

# Research Article

# Experimental and Numerical Investigation of One-Dimensional Electroosmotic Consolidation Based on Segment Inheritance Strategy

# Xiaoyu Yang<sup>(1)</sup>, Meijuan Pei<sup>(1)</sup>, Yongbin Xie<sup>(1)</sup>, Xinhai Zhou, and Xuguang Dong<sup>4</sup>

<sup>1</sup>School of Civil Engineering, Lanzhou Institute of Technology, Lanzhou 730050, China
 <sup>2</sup>School of Civil Engineering, Lanzhou Petrochemical University of Vocational Technology, Lanzhou 730060, China
 <sup>3</sup>School of Civil Engineering, Tianshui Normal University, Tianshui 741000, China
 <sup>4</sup>School of Civil Engineering, Ningxia University, Yinchuan 750021, China

Correspondence should be addressed to Xiaoyu Yang; yangxy@lzit.edu.cn

Received 15 September 2023; Revised 19 December 2023; Accepted 27 December 2023; Published 17 January 2024

Academic Editor: Fujiao Tang

Copyright © 2024 Xiaoyu Yang et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

The attenuation of effective potential is a significant factor leading to the low postdrainage efficiency in the electroosmotic consolidation and drainage methods. In this study, the influence of the attenuation of effective potential on the electroosmotic consolidation process and the problem of solving the governing equations were investigated. A 102.5-hr one-dimensional electroosmotic consolidation test was performed to monitor the changes in the properties of the soil interior and the soil-electrode interface, and the variation curve of the effective potential in the test model was measured. The time-dependent nature of the potential distribution due to the attenuation of effective potential contradicts the assumption of introducing intermediate variables. To address this key issue, the variation curve of the effective potential was linearly segmented, ensuring the validity of introducing intermediate variables within each local state segment. Based on the continuity between state segments, the initial conditions of the governing equations in different state segments were updated, thereby extending the differential iteration within local state segments to the entire electroosmotic time domain. A finite-difference program for this method was developed using the Python language. Calculations and analyses based on the measured potential data were performed, revealing that a decrease in potential leads to a reduction in the effectiveness of electroosmotic drainage and consolidation in terms of the instantaneous distribution of pore pressure and the overall average degree of consolidation. This method can reflect the influence of the attenuation of effective potential on the pore water pressure during the electroosmotic consolidation process. The research findings of this paper can provide theoretical and numerical support for the improvement and engineering application of the electroosmotic consolidation and drainage method.

# 1. Introduction

Electroosmotic consolidation drainage method is a favorable technique for the reinforcement of soft soils. Since its first application in practical engineering in 1938, it has gradually gained attention from the engineers and scholars. Due to the small particle size of soft soils, consolidation drainage through hydraulic permeation is inefficient, and excessive external loading can lead to shear failure of the soft soil. Therefore, the reinforcement of soft soil foundations has always been a challenging engineering problem. Electroosmotic consolidation drainage utilizes the potential gradient generated by applying voltage at both ends of the soil to facilitate drainage. Due to the insensitivity of the electroosmotic coefficient to soil particle size, electroosmotic consolidation drainage has received continuous attention from the researchers in recent years and has become a hot topic. Currently, with the vigorous development of infrastructure, marine engineering, and water conservancy projects in coastal areas, the reinforcement of soft soil foundations has become an unavoidable engineering problem. Therefore, based on the current development needs, electroosmotic consolidation drainage has received increasing attention in recent years.

The main drawback of an electroosmotic consolidation is its relatively low efficiency in the later stages, which means that the applied voltage cannot effectively drive the pore water, resulting in higher energy consumption during the later stages of electroosmosis. In order to investigate the performance characteristics of factors influencing electroosmosis, researchers have conducted experimental studies to identify the factors that affect the effectiveness of electroosmosis. Different chemical additives have different degrees of promotion in the electroosmosis process, and the introduction of external additives can achieve a more uniform distribution of the reinforced soil strength [1, 2]. The concentration of the salt solution also affects the electroosmotic drainage effect, as excessively high concentrations can actually reduce the effectiveness of electroosmotic drainage [3]. Changes in soil properties during the electroosmosis process, as well as electrode corrosion and exothermic chemical reactions, can affect the drainage efficiency and the properties of the reinforced soil, leading to deviations in the theoretical calculation models [4, 5]. Evaluating the efficiency of electroosmotic drainage from the perspective of energy loss is a new approach that incorporates soil thermal energy and interfacial energy into the evaluation model, which can accurately describe the efficiency of electroosmotic drainage [6]. The electrode material, soil moisture characteristic curve, changes in soil resistance, and pH value all affect the efficiency and energy consumption of electroosmosis [7-12]. The electroosmotic permeability coefficient is an important parameter in the electroosmotic consolidation process. The results obtained from the Helmholtz-Smoluchowski model have significant errors, and incorporating the migration of electroosmotic ions into the model can better explain the phenomenon of electroosmosis [13]. Some scholars have combined this method with vacuum preloading and surcharge preloading to form a combined approach. Comparative experiments have also shown that the combined approach can overcome the low efficiency issue in the later stages of electroosmosis, fully utilizing the advantage of highinitial electroosmotic drainage rate. At this point, electroosmosis is more suitable to be described using an axisymmetric model. The presence of smear zones will reduce the consolidation degree and increase the final settlement [2, 14-16].

The electroosmotic consolidation theory is an important topic in the study of electroosmotic drainage. Through theoretical models, the non-steady-state process of electroosmosis can be quantitatively described, thereby obtaining an understanding of the dissipation law of pore water pressure. Esrig [17] first established a one-dimensional electroosmotic consolidation theory. The Esrig theoretical model requires that the potential distribution during the electroosmotic process must be independent of time; otherwise, there will be significant difficulties in solving the equations. Subsequent scholars, based on Esrig's basic ideas, continuously expanded the electroosmotic consolidation theory and combined it with numerical methods for calculations and analysis [18]. The inclusion of more influencing factors into the theoretical model is the main direction of theoretical research. Subsequently, electroosmotic consolidation models were gradually established, considering the large deformation properties of the soil [4, 19, 20], as well as the variations in soil properties [21]. The introduction of soil properties makes the theoretical

model more consistent with the actual conditions. With the emergence and development of coupled methods, corresponding theoretical models have also been developed. Axisymmetric models have been applied to describe this new method. In this case, the conductivity of the smear zone soil has a significant influence on the distribution of pore water pressure. Both vertical and radial flows are included in the theoretical model [16, 22, 23]. The proposed analytical solutions can better help researchers to understand the mechanism of electroosmotic drainage. In addition, the process of soil desiccation leads to changes in electrical conductivity, thereby altering the distribution of the electric field. Of course, the changes in the electric potential field also need to consider the influence of interface properties caused by electrode corrosion. Therefore, it is necessary to consider the impact of electric potential distribution on pore water pressure [8, 24-26]. The introduction of electrode spacing and nonlinear electric potential distribution further improves the rationality and reliability of the electroosmotic consolidation theory model.

However, it is objectively acknowledged that changes occur in the properties of the soil and the soil–electrode interface during the electroosmotic process, and the corrosion of metal electrodes is inevitable due to the electroosmotic mechanism. Consequently, these objective factors significantly affect the effectiveness of electroosmotic consolidation by causing changes in the electrical properties of the model. Existing theoretical models mainly consider the nonlinearity of the soil and the nonlinear characteristics of potential distribution, but they overlook the impact of the reduction in effective potential during the actual process, thus failing to adequately reflect the extent of efficiency decline in the later stages of electroosmosis.

Therefore, this study focuses on the one-dimensional electroosmotic consolidation theory under conditions of effective potential variation and conducts one-dimensional electroosmotic tests to monitor changes in drainage volume and current during the electroosmotic process. To solve the electroosmotic consolidation problem under conditions of effective potential variation, a computational method based on linear segmentation and inheritance is proposed. This method keeps the intermediate variables within their respective local segments and utilizes the continuity of the local segments in the time dimension to update the initial conditions of the local segments. The discrete form of the governing equation is obtained using the finite-difference method, and a dedicated finite-difference numerical calculation program is developed using the Python language. The program is then used to perform one-dimensional electroosmotic consolidation calculations based on measured potential data, and a comparison and analysis are conducted with the classical Esrig's results. This enables the quantitative calculation of the dissipation process of pore water pressure due to the variation in effective potential.

## 2. One-Dimensional Electroosmotic Test

2.1. Test Methods and Materials. The one-dimensional electroosmotic consolidation drainage method was studied using



FIGURE 1: Distribution and photos of experimental models. (a) Geometric dimensions and measurement point distribution of one-dimensional electroosmotic test model; (b) test model photos.

a Miller box apparatus. The electroosmosis model had dimensions of 25 cm in length, 15 cm in width, and 20 cm in height. Isolation grooves were provided on both sides of the model, and a drainage hole was located at the bottom for the collection and discharge of water from the soil. The soil used was silty clay, obtained from a construction site excavation. The soil sample was dried, crushed, and reconstituted before the experiment. During the test, the soil was saturated, and the initial moisture content of the soil before electroosmosis was approximately 34.6%. The electroosmosis power supply used was a DC power supply, model PS-605D, with a maximum supply voltage of 60 V and a maximum current of 5 A. The voltage applied during the electroosmosis test was 25 V, with an electric potential gradient of 1 V/cm. The metal probe had a height of approximately 5 cm and was connected to lead wires for potential measurement. Stainless steel plate electrodes were used, with the anode plate being solid and the cathode plate having drainage holes for the removal of pore water.

The plan layout of the experimental model is shown in Figure 1. Figure 1(a) depicts the plan layout of the electroosmosis model. In order to measure the interface contact voltage at the electrode outlets, metal probes were placed approximately 1 cm away from both the anode and cathode plates. Additional metal probes were placed at intervals of 4.6 cm in the middle section. The potential measurement points were numbered accordingly, with the cathode being labeled as point 0, and the metal probes in the anode direction numbered as 1, 2, 3, 4, 5, 6, with probe 7 being the anode plate. The drainage holes at the bottom of the isolation grooves on both sides had a diameter of 2 cm. Small crushed stones were added to the isolation groove on the cathode side to prevent soil blockage of the drainage holes. A photograph of the experimental model is shown in Figure 1(b).

After allowing the prepared soil sample to settle for 24 hr to ensure a uniform distribution of moisture content, the soil sample is placed into the experimental box. Following the predetermined positions of the metal probes mentioned earlier, the probes are inserted into the soil one by one, and their external wires are secured to prevent probe displacement during measurements. Crushed stones are added to the cathode isolation groove to prevent soil blockage of the drainage

holes caused by the expulsion of pore water. A collection box is placed below the cathode to collect the water discharged during the electroosmosis process. The electrode plates are connected to the DC power supply, and the power is turned on, adjusting the voltage to 25 V, which is maintained throughout the experiment. During the electroosmosis test, the potential values at the probe locations, drainage volume, and current are measured and recorded every hour during the initial stage of the test, only during daylight hours. Measurements are not taken during the night, although the power supply remains on. The total duration of the electroosmosis test is 102.5 hr. Probes 0 and 7 are directly connected to the electrode plates. Therefore, the potential at probe 0 remains at 0 V throughout the experiment, while the potential at probe 7 remains consistent with the power supply voltage at 25 V. The potential values at probes 1-6 are measured using a multimeter, and simultaneously, the reading of the electronic scale under the collection box is recorded.

2.2. Analysis of Test Results. The total duration of electrification was 102.5 hr. The photograph of the soil specimen after electroosmosis is shown in Figure 2. Figure 2(a) displays the entire soil sample after electroosmosis, where a reduction in the overall size of the soil model is observed due to consolidation and drainage. The interface between the cathode plate and the soil has detached, resulting in a loss of applied voltage, as depicted in Figure 2(b). During the electroosmotic consolidation process, the pore water in the soil flows directionally under the potential gradient, from the anode to the cathode. Consequently, the moisture content near the anode decreases significantly, leading to the best reinforcement effect. However, due to water loss, the volume reduction is most pronounced at this location, resulting in a significant decrease in contact between the soil and the anode plate. Additionally, corrosion of the electrode is observed at the anode end, as shown in Figure 2(c). Therefore, the contact between the electrode plate and the soil during electroosmosis is influenced by the consolidation characteristics of the soil, leading to a decrease in contact. Furthermore, the hydrolysis and redox reactions at the anode site cause corrosion of the metal electrode plate, resulting in the generation of corrosive substances that further reduce the contact



FIGURE 2: Photos of model after electroosmosis. (a) Photographs of the soil specimen after electroosmosis; (b) soil sample at the cathode end after electroosmosis; (c) soil sample at the anode end after electroosmosis.



FIGURE 3: Model current variation curve and resistance variation curve.

between the soil and the electrode plate. All these factors have an impact on the applied voltage during electroosmosis.

During the electroosmosis process, the power supply voltage is always maintained at 25 V, and the current value can be displayed in real-time by the power supply. Metal probe P0 is directly connected to the cathode plate, so its potential is 0 V. The potential value of metal probe P1 represents the contact voltage between the cathode and the soil. The cathode interface resistance can be calculated using Ohm's law. Similarly, metal probe P7 is connected to the anode plate, and its potential value is always 25 V. The potential difference between metal probe P6 and the total voltage of 25 V represents the anode interface voltage. Since

this model belongs to a series circuit, the resistance of this part can be directly calculated. The total resistance of the model can be obtained by dividing the total voltage of 25 V by the current value. Figure 3 shows the current variation curve in the electroosmosis model and the calculated model resistance variation curve. The current curve generally shows a decreasing trend. In the first 20 hr, the rate of current decrease is high. Between 20 and 45 hr, the rate of current change decreases, and after 45 hr, the current change is relatively small. The initial current value is 0.412 A, and at the end of the electroosmosis process, the current value is 0.031 A, which represents an overall decrease of 92.5%. Since the power supply voltage is always maintained at 25 V during the electroosmosis process, the change in current indirectly reflects the change in the overall resistance of the model. The overall resistance of the model shows a linear increasing trend. The initial total resistance of the model is  $60.68 \Omega$ , and at the end of the electroosmosis process, the overall resistance reaches 806.45  $\Omega$ . The cathode interface resistance shows the smallest overall change, increasing from  $7.14 \Omega$  at the beginning to 111.29  $\Omega$  at the end, an increase of nearly 15 times. The cathode interface resistance shows a linear change trend overall. The initial anode interface resistance is  $8.11 \Omega$ , and at the end, it is 159.68  $\Omega$ . Before 75 hr of electroosmosis, the cathode interface resistance is higher than the anode interface resistance. After 75 hr, the rate of change in the anode interface resistance is higher. The change in resistance of the model essentially reflects the drainage effect of the soil. The decrease in soil moisture content is reflected as an increase in resistance at the electrical level. The increase in interface resistance will lead to an increase in interface contact voltage, thereby reducing the voltage applied to the soil. The main drawback of the electroosmosis method is its low efficiency in the later stages, and the generation of interface contact resistance is an important factor affecting the efficiency of electroosmosis.



FIGURE 4: Changes in displacement, current, and voltage during one-dimensional electroosmotic testing. (a) Electroosmotic drainage curve and test current variation curve; (b) interface voltage and effective potential variation curve.

Figure 4 illustrates the drainage conditions and voltage variations of the soil during the electroosmosis process. The drainage volume during the electroosmosis process was determined by measuring the mass of the collection box. Figure 4(a)presents the drainage volume curve and drainage rate curve of the model. In the electroosmosis test, the test was terminated when the difference in drainage volume between two consecutive measurements was less than 0.1 g, indicating that the electroosmosis had reached the limit state. The test was terminated at 102.5 hr with a drainage volume of 1,173.9 g. The drainage rate curve is also shown in Figure 4(a), indicating a decreasing trend as the electroosmosis progresses. After 75 hr, the drainage rate stabilizes. The drainage rate was 40.20 g/hr after 1 hr of electrification, ranging from 7 to 10 g/hr during the mid-stage of electroosmosis, and approximately 2.6 g/hr at the end of the process. The overall drainage rate during electroosmosis follows an exponential decay function. The lower drainage rate in the later stage reflects the inefficiency of the electroosmosis method during this period.

Figure 4(b) displays the variations in effective potential and interface voltage during the electroosmosis process. The cathode interface voltage was directly obtained by measuring the potential of metal probe P1. In the initial stage of electroosmosis, the cathode interface voltage initially increased from 2.94 to 8.08 V before decreasing. This phenomenon can be attributed to the high-initial saturation of the soil and the presence of free water in the model. The proximity of the cathode to the drainage boundary results in rapid decrease in soil moisture content at the cathode end due to free seepage. The anode interface voltage, on the other hand, exhibited a linear increasing trend. The initial anode interface voltage was 3.34 V, which increased to 4.95 V at the end of the electroosmosis process. The sum of the cathode and anode interface voltages represents the total interface voltage of the model, which includes the loss voltage during the electroosmosis process. By subtracting this loss voltage from the power supply voltage of 25 V, the effective voltage during the electroosmosis process can be obtained. The effective voltage rapidly decreased from 18.72 to 13.53 V initially, and then slowly increased after 7 hr. At the end of the electroosmosis process, the effective potential value was 16.60 V. Therefore, monitoring the effective potential during electroosmosis indicates that it undergoes changes throughout the process, exhibiting an overall decay. The effective potential values at different time points should be considered in the calculation of electroosmotic consolidation to obtain results that better align with the real-world conditions.

The potential distribution of the soil specimen during the experiment was monitored using preplaced metal probes P1–P6. The arrangement of the metal probes is shown in Figure 1(a), and the potential variations at the measurement points during the electroosmotic process are presented in Figure 5. Figure 5(a) displays the potential variation curves of the six metal probes, with P1 being closer to the cathode. The potential at P1 represents the cathode interface voltage, as previously analyzed. As the measurement points move closer to the anode, the magnitude of the potential increase decreases. The potential at P7 exhibits a linear decreasing trend, starting at 22.16 V and ending at 20.05 V, indicating a decrease of 4.96% after the completion of electroosmosis.

The potential variation curves at P2–P5 indicate that the potential values at these points reach their peak approximately 24 hr after the start of electroosmosis. P2 and P3 show a significant increase in potential. At the initial moment, the potential at P2 is 6.74 V, which reaches a peak of 15.89 V after 24 hr, representing an increase of approximately 135.76%. Subsequently, the potential gradually decreases and reaches 9.12 V at the end of the process. The potential at P3 increases



FIGURE 5: Monitoring point potential changes and soil potential distribution. (a) Potential variation curve at monitoring points; (b) soil potential distribution curve at different times.

by approximately 106.61% after 24 hr, with a final potential value of 11.17 V at the end of electroosmosis.

By using the metal probes, the potential distribution at different locations within the soil specimen can be monitored over time. The variations in potential at different measurement points are mainly attributed to the loss of water content in the soil during the electroosmotic process, which consequently alters the soil's electrical properties. Figure 5(b) presents the potential distribution curves at different time intervals, indicating that the potential distribution within the soil specimen changes over time. However, overall, the distribution follows a linear pattern. The factors influencing the distribution can be summarized into two aspects. First, changes in effective potential, resulting from variations in the boundary conditions of the potential field, directly affect the magnitude of the distribution. Nevertheless, the pattern remains linear, consistent with the phenomenon reflected in Figure 5(b). Second, the decrease in water content due to the drainage of pore water alters the electrical properties of the soil. The nonuniform distribution of water content at different locations leads to an uneven distribution of the electrical properties. Therefore, the potential distribution within the soil during the electroosmotic process does not strictly follow a linear pattern.

Figure 6 depicts the spatiotemporal distribution of the electric potential at the metal probes. This color contour map reflects the variations in potential at different measurement points in both spatial and temporal dimensions. The potential at points P2 and P3 exhibits significant amplitude changes over time, which aligns with the patterns observed in Figure 5(a).

Figure 7 presents violin plots of the electric potential data at each metal probe. In Figure 7(a), the distribution of measured potential data at P1–P3 is displayed. P1, being close to the cathode, exhibits a relatively small range of potential

										- 25
	0 -	0.0	2.9	6.7	8.0	14.0	18.9	21.7	25.0	- 23
Time (hr)	4 -	0.0	7.7	11.6	12.6	16.9	20.3	22.4	25.0	
	∞ -	0.0	8.5	15.8	16.3	19.1	21.2	22.2	25.0	- 20
	12	0.0	7.8	16.1	16.7	19.7	21.5	22.3	25.0	
	16	0.0	8.1	15.8	16.5	19.5	21.3	22.1	25.0	- 15
	20	0.0	7.0	15.9	16.5	19.2	21.3	21.9	25.0	
	24	0.0	6.4	15.2	16.3	18.2	21.2	21.6	25.0	
	- 28	0.0	6.0	14.7	16.1	18.1	21.3	21.7	25.0	- 10
	32	0.0	5.7	14.3	15.7	17.7	21.2	21.6	25.0	
	36	0.0	4.7	12.4	14.1	16.2	20.4	20.8	25.0	- 5
	40	0.0	4.2	11.2	13.2	15.0	19.8	20.1	25.0	
	44	0.0	3.4	9.8	11.9	14.1	19.8	19.9	25.0	0
	(	Cathod	e P1	P 2	P 3	P 4	P 5	P 6	Anode	- 0
					Pro	be				

FIGURE 6: Heatmap of spatiotemporal distribution of potential data at monitoring points.

variations, resulting in a concentrated distribution and a wider color-filled area with a smaller height in the colored region. On the other hand, P2 and P3 probes demonstrate larger variations in potential, with a wider range of data points, leading to a narrower color-filled area but with a greater height. Similarly, in Figure 7(b), the potential data distribution at P4 appears more scattered, while P5 and P6, located closer to the anode, exhibit a relatively concentrated distribution of potential data. Through analysis, it can be observed that the potential variations diminish as the measurement points approach the electrode positions, whereas the potential variations are more pronounced in the intermediate region of the soil.



FIGURE 7: Violin diagram of measured potential data at measuring points. (a) Distribution of potential data at P1, P2, and P3; (b) distribution of potential data at P4, P5, and P6.

#### 3. Calculation Method and Process

3.1. Key Issues in Computing. Esrig proposed a one-dimensional electroosmotic consolidation theory, which posits that the potential distribution remains constant throughout the entire potential field, independent of time. The distribution of potential is derived based on the theory of electrostatic fields, and in one-dimensional space, the potential distribution function is a linear function. Due to the time-invariant nature of the potential distribution term, when solving the governing equations, an intermediate variable can be introduced to transform the equations into standard parabolic partial differential equations (PDEs). This allows for the analytical solution to be obtained under simple boundary conditions. This general approach represents the typical treatment for solving equations in classical electroosmotic theory, and has found widespread application. The governing equation for one-dimensional electroosmotic consolidation is shown in Equation (1) as follows:

$$\frac{m_{\nu}\gamma_{w}}{k_{h}}\frac{\partial u(x,t)}{\partial t} = \frac{\partial^{2}u(x,t)}{\partial x^{2}} + \frac{k_{e}\gamma_{w}}{k_{h}}\frac{\partial^{2}V(x)}{\partial x^{2}},\qquad(1)$$

where u(x, t) is the pore water pressure at position x at time t, a function of time t and spatial coordinates x;  $m_v$  is the soil volume compression coefficient, with units of Pa<sup>-1</sup>;  $\gamma_w$  is the unit weight of water, in units of N/m<sup>3</sup>;  $k_h$  is the hydraulic conductivity of the soil, in units of m/s;  $k_e$  is the electroosmotic permeability coefficient, in units of m  $\cdot$  (V  $\cdot$  sec)<sup>-1</sup>; and V(x) is the potential distribution function, in units of Volt.

Esrig introduced an intermediate variable  $\zeta(x, t)$ , which is expressed as follows:

$$\zeta(x,t) = u(x,t) + \frac{k_e \gamma_w}{k_h} V(x).$$
<sup>(2)</sup>

By introducing the intermediate variable, a parabolic partial differential Equation (3) can be obtained. The solution to this equation can be achieved through analytical or numerical methods, thereby completing the description of the pore water pressure distribution in electroosmotic consolidation.

$$\frac{\partial \zeta(x,t)}{\partial t} = C_{\nu} \frac{\partial^2 \zeta(x,t)}{\partial x^2}, \qquad (3)$$

where  $C_v$  is the coefficient of consolidation of the soil, with units of  $m^2/s$ ,  $C_v = k_h/m_v \gamma_w$ .

During the electroosmotic process, the variation of effective potential is objectively present, and numerous experimental studies have confirmed its existence. Previous experimental research has also indicated the phenomenon of attenuation in the effective potential during the electroosmotic process. In the electroosmotic drainage process, as pore water is discharged, the electrical properties of the soil undergo significant changes. The cathode region is closest to the drainage boundary, while the anode region has the highest voltage, resulting in the most significant changes in the electrical properties at the interface between the electrodes and the soil. The effective potential serves as the boundary condition for the potential field, and the calculation of potential distribution in the electrostatic field belongs to a boundary value problem. Therefore, the variation in boundary conditions determines the magnitude of the potential at each point within the field. Neglecting this variation would lead to an overestimation of the effectiveness of electroosmotic consolidation in the soil. In practical electroosmotic experiments, the main reasons for the variation in effective potential can be summarized as follows:

- During the dewatering process of the soil, the reduction in soil volume leads to a decrease in the contact between the electrodes and the soil. This results in the generation of interface resistance or interface voltage, leading to a decrease in the applied voltage for electroosmosis in the soil;
- (2) The electroosmotic process is a complex electrochemical process. At the electrodes, gas evolution

occurs due to electrolysis reactions. The overflow of gas at the electrode-soil interface reduces the contact between the electrodes and the soil, which is another reason for the decrease in effective potential during the electroosmotic process;

(3) Metal electrodes undergo corrosion during electrochemical reactions. The formation of other substances on the electrode surface due to corrosion also reduces the contact between the electrodes and the soil.

Therefore, the loss of effective potential during the electroosmotic process is inevitable. The voltage applied to the soil ends up being lower than the total voltage. Additionally, it is important to consider that this variation is solely caused by the electroosmotic process itself, making it a timedependent issue. When calculating the electroosmotic consolidation equation considering the variation in effective potential, the main challenge lies in how to introduce an intermediate variable to transform the governing equation into a standard parabolic PDE. The major difficulty arises from the fact that the potential distribution term is timedependent, while the introduction of the intermediate variable requires the potential to be independent of time. These two requirements contradict each other, rendering the solution to this problem infeasible. The existence of this contradiction is the key issue in solving the electroosmotic consolidation equation under the variation of effective potential. Therefore, the critical constraint points are as follows:

- (i) The introduction of the intermediate variable ζ requires that the potential distribution term V(x) must be time independent before the original control equation can be transformed into Equation (1) to complete the solution;
- (ii) The variation of electric potential in electroosmosis is objective, and as the electroosmosis process progresses, the effective electric potential will change. At this point, the electric potential distribution term should be a function of time.

The two questions mentioned above are mutually conflicting. The key issue in solving the calculation of electroosmotic consolidation under effective potential variation lies in how to handle the contradiction between the two. Therefore, new computational methods or approaches need to satisfy the following characteristics.

- Developing a suitable mathematical framework that can reconcile the time-dependent nature of the potential distribution term with the requirement of time-independence for the introduced intermediate variable.
- (2) Finding an appropriate method to accurately model and incorporate the variation in effective potential into the governing equation.
- (3) Developing numerical or analytical techniques that can effectively solve the resulting parabolic PDE, considering the time-dependent nature of the problem.

3.2. Strategies for Linear Segmentation and Inheritance. Through the experiment, it is observed that the voltage applied to the ends of the soil specimen during the electroosmotic process undergoes variations. This variation is primarily caused by changes in the interface properties between the electrodes and the soil. Therefore, the changes in effective potential are induced by electroosmosis. Consequently, the electroosmotic process and the variation in effective potential occur simultaneously and are unified in the temporal dimension. Considering this, the effective potential changes as electroosmosis progresses. The effective potential is a boundary condition of the potential field, and changes in the boundary conditions lead to variations in the potential distribution within the soil specimen. Thus, the potential distribution should be expressed as a function of spatial coordinate *x* and time *t*, denoted as V(x, t). If the variation in effective potential is taken into account in the calculations, the time variable t will be included in the potential distribution term, which can introduce difficulties in solving the equations. This difficulty arises mainly during the introduction of the intermediate variable  $\zeta(x, t)$  and the transformation from Equation (2) to Equation (3) cannot be obtained, resulting in challenges in the solution process.

To address this issue, a linear segmentation approach can be applied to the effective potential variation curve. Within each segment, the effective potential is assumed to be constant, while abrupt changes in potential values can occur between adjacent segments. By using a method similar to Riemann sum integration, the effective potential variation curve can be divided into segments. In this approach, the effective potential within each local segment is considered constant and independent of time. The overall variation is described by the abrupt changes between segments. This approach allows for the description of the variation of effective potential with time while maintaining constant values within each segment, satisfying the requirements for introducing the intermediate variable. Please refer to Figure 8 for an illustration of this approach.

After linearly segmenting  $V_e(t)$ , the effective potential within a specific segment k is denoted as  $V_e^k$ . In a onedimensional model, the potential distribution follows a linear pattern. Since the potential at the cathode is 0 V, the potential distribution function can be represented as a proportional function. Therefore, the potential distribution within segment k can be expressed as Equation (4):

$$V^k(x) = \frac{V_e^k}{L} \cdot x, \tag{4}$$

where *L* represents the distance between the anode and the cathode, and  $V_e^k$  represents the potential value within segment *k*.

By linearly segmenting the effective potential, the introduction of the intermediate variable remains valid within each segment, effectively limiting Esrig's solution method to a single segment. However, the consolidation process during electroosmosis requires calculations over the entire time



FIGURE 8: Schematic diagram of effective potential linear segmentation.

domain, which currently can only be achieved within each segment. Experimental studies and the mechanism of electroosmosis indicate that the variation in effective potential is caused by changes in soil or interface properties during the electroosmotic process. The variation in effective potential is entirely due to the electroosmotic process, indicating a unified time process between the two. Therefore, the pore water pressure at the end of segment *k* must be equal to the initial pore water pressure at the beginning of segment k + 1, satisfying Equation (5) as follows:

$$u^{k}(x, t_{e}) = u^{k+1}(x, 0),$$
(5)

where  $u^k(x, t)$  represents the pore water pressure within the (k) th segment,  $t_e$  represents the duration of that segment, and  ${}^{u}(k+1)(x, t)$  represents the pore water pressure within the (k+1)th segment.

Based on this approach, the introduction of the intermediate variable  $\zeta(x, t)$  remains valid within each segment, as shown in Equation (6):

$$\zeta^k(x,t) = u^k(x,t) + \frac{k_e \gamma_w}{k_h} \frac{V_e^k}{L} \cdot x.$$
(6)

Within segment k, the original governing equation can be transformed into a standard parabolic partial differential Equation (7).

$$\frac{\partial \zeta^k(x,t)}{\partial t} = C_v \frac{\partial^2 \zeta^k(x,t)}{\partial x^2}.$$
(7)

In this case, Equation (7) can be solved using numerical algorithms, given the boundary and initial conditions. After obtaining the solution for the current segment, the calculation for the (k + 1)th segment can be performed by updating the initial conditions within that segment, as shown in Equation (8):

$$\zeta^{k+1}(x,0) = \zeta^k(x,t_e). \tag{8}$$

By ensuring continuity between segments, the initial conditions can be updated and inherited, allowing for the calculation of each segment's results based on the sequential order. This computational approach, compared to the classical Esrig method, shares the commonality of introducing an intermediate variable to transform the original governing Equation (1) into a standard parabolic PDE for solving. The difference lies in the fact that in the Esrig method, the introduction of the intermediate variable spans the entire time domain, without considering the temporal variation of the potential distribution. The proposed method in this paper restricts the introduction of the intermediate variable to local segments, where the effective potential remains constant within each segment and the potential distribution is independent of time. By using different effective potentials within each segment to reflect the overall variation of the effective potential and inheriting the initial conditions between segments, this local non-steady-state calculation is extended to the entire time domain, enabling the calculation of electroosmotic consolidation under the variation of effective potential.

3.3. The Finite-Difference Form of the Equation. In order to analyze the electroosmotic consolidation process under the condition of effective potential attenuation, it is necessary to approximate the variation of potential by segmenting the curve. It is important to ensure continuity between the different segments. Therefore, a numerical method is employed to solve the problem, which can adapt well to the iterative nature of the calculations. In this study, the finite-difference method is used for solving the problem. Since it is a nonsteady-state problem, the stability and convergence of the iterative process must be considered during the differencing iteration. Therefore, the Crank–Nicolson method is chosen as the finite-difference scheme. This method has secondorder accuracy in both spatial and temporal dimensions, and it possesses unconditional stability for iteration.

The length of the computational domain is denoted as L, which is uniformly divided into n segments. The spatial step size is denoted as  $\Delta x$ , resulting in n + 1 spatial nodes, where  $x_i = \Delta x \times i$ . The temporal dimension is also discretized uniformly, with m time segments. The time step size is denoted as  $\Delta t$ , resulting in m + 1 temporal nodes, where  $t_i = \Delta t \times j$ . The finite-difference model is illustrated in Figure 9. In the one-dimensional electroosmotic consolidation model, the cathode end is considered as a drainage boundary, satisfying the homogeneous Dirichlet boundary condition. The anode end is considered as a nondrainage condition. The function values at the points on this boundary are unknown.

To approximate the second derivative on the right-hand side of Equation (7), the average value between time j and time j + 1 is used instead of the original second derivative. The second derivative of the function  $\zeta(x, t)$  at the node  $x_i, t_j$  is given by Equation (9) as follows:



FIGURE 9: One dimensional finite-difference model for electroosmotic consolidation.

$$\frac{\partial^{2} \zeta^{k}}{\partial x^{2}} = \frac{1}{2} \left( \frac{\zeta^{k}(i-1,j) - 2\zeta^{k}(i,j) + \zeta(i+1,j)}{\Delta x^{2}} + \frac{\zeta^{k}(i-1,j+1) - 2\zeta^{k}(i,j+1) + \zeta^{k}(i+1,j+1)}{\Delta x^{2}} \right).$$
(9)

For the time term on the left-hand side of Equation (7), the forward difference scheme is used, as shown in Equation (10):

$$\frac{\partial \zeta^k(i,j)}{\partial t} = \frac{\zeta^k(i,j+1) - \zeta(i,j)}{\Delta t}.$$
(10)

By substituting Equations (9) and (10) into Equation (7), the finite-difference form corresponding to the PDE is determined, as shown in Equation (11):

$$-r\zeta^{k}(i-1,j+1) + (2+2r)\zeta^{k}(i,j+1) - r\zeta^{k}(i+1,j+1)$$
  
=  $r\zeta^{k}(i-1,j) + (2-2r)\zeta^{k}(i,j) + r\zeta^{k}(i+1,j),$   
(11)

where *r* represents the grid ratio,  $r = C_v \Delta t / \Delta x^2$ .

Equation (11) represents the finite-difference form of the governing equation within the k state segment. By using the nodal values at time j to calculate the nodal values at the next time step j + 1, the iterative calculation can be performed using the initial conditions. It is important to note that the Crank–Nicolson form yields an implicit structure for the finite-difference scheme. Therefore, solving a system of linear equations is required at each time step to obtain the results. The finite-difference grid structure is illustrated in Figure 10.



FIGURE 10: Crank-Nicolson difference structure.

By applying the finite-difference grid structure to the discretized region of the model, we can obtain the coefficient matrix of the system of linear equations by traversing from the leftmost point to the rightmost point at each time step. It is important to note that, due to the homogeneous Dirichlet condition at the cathode, the function value at x = 0 is known and does not need to be included in the calculations. The number of free points to be computed is denoted as n, and the size of the coefficient matrix is  $n \times n$ . At the anode end, which satisfies the homogeneous Neumann condition, the value of the derivative of the field function needs to be determined. In this case, a second-order extrapolation formula is used to describe the first derivative difference at the rightmost anode end, as shown in Equation (12):

$$\frac{\partial^k(L,t)}{\partial x} = \frac{-\zeta^k(L-2\Delta x,t) + 4\zeta^k(L-\Delta x,t) - 3\zeta^k(L,t)}{-2\Delta x} = 0.$$
(12)

By directly incorporating the boundary conditions into the finite-difference iteration, the coefficient matrix is modified to obtain the matrix form of the finite-difference equations, as shown in Equation (13)–(15). Equation (13) represents the modified coefficient matrix, where the boundary conditions are included. Equation (14) represents the modified righthand side matrix, which also takes into account the boundary conditions. Finally, Equation (15) represents the system of linear equations in matrix form.

$$\mathbf{A} = \begin{bmatrix} 2+2r & -r & 0 & \cdots & 0\\ -r & 2+2r & -r & \ddots & 0\\ 0 & -r & 2+2r & \ddots & 0\\ \vdots & \vdots & \ddots & \ddots & -r\\ 0 & 0 & -1 & 4 & -3 \end{bmatrix},$$
(13)  
$$\mathbf{B} = \begin{bmatrix} 2-2r & r & 0 & \cdots & 0\\ r & 2-2r & r & \ddots & 0\\ 0 & r & 2-2r & \ddots & 0\\ \vdots & \vdots & \ddots & \ddots & r\\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix},$$
(14)

TABLE 1: Calculation parameters for electroosmotic consolidation.

$C_{v} (m^{2}/s)$	$k_e(\mathrm{m}/(\mathrm{V}\cdot\mathrm{s}))$	$k_h (m/s)$	Total time (hr)
$5 \times 10^{-7}$	$1 \times 10^{-9}$	$1 \times 10^{-9}$	102.5

TABLE 2: Finite difference calculation scheme.

Calculation scheme	S	п	т
A	1	50	60,000
В	5	50	12,000
С	10	50	6,000
D	15	50	4,000

$$\mathbf{A} \cdot \boldsymbol{\zeta}_{i+1}^{k} = \mathbf{B} \cdot \boldsymbol{\zeta}_{i}^{k}, \tag{15}$$

where  $\zeta_j^k$  represents the field function values at each node at time *j*, and  $\zeta_{j+1}^k$  represents the field function values at each node at time j + 1.

The above process describes the finite-difference computation process for the PDE. According to the computational strategy in this paper, the first step is to divide the domain into state segments. The computation starts with the first state segment, where the finite-difference iteration is performed to obtain the solution. Then, utilizing the inheritance property of the initial conditions, the computation proceeds sequentially for the next state segments, thus completing the electroosmotic consolidation calculation over the entire time domain. The finite-difference method is applied to the intermediate variable  $\zeta(x, t)$ , and Equation (6) is used to obtain the corresponding pore water pressure values. For the implementation of the algorithm in this paper, a dedicated electroosmotic consolidation calculation program called PyEc has been developed using the Python programing language. PyEc realizes the aforementioned numerical finite-difference process.

3.4. Example Validation. To validate the rationality of this method and compare it with the results of the classical theory that does not consider changes in electrical potential, numerical solutions were obtained using the self-developed PyEc numerical program based on the calculation process described in Section 3.3. In this calculation, the pore water pressure in the pre-electroosmotic soil is not considered, i.e., the initial pore pressure value is set to 0 kPa. The geometric model and electrical potential parameters of the case are based on the aforementioned one-dimensional electroosmotic consolidation test. The distance from the cathode to the anode is L = 0.25 m, and the total electroosmotic time is 102.5 hr. The relevant parameters of the soil in the case are shown in Table 1.

To investigate the influence of the number of state segments on the calculation results, four schemes were implemented with different finite-difference parameters and segment numbers, as shown in Table 2. In Table 2, s represents the number of state segments, which are 1, 5, 10, and 15, respectively. n represents the number of spatial steps,



FIGURE 11: Experimental measurement of effective potential data and linear segmented curve.

and *m* represents the number of finite difference time steps within each state segment. To ensure consistent time step lengths for the four schemes, different values are assigned to *m*. However, the total number of time steps for the entire electroosmotic process remains constant at 60,000 steps. In all four schemes, the spatial step length is  $\Delta x = 0.005$ m, and the time step length is  $\Delta t = 6.15$  s.

The variation data of the effective potential is obtained from the measured data in the one-dimensional electroosmotic test, and its variation curve is shown in Figure 4(b). For the measured data, a continuous function is fitted first, and then it is divided into state segments. The results after processing are shown in Figure 11. The case with one state segment corresponds to the result of the Esrig theory, which does not consider the later changes in the effective potential. In this case, the initial voltage is used as the calculation parameter throughout the electroosmotic process, so no segmentation is needed. The segmentation results for the other three schemes B, C, and D are shown in Figure 11. In scheme A, the number of state segments is 5. The step-like segments have a relatively poor approximation to the measured effective potential data points. As the number of state segments increases, the approximation effect improves. It is worth noting that excessively increasing the number of state segments will increase the number of computational iterations and reduce the solution speed. Considering that the Crank-Nicolson difference scheme used in this paper is unconditionally stable and does not require the grid ratio r to be kept at a small level, the number of state segments and the number of difference time steps can be adjusted according to the actual situation during the calculation to achieve a balance between accuracy and computational efficiency.

The calculation of the consolidation process was performed using the developed computational program PyEc. The distribution of excess pore water pressure throughout the entire process is shown in Figure 12. Figure 12(a) presents the results obtained using the Esrig theory, where the



FIGURE 12: Color maps of pore water pressure distribution. (a) Color map of pore water pressure distribution at s = 1; (b) color map of pore water pressure distribution at s = 5; (c) color map of pore water pressure distribution at s = 10; (d) color map of pore water pressure distribution at s = 15.

number of stages is 1. This implies that the entire electroosmotic process was conducted under the initial voltage condition of 18.72 V, and the effective potential did not decay. Consequently, the contour lines of pore pressure distribution appear smooth. However, when considering the decay of effective potential and using a segmented approach with s = 5 stages, the contour plot of pore pressure distribution, as shown in Figure 12(b), exhibits discontinuities in the contour lines. These discontinuities arise due to the potential jumps between stages. As the number of stages increases, the degree of discontinuity in the contour lines decreases, and they tend to become smoother, as depicted in Figures 12(c) and 12(d). By improving the approximation of the effective potential curve, the proposed method reduces the magnitude of potential jumps between stages, resulting in smoother results. This approach effectively captures the influence of voltage variations applied at both ends of the soil on the pore water pressure during the electroosmotic consolidation process.

Figure 13 illustrates the variation of pore water pressure at different locations for four different scenarios. The pore pressure changes were monitored at distances of 0.05, 0.10, 0.15, 0.20, and 0.25 m from the cathode. Figure 13(a) represents the results obtained using the Esrig calculation method, where the pore pressure changes at the five monitoring points appear smooth. In Figure 13(b), which corresponds to the segmented approach with five stages, the pore pressure changes are influenced by the discontinuities in the effective potential, resulting in abrupt changes at the boundaries between stages. As the number of stages increases, the curves tend to become smoother, and the degree of abrupt changes decreases, as shown in Figures 13(c) and 13(d). Taking the results at the position of x = 0.15 m as an example, at the end of the electroosmotic process, the pore water pressure values at this location for the four scenarios are -112.32, -98.94, -100.65, and -101.21 kPa, respectively. The decay of effective potential inevitably leads to a decrease in the magnitude



FIGURE 13: Pore water pressure variation curve at different positions. (a) Pore water pressure change curve when s = 1; (b) pore water pressure change curve when s = 5; (c) pore water pressure change curve when s = 10; (d) pore water pressure change curve when s = 15.

of negative pore water pressure during the electroosmotic process, and the proposed algorithm is capable of capturing this phenomenon.

Figure 14 presents the variation of average degree of consolidation for the model. In Figure 14(a), the average degree of consolidation curves are shown for the four scenarios. When the number of stages is s = 1, the overall average degree of consolidation increases smoothly. However, once the segmented approach is applied, the average degree of consolidation curve exhibits discontinuities. Nevertheless, as the number of stages increases, the curve becomes smoother.

Additionally, from the overall trend, it can be observed that higher voltage driving leads to accelerated consolidation of the soil, resulting in an earlier attainment of a degree of consolidation of 1. However, if there is a decrease in the high voltage during this process, the degree of consolidation will decrease. Therefore, in Figure 14(a), the curves for scenarios B, C, and D exhibit rebound phenomena, which also reflect the influence of the decay of effective potential on consolidation.

To compare the differences between the proposed algorithm and the results obtained from classical theory, the deviation of the average degree of consolidation is calculated



FIGURE 14: Average consolidation curve and deviation for different calculation schemes. (a) Average consolidation degree change curve; (b) deviation curve between the average consolidation degree of the segmented inheritance algorithm and the average consolidation degree of Esrig's theory.

by subtracting the results obtained using the Esrig theory from the results obtained with s = 5, s = 10, and s = 15 stages. The deviation curve of the average degree of consolidation is shown in Figure 14(b). Considering the variation of effective potential, the consolidation with effective potential decay is lower than the results obtained using the Esrig theory in the middle and later stages. The degree of consolidation reflects the overall diffusion of pore pressure in the model during the consolidation process. Therefore, the proposed method effectively calculates the influence of potential decay on the electroosmotic consolidation process, avoiding overestimation of the effectiveness of electroosmotic consolidation in soil reinforcement.

# 4. Discussion

Electroosmotic consolidation is a complex physical and chemical process, and electrode corrosion is inevitable. The presence of salt content in the soil and other metal ions can cause variations in effective potential that do not follow a fixed pattern. However, considering the separation at the electrode-soil interface and the corrosion of the electrode surface, the effective potential generally exhibits a decreasing trend. This is the main reason for the low efficiency of electroosmotic consolidation in the later stages using drainage methods. The proposed method in this paper enables the calculation of electroosmotic consolidation under variations in the effective potential. The linear segmentation approach ensures the validity of introducing intermediate variables within local state segments. By utilizing the uniformity and continuity of adjacent state segments in the time domain, the initial conditions are iteratively updated, allowing for the extension of local difference iterative calculations to the

entire time domain. This method is concise and cleverly implemented, which is crucial for evaluating the reinforcement effect in the later stages of electroosmotic consolidation and provides theoretical support for improving the efficiency of this method. The subsequent research direction is to further extend this method to two-dimensional spatial calculations and incorporate additional soil variation factors. This advancement has the potential to promote the application of electroosmotic consolidation.

## 5. Conclusions

This study aims to investigate the influence of variations in effective potential on electroosmotic consolidation. Experimental research was conducted on a one-dimensional electroosmotic consolidation model, and potential variation data were monitored. Theoretical research was carried out to address the equation-solving problem in electroosmotic consolidation under variations in effective potential. A linear segmentation approach and an inheritance strategy were employed to solve the governing equations, and the finitedifference method was used to discretize the equations. A dedicated numerical computing program called PyEc was developed using the Python language. Several important research findings are summarized as follows:

 The findings from the one-dimensional electroosmotic test reveal that the effective potential variation during the electroosmotic process does not consistently decrease throughout the entire duration. However, overall, there is a decreasing trend due to changes in the contact properties between the soil and electrode interface. The metal anode experiences noticeable corrosion, and the reinforced effect is enhanced in the anode region due to its lower water content. Consequently, the volume change in this region is the most significant, and the detachment between the anode and soil is most pronounced. The increase in interface resistance resulting from this phenomenon is a crucial factor contributing to the lower efficiency observed in the later stages of electroosmosis.

- (2) The one-dimensional electroosmotic consolidation theory proposed by Esrig faces a fundamental challenge in directly describing the underlying cause of effective potential variation. This challenge arises from the requirement that the potential distribution term remains independent of time when introducing intermediate variables during the solution process. However, this characteristic contradicts the distribution observed during effective potential variation, limiting the resolution of the problem. Therefore, the key to addressing this issue lies in finding a way to describe the decay of effective potential while maintaining the potential distribution independent of time. Based on the concept of linear segmentation using Riemann summation, the partitioning of effective potential is achieved by maintaining it as a constant within each local segment. The approximation of the entire effective potential curve variation is achieved by utilizing the discontinuity between segments. This approach confines the introduction of intermediate variables and equation transformations within each segment, while incorporating the unity of potential variation and electroosmotic process in the time domain to update the initial conditions within each segment. Consequently, the problem is transformed into iterative calculations of difference within each segment, enabling non-steady-state computations across the entire time domain.
- (3) The proprietary finite-difference numerical program developed in Python language can successfully perform the computational process of the algorithm proposed in this paper. The calculations based on the test data from the one-dimensional electroosmotic experiment demonstrate that the decay of effective potential leads to a reduction in the amplitude of negative pore water pressure during the electroosmotic consolidation process. Increasing the number of segments can decrease the degree of discontinuity in pore water pressure equipotential lines. The algorithm proposed in this paper allows for a quantitative assessment of the decrease in electroosmotic efficiency caused by potential decay, providing theoretical support for addressing the issue of lower efficiency in the later stages of electroosmotic consolidation drainage methods.

#### **Data Availability**

The data used to support the findings of this study are available from the corresponding author upon request.

### **Conflicts of Interest**

The authors declare that they have no conflicts of interest regarding the publication of this paper.

#### Acknowledgments

This research is funded by the Gansu Provincial Youth Science and Technology Fund Program (Grant Nos. 22JR5RA387, 22JR5RE200), the Gansu Provincial Department of Education: Young Doctoral Fund Program (Grant Nos. 2022QB-188, 2022QB-190), the Gansu Provincial Department of Education: Innovation Fund for University Teachers (Grant No. 2023A-202), the Youth Science and Technology Innovation Project of Lanzhou Institute of Technology (Grant No. 2021KJ-03), the Key Research and Development Plan Project of Ningxia Hui Autonomous Region (Grant No. 2021BEG03023).

#### References

- S.-C. Chien, F.-C. Teng, and C.-Y. Ou, "Soil improvement of electroosmosis with the chemical treatment using the suitable operation process," *Acta Geotechnica*, vol. 10, no. 6, pp. 813– 820, 2015.
- [2] W. Qi, Y. Shen, S. Li, and K. Chen, "Study on the interaction between the reduction and remediation of dredged sediments from tai lake based on vacuum electro-osmosis," *Applied Sciences*, vol. 13, no. 2, Article ID 741, 2023.
- [3] S. Gargano, S. Lirer, B. Liguori, and A. Flora, "Effect of the pore fluid salinities on the behaviour of an electrokinetic treated soft clayey soil," *Soils and Foundations*, vol. 60, no. 4, pp. 898–910, 2020.
- [4] S. Ge, W. Jiang, L. Zheng et al., "Theoretical analysis and experimental verification of large deformation electro-osmosis consolidation treatment of dredged slurry," *Engineering Geology*, vol. 312, Article ID 106924, 2023.
- [5] Y. Liu, L. Zheng, and S. Rao, "Experimental study of the development of soil microstructure under the electroosmosis dewatering process," *Advances in Materials Science and Engineering*, vol. 2021, Article ID 6642785, 11 pages, 2021.
- [6] X. Sun, R. Zheng, Y. Wang, X. Jin, and Q. Yu, "A novel evaluation method of consolidation efficiency of electro-osmotic system," in *Géotechnique*, pp. 1–10, ICE Virtual Library, 2023.
- [7] K. Tang, F. Zhang, D. Feng, and X. Lu, "Moisture migration and electric distribution of unsaturated clay under electroosmosis with carbon fiber tape as electrode," *Engineering Geology*, vol. 294, Article ID 106404, 2021.
- [8] L. J. Wang, Y. M. Wang, S. H. Liu, and P. H. Huang, "Analytical investigation of electroosmotic consolidation in unsaturated soils considering the coupling effect and a nonuniform initial water content," *International Journal of Geomechanics*, vol. 21, no. 8, 2021.
- [9] P. Xiang, Y. Cui, and G. Wei, "Study on the effect of lowtemperature anode filled with FeCl3 solution on electroosmotic reinforcement of soft clay," *Applied Sciences*, vol. 12, no. 5, Article ID 2517, 2022.
- [10] H. Wu, L. Hu, and Q. Wen, "Numerical assessment of equivalent radius for electrokinetic geosynthetics electrodes during electroosmotic consolidation," *International Journal of Geomechanics*, vol. 18, no. 5, 2018.
- [11] Z. Xue, X. Tang, and Q. Yang, "Influence of voltage and temperature on electro-osmosis experiments applied on marine clay," *Applied Clay Science*, vol. 141, pp. 13–22, 2017.

- [12] L. Zhang and L. Hu, "Numerical simulation of electro-osmotic consolidation considering tempo-spatial variation of soil pH and soil parameters," *Computers and Geotechnics*, vol. 147, Article ID 104802, 2022.
- [13] J. Zhou, Q. Gan, and Y. Tao, "Electro-osmotic permeability model based on ions migration," *Acta Geotechnica*, vol. 17, no. 6, pp. 2379–2393, 2022.
- [14] Q. Gan, J. Zhou, C. Li, Y. Zhuang, and Y. Wang, "Vacuum preloading combined with electroosmotic dewatering of dredger fill using the vertical-layered power technology of a novel tubular electrokinetic geosynthetics: test and numerical simulation," *International Journal of Geomechanics*, vol. 22, no. 1, 2022.
- [15] J. Peng, H. Ye, and A. N. Alshawabkeh, "Soil improvement by electroosmotic grouting of saline solutions with vacuum drainage at the cathode," *Applied Clay Science*, vol. 114, pp. 53–60, 2015.
- [16] L. Wang, P. Huang, S. Liu, and E. Alonso, "Analytical solution for nonlinear consolidation of combined electroosmosisvacuum-surcharge preloading," *Computers and Geotechnics*, vol. 121, Article ID 103484, 2020.
- [17] M. I. Esrig, "Pore pressures, consolidation, and electrokinetics," *Journal of the Soil Mechanics and Foundations Division*, vol. 94, no. 4, pp. 899–921, 1968.
- [18] R. W. Lewis and C. Humpheson, "Numerical analysis of electro-osmotic flow in soils," *Journal of the Soil Mechanics* and Foundations Division, vol. 99, no. 8, pp. 603–616, 1973.
- [19] Y. Zhou, S. Zhang, S. Feng, and A. Deng, "One-dimensional large strain electro-osmotic consolidation model based on multi-field coupling," *Computers and Geotechnics*, vol. 159, Article ID 105491, 2023.
- [20] J. Yuan and M. A. Hicks, "Large deformation elastic electroosmosis consolidation of clays," *Computers and Geotechnics*, vol. 54, pp. 60–68, 2013.
- [21] X.-D. Zhao, Y. Liu, and W.-H. Gong, "Analytical solution for one-dimensional electro-osmotic consolidation of doublelayered system," *Computers and Geotechnics*, vol. 122, Article ID 103496, 2020.
- [22] Y. Liu, J. J. Zheng, X. Zhao, W. Cao, and Z. Huang, "A closedform solution for axisymmetric electro-osmotic consolidation considering smear effects," *Acta Geotechnica*, vol. 17, pp. 2597– 2609, 2022.
- [23] Y. Liu, J.-J. Zheng, X. Zhao, and L. You, "Analytical model for two-dimensional electro-osmosis-enhanced preloading consolidation of unsaturated soil," *Acta Geotechnica*, vol. 18, no. 2, pp. 1093–1110, 2023.
- [24] Y. Shen, J. Feng, Y. Ma, and H. Liu, "Two-dimensional electroosmotic consolidation theory of nonlinear soil voltage distribution characteristics," *Advances in Civil Engineering*, vol. 2019, Article ID 7354873, 10 pages, 2019.
- [25] X. Yang, Y. Xie, J. Dong, G. Liu, and Y. Zheng, "Study on electroosmosis consolidation of punctiform electrode unit," *Advances in Materials Science and Engineering*, vol. 2021, Article ID 6627331, 12 pages, 2021.
- [26] X. Y. Yang, J. H. Dong, G. S. Liu, and Y. B. Xie, "Study on twodimensional electroosmotic consolidation of punctiform electrode units with symmetric and asymmetric forms," *Chinese Journal of Rock Mechanics and Engineering*, vol. 40, no. 7, pp. 1491–1503, 2021.