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Research Article

Benchmark for Nonlinear Consolidation of a Soil Deposit by Considering Depth-Dependent Initial Effective Pressure

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Analytical solutions for 1D nonlinear consolidation of soils are widely applied to calculate consolidation settlement for their simplicity and convenience. Furthermore, analytical solutions can provide valid benchmark for numerical solutions of nonlinear consolidation. However, the existing analytical solutions for 1D nonlinear consolidation assume the initial effective stress to remain constant in the whole deposit. In practice, the initial effective stress increases with depth in a soil deposit. The existing analytical solutions for 1D nonlinear consolidation are lack of consideration about the variation of initial effective stress along with depth, so there is no benchmark for evaluating the corresponding numerical calculation. In this study, an analytical solution for nonlinear consolidation is developed under a special case, considering the linear increase of initial effective stress with the depth, which provides a benchmark for the corresponding numerical calculation of nonlinear consolidation. To facilitate the verification of the reliability of the numerical calculations of nonlinear consolidation, the pore pressure dissipation curves and settlement curves with time for conditions of PTIB (the case for a permeable top surface and an impermeable bottom surface is referred to as PTIB) and PTPB (the case for a permeable top surface and a permeable bottom surface is named as PTPB) are provided in this paper, thus providing some validation data for future calculations of nonlinear consolidation considering a linear increase in the initial effective stress with depth.

1. Introduction

Both deformation and shear strength are important parts of classical soil mechanics. These two characteristics of soils may change with the dissipation of excess pore water pressure [1, 2], and the dissipation of excess pore water pressure with time can be described by the theory of consolidation. Meanwhile, the temporal and spatial variation in soil properties also can be incorporated by the theory of consolidation [3, 4]. Therefore, the theory of consolidation plays an important role in classical geomechanics. The compressibility and permeability of soils, which have a great influence on the consolidation behavior, decrease with an increase in the effective stress during the process of consolidation, and the consolidation theory with consideration of variable compressibility and permeability is usually called as nonlinear consolidation. As a result, the theory of nonlinear consolidation has been continuously investigated by some researchers.

Since Davis and Raymond [5] developed an analytical solution for 1D nonlinear consolidation by assuming the coefficient of consolidation and initial effective stress to be constant, some studies had been implemented on the theory of 1D nonlinear consolidation of a soil deposit. Most of these studies mainly focused on different numerical solutions for nonlinear consolidation of soils [6–11], and these numerical solutions have provided many references to investigate 1D nonlinear consolidation. However, it must be noted that numerical solutions heavily depend on the reliability of calculation methods, and the reliability of numerical solutions must be evaluated by analytical solutions. Therefore, there is great significance in developing some analytical solutions for some special cases to provide a benchmark for numerical solutions in which more complex factors can be incorporated.

Basing on the same assumptions about the coefficient of consolidation and initial effective stress as Davis and Raymond [5], Xie et al. [12, 13] derived the analytical solutions for 1D nonlinear consolidation of single-layered soil and double-layered soil under time-dependent loading, respectively. Lekha et al. [14] developed an approximate solution for 1D consolidation of soils with variable consolidation coefficient by assuming initial effective stress to be constant. Li et al. [15] obtained an analytical solution for the nonlinear consolidation of a soil deposit subjected to a ramp load with consideration of the variation of consolidation coefficient. In addition, some analytical solutions for rheological consolidation also have been derived [16, 17]. However, it must be noted that the assumption that the initial effective stress remains constant is incorporated in all these aforementioned analytical solutions. In fact, Li and Xie [10] found that the depth-dependent distribution of initial effective stress has great influences on the consolation rate and the final settlement of the soil deposit. Therefore, the depth-dependent increase in initial effective stress has to be considered in nonlinear consolidation theories of an actual soil deposit. However, there seems to be no benchmark to evaluate numerical solutions for nonlinear consolation in the literature when considering the depth-dependent increase in the initial effective stress.

As stated above, there are great significances in the development of analytical solutions for nonlinear consolidation. On the one hand, analytical solutions can be applied to calculate consolidation settlement for their simplicity and convenience. On the other hand, analytical solutions can provide a benchmark to evaluate numerical solutions for nonlinear consolidation. However, there seems to be no analytical solutions that take both the variation of compressibility and permeability and the linear increase in initial effective stress with depth into the theory of nonlinear consolidation in the literature. Thus, there is no theoretical benchmark to evaluate numerical solutions for nonlinear consolidation when considering the variation of initial effective stress. In this paper, an analytical solution for nonlinear consolidation by assuming the initial effective stress to increase linearly with depth is developed under a special increase in the additional stress, and this analytical solution can provide a benchmark to evaluate numerical solutions for nonlinear consolidation when considering the variation of initial effective stress.

2. Presentation of This Problem and Assumptions

As shown in Figure 1, the top surface of soil deposit is permeable, and the bottom one is permeable or impermeable. The case for a permeable top surface and an impermeable bottom surface is referred to as PTIB, and the case for a permeable top surface and a permeable bottom surface is named as PTPB. The thickness of the soil deposit is H. The initial effective stress at the top surface of the soil layer is equal to q_p, and the deformation of the soil deposit under the initial effective stress has been steady. As well known, initial void ratio of the soil, e_0, varies marginally with depth, and it can be initially considered as a constant in the whole soil deposit. So the effective unit weight of soils, γ , remains constant at different depth. Thus, the initial effective stress in the soil deposit increases linearly from q_p at the top surface to $q_p + \gamma' H$ at the bottom surface. The initial effective stress at depth z, $\sigma_0'(z)$, can be expressed as follows:

$$\sigma_0'(z) = q_p + \gamma' z. \tag{1}$$

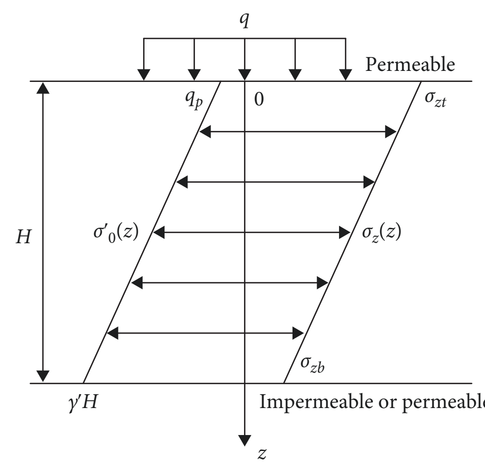

Figure 1: A soil deposit with a linear increase in the initial effective stress.

As noted by Zhu and Yin [18, 19], foundation construction loading or surcharge loading on a soil deposit, q, usually caused a stress increase at depth z. If the area of the foundation was small, the vertical stress increase below the center of the foundation at depth z due to q, $\sigma_z(z)$, would be simplified to decrease linearly with depth. In this case, the consolidation deformation of the soil deposit is not 1D in the vertical direction anymore. In practice, however, the consolidation deformation was still calculated by 1D consolidation theory which was modified by some methods [20]. To develop an analytical solution for nonlinear consolidation with consideration of the linear increase in initial effective stress, the depth-dependent decreases in the additional stress caused by q are assumed to be equal to the depth-dependent increases in the initial effective stress in this study. So, the vertical stress increase caused by q, $\sigma_z(z)$, is considered to change with depth as follows:

$$\sigma_z(z) = \sigma_{zt} - \gamma' z, \tag{2}$$

where σ_{zt} is the vertical additional stress at the top surface of soil deposit, σ_{zb} is the vertical additional stress at the bottom surface of soil deposit, and $\sigma_{zb} = \sigma_{zt} - \gamma' H$. To specify the rationality of this assumption for some cases, a comparison of the vertical additional stress below the center of the strip load of width of B by the above-proposed method with that by Boussinesq's solution is shown in Figure 2. The parameters adopted in the comparison follow as: the width of strip load $B = 5\,\text{m}$, $H = 10\,\text{m}$, $\gamma' = 8\,\text{kN/m}^3$, $q = 110\,\text{kPa}$. It can be noted from Figure 2 that the vertical additional stress below the center of the strip load calculated by the proposed method in this study approached that calculated by Boussinesq's solution. Therefore, this assumption can be suitable for some engineering cases. Under this assumption, the total stress at depth z, $\sigma(z)$, which is sum of the initial effective stress and vertical additional stress, is a constant. That is

$$\sigma(z) = \sigma_0'(z) + \sigma_z(z) = q_p + \sigma_{zt}. \tag{3}$$

As studied by Mesri and Rokhsar [7] and Tavenas et al. [21], the logarithm relation between void ratio and effective

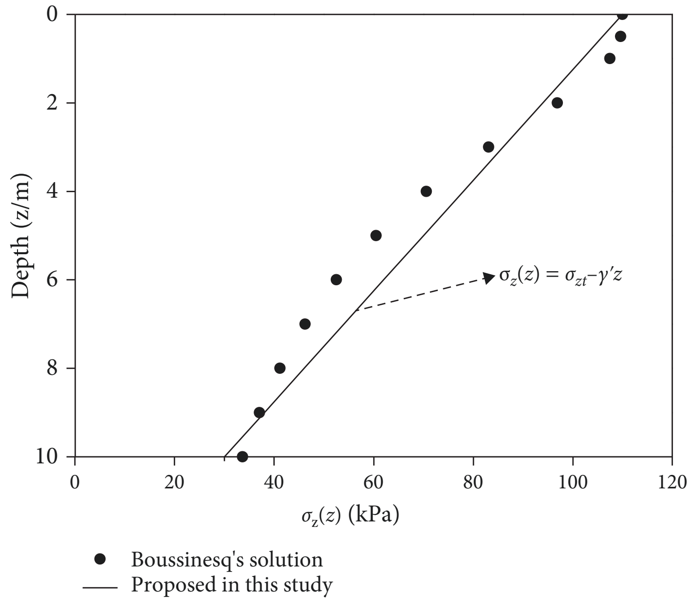

Figure 2: Comparisons of vertical stress increase between by proposed method and Boussinesq's method.

stress and the logarithm relation between the void ratio and permeability are suitable to describe the nonlinearity of compressibility and permeability of most natural soils, and these relationships are:

$$e = e_1 - C_c \log(\sigma'/\sigma'_1), \tag{4}$$

$$e = e_1 + C_k \log(k_v/k_{v1}), \tag{5}$$

where e is the void ratio, σ' is the effective stress, σ'_1 is an appointed effective stress, e_1 is the void ratio corresponding to the appointed effective pressure σ'_1, C_c is the compressibility index, C_k is the permeability index, k_v is the coefficient of permeability, and k_{v1} is the coefficient of permeability corresponding to the void ratio e_1.

3. The Governing Equation

From Equations (4) to (5), the coefficients of permeability and volume compressibility change with the effective pressure as follows:

$$k_v = k_{v1}\left(\frac{\sigma'_1}{\sigma'}\right)^{C_c/C_k}, \tag{6}$$

$$m_v = \frac{1}{1+e_0}\frac{\partial e}{\partial \sigma'} = -\frac{1}{1+e_0}\frac{C_c}{\ln 10}\frac{1}{\sigma'}, \tag{7}$$

where k_v is the coefficient of permeability and m_v is the coefficient of volume compressibility. The coefficient of consolidation, c_v, follows as:

$$c_v = c_{v1}\left(\frac{\sigma'_1}{\sigma'}\right)^{C_c/C_k-1}, \tag{8}$$

where $c_{v1} = (1+e_1)k_{v1}\sigma'_1 \ln 10/C_c\gamma_w$ and γ_w is the unit weight of water. According to Davis and Raymond [5], analytical solutions cannot be obtained under all cases of nonlinear consolidation. The analytical solution for the nonlinear consolidation can be obtained only in case of $C_c/C_k = 1$, and that means the coefficient of consolidation remains constant. Therefore, $C_c/C_k = 1$ is adopted in this note, and the coefficient of consolidation keeps constant during the whole consolidation.

If vertical drainage and small strain assumption are incorporated, the rate of change in the volume of the soil element is equal to the rate of change in the volume of voids during consolidation. According to Darcy's law and the assumptions that the soil grains and waters in void are incompressible, the following equation can be developed as follows:

$$\frac{1}{\gamma_w}\frac{\partial}{\partial z}\left(k_v\frac{\partial u}{\partial z}\right) = \frac{1}{1+e_0}\frac{\partial e}{\partial t}, \tag{9}$$

where u is the excess pore water pressure, z is the vertical coordinate, and t is the time. If the rheological deformation of the soil is ignored, Equation (9) can be rewritten as follows:

$$\frac{1}{\gamma_w}\frac{\partial}{\partial z}\left(k_v\frac{\partial u}{\partial z}\right) = \frac{1}{1+e_0}\frac{\partial e}{\partial \sigma'}\frac{\partial \sigma'}{\partial t}. \tag{10}$$

According to the principle of effective stress, σ' can be expressed as follows:

$$\sigma' = \sigma_0'(z) + \sigma_z(z) - u. \tag{11}$$

Substituting Equations (6), (7), and (11) into Equation (10), the governing equation for 1D nonlinear consolidation is follows:

$$c_{v1}\frac{\sigma'}{\sigma_1'}\frac{\partial}{\partial z}\left[\left(\frac{\sigma_1'}{\sigma'}\right)\frac{\partial u}{\partial z}\right] = \frac{\partial u}{\partial t}. \tag{12}$$

For the case of PTIB, the boundary conditions are as follows:

$$u(0,t) = 0, t>0, \tag{13}$$

$$\left.\frac{\partial u}{\partial z}\right|_{z=H} = 0. \tag{14}$$

For the case of PTPB, the boundary conditions are expressed as follows:

$$u(0,t) = 0, t>0, \tag{15}$$

$$u(H,t) = 0. \tag{16}$$

The initial condition for the consolidation model is follows:

$$u(z,0) = \sigma_z(z) = \sigma_{zt} - \gamma' z. \tag{17}$$

3.1. Solutions for Excess Pore Water Pressure. In order to obtain an analytical solution for Equation (12), let

$$w = \ln\frac{\sigma'}{\sigma_0'(z) + \sigma_z(z)} = \ln\frac{\sigma'}{q_p + \sigma_{zt}}. \tag{18}$$

Substituting Equation (18) into Equation (12), the governing equation is transformed into:

$$c_{v1}\frac{\partial^2 w}{\partial z^2} = \frac{\partial w}{\partial t}. \tag{19}$$

In terms of w, the boundary conditions for the case of PTIB become:

$$w(0,t) = 0, t>0, \tag{20}$$

$$\left.\frac{\partial w}{\partial z}\right|_{z=H} = 0. \tag{21}$$

The initial condition can be expressed as follows:

$$w(z,0) = \ln\frac{\sigma_0'(z)}{q_p + \sigma_{zt}} = \ln\frac{q_p + \gamma' z}{q_p + \sigma_{zt}}. \tag{22}$$

Similar to Terzaghi's solution for 1D consolidation in the case of PTIB, the analytical solution for w can be expressed as follows:

$$w = \sum_{m=1}^{\infty} A_m \sin\frac{Mz}{H}\exp(-M^2 T_v), \tag{23}$$

where $M = (2m-1)/2\pi$, $m = 1, 2, 3, \ldots$, $T_v = c_{v1}t/H^2$. By using the orthogonality of the trigonometric function and the initial condition, the expression for A_m can be determined as follows:

$$A_m = \frac{2}{H}\int_0^H \sin\frac{Mz}{2H}\ln\frac{q_p + \gamma' z}{q_p + \sigma_{zt}}dz. \tag{24}$$

In terms of the dimensionless variable of depth z, Equation (24) can be rewritten as follows:

$$A_m = 2\int_0^1 \sin\left(\frac{MZ}{2}\right)\ln\left(\frac{q_p + \gamma' HZ}{q_p + \sigma_{zt}}\right)dZ, \tag{25}$$

where $Z = z/H$, and it varies between 0 and 1. The interval [0, 1] is divided into N subintervals of each width $\Delta Z = 1/N$. A_m can be obtained by numerical integration. Letting $Z_1(=0)$, Z_2, …, $Z_{N+1}(=1)$ be the endpoints of these subintervals, the endpoints of kth subinterval are $Z_k = (k-1)\Delta Z$ and $Z_{k+1} = k\Delta Z$, respectively. Then A_m can be obtained by the following numerical integration:

$$A_m = \frac{1}{N}\sum_{k=1}^{N}\left[\sin\left(\frac{MZ_k}{2}\right)\ln\left(\frac{q_p + \gamma' HZ_k}{q_p + \sigma_{zt}}\right) + \sin\left(\frac{MZ_{k+1}}{2}\right)\ln\left(\frac{q_p + \gamma' HZ_{k+1}}{q_p + \sigma_{zt}}\right)\right], \tag{26}$$

where $k = 1, 2, 3, \ldots, N$. To ensure the accuracy of numerical integration, $N = 10{,}000$ is adopted in this study. Substituting Equation (23) into Equation (18), the expression for u is follows:

$$u = \left(q_p + \sigma_{zt}\right)\left[1 - \exp\left(\sum_{m=1}^{\infty} A_m \sin\frac{Mz}{H}\exp(-M^2 T_v)\right)\right]. \tag{27}$$

If the bottom surface of the soil deposit is also permeable, the boundary conditions in terms of w become:

$$w(0,t) = 0, t>0, \tag{28}$$

$$w(H,0) = 0, t>0. \tag{29}$$

According to Terzaghi's solution for 1D consolidation, the solution corresponding to the case PTPB can be expressed as follows:

$$w = \sum_{m=1}^{\infty} B_m \sin\frac{m\pi z}{H}\exp(-m^2\pi^2 T_v). \tag{30}$$

Applying orthogonality of trigonometric function and initial condition to Equation (30), B_m can be expressed as follows:

$$B_m = \frac{2}{H}\int_0^H \sin\frac{m\pi z}{H}\ln\frac{q_p+\gamma' z}{q_p+\sigma_{zt}}dz. \tag{31}$$

In terms of the dimensionless variable Z, Equation (31) is rewritten as follows:

$$B_m = 2\int_0^1 \sin(m\pi Z)\ln\left(\frac{q_p+\gamma' HZ}{q_p+\sigma_{zt}}\right)dZ. \tag{32}$$

The same numerical integration as Equation (26) was adopted, and B_n can be calculated by:

$$\begin{aligned} B_m = \frac{1}{N}\sum_{k=1}^{N}\Bigg[&\sin(m\pi Z_k)\ln\left(\frac{q_p+\gamma' HZ_k}{q_p+\sigma_{zt}}\right) \\ &+ \sin(m\pi Z_{k+1})\ln\left(\frac{q_p+\gamma' HZ_{k+1}}{q_p+\sigma_{zt}}\right)\Bigg]. \end{aligned} \tag{33}$$

Substituting Equation (30) into Equation (18), the excess pore water pressure for the case of PTPB is expressed as follows:

$$u = (q_p+\sigma_{zt})\left[1-\exp\left(\sum_{m=1}^{\infty}B_m\sin\frac{m\pi z}{H}\exp(-m^2\pi^2T_v)\right)\right]. \tag{34}$$

3.2. Solutions for Average Degree of Consolidation. The average degree of consolidation in terms of stress, U_{pt}, follows as:

$$\begin{aligned} U_{pt} &= 1-\frac{\int_0^H u dz}{\int_0^H \sigma_z(z)dz} \\ &= 1-\frac{(q_p+\sigma_{zt})}{(\sigma_{zt}-1/2\gamma' H)H}\int_0^H (1-\exp(w))dz. \end{aligned} \tag{35}$$

Applying numerical solutions to Equation (35), U_{pt} can be calculated by a similar method for the calculation of A_m and B_n as follows:

$$\begin{aligned} U_{pt} &= 1 \\ &-\frac{(q_p+\sigma_{zt})}{(\sigma_{zt}-1/2\gamma' H)}\frac{1}{N}\sum_{k=1}^{N}\left[1-\frac{\exp(w(Z_k,t))+\exp(w(Z_{k+1},t))}{2}\right]. \end{aligned} \tag{36}$$

The settlement of soil deposit at time t, S_t, can be expressed as follows:

$$\begin{aligned} S_t = \int_0^H \frac{e_0-e(z,t)}{1+e_0}dz = \frac{C_c}{1+e_0}\int_0^H \big[&\log(q_p+\sigma_{zt}-u) \\ &-\log(q_p+\gamma' z)\big]dz. \end{aligned} \tag{37}$$

Combining Equation (27) with Equation (37), the settlement of soil deposit at time t under the case of PTIB can be obtained.

$$\begin{aligned} S_t = \frac{C_c H}{1+e_0}\Bigg[&\log(q_p+\sigma_{zt}) \\ &-\frac{(q_p+\gamma' H)\ln(q_p+\gamma' H)-q_p\ln q_p}{\gamma' H} \\ &+\frac{1}{\ln 10}+\frac{1}{\ln 10}\sum_{m=1}^{\infty}\frac{A_m}{M}\exp(-M^2T_v)\Bigg]. \end{aligned} \tag{38}$$

Combining Equation (34) with Equation (37), the settlement of soil deposit at time t under the case of PTPB follows as follows:

$$\begin{aligned} S_t = \frac{C_c H}{1+e_0}\Bigg[&\log(q_p+\sigma_{zt}) \\ &-\frac{(q_p+\gamma' H)\ln(q_p+\gamma' H)-q_p\ln q_p}{\gamma' H} \\ &+\frac{1}{\ln 10}+\frac{2}{\ln 10}\sum_{n=1,3,5}^{\infty}\frac{B_n}{n\pi}\exp(-n^2\pi^2T_v)\Bigg]. \end{aligned} \tag{39}$$

When excess pore water pressure completely dissipated, the final settlement of the soil deposit is follows:

$$\begin{aligned} S_\infty = \frac{C_c H}{1+e_0}\Bigg[&\log(q_p+\sigma_{zt}) \\ &-\frac{(q_p+\gamma' H)\ln(q_p+\gamma' H)-q_p\ln q_p}{\gamma' H}+\frac{1}{\ln 10}\Bigg]. \end{aligned} \tag{40}$$

Solutions of the average degree of consolidation in terms of deformation can be expressed as follows:

$$U_{st} = S_t/S_\infty. \tag{41}$$

3.3. Benchmark Problem. A benchmark problem was investigated to provide specific solutions for the excess pore water pressure, settlement, and average degree of consolidation of a soil deposit. These solutions can be used to evaluate the reliability of numerical solutions when the initial effective stress increased linearly with depth. The boundary condition of the soil deposit is PTIB or PTPB, and its thickness is $H = 10$ m. The specific gravity of clay solids $G_s = 2.72$, and the initial void ratio $e_0 = 1.15$ remains constant in the whole deposit. So the effective unit weight of soils $\gamma' = 8\,\text{kN/m}^3$. The compression index, which is the slope of the $e-\log\sigma'$

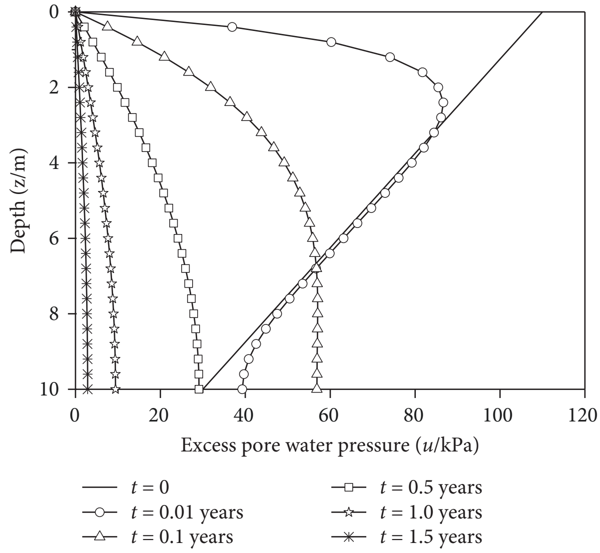

Figure 3: Distribution of excess pore water pressure with depth under the case of PTIB.

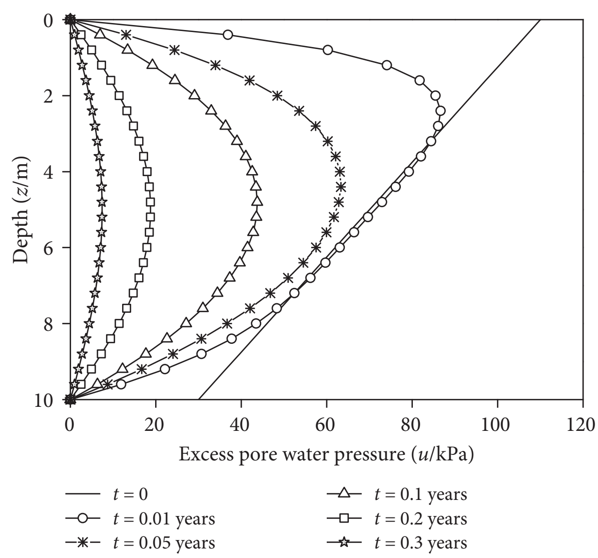

Figure 4: Distribution of excess pore water pressure with depth under the case of PTPB.

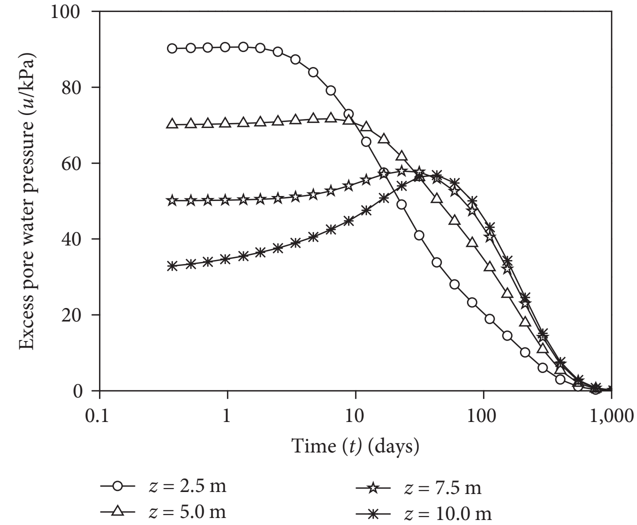

Figure 5: Dissipation of excess pore water pressure with time under the case of PTIB.

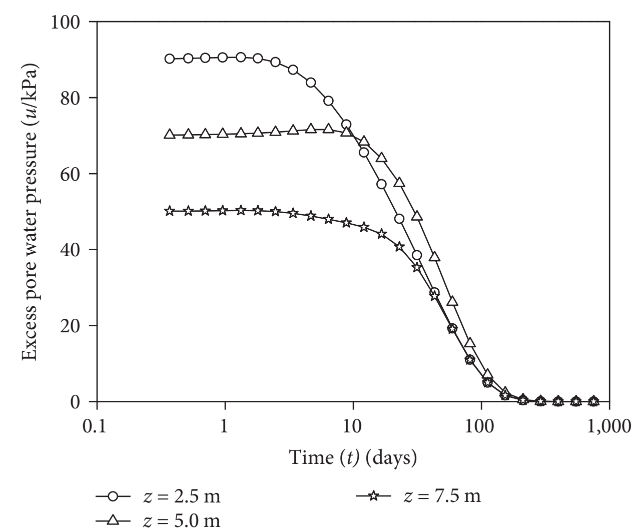

Figure 6: Dissipation of excess pore water pressure with time under the case of PTPB.

plot, $C_c = 0.63$, and $\sigma'_1 = 50\,\text{kPa}$ is the designated effective stress, and $e_1 = 1.15$ is the void ratio corresponding to σ'_1 in the $e - \log \sigma'$ plot. $k_{v1} = 8 \times 10^{-8}\text{m/s}$ is the coefficient of permeability corresponding to e_1 in the $e - \log k_v$ plot, and the permeability index C_k is equal to $C_c = 0.63$. The initial effective stress at the top surface is $q_p = 10\,\text{kPa}$. The increase in the vertical stress caused by the strip load at the top surface of soil deposit is $\sigma_{zt} = 110\,\text{kPa}$, and the increase in the vertical stress caused by the strip load at the bottom surface is $\sigma_{zb} = 30\,\text{kPa}$. Moreover, the increase in the initial effective stress with depth is equal to the decrease in the additional stress.

Applying the developed solutions, the excess pore water pressure, settlement of the deposit and the average degree can be obtained. Figures 3 and 4 show the dissipation of excess pore water pressure at different depth for the case of PTIB and PTPB, respectively. The initial excess pore water pressure decreases linearly with depth, and it gradually dissipates with time. At the beginning time, the excess pore water pressure at the bottom of the soil layer is larger than the initial value for the case of PTIB, which is induced by the nonuniform distributions of initial excess pore water pressure and the bottom boundary condition. When the time exceeds 0.5 year, the excess pore water pressure at the bottom is smaller than the initial value. Figures 3 and 4 can be used to evaluate the dissipation at different depth of the soil layer when the consolidation time remains constant.

Figures 5 and 6 are the variation of excess pore water pressure with time for cases of PTIB and PTPB, respectively. For the case of PTIB, the increase in excess pore water pressure of the bottom layer at the beginning consolidation time

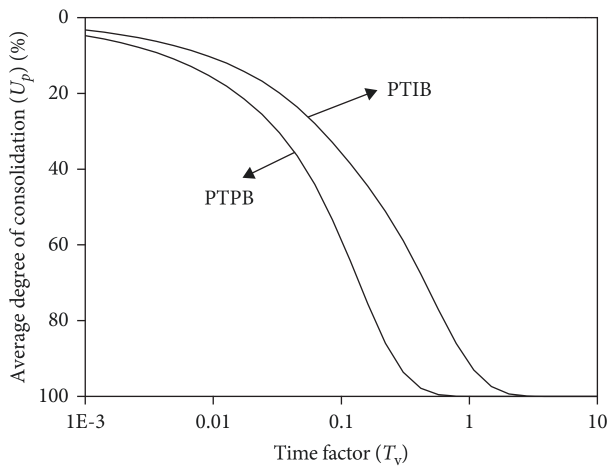

Figure 7: Average degree of consolidation U_p versus time factor T_v.

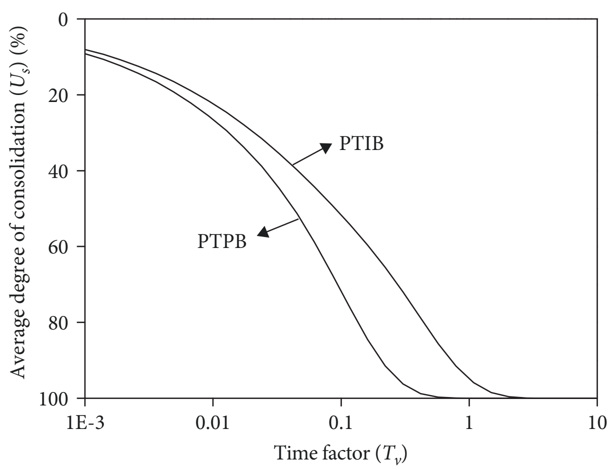

Figure 8: Average degree of consolidation U_s versus time factor T_v.

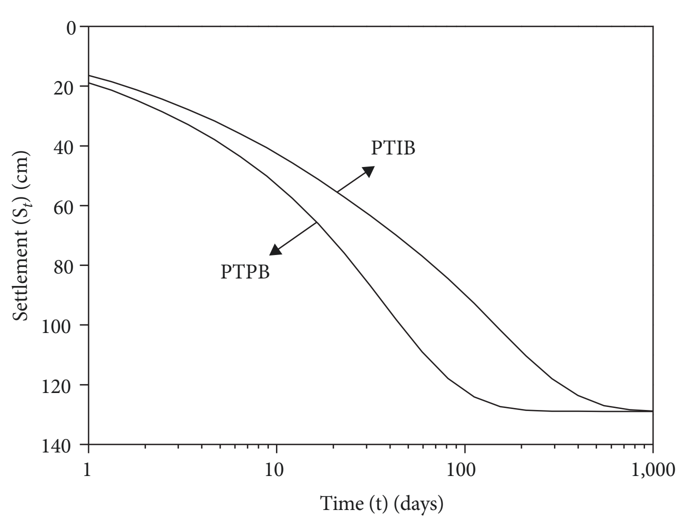

Figure 9: Settlement of the soil deposit S_t versus time t.

can be also found in Figure 5. For the case of PTPB, The excess pore water pressure gradually dissipates from the initial value, and the final excess pore water pressure is equal to 0. Figures 5 and 6 can be used to evaluate the dissipation of excess pore water pressure at different times when the depth remains constant.

Figures 7 and 8 are curves of the average degree of consolidation U_p and U_s versus time factor, respectively. These curves provide the benchmark for the average degree of nonlinear consolidation calculated by the numerical method when the initial effective stress increases with depth. A relationship between the settlement and time is further obtained from Figure 9 under cases of PTIB or PTPB. It is also used to evaluate the settlement of soil layer by numerical method.

4. Conclusions

In this study, if the additional stress varies with depth and soil consolidation, this problem is no longer a 1D consolidation. Although at this time there are 2D or even 3D deformation, but 2D or 3D consolidation calculations will be very complex, and it is not easy for engineers to master. This paper has been shown that the 1D consolidation theory is approximately adopted to carry out consolidation calculations and the error can be accepted by the actual engineering [22].

Some conclusions are as follows:

(1) In this paper, an analytical solution for 1D nonlinear consolidation considering the linear increase in the initial effective stress is developed. This analytical solution can be used to evaluate the reliability of numerical solutions when the initial effective stress is considered to increase linearly with depth.

(2) For the case of PTPB or PTIB, the dissipating process of excess pore water pressure is provided to verify the correctness of numerical solutions.

(3) The curves of settlement versus time under different boundary conditions are provided to verify the reliability of numerical solutions.

Notations

A_m: $\frac{2}{H}\int_0^H \sin\left(\frac{Mz}{2H}\right)\ln\left(\frac{q_p+\gamma' z}{q_p+\sigma_{zt}}\right)dz$

B: Width of the strip load

B_m: $\frac{2}{H}\int_0^H \sin\left(\frac{m\pi z}{H}\right)\ln\left(\frac{q_p+\gamma' z}{q_p+\sigma_{zt}}\right)dz$

c_v: Coefficient of consolidation

c_{v1}: $\frac{(1+e_1)k_{v1}\sigma'_1 \ln 10}{C_c\gamma_w}$

C_c: Compressibility index

C_k: Permeability index

e: Void ratio

e_0: Initial void ratio

e_1: Void ratio corresponding to the appointed effective stress

H: Thickness of the clay layer
k: Positive integer, 1, 2, 3, …, N
k_v: Coefficient of permeability
k_{v1}: Coefficient of permeability corresponding to the void ratio e_1
m: Positive integer, 1, 2, 3, …
m_v: Coefficient of volume compressibility
M: $\frac{(2m-1)\pi}{2}$
N: Total thin layers divided in the numerical integration
q: Foundation construction loading or surcharge loading
q_p: Initial effective stress at the top surface of the soil deposit
S_t: Settlement of the soil deposit at time t
S_∞: Final settlement of the soil deposit
t: Time
T_v: Time factor, and $T_v = \frac{c_{v1}t}{H^2}$
u: Excess pore water pressure
U_{pt}: Average degree of consolidation in terms of excess pore water pressure
U_{st}: Average degree of consolidation in terms of deformation
w: $\ln\left[\frac{\sigma'}{\sigma'_0(z)+\sigma_z(z)}\right]$
z: Depth
Z: z/H
Z_k: $k\Delta Z$
γ_w: Specific weight of water
γ': Effective unit weight of soils
$\sigma(z)$: Total stress at depth z
σ': Vertical effective stress
σ'_1: Appointed effective stress
$\sigma'_0(z)$: Initial effective stress at depth z
$\sigma_z(z)$: Vertical stress increase at depth z due to external loading
σ_{zb}: Vertical stress increase at the bottom surface of soil deposit
σ_{zt}: Vertical stress increase at the top surface of soil deposit
ΔZ: $1/N$.

Data Availability

All data, models, and code generated or used during the study appear in the published article.

Conflicts of Interest

The author declares that there is no conflicts of interest.

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References

[1] Y. Watabe and S. Leroueil, "Modeling and implementation of the isotache concept for long-term consolidation behavior," *International Journal of Geomechanics*, vol. 15, no. 5, Article ID A4014006, 2015.
[2] A. A. Golestani Dariani and M. M. Ahmadi, "Undrained shear strength and in situ horizontal effective stress from piezocone penetration test measurements in clayey soils: new approach," *International Journal of Geomechanics*, vol. 18, no. 9, Article ID 04018097, 2018.
[3] M. D. Liu and J. P. Carter, "Volumetric deformation of natural clays," *International Journal of Geomechanics*, vol. 3, no. 2, pp. 236–252, 2003.
[4] L. C. S. M. Ozelim, J. Camapum de Carvalho, A. L. B. Cavalcante, J. Pereira da Silva, and C. M. G. Muñetón, "Novel approach to consolidation theory of structured and collapsible soils," *International Journal of Geomechanics*, vol. 15, no. 4, Article ID 04014064, 2015.
[5] E. H. Davis and G. P. Raymond, "A non-linear theory of consolidation," *Geotechnique*, vol. 15, no. 2, pp. 161–173, 1965.
[6] T. J. Poskitt, "The consolidation of saturated clay with variable permeability and compressibility," *Geotechnique*, vol. 19, no. 2, pp. 234–252, 1969.
[7] G. Mesri and A. Rokhsar, "Theory of consolidation for clays," *Journal of the Geotechnical Engineering Division*, vol. 100, no. 8, pp. 889–904, 1974.
[8] J. M. Duncan, "Limitations of conventional analysis of consolidation settlement," *Journal of Geotechnical Engineering*, vol. 119, no. 9, pp. 1333–1359, 1993.
[9] Y.-C. Zhuang, K.-H. Xie, and X.-B. Li, "Nonlinear analysis of consolidation with variable compressibility and permeability," *Journal of Zhejiang University-Science A*, vol. 6, no. 3, pp. 181–187, 2005.
[10] C.-X. Li and K.-H. Xie, "One-dimensional nonlinear consolidation of soft clay with the non-Darcian flow," *Journal of Zhejiang University-Science A*, vol. 14, no. 6, pp. 435–446, 2013.
[11] A. Mishra and N. R. Patra, "Long-term response of consolidating soft clays around a pile considering non-Darcian flow," *International Journal of Geomechanics*, vol. 19, no. 6, Article ID 04019040, 2019.
[12] K. H. Xie, B. H. Li, and Q. L. Li, "A nonlinear theory of consolidation under time-dependent loading," in *Proceedings of 2nd International Conference on Soft Soil Engineering*, pp. 193–196, Hohai University Press, Nanjing, China, 1996.
[13] K.-H. Xie, X.-Y. Xie, and W. Jiang, "A study on one dimensional nonlinear consolidation of double-layered soil," *Computers and Geotechnics*, vol. 29, no. 2, pp. 151–168, 2002.
[14] K. R. Lekha, N. R. Krishnaswamy, and P. Basak, "Consolidation of clays for variable permeability and compressibility," *Journal of Geotechnical and Geoenvironmental Engineering*, vol. 129, no. 11, pp. 1001–1009, 2003.
[15] C. Li, J. Huang, L. Wu, J. Lu, and C. Xia, "Approximate analytical solutions for one-dimensional consolidation of a clay layer with variable compressibility and permeability under a ramp loading," *International Journal of Geomechanics*, vol. 18, no. 11, Article ID 06018032, 2018.
[16] R. E. Gibson and K. Y. Lo, "A theory of consolidation for soils exhibiting secondary compression," *Acta Polytechnica Scandinavica, Civil Engineering and Building Construction Series*, vol. 10, pp. 1–16, 1961.
[17] K.-H. Xie, X.-Y. Xie, and X.-B. Li, "Analytical theory for one-dimensional consolidation of clayey soils exhibiting rheological characteristics under time-dependent loading," *International Journal for Numerical and Analytical Methods in Geomechanics*, vol. 32, no. 14, pp. 1833–1855, 2008.

[18] G. Zhu and J.-H. Yin, "Consolidation of soil under depth-dependent ramp load," *Canadian Geotechnical Journal*, vol. 35, no. 2, pp. 344–350, 1998.

[19] G. F. Zhu and J. H. Yin, "Consolidation of double soil layers under depth-dependent ramp load," *Geotechnique*, vol. 49, no. 3, pp. 415–421, 1999.

[20] A. W. Skempton and L. Bjerrum, "A contribution to the settlement analysis of foundations on clay," *Geotechnique*, vol. 7, no. 4, pp. 168–178, 1957.

[21] F. Tavenas, P. Jean, P. Leblond, and S. Leroueil, "The permeability of natural soft clays. Part II: permeability characteristics," *Canadian Geotechnical Journal*, vol. 20, no. 4, pp. 645–660, 1983.

[22] C.-X. Li, K.-H. Xie, and K. Wang, "Analysis of 1D consolidation with non-Darcian flow described by exponent and threshold gradient," *Journal of Zhejiang University-Science A*, vol. 11, no. 9, pp. 656–667, 2010.