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Research Article

Reliability Analysis of Compressive Fatigue Life of Cement Concrete under Temperature Differential Cycling

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Cement concrete, as an extensively used engineering material, is omnipresent in various infrastructure projects such as bridges and roads. However, these structures often need to operate for extended periods under varying and harsh environmental conditions, facing not only complex vehicular loads but also the effects of temperature differential cycling. Consequently, understanding how temperature differential cycling impacts the compressive fatigue life of cement concrete has become a pivotal research topic. In this study, through a comprehensive experimental design, the fatigue life of cement concrete under typical temperature difference conditions (20–60°C) and different number of temperature differential cycling (60, 120, 180, 240, 300) was tested at three stress levels (0.70, 0.75, 0.85). Statistical analysis was conducted to obtain the Weibull distribution parameters of the compressive fatigue life of cement concrete. The P_f–S–N relationship of concrete considering reliability was analyzed, and a fatigue life prediction model under different reliability probabilities was established. The results show that the fatigue life of concrete subjected to temperature differential cycling follows a two-parameter Weibull distribution well. From the P_f–N curve, it can be seen that, regardless of the stress level, the calculated fatigue life under the same reliability probability decreases with the increase of temperature differential cycling times. At a 95% reliability probability, the decrease can reach 77.5%–87.5%. Based on the exponential function, a concrete fatigue life prediction model based on different reliability levels was established. Using this model, the S–lgN curve was plotted, and it was found that, regardless of the temperature differential cycling, an increase in reliability probability could lead to a 7.3%–14.4% reduction in logarithmic fatigue life (lgN). Additionally, this study also defined a fatigue life safety factor related to the number of temperature differential cycling and reliability probability, aiming to provide a theoretical basis for the design of cement concrete materials under the coupled environment of temperature differential cycling and fatigue loading.

1. Introduction

In the recent wave of rapid urbanization and modern infrastructure construction, the development of transportation facilities has undoubtedly become a crucial component. Among a plethora of building and construction materials, cement concrete, which has unique advantages including high rigidity, outstanding flatness, long service life, and relatively abundant raw material supply, is widely used in the construction of roads, bridges, and other transportation infrastructures. However, these transportation facilities must operate under complex environmental conditions, which include, but are not limited to, constantly changing traffic loads and various climatic factors.

Cement concrete structures often need to endure the repeated effects of traffic loads, which could lead to internal damage within the material and affect the concrete's mechanical properties and service life subsequently. The gradual, localized, and irreversible damage process caused by cyclic loads during the service life of concrete is referred to as fatigue [1], which kind of damage is often irreversible and accumulates over time. Simultaneously, the continuous changes of temperature also play a significant role in affecting themechanical properties and service life of concrete. Especially in the northeast and northwest regions of China, of where diurnal temperature differences are large, the concrete material experiences daily temperature cycling [2, 3]. During the temperature differential cycling, the thermal strains produced during temperature changes are different due to the different thermal expansion and conductivity properties of the various components of cement concrete. The discordant deformation among components leads to phenomena such as tension and compression between the constituent materials, subsequently resulting in the formation of microcracks [4]. These microcracks keep forming and expanding with continuous cyclical temperature changes, thereby impacting the long-term structural integrity and mechanical properties of the concrete irreversibly.

The study of concrete fatigue can be traced back to the early 20th century. The importance of researching the fatigue performance of concrete gradually came into focus with the rapid development of the highway system. Furthermore, with the high-strength, lightweight development of concrete structures and the significant increase in working stress, the fatigue performance of concrete materials has garnered widespread attention from scholars both domestically and internationally [5, 6]. Wu et al. [7] investigated the axial compressive fatigue performance of high-strength concrete under constant and varying repeated loads. They provided empirical formulas for fatigue strength, longitudinal strain, and static strain and proposed a formula to determine the fatigue failure of high-strength concrete based on static strain. Zheng [8] studied the influence of the proportion of mineral admixture components on the fatigue performance of concrete as well as the quantitative relationship between composition, structural parameters, and fatigue performance. Sun et al. [9] researched the uniaxial compressive fatigue performance of ultraearly strength composite fiber concrete columns under high temperatures and provided the hysteresis curves and skeleton curves. Xue et al. [10] studied the compressive fatigue performance of rubber powder concrete under freeze-thaw cycling, who conducted a reliability analysis of the concrete fatigue life test results through probabilistic statistical methods and established a fatigue equation for rubber powder concrete under the effect of freeze-thaw cycling based on the equivalent damage theory.

It is well known that fatigue failures often occur in areas with stress concentrations and complex stress states, such as rebars in reinforced concrete structures, concrete itself, riveted joints in steel structures, and weld seams. However, whether such structures will fail under a certain load cycle is also random due to the randomness of structural dimensions, materials, and loads. Therefore, it is necessary to combine probabilistic statistical methods for analysis, namely, reliability analysis [11, 12]. Research on the reliability of concrete fatigue life has always been a topic of interest to scholars both domestically and internationally. Byung [13] studied the impact of varying amplitude fatigue loads on the fatigue life of concrete beams, finding that the fatigue life of concrete conforms to the two-parameter Weibull distribution rule. Additionally, the shape and scale parameters in the Weibull model differ at various stress levels. Li et al. [14] investigated the approximation of two commonly used models simulating the probabilistic distribution of concrete fatigue life: the log-normal distribution model and the Weibull distribution model. They derived a method for estimating parameters for the two-parameter Weibull distribution and verified it with examples. Ou and Sun [15] studied the net bending strength, fatigue strength, and fatigue life of concrete beams with different freeze-thaw damage levels. They conducted a reliability analysis on the bending fatigue life and found that the bending fatigue life of concrete fits the Weibull distribution well, and its fatigue life reliability probability decreases with the increase in freeze-thaw cycle times. Tan et al. [16] conducted compressive fatigue tests to study the fatigue performance of basalt fiber recycled concrete under different volume ratios, length ratios, and stress levels. They established a three-parameter Weibull distribution fatigue model. The research results indicate that the test data fits the three-parameter Weibull distribution well and can accurately produce the *P–S–N* curve.

In summary, current research on the fatigue life and its reliability of concrete mainly focuses on exploring the impact of single factor, such as fixed mechanical loads, environmental temperature, and material composition. And research concerning the temperature factor is often centered on fixed low temperatures, high temperatures, or established temperature histories, with relatively few studies delving into the impact of temperature cycles [17, 18]. In reality, the actual engineering environments subjected to the continuous cyclical changes instead of fixed, due to factors like seasons and sunlight exposure. Especially in the summer in Eastern and Northwestern China, affected by sunlight, the road surface temperature can vary between the normal night temperature (around 20°C) and the high noon temperature (around 60°C), and this variation repeats daily due to the influence of sunlight. Therefore, in order to better align with real-world engineering scenarios and to improve the accuracy of predicting concrete's life span, it is of significant theoretical and practical importance to conduct a comprehensive study on the reliability analysis of compressive fatigue life of cement concrete under temperature cycle conditions.

In order to evaluate the compressive fatigue life and its reliability of cement concrete under temperature cycling more comprehensively and systematically, this study conducted axial compression fatigue tests on cement concrete subjected to fatigue loads at different temperature cycles and stress levels. The two-parameter Weibull distribution model was used for statistical analysis of the fatigue life of the axial compressive specimens. A fatigue life prediction model was established for concrete specimens subjected to fatigue loads at different reliability probabilities, temperature cycles, and stress levels, of which aim is to provide theoretical support for the development of related fundamental research and a scientific basis for more accurate and economical material applications in construction engineering.

2. Materials and Methods

2.1. Raw Materials and Specimen Preparation. The raw materials used in this experiment for concrete mixing include cement (Jidong P·O42.5 grade ordinary portland cement), fly ash of II grade, S95 mineral powder, medium sand, crushed stone (5–31.5 continuous grading), SM-1 pumping admixture, and water. The main physical properties of the selected cement for the experiment are shown in Table 1.

Table 1: Main physical properties of Jidong brand P·O42.5 grade ordinary portland cement.

Specific surface area (m^2/kg)	Setting time (min)		Flexural strength (MPa)		Compressive strength (MPa)	
	Initial setting	Final setting	3d	28d	3d	28d
387	195	242	4.4	8.3	26.5	52.7

Table 2: Mix ratio of concrete used in the experiment (kg/m³).

Cement	Fly ash	Mineral powder	Medium sand	Crushed stone	Water	Pumping agent
300	60	70	698	1,035	165	11.6

Table 3: Temperature and wildlife count in the three areas covered by the study.

Experiment	Size of specimen (mm)	Number of specimens (pieces)
Axial compression strength test	$100 \times 100 \times 300$	54
Axial compression fatigue test	$100 \times 100 \times 300$	54

PWS-200 dynamic and static fatigue testing machine High- and low-temperature alternating test chamber

Figure 1: Experimental equipment.

The design of mix ratio of concrete in the experiment was based on the standard JGJ55-2011 "Specification for mix proportion design of ordinary concrete" [19]. The designed strength grade is C40, with a 28-day axial compressive strength measured value of 30.6 MPa and a slump of 200 mm. The specific mix proportions are shown in Table 2.

In this experiment, we referred to the standards GB/T 50081-2019 "Standard for test methods of concrete physical and mechanical properties" [20] and GB/T 50082-2009 "Standard for test methods of long-term performance and durability of ordinary concrete" [21] to prepare the concrete specimens, sizes, and quantities of which are shown in Table 3.

The mould of concrete was demolded 24 hr after preparation. The specimens were then cured under standard conditions with a temperature of 20 ± 2°C and a relative humidity (RH) greater than 95% for 28 days before conducting the relevant tests [21].

2.2. Equipment and Methods of Experiment

2.2.1. Experimental Equipment. The PWS-200 dynamic and static fatigue testing machine from Jinan Dongce Testing Machine Factory is employed for the axial compressive fatigue test. In order to improve the efficiency of the test, the equipment used for treating the temperature differential cycling is the high- and low-temperature alternating test chamber from Shanghai Yihua Instrument Equipment Corporation (see Figure 1).

2.2.2. Test Method.

(1) Experiment of Temperature Differential Cycling. The temperature 20°C was chosen as the reference temperature in the experiment, and the boundary temperature was set at 60°C (i.e., a temperature difference of 40°C). In this study, the specimen was heated from the reference temperature (20°C) to the boundary temperature of 60°C with the heating rate of environment box of 2°C/min [22]. The specimen was then placed at this boundary temperature for 0.5 hr. Subsequently, the temperature was lowered back to the reference temperature (20°C) with the cooling rate same as that of heating rate (2°C/min). The specimen was then left at this reference temperature for another 0.5 hr, and the process above was repeated. Each sequence of heating from the reference temperature to the boundary

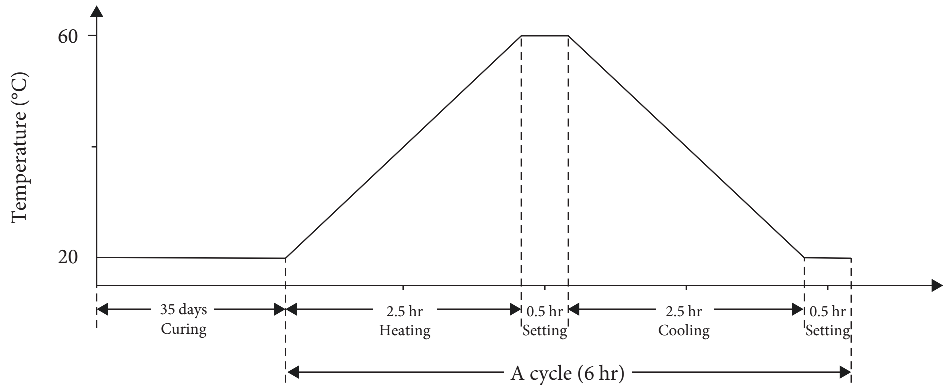

FIGURE 2: System of temperature differential cycling.

temperature and then cooling back to the reference temperature is referred to as one cycle. And there were five different temperature differential cycling conditions set in this study: 60, 120, 180, 240, and 300 cycles. The system of temperature differential cycling is shown in Figure 2.

(2) Experiment of Compressive Fatigue. Three stress levels ($S = 0.70$, 0.75, and 0.85) were selected in the experiment, under which axial compressive fatigue tests were performed on concrete specimens after different temperature differential cycling conditions. Here, the stress level S is defined as the ratio of the maximum fatigue stress f_{max} to the static compressive strength f_r ($S = f_{max}/f_r$). The experiment adopted a constant amplitude loading with a loading frequency set at 4 Hz. The stress ratio Q was set at 0.1, where Q represents the ratio of the minimum to maximum fatigue stress ($Q = f_{min}/f_{max}$) [23]. The number of cycles that the concrete specimen undergoes from the start of loading to complete failure is termed as the fatigue life (N). Furthermore, to improve the efficiency of the experiment, a fatigue life (N) reaching 2 million cycles was set as the stopping criterion for the experiment, which means that the test would be terminated either upon complete failure of the concrete specimen or when its fatigue life reaches 2 million cycles. The residual fatigue life will then be tested if the concrete specimen does not fail completely within 2 million cycles.

TABLE 4: Compressive fatigue life of concrete specimens undergoing temperature differential cycling.

Number of temperature differential cycling (T)	Compressive fatigue life (N)		
	$S = 0.70$	$S = 0.75$	$S = 0.85$
	765,849	481,528	15,692
0	897,656	496,386	16,446
	1,876,975	1,102,543	33,247
	678,593	362,417	10,082
60	739,652	385,462	12,513
	1,556,487	862,573	24,638
	587,636	316,248	8,765
120	652,316	335,279	7,986
	1,248,567	769,249	19,548
	307,685	235,234	4,468
180	359,647	260,348	5,965
	806,943	584,627	12,463
	224,682	179,975	3,175
240	255,298	199,657	3,341
	736,347	452,432	7,999
	230,248	108,678	2,398
300	250,264	126,428	2,547
	629,534	284,682	6,283

3. Experimental Results and Analysis

3.1. Fitting of Compressive Fatigue Life Distribution. The compressive fatigue life of the concrete specimens subjected to different number of temperature differential cycling under different stress levels is shown in Table 4.

According to Table 4, regardless of the stress level, the compressive fatigue life of concrete gradually decreases with the progress of temperature differential cycling. The possible reason is, since concrete consists of many components with different thermal properties, they exhibit different thermal deformations when facing temperature changes. This leads to the generation of microcracks at the interfaces of the components. Moreover, these microcracks continuously initiate and develop under the cyclic action of temperature, gradually destroying the integrity of the concrete and ultimately leading to the deterioration of its fatigue performance. Additionally, under the combined effects of temperature differential cycling and fatigue loading, the damage mode of concrete is not a simple superposition of the two influences. According to the law of initiation and development of microcracks in concrete, the deterioration speed of concrete accelerates as the internal microcracks extend and connect, and the damage mode of the concrete also shows a trend of accelerated development.

It is all known that the compressive fatigue life represents the maximum number of cycles that cement concrete can withstand before failure under repeated loading and unloading conditions and the distribution characteristics of which are key factors in evaluating the long-term performance and reliability of concrete. However, the fatigue life is affected by various factors, including load level, temperature cycling, and material heterogeneity. As such, the fatigue life of concrete may still exhibit considerable variability even under strictly controlled test conditions and methods, which is also reflected in Table 4. Therefore, to clearly describe and further explore the distribution of cement concrete's fatigue life, probabilistic statistical methods need to be introduced.

As a continuous distribution, the Weibull distribution is widely used in various life testing data processing due to its derivation of convenience and straightforward. The Weibull model is essentially based on the weakest link model or the serial model, which can fully reflect the impact of material defects or stress concentration on material fatigue life. Coupled with its characteristic of increasing failure probability, it is very suitable for describing the fatigue life distribution model of materials. Studies by Kai et al. [23], Zhou et al. [24], and Li [25], and Tao et al. [26] have proven that the fatigue life distribution of polypropylene fiber concrete, SFRC, SICRC, and plain concrete all conform to the two-parameter Weibull distribution. Therefore, the two-parameter Weibull model is chosen in this study to statistically analyze the fatigue life of concrete and establish the fatigue life distribution model of cement concrete after temperature cycling.

The distribution function of the two-parameter Weibull distribution is as follows [27]:

$$F(x) = 1 - \exp\left\{-\left(\frac{x}{\eta}\right)^{\beta}\right\} \quad x > 0, \beta > 0, \eta > 0. \tag{1}$$

In the given context, β and η are parameters of the Weibull distribution. β is known as the shape parameter and η is the scale parameter, x represents the random variable for the target object being studied, denoted as $X \sim \text{Weibull}\ (\eta,\beta)$.

The probability density function is then expressed as:

$$f(x) = \left(\frac{\beta}{\eta}\right)\left(\frac{x}{\eta}\right)^{\beta-1}\exp\left\{-\left(\frac{x}{\eta}\right)^{\beta}\right\}. \tag{2}$$

The reliability function (also known as the survival function) for the Weibull distribution is given by:

$$R(x) = 1 - F(x) = \exp\left\{-\left(\frac{x}{\eta}\right)^{\beta}\right\}. \tag{3}$$

The failure probability for a Weibull distribution is given by the complement of the reliability function, expressed as:

$$r(x) = \frac{f(x)}{R(x)} = \frac{\beta}{\eta}\left(\frac{x}{\eta}\right)^{\beta-1}. \tag{4}$$

It can be seen from the density function curve form of the Weibull distribution (Formula 2) that, in the two-parameter Weibull model, the shape parameter β determines the basic shape of the curve, while the scale parameter η acts to amplify or diminish the curve without changing its shape.

The moment estimation method [28], which uses sample moments to estimate population moments, is a method where continuous functions of sample moments are used to estimate the continuous functions of population moments. As it does not require complex equations or iterative calculations, the method of moments is simple and easy to use, and hence, it is widely applied in parameter estimation. In this study, the method of moments is chosen to estimate the parameters of the Weibull distribution for the life of cement concrete subjected to temperature differential cycling.

Let a random variable X follow the Weibull distribution, denoted as $X \sim \text{Weibull}\ (\eta,\beta)$, and let $X_1, X_2, \ldots, X_n$ be a complete sample data from X. The moment expression based on the moment estimation method for this Weibull distribution can be obtained as:

$$\begin{cases} E[X] = \eta\Gamma\left(1 + \dfrac{1}{\beta}\right) = \dfrac{1}{n}\sum\limits_{i=1}^{n} X_i \\ E[X^2] = \eta^2\Gamma\left(1 + \dfrac{2}{\beta}\right) = \dfrac{1}{n}\sum\limits_{i=1}^{n} X_i^2 \end{cases}. \tag{5}$$

where $E[\cdot]$ represents the sample expectation, and $\Gamma(\cdot)$ represents the gamma function. Then, $E[X]$ is the sample mean, which is the standard value of the fatigue life of cement concrete under a certain stress level.

Let the sample variance, which represents the variance of the fatigue life of cement concrete under a certain stress level, be denoted as σ:

$$\sigma^2 = E[X^2] - E^2[X]. \tag{6}$$

Considering the actual physical meaning, the standard deviation of the fatigue life of the cement concrete specimen under a certain stress level, relative to its expected value, is defined as the coefficient of variation of the fatigue life of cement concrete, denoted as α:

$$\alpha = \frac{\sigma}{E[X]} = \frac{\Gamma\left(1 + \frac{2}{\beta}\right)}{\Gamma\left(1 + \frac{1}{\beta}\right)} - 1. \tag{7}$$

The parameter estimation expressions for the two-parameter Weibull distribution can be derived as based on Formulas 5–7:

$$\beta = (\alpha)^{-1.08} \tag{8}$$

$$\eta = \frac{E[X]}{\Gamma\left(1 + \frac{1}{\beta}\right)}. \tag{9}$$

The parameters (shape parameter β and scale parameter η) of the Weibull distribution for the compressive fatigue life of cement concrete subjected to different number of temperature differential cycling (T) in this study were estimated. The results are shown in Table 5.

Considering that the method of moments only calculates the first few moments and ignores the higher order moments, it may lead to some bias in the results. Therefore, it is necessary to

Table 5: Estimated parameters for the Weibull distribution of compressive fatigue life of cement concrete after different temperature differential cycling.

T	β			η		
	$S=0.70$	$S=0.75$	$S=0.85$	$S=0.70$	$S=0.75$	$S=0.85$
0	2.55214	2.57067	2.91106	1,329,399	781,024	24,439
60	2.32551	2.49146	2.65873	1,066,447	605,076	17,714
120	2.29761	2.41667	2.61327	872,374	534,167	13,921
180	2.15702	2.41571	2.34238	537,966	406,128	8,613
240	2.03662	2.38430	2.30171	480,200	312,908	5,461
300	2.01112	2.33166	2.20821	408,906	195,541	4,226

Table 6: *K–S* test for the distribution of fatigue life of cement concrete specimens at $S=0.75$, $T=0$.

Test serial number (i)	x_i	$F_n(x)$	$P_n(x)$	$\|F_n(x_i)-P_n(x_i)\|$	D_{max}	D_C
1	481,528	0.3333	0.2506	0.0828		
2	496,386	0.6667	0.2679	0.3987	0.3987	0.708
3	1,102,543	1.0000	0.9116	0.0884		

verify the accuracy of the obtained parameters, which means conducting a goodness-of-fit test between the experimentally obtained concrete fatigue life distribution and the calculated two-parameter Weibull distribution. In this study, the *K–S* (Kolmogorov–Smirnov) test method is selected. The essence of the *K–S* test method is to reflect the goodness of fit between two distributions by quantitatively describing the distance between the empirical function of a sample distribution and the cumulative distribution function of a reference distribution. It is also one of the most commonly used nonparametric methods currently available [29].

Let $F_n(x)$ denote the cumulative distribution function of observed values from a random sample of size n:

$$F_n(x_i) = i/n. \quad (10)$$

Where i represents the sample order, and n is the total number of samples.

Let $P_n(x)$ represent the cumulative probability distribution function of the theoretical reference distribution. The *K–S* test is thus conducted by comparing the cumulative distribution function $F_n(x)$ of the sample with the cumulative distribution function $P_n(x)$ of the reference theoretical distribution. The test statistic D_{max} is represented by the maximum deviation between $F_n(x)$ and $P_n(x)$:

$$D_{max} = \max_{0\le i\le n} |F_n(x_i) - P_n(x_i)|. \quad (11)$$

In this study, $F_n(x)$ represents the cumulative distribution function of the fatigue life of the concrete specimens obtained from the experiments, and $P_n(x)$ represents the cumulative distribution function of the two-parameter Weibull model.

$$P_n(x_i) = 1 - \exp\left(-\frac{x_i^{\beta}}{\eta^{\beta}}\right). \quad (12)$$

Taking the stress level of $S=0.75$ and the number of temperature differential cycling $T=0$ as an example in this study, the *K–S* test procedure for its fatigue life distribution is shown in Table 6. It can be seen that the calculated value of D_{max} is 0.397. For an experiment with three repetitions at a 5% significance level, the critical value is $D_C=0.708$ according to the table of standard critical value of *K–S* test [30]. Since $D_{max} < D_C$, the Weibull distribution of the fatigue life of the concrete at this time passes the *K–S* test, which means that the distribution of the fatigue life of the cement concrete specimens at a stress level of $S=0.75$ and the number of temperature differential cycling $T=0$ do follow the two-parameter Weibull distribution.

The *K–S* test was performed on the fatigue life of the concrete specimens under all working conditions using the aforementioned calculation method. The results are presented in Table 7.

It is evident from the available information that, under all stress levels and temperature differential cycling conditions, the distribution of fatigue life of the cement concrete specimens passes the *K–S* goodness-of-fit test, which indicates that the distribution of fatigue life of the concrete specimens obtained from this experiment aligns well with the two-parameter Weibull distribution model determined by the moment estimation method used in this study. This also suggests that adopting the two-parameter Weibull model to describe the compressive fatigue life distribution of cement concrete after temperature differential cycling is acceptable and reasonably accurate at a 5% significance level.

3.2. Reliability Analysis of Compressive Fatigue Life. Establishing a fatigue equation, which describes the relationship between fatigue stress level (S) and the number of load applications (N), is among the clearest ways to portray material fatigue properties. Moreover, it is the primary basis for predicting material fatigue life in practical engineering. In early researches, there was a tendency to directly establish the *S–N*

Table 7: *K*–*S* test for the fatigue life of cement concrete specimens under different stress levels and the number of temperature differential cycling.

S	D_{max}					
	$T=0$	$T=60$	$T=120$	$T=180$	$T=240$	$T=300$
0.70	0.359431	0.395871	0.391778	0.388951	0.373955	0.355646
0.75	0.398742	0.389076	0.389575	0.377305	0.376612	0.363130
0.85	0.395955	0.339070	0.393838	0.321762	0.390879	0.387819

curve through experiments, while taking the reliability probability P_f into account rarely. In fact, the reliability analysis of the compressive fatigue life of cement concrete is a pivotal step in ensuring the structural safety and durability and serves as a strong decision-making foundation in engineering design, maintenance, and risk assessment. Accordingly, this study considers the reliability, probability P_f and aims to establish a relationship between P_f–S–N thereby providing a scientific foundation for concrete design in actual engineering.

Define the reliability function based on the two-parameter Weibull distribution:

$$R = 1 - P_f. \tag{13}$$

Determine the fatigue life calculation values of cement concrete under different reliability probabilities using the formula mentioned earlier (refer to Formula 3):

$$N_{P_f} = \eta \left(\ln \frac{1}{P_f} \right)^{\frac{1}{\beta}}. \tag{14}$$

We computed the fatigue life of cement concrete at various reliability probabilities P_f basing on the Weibull distribution model derived from the above calculations and plotted the P_f–N curve for different stress levels S, as depicted in Figure 3, to observe and discuss the relationship among P_f–S–N.

It can be seen from Figure 3 that, regardless of the stress level, the calculated compressive fatigue life of cement concrete decreases with the increasing reliability probability, which implies that, as the demand for higher reliability grows, there is an increasing demand for longer fatigue life of cement concrete. It is also observed that the fatigue life of cement concrete decreases with the increase in number of temperature differential cycling at a given reliability probability. This suggests that temperature differential cycling does lead to a reduction in the compressive fatigue life of cement concrete. Specifically, when the reliability probability is at 95%, the decrease in concrete fatigue life reaches its maximum, 87.51%, at $S=0.70$ and $T=300$. This confirms that temperature cycling indeed has a significant impact on the compressive fatigue performance of cement concrete, and this influence is considerable and cannot be overlooked.

Furthermore, by observing the concrete fatigue life under a 95% reliability probability, it can be seen that assuming the number of temperature differential cycling increases by 60 as one gradient, and assuming the stress level as $S=0.70$–$S=0.75$ as the first gradient, and $S=0.75$–$S=0.85$ as the second gradient. Then, for each gradient increase in the number of temperature differential cycling, the average fatigue life decreases by 25.1%–33.5%; whereas for each gradient increase in stress level, the average rate of decrease in fatigue life reaches 31.2%–97.45%. It is evident that both temperature differential cycling and stress level have a certain degree of impact on the fatigue life of concrete. Moreover, compared to the effect of temperature differential cycling, changes in stress level have a more significant impact.

Choosing an exponential function to fit the relationship between reliability probability and fatigue life based on the experimental results and calculated fatigue life of concrete specimens at different reliability probabilities in this study. This aims to quantitatively characterize the impact of stress level S, number of temperature differential cycling T, and fatigue life N on the reliability probability P_f of cement concrete. The fitting formula is referred to as Formula 15, and the curve is shown in Figure 3.

$$P_f = \exp(\mathrm{a} + \mathrm{b}N + \mathrm{c}N^2), \tag{15}$$

where a, b, and c are constants related to the stress level and the number of temperature differential cycling. The specific values are given in Table 8.

In order to evaluate the fit of the curve, we determined its goodness of fit and standard deviation, as shown in Table 9. It is seen that the goodness of fit of the chosen formula is more than 0.95, and the standard deviation is less than 0.10 for all conditions studied which indicates an excellent fit. And it is suggested that Formula 15 can accurately reflect the impact of stress level, number of temperature differential cycling, and fatigue life on the reliability probability of cement concrete, which provides a theoretical foundation for calculating the reliability of cement concrete structures in actual engineering projects subjected to temperature cycling and fatigue load coupling.

We derived the fatigue life equation for cement concrete under different temperature differential cycling at various reliability probabilities by analyzing the calculation values of fatigue life of cement concrete subjected to different temperature differential cycling conditions at a certain reliability probability. The P_f–S–N curve (Figure 4) was then plotted to more intuitively describe the relationships among the number of temperature differential cycling (T), reliability probability (P_f), stress level (S), and fatigue life (N).

It can be seen upon observing Figure 2 that, regardless of the temperature differential cycling applied, the fatigue life of cement concrete gradually decreases as the stress level increases at a given reliability probability. This suggests that an elevated fatigue load stress level has a detrimental effect on the compressive

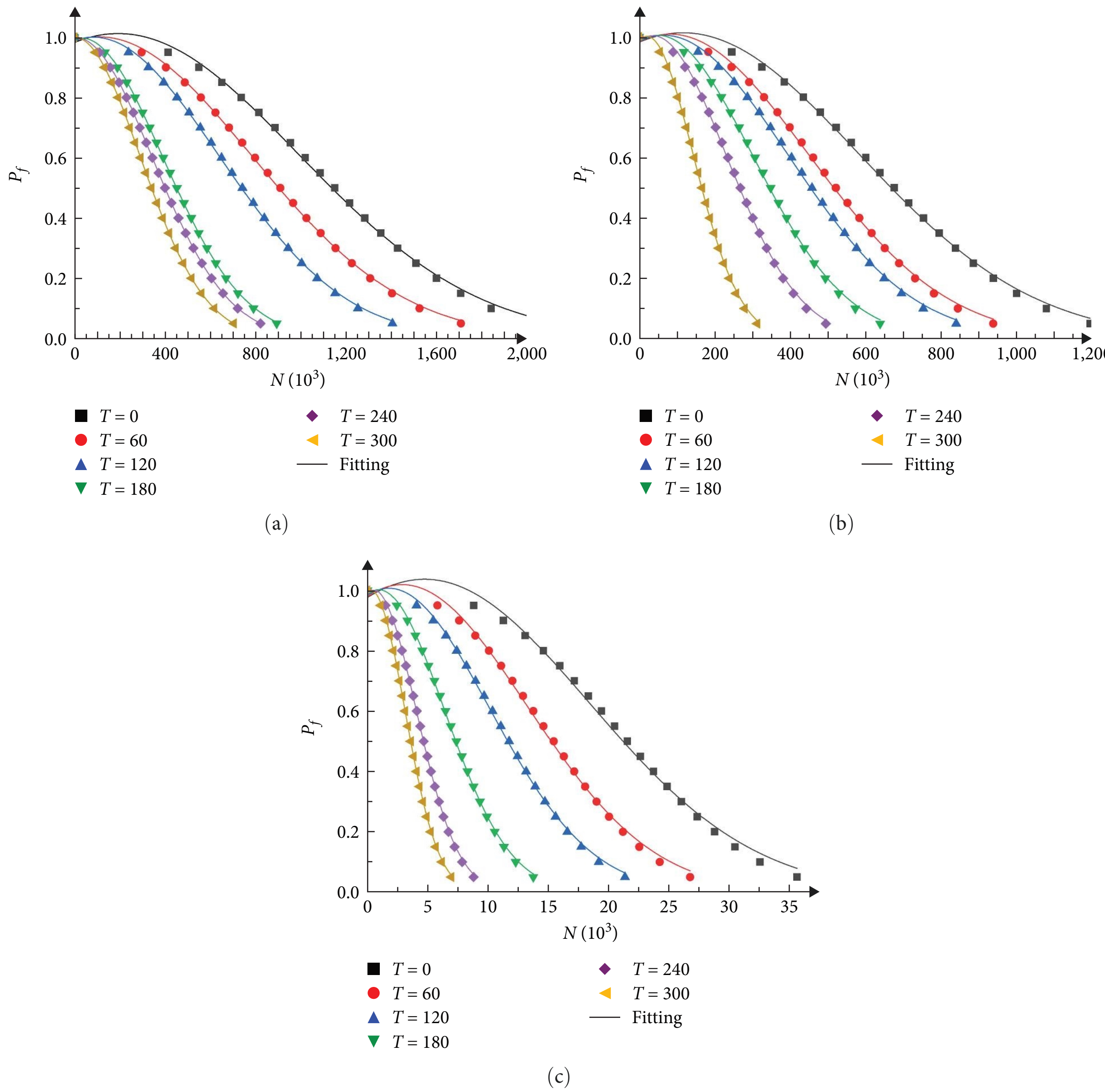

Figure 3: Relationship curve of reliability probability and fatigue life of cement concrete under different stress levels: (a) $S = 0.70$, (b) $S = 0.75$, and (c) $S = 0.85$.

Table 8: Constants for the reliability probability fitting curve of cement concrete.

T	$S = 0.70$			$S = 0.75$			$S = 0.85$		
	a × (−1)	b (10^3)	c (10^6)	a × (−1)	b (10^3)	c (10^6)	a × (−1)	b (10^3)	c
0	0.0169	0.3057	−0.7905	0.0173	0.5355	−2.3090	0.0233	25.340	−0.0027
60	0.0110	0.2376	−1.0972	0.0155	0.6067	−3.7110	0.0192	26.700	−0.0047
120	0.0102	0.2675	−1.6140	0.0136	0.5934	−4.5905	0.0144	24.920	−0.0072
180	0.0056	0.2375	−3.8877	0.0135	0.7789	−7.9374	0.0115	30.820	−0.0170
240	0.0014	0.0641	−4.4676	0.0127	0.9420	−13.1562	0.0103	43.270	−0.0413
300	0.0004	0.0230	−6.0359	0.0112	1.3200	−32.7465	0.0073	39.550	−0.0652

Table 9: Fit evaluation of the reliability probability-fatigue life curve.

T	$S = 0.70$		$S = 0.75$		$S = 0.85$	
	Fitting degree	Standard deviation (max)	Fitting degree	Standard deviation (max)	Fitting degree	Standard deviation (max)
0	0.9802	0.0121	0.9795	0.0125	0.9674	0.0206
60	0.9885	0.0067	0.9824	0.0106	0.9763	0.0146
120	0.9895	0.0060	0.9851	0.0088	0.9583	0.0191
180	0.9946	0.0030	0.9852	0.0088	0.9879	0.0071
240	0.9988	0.0007	0.9863	0.0080	0.9894	0.0061
300	0.9996	0.0002	0.9883	0.0069	0.9927	0.0041

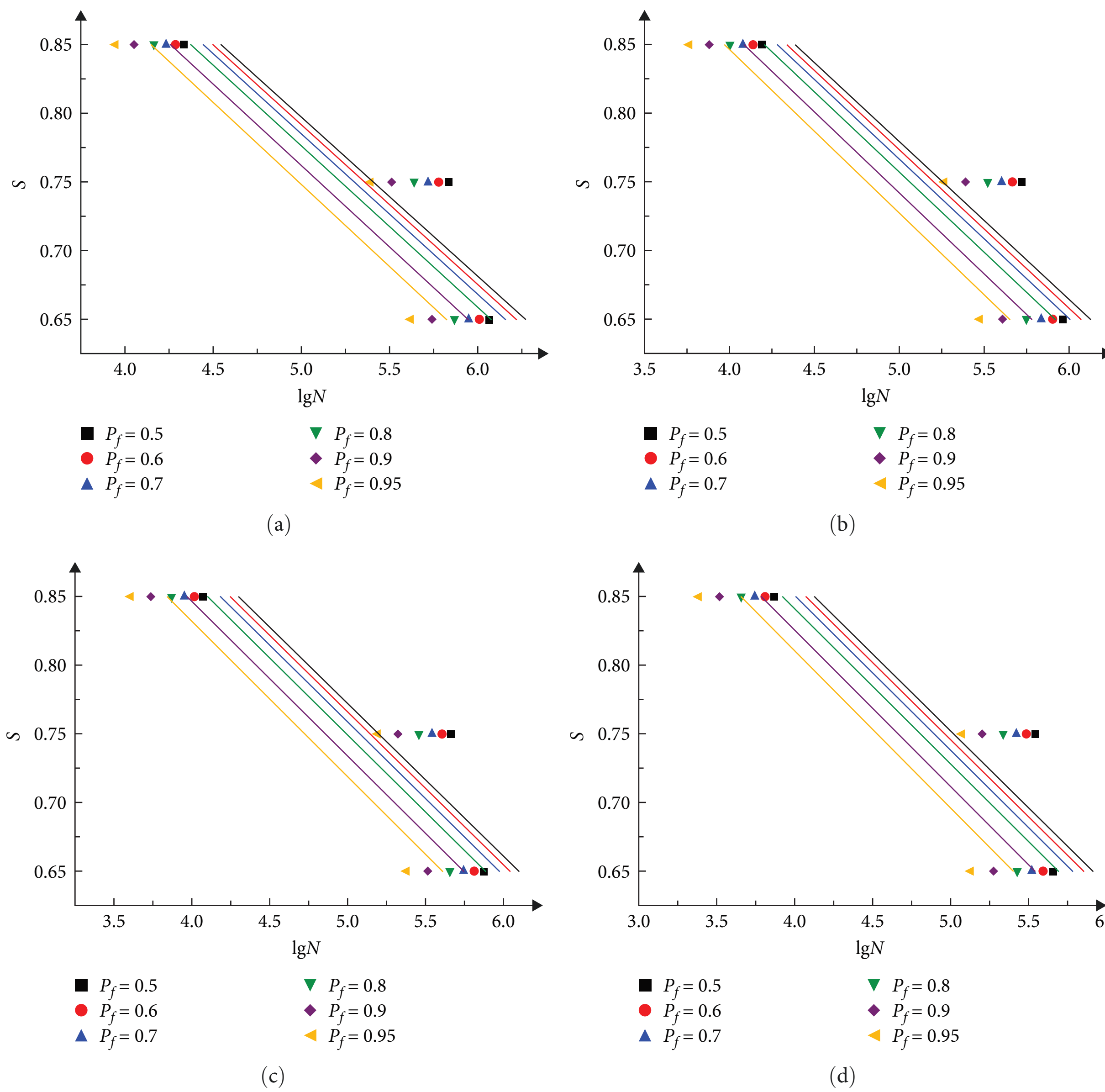

Figure 4: Continued.

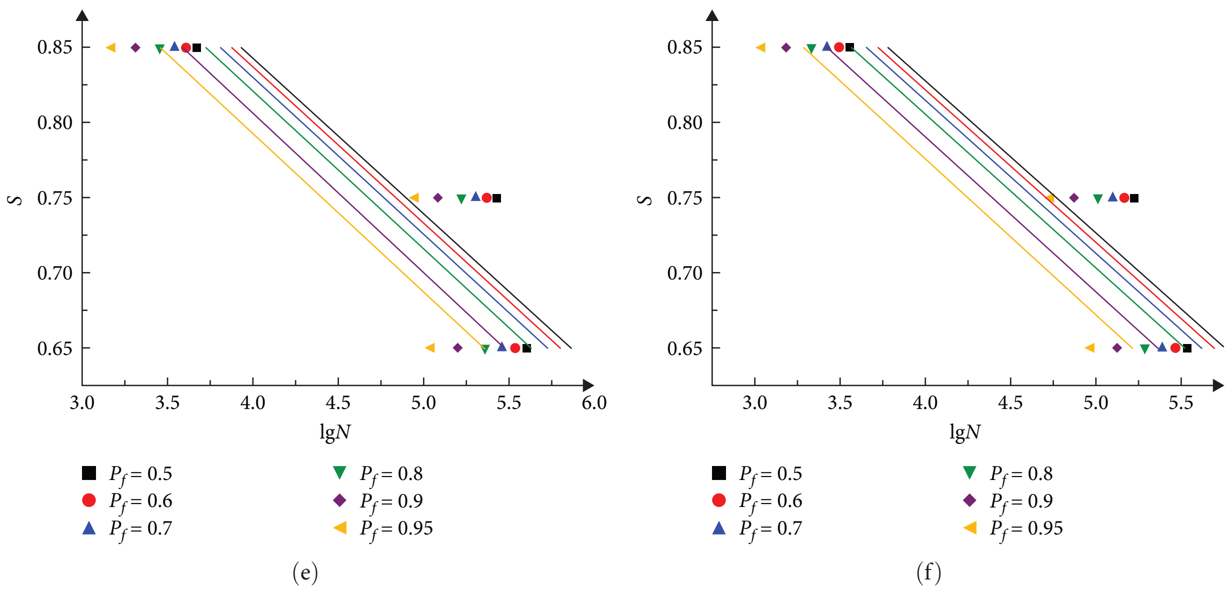

FIGURE 4: P_f–S–N relationship of cement concrete after different temperature differential cycling. (a) $T = 0$, (b) $T = 60$, (c) $T = 120$, (d) $T = 180$, (e) $T = 240$, and (f) $T = 300$.

fatigue performance of cement concrete. Concurrently, it can also be discerned that an increase in reliability probability leads to a decrease in the compressive fatigue life of cement concrete at a uniform stress level. The maximum decrease in concrete fatigue life (logN) can range from 7.31% to 14.41%. This indicates that a higher reliability probability corresponds to increasingly stringent demands on the compressive fatigue performance of cement concrete.

Additionally, one can note by observing the fatigue life curves of cement concrete at the same reliability probability but under different condition of temperature differential cycling that, with the rise in the number of temperature differential cycling, the P_f–S–N curve for cement concrete shows an overall leftward shift, which means that the fatigue life of cement concrete is significantly reduced. Specifically, at a reliability probability of $P_f = 95\%$ and a load level of $S = 0.75$, the fatigue life of concrete with $T = 300$ temperature differential cycling (lgN) is reduced by as much as 12.11% compared to when $T = 0$. This reiterates the profound influence of temperature differential cycling on the compressive fatigue life span of cement concrete. Consequently, for more scientific and accurate design and prediction of the fatigue life of cement concrete in practical engineering design, it is essential not only to consider the environmental stress level (S) and temperature differential cycle characteristics (T) but also to judiciously choose the appropriate reliability probability (P_f).

In order to describe the requirements of fatigue performance for cement concrete under different temperature differential cycling and reliability probabilities more succinctly and provide a scientific basis for engineering design, we define the fatigue life safety factor as α_{P_f} in Formula 16:

$$\alpha_{P_f} = \frac{\Gamma\left(1 + \frac{1}{\beta}\right)}{\left(\ln \frac{1}{P_f}\right)^{\frac{1}{\beta}}}. \tag{16}$$

From Formula 14, it can be transformed to:

$$N_{P_f} = \eta \left(\ln \frac{1}{P_f}\right)^{\frac{1}{\beta}} = \frac{\mathrm{EX}\left(\ln \frac{1}{P_f}\right)^{\frac{1}{\beta}}}{\Gamma\left(1 + \frac{1}{\beta}\right)}. \tag{17}$$

Therefore, α_{P_f} can be expressed as:

$$\alpha_{P_f} = \frac{E[X]}{N_{P_f}}. \tag{18}$$

The fatigue life safety factor of cement concrete under different temperature differential cycling, different stress levels of fatigue loads, and varying reliability probabilities is presented in Table 10.

It can be observed from Table 10 that the compressive fatigue life safety factor of cement concrete increases with the rise in reliability probability under the same stress level. This means that the higher the reliability probability is, the higher the requirement for the fatigue performance of cement concrete would be, which is consistent with reliability theory. Meanwhile, it can be observed that the fatigue life safety factor increases as the number of temperature differential cycling increases under the same stress level and reliability probability. That is, the more the number of temperature differential cycling is, the higher the requirements for the compressive

Table 10: Fatigue life safety factor α_{P_f} for cement concrete after different temperature differential cycling.

P_f	$T=0$	$T=60$	$T=120$	$T=180$	$T=240$	$T=300$
Stress level $S=0.70$						
0.5	1.025	1.037	1.039	1.050	1.061	1.063
0.6	1.155	1.183	1.187	1.209	1.232	1.238
0.7	1.330	1.380	1.388	1.428	1.470	1.480
0.8	1.598	1.689	1.702	1.775	1.850	1.868
0.9	2.144	2.332	2.359	2.514	2.675	2.713
0.95	2.843	3.178	3.227	3.510	3.809	3.881
Stress level $S=0.75$						
0.5	1.024	1.028	1.032	1.032	1.034	1.037
0.6	1.153	1.162	1.171	1.171	1.175	1.182
0.7	1.326	1.342	1.358	1.359	1.366	1.379
0.8	1.591	1.620	1.649	1.650	1.663	1.686
0.9	2.131	2.189	2.250	2.251	2.278	2.326
0.95	2.819	2.923	3.030	3.032	3.081	3.167
Stress level $S=0.85$						
0.5	1.011	1.020	1.022	1.036	1.039	1.046
0.6	1.123	1.144	1.149	1.180	1.186	1.201
0.7	1.271	1.310	1.318	1.376	1.386	1.413
0.8	1.493	1.563	1.577	1.681	1.700	1.747
0.9	1.932	2.072	2.102	2.316	2.355	2.454
0.95	2.474	2.716	2.768	3.149	3.220	3.399

fatigue performance of cement concrete material to ensure reliability. The calculation of the fatigue life safety factor of cement concrete further confirms the impact of temperature differential cycling on the fatigue performance of cement concrete material. Moreover, it can provide a theoretical basis for the design of cement concrete material in actual engineering which considering temperature differential cycling and the combined with the effect of fatigue loads.

4. Conclusions

(1) Temperature differential cycling indeed affects the compressive fatigue performance of cement concrete. Under a reliability probability of 95%, the reduction in concrete fatigue life reaches its maximum at $S=0.85$, $T=300$, amounting to 87.51%. Additionally, the compressive fatigue life of cement concrete affected by temperature differential cycling conforms to the two-parameter Weibull distribution model.

(2) Regression analysis of the fatigue life of cement concrete under different temperature differential cycling and varying stress levels of fatigue loads shows that, from the P_f–N curve, regardless of the stress level, the calculated fatigue life under the same reliability probability decreases with the increase in the number of temperature difference cycles. At a 95% reliability probability, the decrease can range from 77.5% to 87.5%.

(3) Based on the exponential function, a concrete fatigue life prediction model based on different reliability levels was established, analyzing the P_f–S–N relationship of concrete considering reliability. It was found that, regardless of the temperature differential cycling, an increase in reliability probability could lead to a reduction of 7.3%–14.4% in logarithmic fatigue life (lgN).

(4) Defined fatigue life safety factor (α_{P_f}), which is the ratio of the expectation of experimental fatigue life to the calculated fatigue life. While taking into account the number of temperature differential cycling and stress level, the influence of reliability probability is also taken into account. In order to provide a theoretical basis for practical engineering projects with different reliability requirements and coupled effects of temperature differential cycling and fatigue loads.

Data Availability

The data used to support the findings of this study are available from the authors upon request.

Conflicts of Interest

The authors declare that there is no conflicts of interest regarding the publication of this paper.

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