analytical analysis of stratum deformation, which can well reflect the law of stratum deformation. Under the condition of reasonable parameter selection, the fitting error of the maximum surface settlement can be controlled within 5.0%.

(3) Due to the lack of lateral constraint on displacement of undetermined parameters obtained by the approximate method, tunnel distortion caused by tunnel pure deformation parameters cannot be well estimated when the parameter selection is unreasonable, which may cause large errors in local positions. However, when there is no field measured value or insufficient measured data, such as at the beginning of the design, it can be used to estimate the formation deformation.

(4) When other conditions are the same, the conclusion of formation deformation obtained by the accurate solution is slightly smaller than that obtained by the approximate solution. For the deep-buried tunnel with $r/h<0.2$, the difference between the two can be ignored.

(5) Although the approximate solution and exact solution are complicated, they can be calculated by software programming using the existing analytical conclusions in practice. Combined with the parameter determination method proposed in this paper, the analytical method can obtain the whole stratum deformation and displacement field through simple calculation, which has certain engineering application value.

Data Availability

All data, models, and code generated or used during the study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

Xinlong Li performed the data analyses and wrote the manuscript; Shiyong Zhao established the numerical model and analyzed the results; Jiale Zhao contributed significantly to data analysis and manuscript preparation; Dongjiao Cao and Zhenhui Hu reviewed and edited the manuscript; Yang Liu contributed to the conception and methodology of the study. All authors have read and agreed to the published version of the manuscript.

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