

Research Article

Expression and Analysis of Uncertainty in Deep Foundation Pit Design Scheme Decision-Making

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The burgeoning urbanization of major cities has precipitated a critical examination of deep foundation pit projects, with escalating costs, protracted construction phases, complex site conditions, and specialized technical requirements. Selecting the optimal design scheme from multiple alternatives in a multiattribute decision-making environment poses a significant challenge. This study presents a novel model tailored for the design of deep foundation pits in design-build (DB) contracting projects. The model combines multiattribute ideal point theory with the analytic hierarchy process to evaluate 22 key factors and their uncertainties. It computes the deviations of potential design schemes from ideal benchmarks across all considered attributes. By employing the lexicographic hierarchy aggregation operator, the model aggregates group-level deviations and linguistically weighted evaluations to calculate a comprehensive score for each design scheme. This approach aids in identifying the most suitable design to meet the deep foundation requirements of DB projects. The effectiveness of the model is demonstrated through its application in the decision-making process for a commercial hotel's deep foundation pit design scheme. The empirical findings affirm the model's ability to identify critical factors and accurately assess their impact on engineering design decisions in DB contracting projects. Among the four evaluated designs, the continuous retaining wall scheme achieved the lowest group deviation score, marking it as the preferred option. Consequently, this research offers a robust framework for making informed decisions in the design of deep foundation pits within DB contracting projects, effectively handling the complexities of uncertain linguistic evaluations and the collaboration of multiple attributes.

1. Introduction

The burgeoning urbanization of China's major cities has precipitated a critical examination of housing supply strategies and has caused the densification of living spaces to emerge as a pivotal element in contemporary urban planning discourse [1]. This trend has necessitated the advancement of deep foundation pit support engineering, a sector that has burgeoned in response to the escalating demands of urban infrastructure development. The inherent complexities of these projects, characterized by substantial investment, intricate designs, and formidable technical challenges, are compounded by protracted construction timelines and a plethora of operational uncertainties.

In the realm of execution, deep foundation pit support endeavors are susceptible to a myriad of factors that can precipitate systemic failures and catastrophic events, casting a shadow over the safety and fiscal viability of such projects [2]. The Chinese construction sector, marked by its compartmentalized professional structure and labor division, further exacerbates these challenges through restricted coordination and information exchange between the design and construction phases, thereby hindering effective communication and collaboration. Against this backdrop, our study advocates for the integration of the design-build (DB) contract model, a paradigm that amalgamates design and construction responsibilities within a single entity, thereby fostering a more agile

and responsive approach to on-site construction challenges and dynamic project landscapes.

Our inquiry delves into emblematic case studies to elucidate the synergistic interplay between design and construction efficacy in deep foundation pit projects under the auspices of the DB contract model. This exploration endeavors to furnish a novel framework and methodology for the adjudication of holistic, proficient, and cost-effective design paradigms for deep foundation pit undertakings, thereby propelling the urban deep foundation pit engineering industry forward and enriching the corpus of knowledge and practical wisdom in this domain.

The crux of this article is the formulation of an optimal decision-making paradigm for the design and support mechanisms of deep foundation pit engineering within DB contract ventures. This model synergizes the multiattribute decision-making (MADM) theory predicated on positive ideal points with the analytic hierarchy process (AHP), accommodating the inherent uncertainties of the myriad factors influencing design and support decisions. A compendium of 22 pivotal factors is distilled, informing the decision-making process under the DB framework. By pinpointing the positive ideal points for each determinant, the model quantifies the divergence of potential solutions from these ideals.

Subsequently, the lexicographic hierarchy aggregation (LHA) operator is employed to synthesize group deviations and weighted linguistic assessments for each design support scenario, culminating in the ascertainment of the quintessential support strategy for the deep foundation pit project under DB contractual governance. The main contributions of this paper are twofold:

- (1) The paper pioneers a decision-making model that integrates positive ideal points with the AHP, tailored to the design and support dynamics of deep foundation pit engineering within DB contracts. This model adeptly navigates the inherent uncertainties in such projects, pinpointing the most advantageous design support strategy.
- (2) Our approach innovatively augments the uncertain MADM methodology with an objective analysis weight phase, which mitigates decision-maker biases. This enhancement diversifies the repertoire of fuzzy MADM techniques and paves the way for future explorations into alternative uncertain multiattribute linguistic aggregation operators.

2. Literature Review

2.1. Deep Foundation Support. The exploration of support schemes for deep foundations is a significant research area in geotechnical engineering. Advancements have been made in understanding the intricate interactions between support systems and the geomechanical behaviors of surrounding rock masses. For instance, Zhang et al. [3, 4] employed geological mechanics model experiments, offering insights into the collaborative load-bearing capacities of rock-support systems in deep-buried tunnels across challenging geological

settings. Their findings are pivotal for refining construction methodologies and support design paradigms.

Rock mechanics has become a fundamental discipline for ensuring the structural integrity of hydraulic engineering projects. Significant works, such as those by Feng et al. [5], have synthesized the contributions of rock mechanics in advancing hydraulic engineering by assessing in-situ stress profiles and designing deep excavation strategies. Ou et al. [6] examined deformations in foundation support systems near high embankment railways, proposing novel support schemes to enhance stability during excavation. Furthermore, Zhang et al. [7] addressed squeezing deformations in the Jinping deep-soft rock tunnel, integrating numerical simulations and field observations to suggest supplemental support strategies. Similarly, Gao et al. [8] presented innovative approaches for modeling spatiotemporal rock damage characteristics, which are crucial for optimizing support systems in tunnel environments.

Case studies like those of the Men Keqing coal mine by Ma et al. [9] and the Shandong Anju coal mine by Zhang et al. [10] serve as benchmarks, contributing to the development of effective excavation and support strategies under diverse geological conditions. Additionally, studies on foundation pile-anchor systems and the behavior of deep soft rock, such as those conducted by Li et al. [11] and Xuyang et al. [12], have provided valuable preventive strategies and insights into support design for urban construction and mining applications.

These studies have contributed to advancing the understanding of support schemes for deep foundations, addressing challenges in various geological settings. However, there are still limitations and areas for further improvement. Some potential disadvantages or limitations include the complexity of geological conditions, the accuracy of numerical simulations and modeling techniques, and the need for more comprehensive field data and case studies. Continuous research and collaboration between academia and industry are necessary to overcome these challenges and develop more robust and efficient support systems for deep foundations.

2.2. Uncertainty in Multiattribute Theory. Decision-making within complex engineering projects like deep foundation pits necessitates navigating a labyrinth of variables replete with uncertainties. The crux of MADM is the simplification of these intricate variables into decipherable metrics, allowing for guided selections among various options. In the realm of MADM, innovative strides have been made, particularly in augmenting the robustness of decision-making under uncertain conditions.

Zhang et al. [13] enriched the corpus of MADM by integrating evidence theory with J-divergence in a fuzzy group setting, thereby enhancing model adaptability in diverse decision landscapes. Additionally, handling the uncertainties prevalent in investment decisions, Liu et al. [14] introduced a hesitation fuzzy method aligned with regret theory to address stakeholder-bounded rationality and decision complexities faced by risk-averse investors.

Attribute weighting and manipulation, a focal research consideration, have seen distinctive advancements. Darko

and Liang [15] refined weights using the best–worst method, while others like Ji et al. [16] and Jin et al. [17] employed uncertainty and robust optimization theories, respectively, to tailor strategic weight adjustments, considering cost uncertainties. Further advancing weights management under uncertainty, Garg et al. [18] presented interval-valued picture uncertain linguistic sets to tackle the gradations of indecision in attributes.

Moreover, to confront the challenges of complex decision matrices, Liu and Zhang [19] brought forth the MABAC method, using borderline approximation area comparisons for nuanced differentiation between options. Huang et al. [20] enhanced MAGDM via triangular fuzzy numbers, and Su et al. [21] integrated prospect theory into their evaluation model, emphasizing the management of probabilistic and uncertain linguistic assessments.

Conclusively, the diversification of clustering methods to categorize uncertain data, exemplified by Uddin et al. [22] applying rough set and information theories and Wen [23] utilizing weighted hesitant fuzzy soft sets in group contexts, underscores the progress in aggregating disparate decision agents to forge communal resolutions.

This body of research reaffirms the exponential growth of MADM methods in addressing the omnipresent uncertainties in robust engineering projects like deep foundation pits, with key implications for improving decision confidence and project outcomes.

2.3. AHP. AHP continues to play a vital role in resolving the multifaceted challenges encountered in construction management, offering an established path to assess conflicting criteria in MADM scenarios. Its application, spanning over a decade, has been meticulously examined by Darko et al. [24], uncovering prevalent application areas like risk management and sustainable construction. The evolution of AHP is marked by Chan et al.'s [25] compelling introduction of D-AHP, which enriches AHP's traditional model with D numbers, enhancing the reliability of weight derivations and alternative rankings in multiple criteria decision-making (MCDM) processes.

Focusing on the efficacy of information credibility, Deng and Deng [26] further illuminate the impact of D-AHP on MCDM outcomes, asserting the criticality of trustworthy data. Simplification and methodological refinement in AHP are also noteworthy, as evidenced by Leal's [27] streamlined approach that scales down the number of comparative judgments required, thereby expediting the prioritization process. Liu et al. [28], on the other hand, underscore the burgeoning research on the employment of fuzzy comparison matrices within AHP while signaling a need for a comprehensive discourse on the relative merits of such approaches.

Breaking new ground, Kutlu Gündoğdu and Kahraman [29] explore the territory of generalized fuzzy set theory, extending it to spherical fuzzy sets and adapting the AHP framework into spherical fuzzy AHP. This innovative expansion demonstrates its success in applied scenarios like renewable energy location selection, showcasing its potential against other fuzzy methodologies like neutrosophic AHP.

In summary, AHP's invaluable contribution to weight determination in the uncertain domain of deep foundation

pit design scheme decision-making remains unrivaled. Its systematic approach to deconstructing and evaluating multiple objectives addresses the subjective nuances and fuzziness inherent in judgment, thereby underpinning a sound and transparent decision-making framework.

2.4. Decision Application of Deep Foundation Support Schemes. The arena of deep foundation support schemes has witnessed a surge in research, covering crucial areas such as design theory, construction management, and technological breakthroughs. Distilling substantial contributions from a wide array of studies reveals a multifaceted approach toward tackling the overarching challenges in this field.

A strand of research has concentrated on decision-making tools that employ fuzzy logic to refine choice selection in development scenarios. Król-Korczak and Brzychczy [30] introduced the fuzzy system decision rule to navigate postexploitation phases of open gravel and sand aggregate mines, significantly augmenting decision-making accuracy in such contexts.

Another focus has been on the predictive analysis of deformations in support structures. Gao et al. [31] utilized monitoring data to profile deformation characteristics in foundation pit excavations and circular walls, thereby shedding light on underlying support structure deformation patterns. This aligns with the work of Sun et al. [32], who provided a comprehensive model for the establishment process, emphasizing the importance of recognizing building and support structure deformations. Tackling a similar challenge from a geological perspective, Lei and Gong [33] tailored the mobilizable strength design theory specific to Jinan's geological features, while Zhao et al. [34] offered nuanced insights into the varied, complex deep foundation pits excavated in distinct sections, contributing toward a repository of knowledge for similar engineering endeavors.

Explorations on optimizing deep foundation pit support schemes have been pivotal as well. Liu et al. [35] devised a novel method based on fuzzy logic and gray relational analysis, targeting the enhancement of support scheme selection. Chen et al. [36] adopted an improved AHP tailored for railway station foundation pits, facilitating practical decision-making in support type selection. On a technological front, Ding [37] introduced a deformation detection model grounded in neural networks and wireless communication, pushing the envelope of real-time monitoring of high-rise building foundation pit support structures. Complementing these advances, Yang et al. [38], by leveraging finite element analysis and on-site monitoring data, offered a thorough examination of deformation patterns in large deep foundation pits located in soft soil regions, laying a solid groundwork for subsequent designs.

Collectively, these scholarly endeavors serve not only to expand the decision-making corpus in the realm of deep foundation pit support schemes but also to impart empirical wisdom and forward-thinking strategies to the construction industry at large.

2.5. Research Gaps. Based on a comprehensive review of international research in the field of deep foundation pit design schemes, the following research gaps can be identified:

- (1) In the field of deep foundation pit design schemes, there is limited integration of decision-making research.
- (2) Most researchers primarily analyze deep foundation pit design schemes from the perspective of real estate or construction companies, with limited exploration of decision applications under the DB contract mode.
- (3) Early research in deep foundation pit design relied heavily on environmental and soil condition factors, using the AHP for decision-making and factor selection. However, the AHP method is now considered insufficient to meet the complex and evolving decision requirements.
- (4) Achieving a balance between safety and cost-effectiveness in design schemes remains a significant challenge, leading to limited precision in decision-making.

In summary, based on the above research gaps, the article proposes an optimal decision model for deep foundation pit design schemes in DB contracting projects based on the ideal point multiattribute decision theory and the AHP. This model accurately addresses the evaluation of linguistic uncertainty and the decision-making problem of multiple schemes in a collaborative environment of multiple attribute factors. It provides an effective approach for decision-making regarding deep foundation pit design schemes in DB contracting projects.

The rest of the paper is organized as follows: Section 3 introduces the proposed decision model. Section 4 outlines the experimental results of the proposed method. Section 5 provides a comprehensive exploration of the deep foundation pit design decision model through parameter sensitivity analysis. The model's innovativeness is validated through a case study, and a thorough discussion on its limitations is presented. Finally, Section 6 presents the conclusion of this work.

3. Methodology

3.1. Determination of Decision Model Algorithm. In the process of selecting a solution, the evaluation of deep excavation support is complex and cannot be approached from a singular perspective. It necessitates a comprehensive consideration of multiple characteristics in proposed solutions. This drives deeper requirements for the decision model, demanding the integration of evaluations for various characteristics to form a quantifiable value, facilitating intuitive comparisons. In different scenarios, decision-makers may not entirely focus on the same aspects. Some situations prioritize attribute weights, while others emphasize relationships between attributes. Considering different forms of attribute values, such as real numbers, interval numbers, intuitionistic fuzzy numbers, or even natural language, diverse integration methods are needed to accommodate these distinct forms. Therefore, achieving the selection of the optimal solution for deep excavation support requires ranking various design solutions using specific algorithms. Key steps in the decision process involving fuzzy information [39] include the following:

Data Collection: This involves two types of data: attribute weights and attribute values. Attribute weights come in three scenarios: known, partially determined, and unknown. Attribute values are categorized into linguistic, interval, and real number types. Construct a decision matrix and normalize it, obtaining the matrix as shown in Formula (1).

$$R_k(r_{ij}^{(h)})_{\tilde{n} \times \tilde{m}} \quad (1)$$

The formula represents a decision matrix of size $\tilde{n} \times \tilde{m}$, where $r_{ij}^{(h)}$ is the value of the element in the i th row and j th column of the decision matrix under the h th attribute.

Compute Comprehensive Attribute Values: In MADM, it is essential to consider the interaction of complex variables and uncertain factors, emphasizing the degree of importance in describing the objectives to ensure accurate assessment reports. Due to differences and varying importance among factors, estimating the relative importance of factors is necessary, constructing a weighted combination. Weights serve as indicators of the system's goal aspects, reflecting the subjective and objective aspects of the environmental physical characteristics. Attribute weights are calculated using the AHP.

Various operators are then employed to integrate and compute different types of attribute values, resulting in comprehensive attribute values. Typical methods for calculating comprehensive attribute values include the ordered weighted averaging (OWA) operator, weighted average aggregation operator, weighted geometric average aggregation (WGA) operator, and LHA operator.

Among these, the LHA operator, by constructing positive and negative ideal points and considering the relative relationships between schemes, enhances the rationality of the scheme ranking results and strengthens the stability and reliability of the computed results. Therefore, this study adopts the LHA operator. By calculating the distance between positive and negative ideal points and normalizing it, more accurate comprehensive attribute values are obtained, providing a reliable basis for subsequent scheme ranking. The LHA operator excels in handling the uncertainty of attribute values and is widely applied in decision-making for deep excavation design schemes, which is considered an ideal tool.

Utilize Comprehensive Attribute Values to Prioritize and Rank Various Schemes: In the design-construction general contracting mode, where numerous uncertain and complex influencing factors exist in deep excavation design schemes, this study comprehensively considers the characteristics of decision indicators. It constructs decision models and frameworks to enable rapid, comprehensive ranking, and selection of optimal design schemes when decision-makers possess a thorough understanding of various decision indicators.

By combining the subjective weighting method with the objective AHP to determine decision factors and weights, and establishing an evaluation model based on the uncertain MADM theory, positive ideal points, and the LHA operator, quantitative analysis of deep excavation design schemes can be effectively carried out. This approach aims to achieve the optimization of decision-making in selecting schemes.

3.2. Relevant Definitions of the Decision Model

Definition 1. Let n denote the number of judgment criteria. The judgment matrix C is defined as follows:

$$C = \begin{bmatrix} C_{11} & C_{12} & \dots & C_{1n} \\ C_{21} & C_{22} & \dots & C_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ C_{n1} & C_{n2} & \dots & C_{nn} \end{bmatrix}. \quad (2)$$

This is an $n \times n$ square matrix, where the element C_{ij} represents the importance of the i th criterion relative to the j th criterion. To compare the relative importance of criteria, experts conduct pairwise comparisons for each criterion and provide scores indicating their relative importance. The elements C_{ij} and C_{ji} of the judgment matrix C are related and satisfy the following conditions [40]:

- (1) Nonnegativity: $C_{ij} \geq 0$, indicating that the values of relative importance are nonnegative.
- (2) Consistency: If C_{ij} represents the importance value of the i th criterion relative to the j th criterion as imp, then C_{ji} represents the importance of the j th criterion relative to the i th criterion as $1/\text{imp}$. This ensures bidirectional comparison and considers directionality. By constructing the judgment matrix C , experts can provide a quantitative assessment of the relative importance between criteria. These assessments can be used for decision-making and problem analysis, providing a reference for decision-makers.

Definition 2. Given a judgment matrix C , let ξ_{\max} be the maximum eigenvalue of the matrix C . For each criterion, define the weight vector $\beta = (\beta_1, \beta_2, \dots, \beta_n)$, where β_i represents the weight of the i th criterion. Then, the relative weights can be expressed as follows [40]:

$$D_\beta = \xi_{\max} \times \beta. \quad (3)$$

This implies that by multiplying the maximum eigenvalue of the judgment matrix with the weight vector, a vector D_β is obtained. This vector represents the relative weights of importance among the criteria.

Definition 3. The consistency ratio (CR) of a judgment matrix is the ratio obtained by dividing the consistency index (CI) by the random index (RI) [40] as follows:

$$\text{CR} = \frac{\text{CI}}{\text{RI}}. \quad (4)$$

TABLE 1: Judgment matrix consistency evaluation [40].

n	RI
1	0
2	0
3	0.52
4	0.89
5	1.12
6	1.26
7	1.36
8	1.41
9	1.46

Here,

$$\text{CI} = \frac{\lambda_{\max} - n}{n - 1}. \quad (5)$$

Subsequently, one can refer to the RI table based on the matrix dimension to find the corresponding RI. The numerical value of CR indicates the degree of consistency in the judgment matrix. Generally, if CR is less than or equal to 0.1, the consistency of the judgment matrix is considered acceptable; otherwise, further adjustments or corrections are needed. Assuming the RI for a dimension of 9 is provided in Table 1. If CI is equal to 0 or CR is less than or equal to 0.1, it can be concluded that the consistency analysis of the judgment matrix is acceptable.

Definition 4. Let the set of deep excavation support schemes be denoted as $X = \{x_1, x_2, \dots, x_i\}$, $i \in \mathbb{N}$, where x_i represents one scheme in the set of deep excavation support schemes. If there exists $\exists x_i \in X$ such that the scheme satisfies the evaluation criterion “meets safety requirements while achieving optimal economy,” then x_i is referred to as the optimal solution, denoted as $\text{Opt}(x_i)$.

Definition 5. Let $\tilde{S} = [\tilde{s}_0, \tilde{s}_1, \dots, \tilde{s}_{10}] = \{\text{extremely poor, very poor, relatively poor, slightly poor, moderate, slightly good, relatively good, good, very good, extremely good}\}$ be the fuzzy evaluation standard vector for the questionnaire, and let $S = [s_0, s_1, \dots, s_{10}] = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$ be a vector with elements symmetrically arranged around zero. For computational convenience, this paper designs a mapping function $f: \tilde{S} \rightarrow S$, such that for every $\forall \tilde{s}_i \in \tilde{S}$, it satisfies $f(\tilde{s}_i) = s_i$.

Definition 6. Let the set of decision attribute weights be denoted as $\text{AW} = \{aw_1, aw_2, \dots, aw_{na}\}$, where $\forall aw_j \in \text{AW}$, $j \in \{1, 2, \dots, na\}$, and aw_j represents one attribute weight in the set of decision attributes for deep excavation support schemes. Attribute weights signify the relative importance

of each attribute in the decision-making process. The values of attribute weights range from 0 to 1, and they satisfy the condition that the sum of attribute weights equals 1, i.e., $\sum_{j=1}^{na} aw_j = 1$. This set definition ensures the normativity of attribute weights and consistency in the sum of weights, facilitating the accurate consideration of the importance of each attribute in the decision process for deep excavation support schemes.

Definition 7. Let the set of decision-maker weights be denoted as $DW = \{dw_1, dw_2, \dots, dw_{nd}\}$, where $\forall dw_j \in DW$, $j \in \{1, 2, \dots, nd\}$, and dw_j represents one decision-maker weight in the set of decision-makers for deep excavation support schemes. Decision-maker weights signify the relative importance of each decision-maker in the decision-making process. The values of decision-maker weights range from 0 to 1, and they satisfy the condition that the sum of weights equals 1, i.e., $\sum_{j=1}^{nd} dw_j = 1$. This set definition ensures the normativity of decision-maker weights and consistency in the sum of weights, facilitating the accurate consideration of the importance of each decision-maker in the decision process for deep excavation support schemes.

Definition 8. LHA operator weights are denoted as the weight proportion of deviation for each decision-maker. Let LW be the set of LHA operator weights. The number of decision-makers is j , and the upper limit for decision-maker participation is nl . The calculation formula for LHA operator weights is as follows [41]:

$$\text{LHA} = \frac{\text{Distance to NIS}}{\text{Distance to PIS} + \text{Distance to NIS}}. \quad (6)$$

Here, the concepts are explained as follows:

- (1) Negative ideal solution (NIS): The NIS refers to the solution in multiobjective decision-making where, for each target indicator, a smaller value is desired. The NIS is the solution where, considering all target indicators, each indicator takes its maximum value.
- (2) Positive ideal solution (PIS): The PIS refers to the solution in multiobjective decision-making where, for each target indicator, a larger value is desired. The PIS is the solution where, considering all target indicators, each indicator takes its minimum value.
- (3) Distance to NIS: Represents the distance from a solution to the NIS, i.e., the difference between the solution and the NIS on each target indicator.
- (4) Distance to PIS: Represents the distance from a solution to the PIS, i.e., the difference between the solution and the PIS on each target indicator.

According to the literature [41], the set of LHA operator weights is denoted as $LW = \{lw_1, lw_2, \dots, lw_{nl}\}$, where j can be any positive integer, and it satisfies $\sum_{j=1}^{nl} lw_j = 1$.

Definition 9. Let $R = (r_{ij})_{\bar{m} \times \bar{n}}$ be the fuzzy evaluation decision matrix, and there must be a positive ideal point of the scheme in the matrix, denoted as $x^+ = (r_1^+, r_2^+, \dots, r_m^+)$, representing the optimal evaluation of the influencing factors obtained from the fuzzy evaluation decision matrix R . This has special significance in decision evaluation because, based on the positive ideal point, the optimal values of the influencing factors can be inferred and speculated as the optimal solution for all attribute schemes. Each element r_i^+ of the new vector corresponds to the value of the i th decision attribute in the positive ideal point of the original decision matrix. The positive ideal point generated by the fuzzy evaluation decision matrix will also naturally have fuzzy evaluation; that is, there are fuzzy lower and upper limits, denoted as follows:

$$\nu = [r_i^{+L}, r_j^{+U}]. \quad (7)$$

Here, $r_i^{+L} = \max_i \{r_{ij}^{+L}\}$ and $r_j^{+U} = \max_j \{r_{ij}^{+U}\}$, r_i^{+L} , and r_j^{+U} represent the lower and upper bounds of this positive ideal point, respectively.

Definition 10. To verify the extent of the deviation between the actual decision variables and the positive ideal point of the scheme, calculate the actual deviation of uncertain linguistic variables. Let $\mu = [s_a, s_b]$ and $\nu = [s_c, s_d]$ be two fuzzy evaluation variables, where $c \geq a$, $d \geq b$, $(a, b, c, d) \in [0, 10]$ and are integers. Define the deviation function as follows:

$$\mathcal{D}(\mu, \nu) = \frac{1}{2} (s_{c-a} \oplus s_{d-b}). \quad (8)$$

Here, \oplus denotes the deviation calculation between fuzzy sets. This definition is used to quantify the differences between two fuzzy evaluation variables on a given decision attribute.

Definition 11. In order to manifest the collective deviation of the entire decision-making group in this decision, the deviations of each decision-maker's influencing factors are, respectively, aggregated with their corresponding attribute weights. This aggregation is defined as the group deviation $D(x^+, x_l)$ as follows [41]:

$$D(x^+, x_l) = a\omega_1 \mathcal{D}(\mu, \nu)^{(1)} \oplus a\omega_2 \mathcal{D}(\mu, \nu)^{(2)} \oplus \dots \oplus a\omega_j \mathcal{D}(\mu, \nu)^{(j)}. \quad (9)$$

This enhancement incorporates the weight and deviation of each decision-maker for each influencing factor into consideration, providing a more comprehensive reflection of the collective impact of the entire group on the scheme.

Definition 12. The decision model for deep excavation design under the DB mode is represented as $DP[\text{LHA}_{AW, DW}(\hat{v}_i^{(k)})]$. In this model, $\text{LHA}_{AW, DW}$ denotes the uncertain evaluation value of the scheme by decision-makers based on the attribute weight vector $a\omega$ using the LHA function. The term $D(x^+, x_l^{(k)})$ signifies the group deviation between the scheme

x_i and its positive ideal point, and $\tilde{v}_i^{(k)}$ represents a set of weighted linguistic variables in the form of [41]

$$\left(tdw_1 Dx^+, x_1^{(1)}, tdw_2 Dx^+, x_1^{(2)}, \dots, tdw_k Dx^+, x_1^{(k)} \right), \quad (10)$$

for the k th group, where $k \in N$. The balancing factor t for linguistic variables is equal to the number of decision-makers. Each decision-maker needs to assign a fuzzy linguistic variable for each attribute or criterion for use in the fuzzy MADM model. The one-to-one correspondence between the number of decision-makers and linguistic variables ensures that each attribute is appropriately considered. In the process of weight allocation and fuzzy resolution, each attribute has corresponding parameters, ensuring proper weight and fuzzy processing for each attribute or criterion, facilitating fuzzy MADM. This balance contributes to improving the accuracy of the model, aligning the final decision more closely with the preferences and goals of decision-makers. Therefore [41],

$$LHA_{AW,DW}(\tilde{v}_i^{(k)}) = lw_1 \tilde{v}_1^{(1)} \oplus lw_2 \tilde{v}_2^{(2)} \oplus \dots \oplus lw_p \tilde{v}_i^{(k)}. \quad (11)$$

This calculation is performed sequentially to compute the decision model for deep foundation pit design under the DB mode. The decision model is denoted as $DP[LHA_{AW,DW}(\tilde{v}_i^{(k)})]$, where DP stands for DB mode, LH stands for linguistic hesitant (LH) method, $A_{AW,DW}$ represents the attribute weights based on the AHP and the analytic hierarchy gray relational analysis, and $(\tilde{v}_i^{(k)})$ denotes the weighted linguistic variable of the i th attribute of the k th scheme.

This method advances the field by introducing a multi-attribute ideal point theory. Specifically, the method integrates disparate decision-making processes into a singular, coherent framework, effectively bridging the gap in integrated decision-making research.

3.3. Decision Model Factors. For delineating the factors influencing decision-making, the primary step involves a meticulous analysis and consolidation of the characteristics inherent in deep foundation pit projects. Cui [42] has already conducted a detailed analysis and identification of the influencing factors concerning the design schemes for supporting deep foundation pits. This analysis encompasses various aspects, including project owner requirements, contractor conditions, technical specifications, project attributes, and local policy conditions. A discernible pattern emerges, indicating that within the traditional contracting model, design schemes are exclusively developed by design units. Despite potential input from contracting companies, design enterprises often fall short in fully incorporating considerations for the contractor's experience, construction capabilities, equipment advantages, funds, and management capabilities. Moreover, they may neglect predictions regarding on-site environmental conditions, resident impacts, climate, and

hydrological changes. Notably, they are unlikely to address the financial considerations of the contractor's construction project. Consequently, design decisions under this model struggle to achieve a harmonious balance between safety and economic viability. In contrast, under the DB model, the design scheme is orchestrated by the general contracting enterprise, striving to circumvent issues arising from on-site factors, thereby yielding superior project benefits.

Drawing insights from pertinent literature on deep foundation pit support schemes and synthesizing the distinctive features of engineering general contracting models impacting the design of foundation pit projects, the factors influencing decision-making can be systematically classified into five primary categories: overall project factors, construction unit factors, general contracting enterprise factors, project factors, and environmental factors. Adopting the target design method during the preliminary identification of specific influencing factors involves correlating and implementing the goals of different project stages, including safety, quality, duration, cost, and social impact, across every typical structure of the project. This comprehensive analysis facilitates the identification of decision-making factors for deep foundation pit design schemes [43].

Finally, by referencing the logic of cost compilation classification and aligning with attribute features, these factors can be systematically grouped into five major categories: overall project factors, construction unit factors, general contracting enterprise factors, project factors, and environmental factors. Following a holistic examination of contradictions arising in the design schemes of deep foundation pits in special regions, consulting on specialized issues related to large-scale deep foundation pit design schemes in the region, conducting interviews with industry experts, relying on their theoretical analysis and practical experience, and considering the objective conditions of regional project implementation, a total of 22 decision-making factors for deep foundation pit support design schemes under the DB general contracting model were identified [44], representing specific aspects, as shown in Table 2.

3.4. Model Assumptions

Assumption 1. *Decision Environment: The deep foundation pit project has been determined to adopt the DB contracting mode.*

Assumption 2. *Identifying Decision Attributes: Conducting a literature review and engaging experts in the fields of design research, construction technology, and relevant academic scholars from universities, the selected schemes possess a certain level of authority. Assuming four common deep foundation pit design schemes based on the DB model [2] are as follows:*

- (1) x_1 —Gravity Retaining Wall Scheme: Suitable for shallow foundation pits with favorable geological conditions, utilizing a flexible structural system for support. Its characteristics include lower environmental requirements, making it suitable for situations where

TABLE 2: Decision attributes for deep foundation pit support design schemes under the DB general contracting model.

Engineering classification	Overall project	General contracting enterprise factor	Environmental effect	Project factors
Key influencing factors	μ_1 : Construction coordination and overlapping	μ_5 : Experience in similar projects in contracted enterprises	μ_{12} : Influence of construction on surrounding residents	μ_{17} : Complexity of project technology
	μ_2 : Application of new materials and technologies	μ_6 : Construction safety capability of supporting system	μ_{13} : Influence of construction on surrounding buildings and underground equipment	μ_{18} : Project design depth and service requirements
	μ_3 : Environmental safety around the construction site	μ_7 : Main equipment advantages of enterprises	μ_{14} : Narrow space	μ_{19} : Expected profit of project
	μ_4 : Standardization of economic and technical indicators	μ_8 : Design capability of contracting enterprises	μ_{15} : Building density and strength	μ_{20} : Project scale
	—	μ_9 : Construction technical ability	μ_{16} : Climatic and hydrological conditions of the project	μ_{21} : Project duration requirements
	—	μ_{10} : Enterprise governance ability	—	μ_{22} : Construction site convenience conditions
	—	μ_{11} : Financial ability of contracting enterprises	—	—

a certain horizontal displacement and ground settlement are permissible at the top of the pit.

- (2) x_2 —Soil Nailing and Shotcrete Support Scheme: This is a commonly used technique for foundation pit support, offering advantages of speed and economy. However, its drawback is the tendency for a certain amount of horizontal displacement and ground settlement. Therefore, it requires a larger peripheral area at the top of the pit and strict control over pit deformation.
- (3) x_3 —Sheet Pile Support Scheme: This belongs to a rigid support scheme applicable to deep and geologically complex foundation pit projects. While it comes with higher costs, it excels in specific geological conditions, such as the presence of a thick layer of circular gravel, where the prevention of fine sand and rounded gravel being squeezed out of the support piles is necessary.
- (4) x_4 —Underground Diaphragm Wall Support Scheme: Also a rigid support scheme suitable for deep and geologically complex foundation pit projects. This scheme has a higher reinforcement content and excellent sealing performance and is suitable for applications demanding superior soil retention and water-stopping effects. Additionally, it can be used as an exterior wall for permanent underground structures.

Assumption 3. *Determination of Decision-Maker's Weight Vector: Utilizing the uncertainty algorithm based on the positive ideal point and LHA operator, it is necessary to first determine the relative weight vector of decision-makers. The fairest approach is to assume equal weights for decision-makers, i.e., the weight vector is (0.3333, 0.3333, 0.3333).*

Therefore, according to the definition: $dw_1 = 0.3333$, $dw_2 = 0.3333$, $dw_3 = 0.3333$.

Assumption 4. *Professor Jüger, in his research on operators, indicated that the interval of decision-maker weights required by operators varies systematically based on the number of decision participants and the deviation among decision-makers. The larger the base number of decision-makers or the deviation in decision opinions, the larger the maximum value of the weight interval range. It is generally divided into three types: (0.3, 0.8), (0.2, 0.7), and (0, 0.5). In this study, three industry experts, all holding senior positions, are assumed to be decision-makers. Therefore, the decision-maker weight interval chosen is (0, 0.5) [45].*

3.5. Model Establishment. Based on the aforementioned model assumptions, establish the set of options, attribute set, and language term set (specific settings refer to Section 2.2). The priority relationship among attributes is represented as $x_1 > x_2 > x_3 > \dots > x_i$. The decision model derivation and calculation steps are as follows:

- Step 1: Calculate the weight vector of decision attributes; select nd experts in deep foundation engineering for investigation. Through the distribution of electronic questionnaires and utilizing the received valid responses, construct the judgment matrix C based on the Saaty 1–9 scale method. Organize the decision attributes at various levels of the deep foundation project design, then, using Formula (3), normalize the matrix data at higher levels and calculate the maximum eigenvalue and relative weights. Apply Formula (4) to conduct a rationality analysis of the relative weights of elements at the current

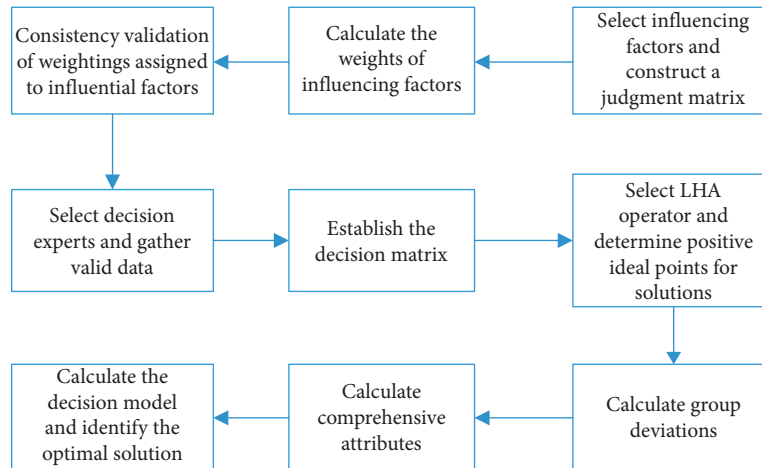


FIGURE 1: Schematic diagram of the modeling and analysis process.

level and eliminate any unreasonable questionnaire data.

- Step 2: Establish the decision matrix; Request the decision group to provide solutions. Conduct nd valid questionnaire surveys with renowned professors and experts through electronic questionnaires, constructing decision matrices $R = (r_{ij})_{m \times n}$ for each of the nd decision-makers.
- Step 3: Identify the positive ideal point of the solutions. Based on the decision-makers' matrices, identify the positive ideal point $\nu = [r_i^{+L}, r_j^{+U}]$ for each influencing factor of the solutions.
- Step 4: Solving the deviation between the solution and the positive ideal point, and aggregating based on their corresponding attribute weights to obtain group deviation: Utilizing Formula (9), perform deviation aggregation on the deviation $\mathcal{D}(\mu, \nu)$ corresponding to the i th decision-maker, calculating the group deviation $D(x^+, x_i)$, and then determining the weighted linguistic variables for each group $\tilde{v}_i^{(k)}$;
- Step 5: Develop the decision model for deep foundation pit support schemes under the DB mode using the LH method. The decision model is represented as DP $[LHA_{AW, DW}(\tilde{v}_i^{(k)})]$. The LH method relies on the calculated weighted linguistic variables $\tilde{v}_i^{(k)}$. Acquire the scheme set $X = \{x_1, x_2, \dots, x_i\}$. Establish the scheme set and perform ranking and selection. Determine the group with the lowest comprehensive score as the optimal choice $Opt(x_i)$ in the decision-making process for deep foundation pit design (refer to Figure 1).

4. Results

4.1. *Project Overview.* X Investment Management Co., Ltd., plans to construct a commercial hotel project in X district, X

city. The project is planned with a total land area of 6,870 square meters, a total construction area of 36,190.2 square meters, and a total building footprint area of 2,197.5 square meters. The proposal includes the construction of a 19-story hotel complex building with a height of 75.9 m above ground, featuring three basement levels. The site's elevation is ± 0.00 , at 75.5 m, and the surrounding environment is complex. The northwest and southeast sides are close to the main street with heavy traffic, and numerous pipelines surround the site. On the northeast side, there is a neighboring food building, while on the southwest side, there is a neighboring group building. The anticipated excavation depth for the project's foundation pit is 14 m, and the safety level of the foundation pit is categorized as Level I. In accordance with engineering requirements and design specifications, four deep foundation pit design schemes have been formulated, namely gravity retaining wall scheme (x_1), soil nailing and shotcrete support scheme (x_2), sheet pile support scheme (x_3), and underground diaphragm wall support scheme (x_4).

The objective of the study is to utilize a scientific multi-attribute decision model to screen and identify the most superior solution among these various deep foundation pit design schemes under specific engineering conditions. This aims to comprehensively consider the requirements of engineering safety and economic feasibility.

4.2. Decision Model Calculation

Step 1: Calculate the weight vector of decision attributes.

Engage three experts in deep foundation pit engineering for a survey. Distribute electronic questionnaires and utilize the received valid responses. Based on the Saaty 1–9 scale method, construct judgment matrices C by averaging the data from each expert's opinion. Organize the AHP hierarchical analysis model for deep foundation pit project design (as illustrated in Figure 2).

Subsequently, normalize the data of each matrix at various levels and compute the relative weights of each

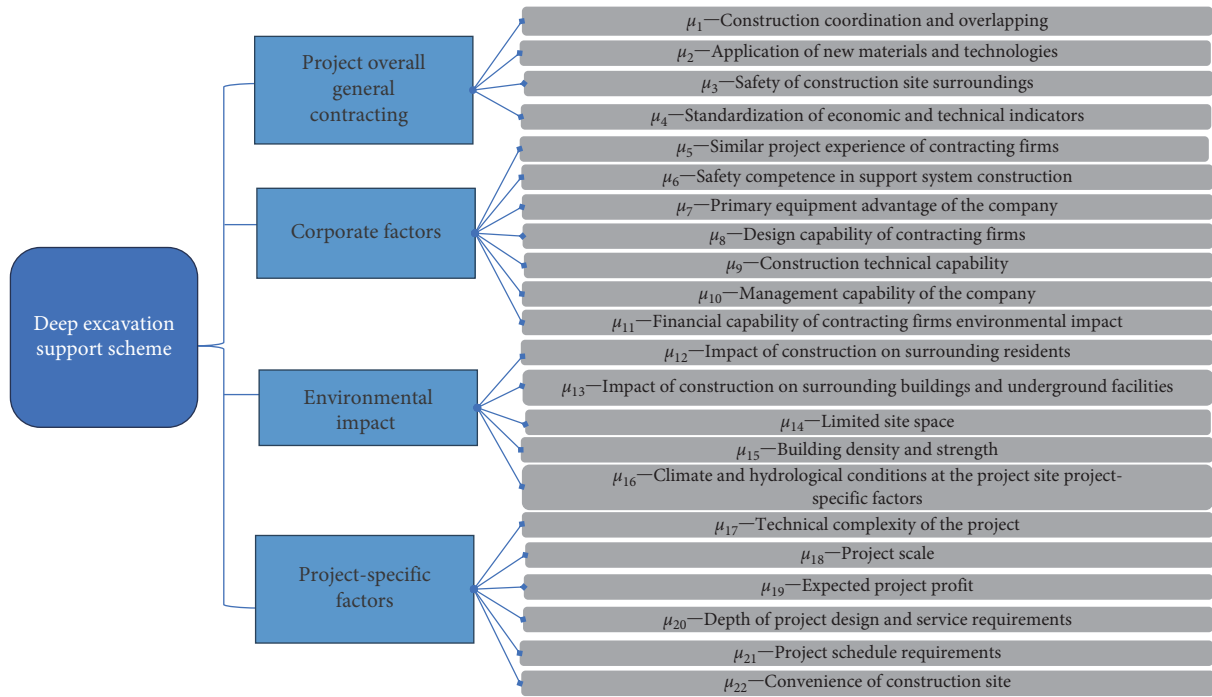


FIGURE 2: AHP hierarchical analysis model for deep foundation pit support schemes.

influencing factor using Formula (3) (refer to Figure 3). In Figure 3, it is apparent that among the 22 indicators, the one with the highest weight is “Construction coordination and overlapping,” scoring 0.211, while the lowest is “Construction site convenience conditions,” scoring 0.0065.

Performing a one-time consistency check using Formula (4), all CR values are less than 0.1, indicating a satisfactory result.

Step 2: Establish a decision matrix.

For the deep foundation pit engineering design schemes under the DB contracting mode, we adopted a questionnaire survey to collect expert opinions to ensure the scientific nature of the decision-making process. In this survey, three experts with advanced professional titles and extensive experience in the field of deep foundation pit engineering were invited to participate. They all have profound theoretical knowledge and practical experience in this field. We used

the quantitative evaluation method of the Satty 1–9 scale to score various indicators of each scheme, aiming to reduce the impact of subjective factors on the results. The three experts filled out electronic questionnaires, presenting a comprehensive view of expert assessment results with high authority and representativeness. All three valid questionnaires collected were compiled to form the decision evaluation matrix (as shown in Tables 3–5). Here, x_1 to x_4 represent different deep foundation pit engineering schemes, and μ_1 to μ_{22} denote the relative weights of various influencing factors for deep foundation pit support schemes.

Step 3: Identifying the positive ideal points for the solutions.

Based on the decision matrices of the three decision-makers, the positive ideal points for each influencing factor of the solutions (refer to Figure 4) are determined as follows:

$$\nu = \begin{bmatrix} [4, 5] & [4, 5] & [4, 5] & [3, 4] & [4, 5] & [4, 5] & [4, 5] \\ [4, 5] & [4, 5] & [4, 5] & [4, 5] & [4, 5] & [4, 5] & [3, 4] \\ [4, 5] & [4, 5] & [4, 5] & [3, 4] & [3, 4] & [3, 4] & [3, 4] & [3, 4] \end{bmatrix}. \tag{12}$$

Figure 4 illustrates the positive ideal points for various influencing factors derived from the decision matrices of the three decision-makers. The horizontal axis (X -axis) represents different influencing factors, labeled from μ_1 to μ_{22} , while the vertical axis (Y -axis) represents the positive ideal points for attribute values, ranging from 3 to 5. The red dots in the chart

indicate the upper bounds for each factor, and the blue dots represent the lower bounds. These data form the basis for calculating the deviation components of different decision-makers from the positive ideal points when choosing different schemes. This information is valuable for assessing the strengths and weaknesses of each scheme and making optimal decisions.

Deep excavation support scheme		
Project overall general contracting	μ_1 —Construction coordination and overlapping	0.2110
	μ_2 —Application of new materials and technologies	0.0908
	μ_3 —Safety of construction site surroundings	0.1292
	μ_4 —Standardization of economic and technical indicators	0.0494
Corporate factors	corporate factors	
	μ_5 —Similar project experience of contracting firms	0.0581
	μ_6 —Safety competence in support system construction	0.0345
	μ_7 —Primary equipment advantage of the company	0.0184
	μ_8 —Design capability of contracting firms	0.0504
	μ_9 —Construction technical capability	0.0388
	μ_{10} —Management capability of the company	0.0255
Environmental impact	μ_{11} —Financial capability of contracting firms	0.0205
	environmental impact	
	μ_{12} —Impact of construction on surrounding residents	0.0557
	μ_{13} —Impact of construction on surrounding buildings and underground facilities	0.0421
	μ_{14} —Limited site space	0.0266
	μ_{15} —Building density and strength	0.0352
Project-specific factors	μ_{16} —Climate and hydrological conditions at the project site	0.0362
	project-specific factors	
	μ_{17} —Technical complexity of the project	0.0257
	μ_{18} —Project scale	0.0136
	μ_{19} —Expected project profit	0.0114
	μ_{20} —Depth of project design and service requirements	0.0111
	μ_{21} —Project schedule requirements	0.0093
μ_{22} —Convenience of construction site	0.0065	

FIGURE 3: Relative weights of various influencing factors in deep foundation pit support schemes.

TABLE 3: Data table of survey on impact factor weights (A).

Survey on impact factor weights	μ_1	μ_2	μ_3	μ_4	μ_5	μ_6	μ_7	μ_8
x_1	[- 3, - 2]	[- 3, - 2]	[4, 5]	[- 4, - 3]	[- 1, 0]	[1, 2]	[- 4, - 3]	[- 1, 0]
x_2	[2, 3]	[0, 1]	[3, 4]	[3, 4]	[3, 4]	[1, 2]	[2, 3]	[2, 3]
x_3	[3, 4]	[0, 1]	[2, 3]	[2, 3]	[3, 4]	[2, 3]	[4, 5]	[2, 3]
x_4	[4, 5]	[3, 4]	[2, 3]	[2, 3]	[4, 5]	[3, 4]	[4, 5]	[2, 3]
	μ_9	μ_{10}	μ_{11}	μ_{12}	μ_{13}	μ_{14}	μ_{15}	μ_{16}
x_1	[0, 1]	[3, 4]	[1, 2]	[4, 5]	[4, 5]	[3, 4]	[3, 4]	[3, 4]
x_2	[2, 3]	[3, 4]	[1, 2]	[3, 4]	[3, 4]	[3, 4]	[3, 4]	[4, 5]
x_3	[3, 4]	[3, 4]	[1, 2]	[3, 4]	[3, 4]	[1, 2]	[1, 2]	[3, 4]
x_4	[4, 5]	[3, 4]	[1, 2]	[1, 2]	[1, 2]	[1, 2]	[1, 2]	[2, 3]
	μ_{17}	μ_{18}	μ_{19}	μ_{20}	μ_{21}	μ_{22}		
x_1	[1, 2]	[- 4, - 3]	[1, 2]	[1, 2]	[2, 3]	[2, 3]		
x_2	[2, 3]	[- 4, - 3]	[2, 3]	[2, 3]	[2, 3]	[1, 2]		
x_3	[2, 3]	[- 3, - 2]	[2, 3]	[2, 3]	[2, 3]	[2, 3]		
x_4	[4, 5]	[- 1, 0]	[1, 2]	[2, 3]	[1, 2]	[3, 4]		

Step 4: Calculate the deviation between the schemes and the positive ideal points and aggregate the deviation components using the LHA operator.

As per Definition 10, the deviation calculation formula for μ and ν is given by $\mathcal{D}(\mu, \nu) = \frac{1}{2} (s_{c-a} \oplus s_{d-b})$. Utilizing this formula, we can compute the deviation components of

TABLE 4: Data table of survey on impact factor weights (B).

Survey on impact factor weights	μ_1	μ_2	μ_3	μ_4	μ_5	μ_6	μ_7	μ_8
x_1	[2, 3]	[-2, -1]	[-4, -3]	[0, 1]	[3, 4]	[0, 1]	[3, 4]	[3, 4]
x_2	[4, 5]	[0, 1]	[-4, -3]	[3, 4]	[3, 4]	[-1, 0]	[4, 5]	[4, 5]
x_3	[4, 5]	[3, 4]	[3, 4]	[3, 4]	[4, 5]	[4, 5]	[4, 5]	[4, 5]
x_4	[4, 5]	[4, 5]	[3, 4]	[-3, -2]	[3, 4]	[3, 4]	[3, 4]	[2, 3]
	μ_9	μ_{10}	μ_{11}	μ_{12}	μ_{13}	μ_{14}	μ_{15}	μ_{16}
x_1	[3, 4]	[3, 4]	[4, 5]	[0, 1]	[-3, -2]	[-4, -3]	[-4, -3]	[2, 3]
x_2	[4, 5]	[4, 5]	[3, 4]	[0, 1]	[-4, -3]	[-4, -3]	[-4, -3]	[-2, -1]
x_3	[3, 4]	[3, 4]	[3, 4]	[1, 2]	[1, 2]	[3, 4]	[2, 3]	[3, 4]
x_4	[4, 5]	[4, 5]	[3, 4]	[1, 2]	[3, 4]	[3, 4]	[3, 4]	[3, 4]
	μ_{17}	μ_{18}	μ_{19}	μ_{20}	μ_{21}	μ_{22}		
x_1	[2, 3]	[2, 3]	[0, 1]	[0, 1]	[0, 1]	[-3, -2]		
x_2	[3, 4]	[-2, -1]	[-3, -2]	[-3, -2]	[3, 4]	[1, 2]		
x_3	[3, 4]	[1, 2]	[1, 2]	[2, 3]	[2, 3]	[3, 4]		
x_4	[2, 3]	[3, 4]	[1, 2]	[3, 4]	[0, 1]	[2, 3]		

TABLE 5: Data table of survey on impact factor weights (C).

Survey on impact factor weights	μ_1	μ_2	μ_3	μ_4	μ_5	μ_6	μ_7	μ_8
x_1	[-3, -2]	[-4, -3]	[-5, -4]	[1, 2]	[3, 4]	[-5, -4]	[3, 4]	[0, 1]
x_2	[-4, -3]	[-4, -3]	[-3, -2]	[1, 2]	[3, 4]	[-4, -3]	[3, 4]	[3, 4]
x_3	[3, 4]	[0, 1]	[2, 3]	[-1, 0]	[3, 4]	[2, 3]	[3, 4]	[3, 4]
x_4	[3, 4]	[0, 1]	[4, 5]	[-1, 0]	[3, 4]	[4, 5]	[3, 4]	[3, 4]
	μ_9	μ_{10}	μ_{11}	μ_{12}	μ_{13}	μ_{14}	μ_{15}	μ_{16}
x_1	[3, 4]	[0, 1]	[2, 3]	[-5, -4]	[-5, -4]	[-5, -4]	[-5, -4]	[-5, -4]
x_2	[3, 4]	[1, 2]	[2, 3]	[-5, -4]	[-5, -4]	[-5, -4]	[-5, -4]	[-5, -4]
x_3	[3, 4]	[3, 4]	[2, 3]	[1, 2]	[1, 2]	[0, 1]	[3, 4]	[2, 3]
x_4	[3, 4]	[3, 4]	[2, 3]	[2, 3]	[2, 3]	[0, 1]	[4, 5]	[2, 3]
	μ_{17}	μ_{18}	μ_{19}	μ_{20}	μ_{21}	μ_{22}		
x_1	[0, 1]	[0, 1]	[-1, 0]	[0, 1]	[-3, -2]	[3, 4]		
x_2	[0, 1]	[0, 1]	[0, 1]	[0, 1]	[-4, -3]	[3, 4]		
x_3	[0, 1]	[0, 1]	[2, 3]	[0, 1]	[3, 4]	[3, 4]		
x_4	[0, 1]	[0, 1]	[3, 4]	[0, 1]	[2, 3]	[3, 4]		

different decision-makers from the positive ideal points when selecting various schemes. Figure 5 visually presents these deviation components in a heatmap format, with the horizontal axis representing “influencing factors” and the vertical axis representing “deviation from the PIS,” color-coded from deep blue to yellow, where yellow indicates a larger deviation. These computed results will be utilized in the subsequent LHA operator processing to aggregate the group deviations of different decision-makers. This process is crucial for evaluating the strengths and weaknesses of each scheme and making the final optimal decision.

Step 5: Construct the decision model for deep foundation pit support schemes under the DB mode, utilizing Formula (9) to aggregate the deviations $\mathcal{D}(\mu, v)$ corresponding to the i -th decision-maker and calculate the group deviation $D(x^+, x_i)$.

$$D(x^+, x_i) = a\omega_1\mathcal{D}(\mu, v)^{(1)} \oplus a\omega_2\mathcal{D}(\mu, v)^{(2)} \oplus \dots \oplus a\omega_j\mathcal{D}(\mu, v)^{(22)}. \quad (13)$$

Next, by computing the weighted linguistic variables $\tilde{v}_i^{(22)}$, obtain the comprehensive scores and compare them.

By coding and using MATLAB 2016b software to process the model data, we derived the group deviation values for each scheme (For detailed code implementation, please refer to the Supplementary Material). Figure 6 illustrates the group deviation values for three decision-makers (A, B, and C) across four different scheme options (Option 1 to Option 4), analyzing the differences between each scheme and the positive ideal solution regarding the target indicators. The x -axis represents the four alternative options, i.e., Option 1 to Option 4, while the y -axis displays the group deviation values, indicating the proximity of each scheme to the positive ideal solution

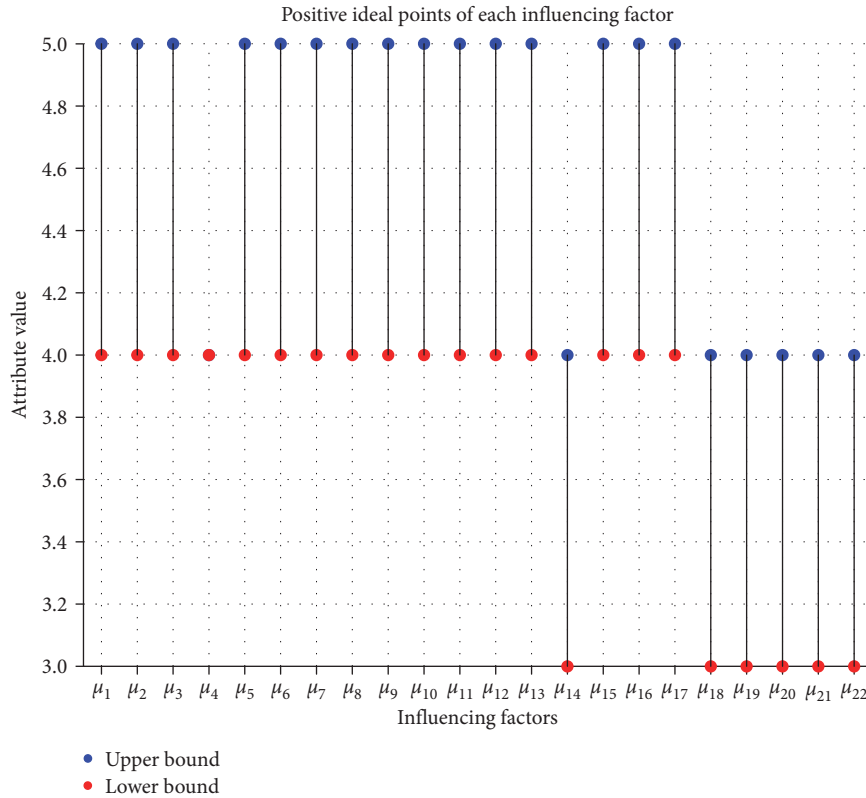


FIGURE 4: Positive ideal points for the solutions.

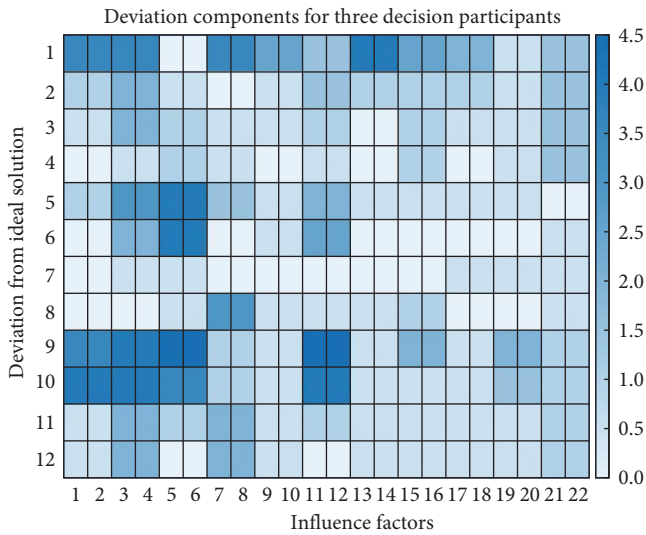


FIGURE 5: Deviation components of each solution from the positive ideal points.

regarding the target indicators. The analysis shows that Option 4 generally has lower group deviation values, being closest to the ideal value, and is thus identified as the optimal solution. This result emphasizes the importance of evaluating the proximity of schemes to the positive ideal solution during the decision-making process.

Furthermore, the analysis of the comprehensive scores for different decision-makers under the four options was conducted by calculating the weighted linguistic variables

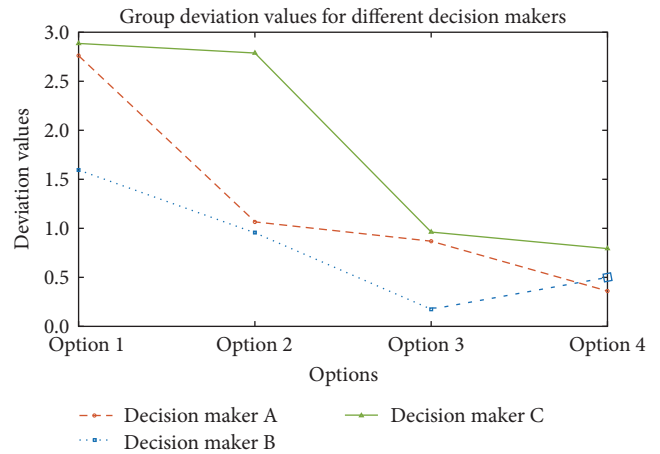


FIGURE 6: Illustrates the collective deviations of each scheme.

$\tilde{v}_i^{(22)}$. In Figure 7, the x -axis represents the design scheme options and y -axis represents the scores for the schemes, measuring their performance across various target indicators. The calculated results indicate that the comprehensive scores for options $x_1, x_2, x_3,$ and x_4 are 2.4489, 1.5495, 0.6884, and 0.5131, respectively, resulting in the ranking $x_1 > x_2 > x_3 > x_4$. This implies that the comprehensive score for Option x_4 is the lowest, indicating it as the optimal solution $\text{Opt}(x_i)$ because it exhibits the smallest deviation from the positive ideal solution across various target indicators, i.e., it is closest to the positive ideal solution. Based on literature and similar case studies, where options like underground continuous wall support are

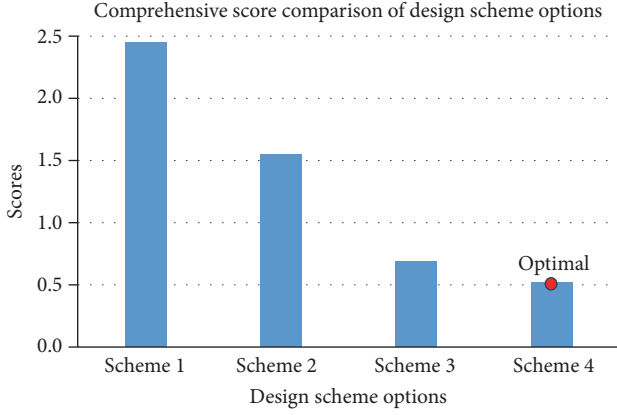


FIGURE 7: Presents a comparative analysis of the optimal schemes.

commonly adopted [46], the decision result aligns with the actual engineering schemes, demonstrating the applicability of the model.

4.3. Comparative Methodology. To demonstrate the superiority of the proposed LHA operator, this section conducts a comparative analysis with three existing methods: the OWA operator employing the OWA method, the aggregated average (AA) operator utilizing the weighted averaging aggregation method, and the WGA operator employing the weighted geometric averaging aggregation method. The calculation methods for the other three approaches are outlined as follows:

- (1) OWA operator for hesitant uncertain MADM:
 - (i) Arrange the attributes based on the ranking function S , sorting them in the order of s_1 to s_n ;
 - (ii) Rearrange the weights of the attributes according to the sorted order, expressed as follows: $AW = [aw_{s_1}, aw_{s_2}, \dots, aw_{s_n}]$;
 - (iii) Calculate the OWA value, which is the weighted average of the attribute values $X = [x_1, x_2, \dots, x_n]$ based on the reordered weights AW .

$$OWA = \sum_{i=1}^n aw_i \cdot x_i. \quad (14)$$

- (2) The AA operator is an aggregation method used for MADM. It summarizes the values of each attribute through weighted averaging. AA stands for “aggregated average,” and the calculation process is as follows:
 - (i) Given a set of attributes $X = [x_1, x_2, \dots, x_n]$ and their respective weights $AW = [aw_{s_1}, aw_{s_2}, \dots, aw_{s_n}]$.
 - (ii) Multiply each attribute value by its corresponding weight and sum all these products to obtain the final aggregated value.

TABLE 6: Score table of integrated results for each operator.

Scheme	LHA	OWA	AA	WGA
x_1	2.4489	2.3393	1.1232	2.1519
x_2	1.5495	1.3566	0.6652	1.5046
x_3	0.6884	2.4577	1.0286	2.0326
x_4	0.5131	0.5657	0.2727	1.1627

$$AA = \sum_{i=1}^n aw_i \cdot x_i. \quad (15)$$

- (3) The WGA operator is an aggregation method for MADM that uses weighted geometric averaging to summarize the values of each attribute. WGA stands for “weighted geometric average,” and the calculation process is as follows:
 - (i) Given a set of attributes $X = [x_1, x_2, \dots, x_n]$ and their respective weights $AW = [aw_{s_1}, aw_{s_2}, \dots, aw_{s_n}]$.
 - (ii) Apply the geometric average to each attribute value, then weight these geometric averages using the weights to obtain the final aggregated value.

$$WGA = \left(\prod_{i=1}^n x_i^{aw_i} \right)^{\frac{1}{\sum_{i=1}^n aw_i}}. \quad (16)$$

Here, x_i denotes the attribute value after sorting, and aw_i corresponds to the respective weight. Please refer to [47–49] for a detailed explanation of the calculation process. Utilizing various operators with the case data for computation, the comparative results are summarized in Table 6.

The results of the four methods are combined and compared using MATLAB 2021b, and the ranking of schemes is displayed in Figure 8. Figure 8 presents the evaluation results of the LHA operator, OWA operator, AA operator, and WGA operator for the four design schemes. The x-axis represents different schemes, while the y-axis represents the overall scores, i.e., the performance evaluation of each method for a specific scheme. The results indicate that in the LHA operator, the score for Scheme 4 is 0.5131, significantly lower than Scheme 1, with a score of 2.4489. Under the OWA operator, Scheme 4 only scores 0.5657, also substantially lower than Scheme 1 with a score of 2.3393. When using the AA operator, Scheme 4’s score of 0.2727 is similarly lower than Scheme 1’s score of 1.1232. The same trend is observed with the WGA operator, where Scheme 4’s score of 1.1627 is lower than Scheme 1’s score of 2.1519. It’s worth noting that there are differences in the scoring rankings of the four operators for the other three schemes. For example, in Scheme 1, the LHA operator gives the highest score of 2.4489 compared to the other three operators, while in Scheme 3, the OWA operator’s 2.4577 points are the highest.

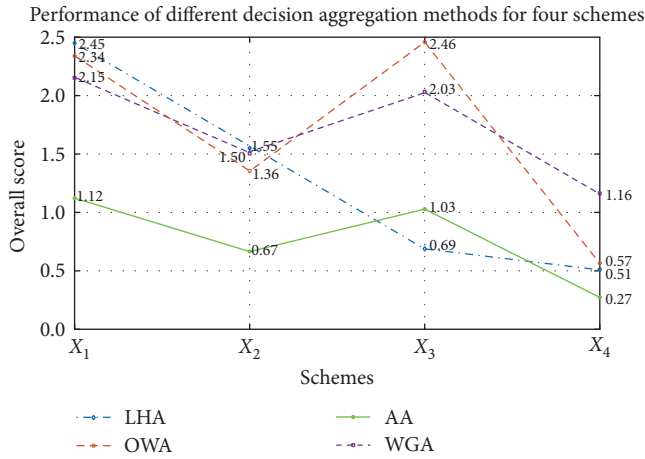


FIGURE 8: Scheme ranking chart for each operator.

The scoring differences among the four operators stem from their respective calculation mechanisms. The OWA operator considers uncertainty between attributes through attribute sorting and weight rearrangement. The AA operator performs a simple linear weighted average. The WGA operator uses a geometric average and adjusts the influence of each attribute by weight. The LHA operator explicitly evaluates based on the relative distance between the positive ideal point and the evaluation point. Despite the different mechanisms, all four operators in this case consider Scheme 4 superior to the other schemes. This is closely related to the specific conditions of the project, such as a large excavation area, significant depth, and a complex surrounding environment. This demands design schemes with strong rigidity and high waterproof performance to meet the requirements of the complex site.

Combining the specific analysis of the four design schemes in the previous text, this aligns with the high rigidity support advantage of Scheme 4. As mentioned earlier, the underground continuous wall in Scheme 4 has the ability to adapt to complex terrain, with high strength to deal with complex geological conditions and excellent waterproof performance. In contrast, the other three schemes have certain limitations in applicable conditions, with weaker advantages in the current site. Furthermore, from an investment decision perspective, deep foundation pit projects have a long investment cycle, a large amount of capital, and high risks. Selecting the optimal scheme is crucial for project success. The decision model constructed in this study considers the mutual constraints and correlations of many influencing factors, providing effective decision support for investors in a complex environment. Comparatively, the LHA operator, by utilizing the relative distance between positive and negative ideal points, can more comprehensively reflect the strengths and weaknesses of each scheme, offering more reliable decision support.

In conclusion, this study, based on data analysis and comprehensive evaluation, validates the necessity of selecting the LHA operator and Scheme 4. This provides an important reference for investment decisions in deep foundation pit projects in similar complex environments, with significant practical application value.

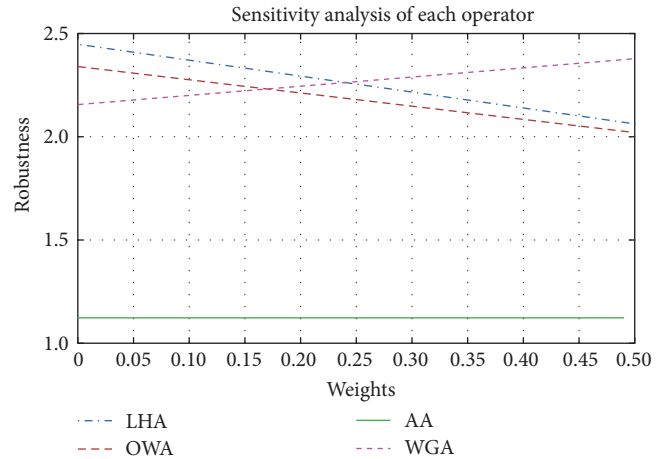


FIGURE 9: Sensitivity analysis graph of weight factors for each operator.

5. Discussion

5.1. Parameter Sensitivity Analysis. Next, we will conduct a detailed analysis of the influence of parameter variations on the decision results. In this case, for the sake of fairness, we assume that the weights among decision-makers are equal, i.e., $dw = (0.3333, 0.3333, 0.3333)$. According to the theorem, the balance factor “ t ” of linguistic variables is determined by the number of decision-makers, in this case, $t = 3$. After analyzing, we have four alternative schemes, denoted as $X_i = [x_1, x_2, x_3, x_4]$. Therefore, we will focus on analyzing a crucial independent parameter in the proposed method: attribute weights (aw). These weights play a significant role in the calculation results. Different values of “ aw ” may lead to variations in the score functions of the alternative schemes obtained through integration, thus affecting the ranking results. Assuming other parameter values remain constant, we will vary the “ aw ” weights from 0 to 0.5 and perform calculations using different integration operators. We will observe the impact of “ aw ” variations on the score functions and the ranking results. Refer to Figure 9 for details.

To gain a deeper understanding of the characteristics of various operators, this study conducts a sensitivity analysis for four operators under changes in attribute weights. The results indicate that the LHA and AA operators exhibit relative stability when weights vary, OWA shows a certain level of adaptability, and WGA demonstrates the highest sensitivity.

Specifically, as attribute weights change, the optimal solution and scores for LHA remain constant, showcasing its robustness to weight variations. OWA shows a minor response to changes in weights, maintaining a relatively stable performance. AA consistently retains the same scores but fails to reflect the impact of different weights on scheme rankings. WGA, on the other hand, exhibits noticeable fluctuations in optimal solution selection and scores with changing weights.

To validate these findings, the study also compares two scenarios: uniform weight distribution and removal of low-weight attributes. The results confirm that, regardless of how weights are set, Scheme 4 consistently maintains its optimal status, affirming its stable advantage. Simultaneously, subtle

differences in the response of different operators to weight changes are revealed. For instance, after streamlining factors, the relative rankings of schemes, excluding Scheme 4, experience slight adjustments due to the reinforced influence of remaining factors.

In summary, Sensitivity analysis helps understand the characteristics of various operators and provides a more comprehensive basis for integrated decision-making. The LHA operator is more stable and can adapt to weight variations. Its relative stability under different attribute weights makes it suitable for complex decision scenarios with uncertain influencing factors. The WGA operator, however, is very sensitive to weight changes and needs careful consideration of the impact of weight selection. Choosing the appropriate operator based on the actual situation can improve the reliability of decision results.

5.2. Model Innovation and Validation. Addressing the gap highlighted in Section 2.5 concerning “Limited Integration of Decision-Making Research,” this study adopts the AHP for weight analysis. It also introduces a refined approach that integrates the positive ideal point with the LHA operator. This methodological advancement harmonizes AHP with linguistic uncertainty and multiattribute group decision-making, offering a cohesive solution that mitigates the fragmentation observed in decision-making research. When applied to the design decision-making processes for deep foundation pits within the DB contract framework, this innovative method demonstrates enhanced accuracy and adaptability in decision-making, surpassing the performance of conventional models.

To validate the efficacy of the proposed method, we executed a decision-making application utilizing the enhanced algorithm through MATLAB. This implementation showcased significant improvements in decision-making efficiency, convenience, and speed. Our methodology addresses the “Scarce DB Contract Perspective” gap by incorporating distinct attributes of DB contracting into the evaluation framework, thereby ensuring that the method’s technical applicability is in harmony with the specific requirements of DB contracting. The pioneering contributions of this study pave new avenues and establish novel perspectives for both scholarly research and practical application in the field of deep foundation pit design.

5.3. Case Analysis. This section applies the proposed method to a prominent commercial hotel project to discuss its empirical validation and demonstrate its real-world engineering applicability. MATLAB 2016b aided computation yields group deviations for each evaluated scheme ($x_1 = 2.4489$, $x_2 = 1.5495$, $x_3 = 0.6884$, and $x_4 = 0.5131$), with the underground continuous wall design scheme emerging as the most optimal due to its minimal deviation from the positive ideal point across diverse targets. This not only substantiates the model’s practical feasibility but also conclusively validates its proficiency in decision-making for complex engineering scenarios.

The positive correlation of the model’s output with the genuine engineering solution harnessed strengthens its

scientific foundation as a decision-making reference that elevates the precision and reliability of engineering judgments. Such alignment undeniably tackles the gaps identified in Section 2.5, demonstrating how our systematized method attends to the intricacies of DB contracting constraints and augments integrative decision-making research, thus infusing the deep foundation pit design discipline with robust, innovative approaches.

5.4. Limitations of the Model. We recognize some limitations in our study of the deep foundation pit design decision model that may affect its comprehensiveness and applicability. First, we need to consider the limitations of the number and types of cases. We selected a commercial hotel project as a case, which provided a practical context for deep foundation pit design. However, this case did not cover all possible scenarios in this field. Future research could include more cases of different types and scales to test the model under various conditions.

Second, we relied on expert opinions to set weights, which introduced some subjectivity. We used a subjective weighting method with the AHP to determine the weights of different influencing factors. However, these weights may not reflect universal standards, because they depended on expert judgments and experiences [50]. To address this issue, we suggest creating a recognized system of influencing factors that incorporates risk assessment. Moreover, using big data research could offer new ways of analyzing weights, which could improve their objectivity and universality.

Third, we did not fully consider the impact of the multiparty participation mechanism in the decision-making process under the DB mode. Deep foundation pit design and construction require collaboration among multiple parties, and our model did not account for the influence of the multiparty participation mechanism under the DB mode. Future research could use a systems engineering approach to explore the framework and interaction mechanisms of multiparty decision-making systems under the DB mode, which could provide a deeper understanding of the complexity of deep foundation pit design decision-making.

In addition to these concerns, we must also acknowledge the absence of geological and geotechnical uncertainties in our analysis, which are intrinsic and can notably affect the performance and safety of geotechnical structures, as highlighted in prior studies [51]. The complex nature of subsurface conditions and their unpredictable variability present significant challenges that were not encapsulated in the 22 types of uncertainties we considered. This exclusion represents a limitation of our research that may impact the model’s predictive capability regarding the behavior of actual geotechnical systems under varied field conditions [52]. Addressing this limitation might involve integrating advanced geostatistical methods to better capture the stochastic nature of soil properties, thereby enhancing the model’s robustness and making it a more comprehensive tool for risk management in deep foundation pit design. This added dimension would certainly augment the accuracy with which our model can simulate real-world conditions and project outcomes.

In conclusion, despite the innovations our research presents in the decision model for deep foundation pit design, it also possesses discernible limitations, from case selection and expert-derived weighting to multiparty participation and the critical yet omitted aspect of geotechnical uncertainty. Future investigations should strive to incorporate these parameters, thus refining our model to better align with the complexities and nuances of this engineering domain, elevating both its practical and academic applicability.

6. Conclusion

Drawing upon the theory of ideal point MADM and leveraging the AHP, an optimal decision-making model has been introduced for deep foundation pit design schemes within the framework of DB contracting projects. Applied to the assessment of four design alternatives for the deep foundation pit of a commercial hotel project under DB contracting, the following key observations emerge:

- (1) The model selected the optimal design scheme among the four options for a deep foundation pit in a DB contracting project, showing its effectiveness.
- (2) The model identified the key factors influencing the decision and calculated the weights of different influencing factors for the design schemes in a deep foundation pit project under DB contracting.
- (3) The design scheme with an underground continuous wall had the smallest group deviation score, meaning it was the optimal design scheme.

Data Availability

The data (questionnaire survey) used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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Supplementary Materials

MATLAB code for evaluating deep foundation pit design schemes using fuzzy multiple attribute decision-making. The code includes model assumptions, model establishment steps, and optimization for selecting the optimal scheme with the minimum deviation. (*Supplementary Materials*)

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