

Research Article

Prediction of Reservoir Bank Collapse Based on the Limit Equilibrium Theory

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The prediction of reservoir bank collapse width is a unique problem encountered in the construction of hydropower projects. Existing empirical graphic methods are based on the final state of bank collapse and can be used to predict only bank collapse width, and they thus do not adequately reflect the characteristics of the bank collapse process. To solve this problem, a prediction model for bank collapse width based on the limit equilibrium theory was established, and the key parameters and bank collapse process of the model were analyzed. The results reveal the necessity of selecting a reasonable underwater accumulation rate when predicting the bank collapse width using the limit equilibrium theory. At a constant ratio of water depth to bank slope height, the underwater accumulation rate increases with increasing bank slope height, with a linear relationship between them. In contrast, the bank slope angle has little impact on bank collapse width. Specifically, it mainly affects the width of the first bank collapse but has little effect on the width of subsequent bank collapses. With an increase in the bank slope angle, the bank collapse width fluctuates and rises. The prediction model of bank collapse width based on the limit equilibrium theory can better explain the time-dependent behavior of bank collapse. The research results are of high significance for the prediction of loess bank collapse.

1. Introduction

Reservoir impoundment changes the geological conditions of reservoirs. In particular, the bank slope within the backwater area will recede under the influence of water, which may lead to secondary disasters such as reservoir siltation and swelling. Therefore, the prediction of bank slope stability and bank collapse width has been of interest to many scholars [1–4]. Taking the dead water level as the starting point, Kachugin [5], who laid the foundation of the graphical method of bank collapse prediction, divided the bank slope into above-water and below-water parts and predicted bank collapse width by combining the stable angle of the bank slope. Then, Kondratjev [6] considered the impact of waves and revised the Kachugin method to improve the prediction ability. Wang et al. [7] further revised

the starting point to the historical maximum flood level of the river channel and established the two-stage Method. The abovementioned methods are applicable to the prediction of a homogeneous bank slope collapse. Peng [8] believed that the prediction of bank collapse requires the consideration of deposition and put forward the balanced alluvial accumulation approach, which has been widely used in the Three Gorges Reservoir. Based on the investigation of soil bank collapse in southwest China, Liao [9] proposed the three-stage method, which takes the low water level and high water level as demarcation points, and pointed out that collapses migrate towards the center of the river as far as possible. But the stage characteristics of bank collapse are difficult to reflect using only the empirical graphical method, which considers only the characteristics of the final state of bank collapse.

At the same time, existing research studies on the evaluation of bank slope stability focus on the influence of various factors on the bank slope. In addition, the coupling effect of unsaturated seepage and fluid mechanics has been combined. According to the model test, Lindow et al. [10] reported a correlation between self-weight, seepage force, and bank slope stability and pointed out that lateral seepage should be considered in the prediction of bank slope stability. Wang [11] indicated that coarse particles are more susceptible to wind and waves and that the strength and deterioration ranges of gravel soil decrease with periodic seepage and eventually tend to stabilize. Wang and Zhou [12] indicated that the bank slope of the subgrade near the water will not slip along the interface after dynamic compaction, but the underwater slope will form a slip surface. Zhou et al. [13] modified the calculation of the seepage force and phreatic surface and calculated the factor of safety using the trust region reflective algorithm; they also indicated that slope stability is affected by the drop speed of the water level, slope height, permeability coefficient, and aquifer thickness. Ma et al. [14] proposed a machine learning approach to the slope stability prediction and landslide displacement prediction, which considerably enriches the prediction method of landslides. Therefore, bank slope stability is analyzed and used only to determine whether the bank slope will collapse.

Although a relationship between bank collapse width and stability is expected, it remains unclear. Therefore, in order to establish a more comprehensive prediction method for bank collapse width, which can reflect the stage characteristics of bank collapse, this study applied the limit equilibrium theory (M-P method) to predict bank collapse width on a loess bank slope.

2. Methodologies

2.1. Reference Value of Bank Collapse Width. As field measured data of bank collapse width are generally lacking, the prediction results of the empirical graphic method were used as the reference value of bank collapse width. As shown in Figure 1, the Kachugin method takes the intersection point O of the dead water level and the original bank slope as the starting point and uses the underwater stability angle α as the inclination angle to draw the ray intersecting with the constant water level line at A . Then, the overwater stability angle β is used as the inclination angle to draw the ray intersecting with the bank slope line at B . The horizontal distance between B and C is the bank collapse width.

In the Kachugin method for predicting loess bank collapse width, the overwater stability angle β is 60° , the underwater stability angle α is 18° , and the correction coefficient is 0.6 [8, 15]. However, studies have shown that the Kachugin method is more accurate only in the downstream of the reservoir (where the ratio of the water depth and slope height is large) [16]:

$$BCW = \frac{H_1}{\tan \alpha} + \frac{H_2}{\tan \beta}, \quad (1)$$

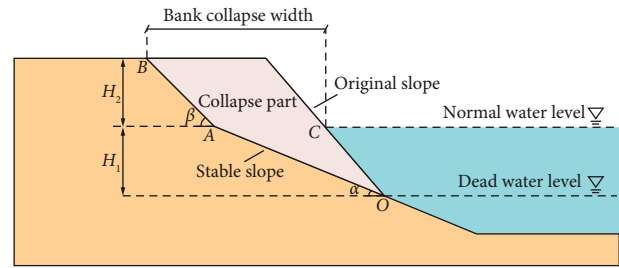


FIGURE 1: Schematic of the Kachugin method.

where BCW is the bank collapse width, H_1 is the height between the dead water level and normal water level, H_2 is the height of the above-water slope, α is the stable angle of the underwater bank slope, and β is the stable angle of the shore slope above water.

2.2. Key Parameters of the Model. As shown in Figure 2, the most dangerous sliding surface and corresponding sliding mass volume were obtained after stability calculation of any bank slope using the M-P method. The M-P method is a more rigorous method among limit equilibrium methods, which can be applied to sliding surfaces of any shape, meeting the static equilibrium conditions of the zero principal vector and the zero principal moment for each soil force. The sliding part was piled up at the foot of the slope in a certain proportion, and the rates of the accumulation part and slip part were collectively termed as the underwater accumulation rate. Then, the abovementioned steps were repeated until the slope stability factor was larger than a certain standard value (factor of safety >1.0), at which the bank collapse was considered to be terminated. The collapse width, mass volume, and stability factor of the bank slope were updated at each step. It is worth noting that, similar to the Kachugin method, the bank slope below the dead water level remained undamaged when the limit equilibrium theory was used to predict bank collapse.

The underwater accumulation rate and angle affected the prediction results of bank collapse width. As the range of the underwater accumulation angle of loess was small, only the influence of the underwater accumulation rate on the prediction results of the bank collapse width was studied. The experiment plan is as follows:

- (1) The model bank slope height was set to 10 m, 20 m, and 30 m.
- (2) The slope gradient of the model bank slope was set to 60° , 65° , 70° , 75° , 80° , 85° , and 90° .
- (3) The water depth of the model was set to 0.8 times the bank slope height; i.e., the ratio of water depth to bank slope height is 0.8.
- (4) After the bank collapse, the accumulation angle of the underwater accumulation part was set to 18° , and the underwater accumulation rates were 0, 0.1, 0.3, 0.5, 0.7, 0.9, and 1, respectively.

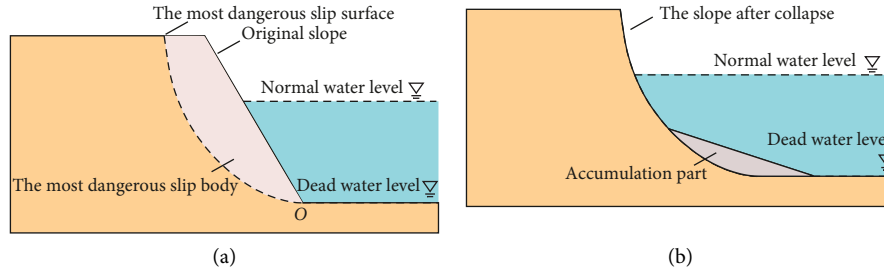


FIGURE 2: Prediction of the bank collapse process based on the limit equilibrium theory: (a) calculation of the potential slip; (b) bank slope after accumulation.

It is generally believed that the slope is in a stable state when the factor of safety is greater than 1. Therefore, the bank collapse is considered to be terminated when the stability factor exceeds 1. The physical and mechanical indexes of the bank slope soil are shown in Table 1, which were obtained through direct shear tests.

3. Results and Analysis

3.1. Determination of the Underwater Accumulation Rate. The relationship between the predicted bank collapse width and slope under different slope heights and underwater accumulation rates is shown in Figure 3. As shown in the figure, the predicted bank collapse width increased with the increasing bank slope angle under different underwater accumulation rates. The predicted bank collapse width decreased gradually with the increasing underwater accumulation rate. In contrast, the predicted bank collapse width increased gradually with increasing bank slope height. Similarly, the reasonable value of the underwater accumulation rate also increased with increasing bank slope height. Therefore, when the limit equilibrium theory is used to predict the loess bank collapse width, it is necessary to select different underwater accumulation rates for different bank slope heights and water depths. Within the range of bank slope height studied in this paper, when the ratio of water depth to bank slope height is 0.8, the relationship between the bank slope height and underwater accumulation rate is shown in the following equation:

$$a = 0.02 \times H - 0.1, \quad (2)$$

where a is the underwater accumulation rate and H is the bank slope height, m .

At a constant ratio of water depth to slope height, the relationship between the underwater accumulation rate and bank slope height was found to be linear, and the underwater accumulation rate increased with increasing bank slope height.

It is worth noting that, in the previous model test of the influence of the bank slope on bank collapse, due to the excessive interval setting of the slope angle, the bank collapse width increased monotonously with an increase in the slope angle. Ji et al. [17] pointed out that the starting point of bank collapse changes with the slope angle, which will lead to the fluctuation and upward trend of bank collapse width with

TABLE 1: Physical and mechanical indexes of the soil.

	Weight $\gamma/(\text{kN}/\text{m}^3)$	Cohesion c/kPa	Internal friction angle $\phi/^\circ$
Above water	15.6	30	30
Under water	19.0	5	20

the slope angle. This is consistent with the variation of the bank collapse width according to the limit equilibrium theory. However, the interval between the values of the underwater accumulation rate in this study is still large and can be further optimized.

3.2. Variation Trend of the Stability Factor. The variation trend of the stability factor with the number of bank collapses under different heights is shown in Figure 4. With the development of bank collapse, the stability factor of bank collapse increases linearly, which indicates that bank collapse is a process from instability to stability. The increase in the stability factor also indicates that bank collapse takes longer time than before. Meanwhile, the number of bank collapses increase continuously with increasing bank slope height. This phenomenon is consistent with the results of the field observation and model test. It indicates that the prediction of the bank collapse width based on the limit equilibrium theory is more representative of the bank collapse process, which is advantageous over the empirical graphic methods.

3.3. Variation Trend of Bank Collapse Width. The temporal variation trend of the bank collapse width under different slope heights is shown in Figure 5. A linear relationship was observed between the bank collapse width and the number of bank collapses, and the slope of the line remained essentially constant, which indicates that the bank slope angle has a significant impact only on the width of the first bank collapse. Model tests in other studies have provided similar results [18, 19]. Meanwhile, the single collapse width predicted using the limit equilibrium theory was close to the measured bank collapse width and the spacing of cracks on the loess plateau [20, 21]. Therefore, the limit equilibrium theory can also reflect the stage of bank collapse, a feature missing in the empirical graphic methods.

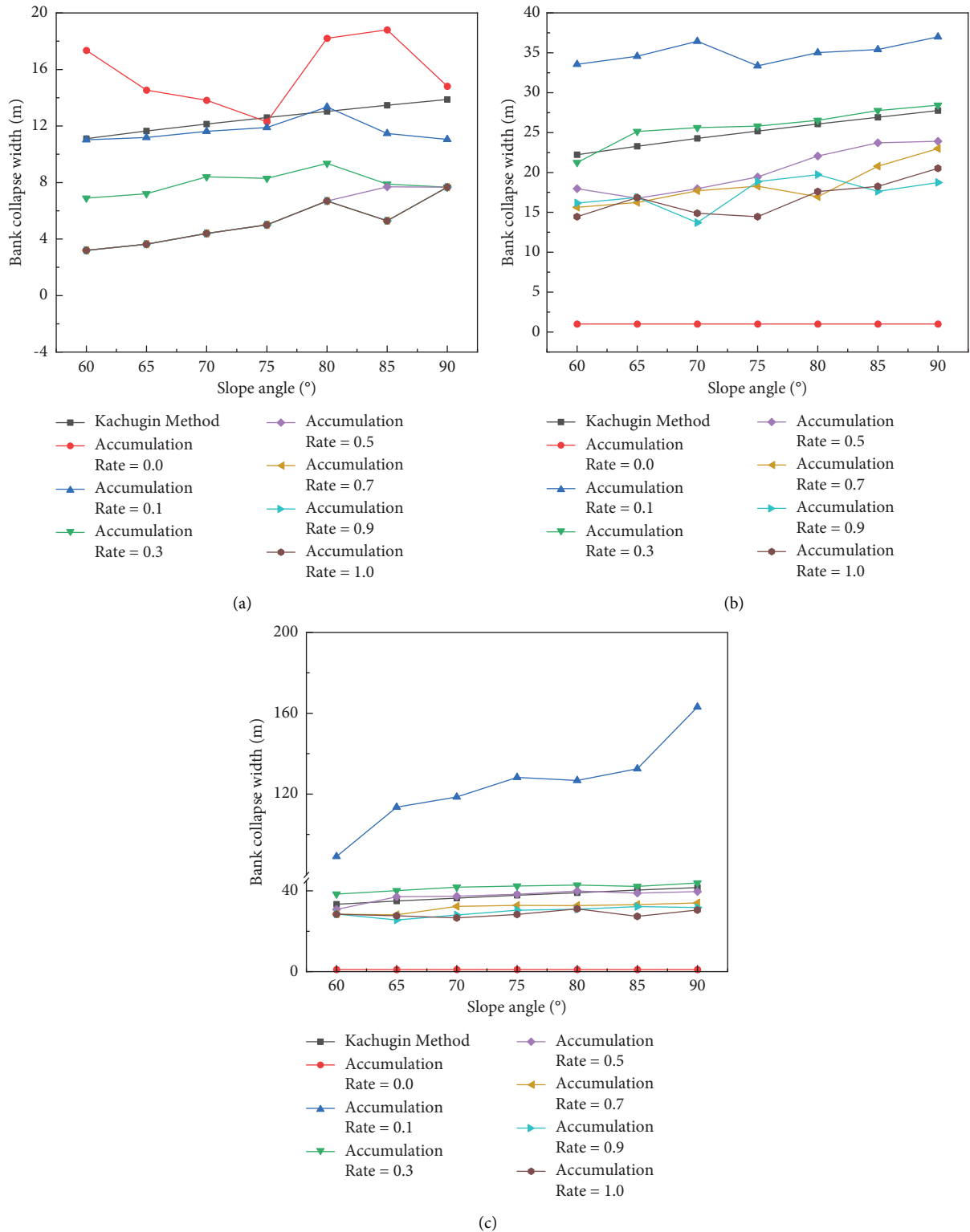


FIGURE 3: Relationship between bank collapse width and slope: (a) slope height = 10 m; (b) slope height = 20 m; (c) slope height = 30 m.

3.4. *Time-Dependent Behavior of Bank Collapse.* The time-dependent behavior of bank collapse has always been difficult to predict. The empirical graphic methods are based on the final steady state, and they cannot reflect the characteristics of the bank collapse process [7]. The M-P method

predicts the bank collapse width considering the initial state of the bank slope, and it can reflect more characteristics of the bank collapse process. Therefore, a basic model of the time-dependent behavior of bank collapse was established on the basis of the limit equilibrium theory.

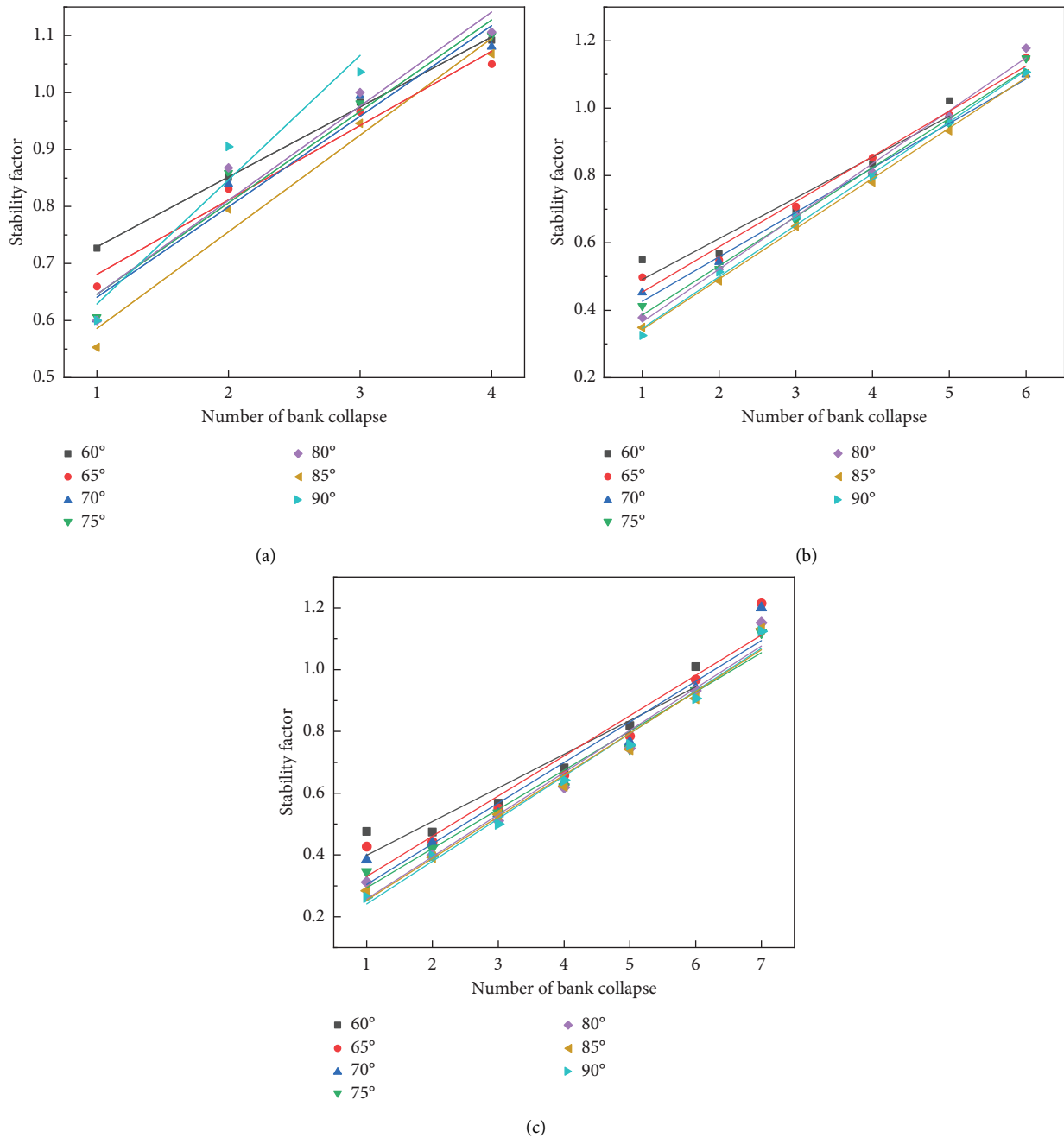


FIGURE 4: Relationship between the stability factor and number of bank collapses: (a) slope height = 10 m; (b) slope height = 20 m; (c) slope height = 30 m.

As mentioned earlier, the slope stability factor and the bank collapse width increase with the increasing number of bank collapses, with both showing linear relationships with the number of bank collapses. Any bank collapse can be simplified as a process of deformation, slip, and stability [22]. As shown in Figures 4 and 5, the number of bank collapses predicted by the M-P method is the same as the measured number of collapses. The field investigation showed that the time interval from the beginning of loess

bank collapse to stability is also consistent. Meanwhile, the time of the first bank collapse is also relatively concentrated [23–25]. Therefore, it is feasible to reflect the time-dependent behavior of bank collapse based on the numbers of bank collapses. The following assumptions were made to simplify the model:

- (1) The sliding body is regarded as a rigid body in the process of sliding.

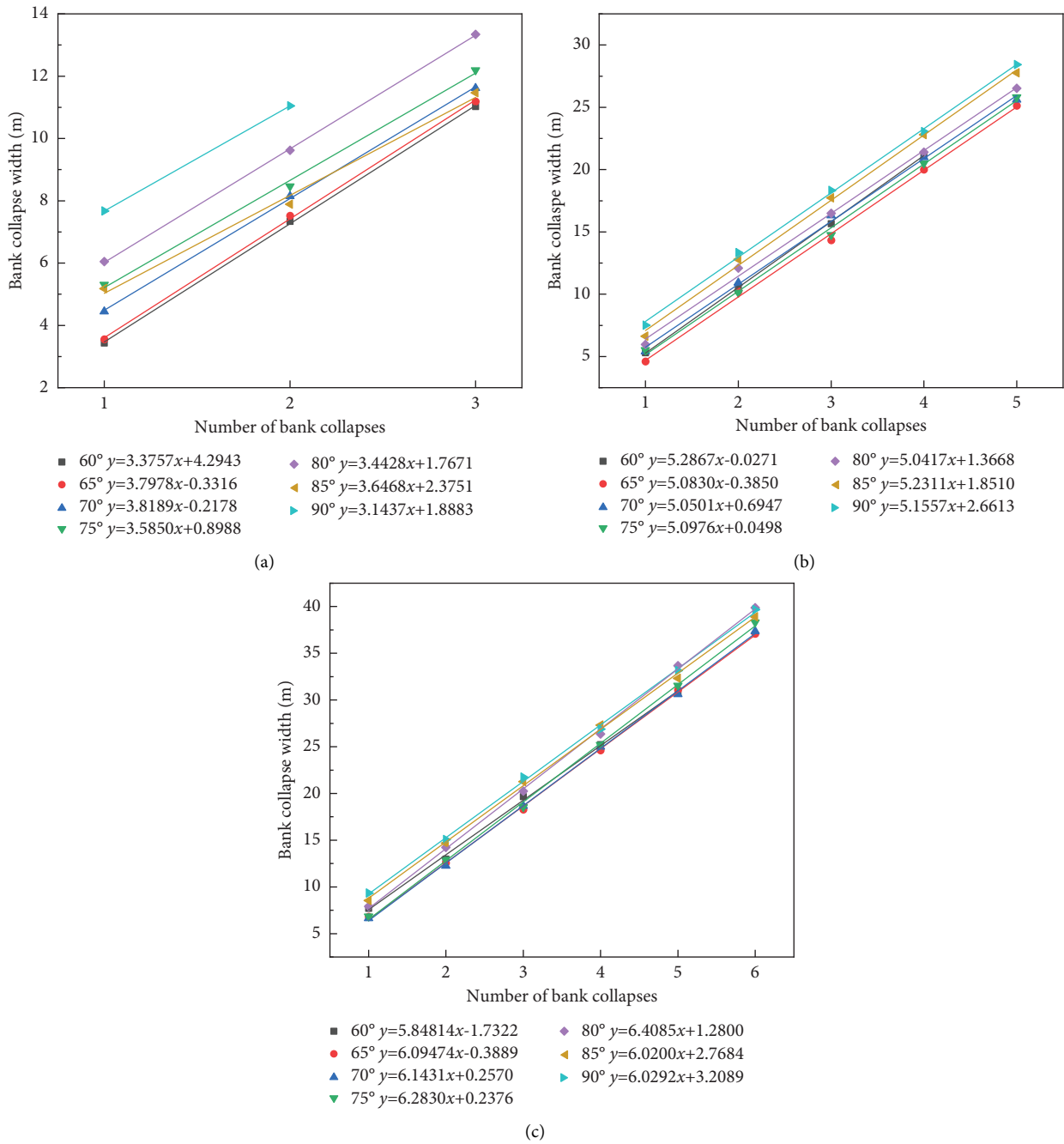


FIGURE 5: Relationship between the bank collapse width and number of bank collapses: (a) slope height = 10 m; (b) slope height = 20 m; (c) slope height = 30 m.

- (2) The horizontal movement distance of the sliding body at any stage is equal to the total width of the bank collapse at that stage.
- (3) The height of the center of gravity of the sliding body changes little at any stage.
- (4) The slip process path assumes a polygonal line; in other words, the sliding body first falls vertically under the action of self-weight and then decelerates uniformly in the horizontal direction until the speed is zero.

- (5) The time during the falling of the sliding body is ignored.

Based on the abovementioned assumptions, for any sliding body, the relationship between the final velocity v_0 and the acceleration a in the process of vertical falling is

$$v_0 \sim \sqrt{a}. \tag{3}$$

In the deceleration stage, the relationship between the deceleration time t and the total bank collapse width S and the final velocity v_0 is as follows:

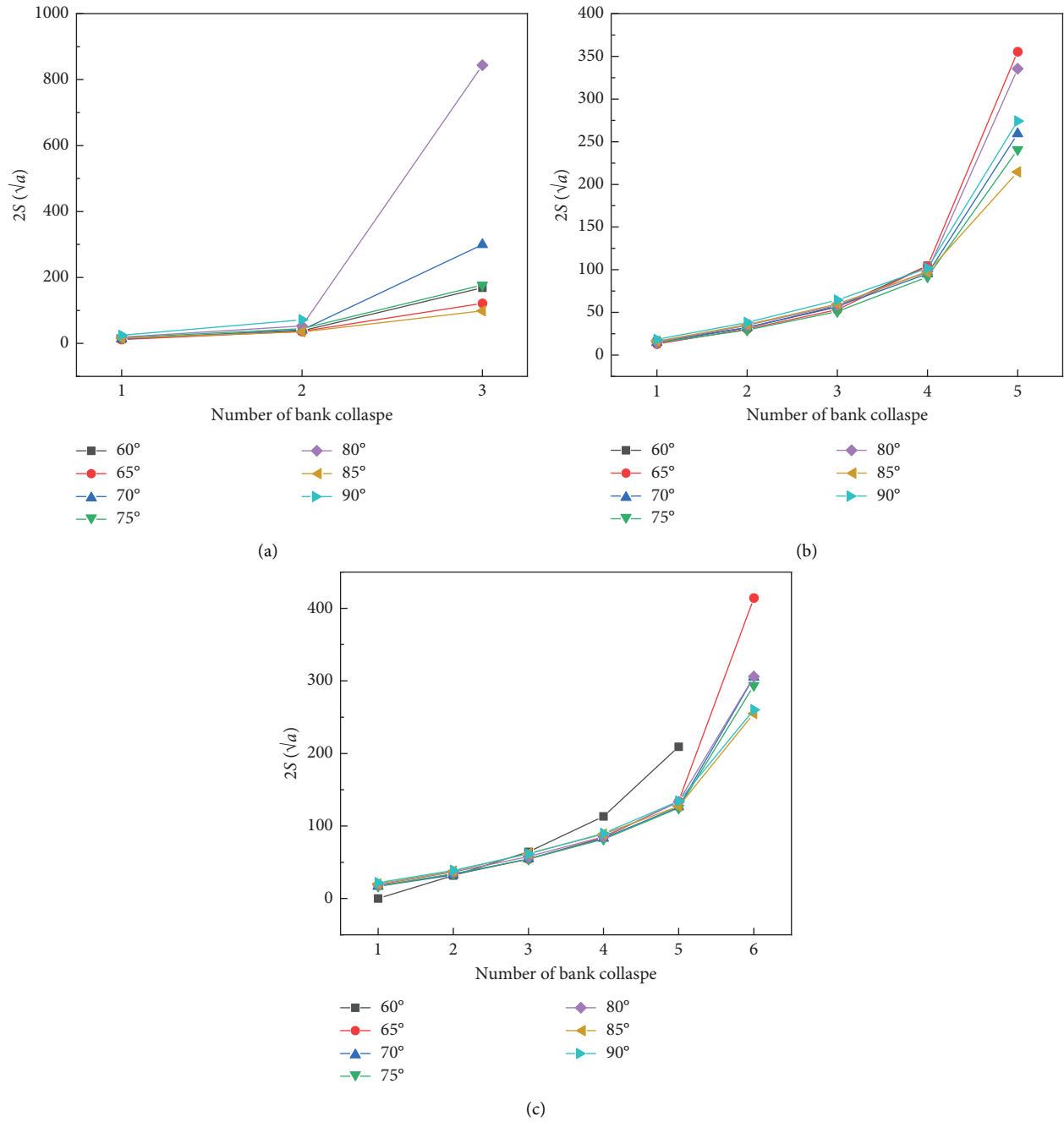


FIGURE 6: Time-dependent behavior of bank collapse based on the limit equilibrium theory: (a) slope height = 10 m; (b) slope height = 20 m; (c) slope height = 30 m.

TABLE 2: Comparison of the M-P method and the empirical graphic methods.

	M-P method	Empirical graphic methods
Prediction of the final width of bank collapse	○	○
Characterization of the final width of bank collapse	○	×
Consideration of the physical and mechanical characteristics of slope soil	○	×

*○ indicates yes, and × indicates no.

$$t = \frac{2S}{v_0}. \quad (4)$$

Then,

$$t \sim \frac{2S}{\sqrt{a}}, \quad (5)$$

where t is the time interval between the two bank collapses, S is the bank collapse width, and a is the acceleration during in the falling process.

As shown in equation (5), the time interval t of two bank collapses is related to the total bank collapse width S and acceleration a at this stage.

Based on the definition of the slope stability factor according to the limit equilibrium theory, the relationship between acceleration a and the stability factor is

$$a \sim 1 - F. \quad (6)$$

According to (3)–(6), then

$$t \sim \frac{2S}{\sqrt{1-F}} \sim \frac{2S}{\sqrt{a}}. \quad (7)$$

The time-dependent behavior of bank collapse based on the limit equilibrium theory is shown in Figure 6. As shown in the figure, the duration of the bank collapse process was essentially equal under different slope angles. It means that, under the same bank slope conditions, the time from the beginning of collapse to stability is the same. Meanwhile, with the development of bank collapse, the duration of bank collapse gradually increased and the bank collapse width increased linearly, indicating a step functional relationship between the bank collapse width and time, which agree with the results of field observations and model tests [18, 19, 22]. However, the empirical graphic methods cannot reflect this characteristic. It is worth noting that, due to the lack of measured data, it is impossible to establish the time-dependent behavior of bank collapse. Although the limit equilibrium theory presents a potential solution, a large number of observation data are still required to determine the calculation parameters of equation (7).

Compared with the empirical graphic methods, the model based on the limit equilibrium theory provides accurate predictions of bank collapse width. Second, the M-P method can better reflect the characteristics of bank collapse [26, 27], as shown in Table 2. Nevertheless, a large amount of field investigation and experimental data are still required to further improve the model.

4. Conclusion

A prediction model for the bank collapse width was established based on the limit equilibrium theory. The key parameters of the model were set, and the variation trends of the bank slope stability factor and bank collapse width were analyzed. Finally, the time-dependent behavior of bank collapse was analyzed. From the results, the following conclusions can be drawn:

- (1) It is necessary to select a reasonable underwater accumulation rate when the limit equilibrium theory is used to predict bank collapse width. The underwater accumulation rate is related to the water depth and bank slope height.
- (2) The bank slope angle mainly affects the width of the first bank collapse and has little effect on the width of subsequent bank collapses. With the increasing bank slope angle, the bank collapse width fluctuates and rises.
- (3) The prediction of the bank collapse width based on the limit equilibrium theory is more representative of the characteristics of the bank collapse process compared to empirical graphic methods.

It is worth noting that, due to the lack of measured data, it is difficult to improve the prediction model. The parameters of the model such as underwater accumulation angle, water depth, and bank slope height remain to be studied.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

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