

Research Article

An Algorithm for Extracting Intuitionistic Fuzzy Shortest Path in a Graph

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We consider an intuitionistic fuzzy shortest path problem (IFSP) in a directed graph where the weights of the links are intuitionistic fuzzy numbers. We develop a method to search for an intuitionistic fuzzy shortest path from a source node to a destination node. We coin the concept of classical Dijkstra's algorithm which is applicable to graphs with crisp weights and then extend this concept to graphs where the weights of the arcs are intuitionistic fuzzy numbers. It is claimed that the method may play a major role in many application areas of computer science, communication network, transportation systems, and so forth. In particular to those networks for which the link weights (costs) are ill defined.

1. Introduction

Graphs [1–4] are a very important model of networks. There are many real-life problems of network of transportation, communication, circuit systems, and so forth, which are modeled into graphs and hence solved. Graph theory has wide varieties of applications in several branches of engineering, science, social science, medical science, economics, and so forth, to list a few only out of many. Many real-life situations of communication network, transportation network, and so forth cannot be modeled into crisp graphs because of the reason that few or all of the arcs/links have the cost/weight which is ill defined. The weights of the arcs are not always crisp but intuitionistic fuzzy (or fuzzy).

One of the first studies on fuzzy shortest path problem (FSPP) in graphs was done by Dubois and Prade [5] and then by Klein [6]. However, few more solutions to FSPP proposed in [7–10] are also interesting. Though the work of Dubois and Prade [5] was a major breakthrough, that paper lacked any practical interpretation even if fuzzy shortest path is found, but still this may not actually be any of the path in the corresponding network for which it was found. There are very few works reported in the literature on finding an intuitionistic fuzzy shortest path in a graph. Mukherjee [11] used a heuristic methodology for solving the IF shortest

path problem using the intuitionistic fuzzy hybrid geometric (IFHG) operator, with the philosophy of Dijkstra's algorithm. In [12], Karunambigai et al. in a team work with Atanassov, present a model based on dynamic programming to find the shortest paths in intuitionistic fuzzy graphs. Nagoor Gani and Mohammed Jabarulla in [13] also developed a method on searching intuitionistic fuzzy shortest path in a network. But all these algorithms have both merits and demerits (none is absolutely the best), as all these are greedy algorithms. In this paper we solve the intuitionistic fuzzy shortest path problem (IFSP) for a graph where the arc weights are intuitionistic fuzzy numbers, and then we reduce the method to the case of finding fuzzy shortest path in a graph. Our work here does also have the same kind of demerits (as in [12, 13]), but the major significance lies in the fact that we follow the concept of classical Dijkstra's algorithm which is applicable to graphs with crisp weights and then extend this concept to graphs where the weights of the arcs are intuitionistic fuzzy numbers.

2. Preliminaries

A graph G is an ordered pair (V, E) which consists of two sets V and E , where V or $V(G)$ is the set of vertices (or, nodes), and E or $E(G)$ is the set of edges (or, arcs/links). Throughout

in our work here we consider those graphs which are without loops. Graphs may be of two types: undirected graphs and directed graphs. In an undirected graph the edge (i, j) and the edge (j, i) , if exist, are obviously identical unlike that in the case of directed graph. For a fast visit on the theory of graphs, one could see [1–4].

The intuitionistic fuzzy set theory of Atanassov [14–16] is now a well-known powerful soft computing tool to the world scientists. If X is a universe of discourse, an intuitionistic fuzzy set A in X is a set of ordered triplets $A = \{\langle x, \mu_A(x), \nu_A(x) \rangle : x \in X\}$, where $\mu_A, \nu_A : X \rightarrow [0, 1]$ are functions such that $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for all $x \in X$. For each $x \in X$ the values $\mu_A(x)$ and $\nu_A(x)$ represent the degree of membership and degree of nonmembership of the element x to $A \subset X$, respectively, and the amount $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called the hesitation part. Of course, a fuzzy set is a particular case of the intuitionistic fuzzy set if $\pi_A(x) = 0$, for all $x \in X$. For details of the classical notion of intuitionistic fuzzy set theory, one could see the book authored by Atanassov [15]. The concept of an intuitionistic fuzzy number is of importance for quantifying an ill-known quantity. Intuitionistic fuzzy numbers are the more generalized form of fuzzy numbers involving two independently estimated degrees: degree of acceptance and degree of rejection. In our work here throughout, we use the notion of triangular intuitionistic fuzzy numbers and the basic operations like IF addition \oplus , IF subtraction \ominus , “ranking” of intuitionistic fuzzy numbers, and so forth. Trivially, any crisp real number can be viewed as a fuzzy number or as an intuitionistic fuzzy number. There is no unique method for ranking the number of intuitionistic fuzzy numbers, because all the existing methods [17–19] are soft computing methods. Each method has got merits and demerits depending upon the properties of the application domains and the problem under consideration. However, if $A_1, A_2, A_3, \dots, A_n$ are n intuitionistic fuzzy numbers sorted in IF ascending order (in fact it is a kind of nonascending order, assuming that the IF equal intuitionistic fuzzy numbers take corresponding positions at random if there is no loss of generality) by any good predecided method, that is, if $A_1 < A_2 < A_3 < \dots < A_n$, then A_1 and A_n are called, respectively, the IF-min and IF-max of these n intuitionistic fuzzy numbers. Almost all the existing methods [17–19] of ranking intuitionistic fuzzy numbers were developed independently, that is, not as extensions of the existing methods [20–26] of ranking of fuzzy numbers [27]. Although several authors [20–26] have reported several ranking methods of fuzzy numbers, all are having limitations too, that is, not an absolute method suitable for every application domain. However, if $A_1, A_2, A_3, \dots, A_n$ are n fuzzy numbers sorted in fuzzy ascending order by a predecided method, that is, $A_1 < A_2 < A_3 < \dots < A_n$, then A_1 and A_n are called, respectively, the fuzzy-min and fuzzy-max of these n fuzzy numbers.

3. Graphs with IF Weighted Arcs

In most of the real-life problems of networks, be it in a communication model or transportation model, the weights

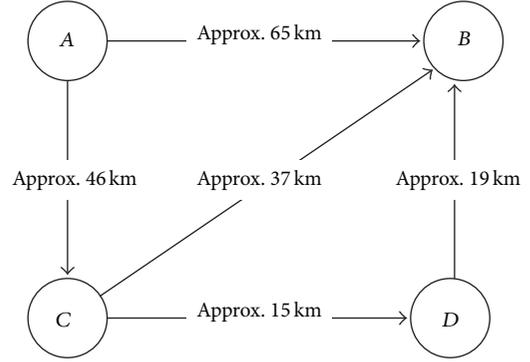


FIGURE 1: A graph G with IF weights of arcs.

of the arcs are not always crisp but intuitionistic fuzzy numbers (or, at best fuzzy numbers). For example, Figure 3 shows a public road transportation model for a traveler where the cost parameter for traveling each arc has been available to him as an intuitionistic fuzzy number.

But for such type of ill graph, there is no attempt made so far in the literature for searching an IF shortest path. In our method here, we solve this intuitionistic fuzzy shortest path problem (IFSPP) for graphs where we also use the notion of Dijkstra’s algorithm but with simple soft-computations without using any hybrid geometric operators, using only basics of Atanassov’s operators [15].

4. Intuitionistic Fuzzy Shortest Path in a Graph

In this section we solve the IFSPP for graphs where we use the philosophy of Dijkstra’s algorithm but with simple soft-computations with IF data. Consider a directed graph G where the arcs are of intuitionistic fuzzy weights (intuitionistic fuzzy numbers), as shown in Figure 1. Suppose that the subalgorithm IFW(G) returns the intuitionistic fuzzy weight set W corresponding to each arc.

4.1. IF Shortest Path Estimate $d[v]$ of a Vertex v in a Directed Graph. As shown in Figure 2, suppose that s is the source vertex and the currently traversed vertex is u . The IF estimate $d[v]$ in graph G is computed using IF addition as follows:

$$\begin{aligned}
 & \text{(IF shortest path estimate of vertex } v) \\
 &= \text{(IF shortest path estimate of vertex } u) \\
 & \oplus \text{(intuitionistic fuzzy number weight} \quad (1) \\
 & \quad \text{corresponding to the arc from} \\
 & \quad \text{the vertex } u \text{ to the vertex } v),
 \end{aligned}$$

or

$$d[v] = d[u] \oplus w_{uv}. \quad (2)$$

4.2. Intuitionistic Fuzzy Relaxation of an Arc in a Directed Graph. We extend the classical notion of relaxation to the

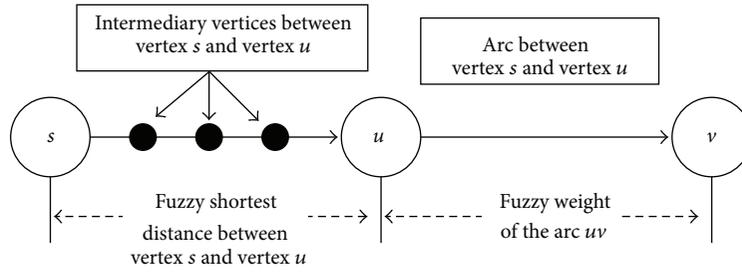


FIGURE 2: IF estimation procedure for $d[v]$ in a graph G .

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IFISS ( $G, s$ )
(1) For each vertex  $v \in V[G]$ 
(2)    $d[v] = \infty$ 
(3)    $v \cdot \pi = \text{NIL}$ 
(4)    $d[s] = 0$ 
    
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ALGORITHM 1

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IF-RELAX ( $u, v, W$ )
(1) IF  $d[v] > d[u] \oplus w_{uv}$ 
(2)   THEN  $d[v] \leftarrow d[u] \oplus w_{uv}$ 
(3)    $v \cdot \pi \leftarrow u$ 
    
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ALGORITHM 2

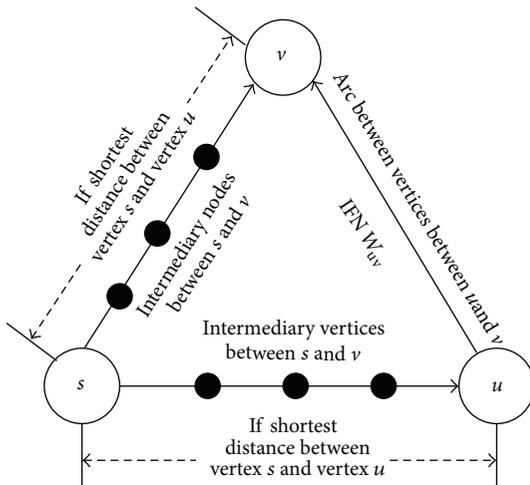


FIGURE 3: Diagram showing how the IF-RELAX algorithm works in a graph.

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IFSP ( $G, s$ )
(1) IFISS ( $G, s$ )
(2)  $W \leftarrow \text{IFW}(G)$ 
(3)  $S \leftarrow \emptyset$ 
(4)  $Q \leftarrow V[G]$ 
(5) WHILE  $Q \neq \emptyset$ 
(6)   DO  $u \leftarrow \text{EXTRACT-IF-MIN}(Q)$ 
(7)    $S \leftarrow S \cup \{u\}$ 
(8)   FOR each vertex  $v \in \text{Adj}[u]$ 
(9)     DO IF-RELAX ( $u, v, W$ )
    
```

ALGORITHM 3

case here with intuitionistic fuzzy number weights. We call it “IF relaxation.” For this, first of all we initialize the graph along with its starting vertex and IF shortest path estimate for each vertices of the graph G . The “INTUITIONISTIC-FUZZY-INITIALIZATION-SINGLE-SOURCE” algorithm IFISS will do what is shown in Algorithm 1.

After the IF initialization, the process of IF relaxation of each arc begins, as shown in Figure 3. The subalgorithm IF-RELAX plays the vital role to update $d[v]$, that is, the IF shortest distance value between the starting vertex s and the vertex v , which is a neighbour of the current traversed vertex u (see Algorithm 2), where $w_{uv} \in W$ is the IF weight of the arc from the vertex u to the vertex v and $v \cdot \pi$ denotes the parent node of vertex v .

4.3. IF Shortest Path Algorithm (IFSP Algorithm) in a Graph.

We now present our main algorithm to find single source IF shortest path in a graph. We name this “intuitionistic fuzzy shortest path algorithm,” that is, in short by the title IFSP algorithm. In this algorithm we use the previously designed above subalgorithms and also the subalgorithm EXTRACT-IF-MIN (Q) which extracts the node u with minimum key using a predecided IF ranking method and updates Q (see Algorithm 3).

Example 1 (an example). Consider the following directed graph G where the IF weights (here they are intuitionistic fuzzy numbers) are shown against each link, as shown in Figure 4. We want to solve the single-source IF shortest path problem (IFSP) taking the vertex A as the source vertex and the vertex D as the destination vertex.

Our algorithm computes the following results:

- (1) $W = \{w_{AB} = \tilde{15}, w_{AC} = \tilde{3}, w_{BC} = \tilde{1}, w_{CB} = \tilde{4}, w_{CD} = \tilde{6}, w_{BD} = \tilde{2}\}$, and then

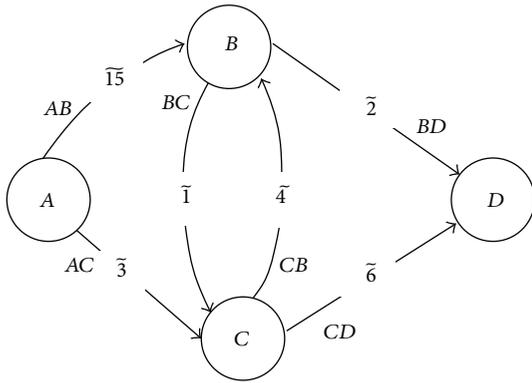


FIGURE 4: A graph G with IF weights of arcs.

(2) $S = \{A, C, B, D\}$; that is, the IF shortest path from the source vertex A is

$$A \longrightarrow C \longrightarrow B \longrightarrow D. \quad (3)$$

(3) with d -values, that is, IF shortest distance estimate values of each vertex from the starting vertex A are

$$\begin{aligned} d[A] &= 0, & d[C] &= \text{TFN } \tilde{3}, & d[B] &= \text{TFN } \tilde{7}, \\ d[D] &= \text{TFN } \tilde{9}. \end{aligned} \quad (4)$$

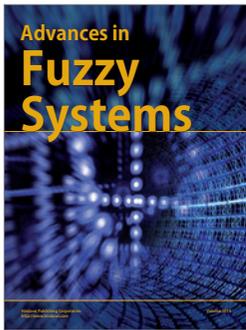
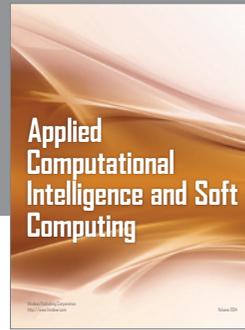
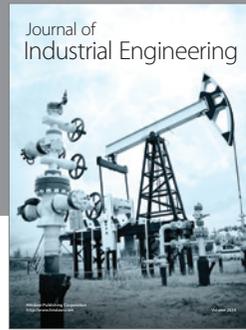
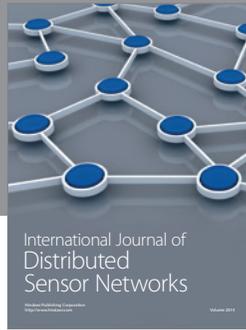
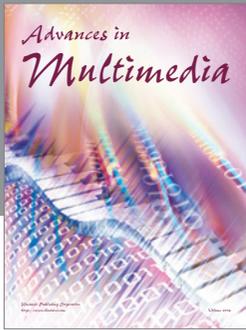
5. Conclusion

There are many real-life problems in the networks of transportation, communication, circuit systems, and so forth which are initially modeled into graphs and hence solved. In many of these directed graphs, in reality, the weights of the arcs are not always crisp but fuzzy numbers. In this paper we develop a new method to solve the intuitionistic fuzzy shortest path problem (IFSPP) from a source vertex to a destination vertex in a directed graph. The importance of our method lies in its potential to give solution in intuitionistic fuzzy environment, unlike any of the existing algorithms of IFSPP. Obviously, our algorithm does also work in case few or all of the weights are fuzzy numbers or crisp numbers, as a special case of IF numbers.

References

- [1] V. K. Balakrishnan, *Graph Theory*, McGraw-Hill, New York, NY, USA, 1997.
- [2] B. Bollobas, *Modern Graph Theory*, Springer, New York, NY, USA, 2002.
- [3] R. Diestel, *Graph Theory*, Springer, 2000.
- [4] F. Harary, *Graph Theory*, Addison Wesley, Boston, Mass, USA, 1995.
- [5] D. Dubois and H. Prade, *Fuzzy Sets and Systems*, Academic Press, New York, NY, USA, 1980.
- [6] C. M. Klein, "Fuzzy shortest paths," *Fuzzy Sets and Systems*, vol. 39, no. 1, pp. 27–41, 1991.
- [7] S. Okada and T. Soper, "A shortest path problem on a network with fuzzy arc lengths," *Fuzzy Sets and Systems*, vol. 109, no. 1, pp. 129–140, 2000.
- [8] L. Sujatha and S. Elizabeth, "Fuzzy shortest path problem based on similarity degree," *Applied Mathematical Sciences*, vol. 5, no. 66, pp. 3263–3276, 2011.
- [9] J.-S. Yao and F.-T. Lin, "Fuzzy shortest-path network problems with uncertain edge weights," *Journal of Information Science and Engineering*, vol. 19, no. 2, pp. 329–351, 2003.
- [10] J.-R. Yu and T.-H. Wei, "Solving the fuzzy shortest path problem by using a linear multiple objective programming," *Journal of the Chinese Institute of Industrial Engineers*, vol. 24, no. 5, pp. 360–365, 2007.
- [11] S. Mukherjee, "Dijkstra's algorithm for solving the shortest path problem on networks under intuitionistic fuzzy environment," *Journal of Mathematical Modelling and Algorithms*, vol. 11, no. 4, pp. 345–359, 2012.
- [12] M. G. Karunambigai, P. Rangasamy, K. Atanassov, and N. Palaniappan, "An intuitionistic fuzzy graph method for finding the shortest paths in networks," *Advances in Soft Computing*, vol. 42, pp. 3–10, 2007.
- [13] A. Nagoor Gani and M. Mohammed Jabarulla, "On searching intuitionistic fuzzy shortest path in a network," *Applied Mathematical Sciences*, vol. 4, no. 69, pp. 3447–3454, 2010.
- [14] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 20, no. 1, pp. 87–96, 1986.
- [15] K. Atanassov, *Intuitionistic Fuzzy Sets: Theory and Applications*, Physica, 1999.
- [16] K. T. Atanassov, "More on intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 33, no. 1, pp. 37–45, 1989.
- [17] D.-F. Li, "A ratio ranking method of triangular intuitionistic fuzzy numbers and its application to MADM problems," *Computers and Mathematics with Applications*, vol. 60, no. 6, pp. 1557–1570, 2010.
- [18] D. F. Li, J. X. Nan, and M. J. Zhang, "A ranking method of triangular intuitionistic fuzzy numbers and application to decision making," *International Journal of Computational Intelligence Systems*, vol. 3, no. 5, pp. 522–530, 2010.
- [19] H. B. Mitchell, "Ranking-intuitionistic fuzzy numbers," *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, vol. 12, no. 3, pp. 377–386, 2004.
- [20] S. Abbasbandy, "Ranking of fuzzy numbers, some recent and new formulas," in *Proceedings of the IFSA-EUSFLAT*, pp. 642–646, Lisbon, Portugal, July 2009.
- [21] T. Allahviranloo, S. Abbasbandy, and R. Saneifard, "A method for ranking of fuzzy numbers using new weighted distance," *Mathematical and Computational Applications*, vol. 16, no. 2, pp. 359–369, 2011.
- [22] L. Q. Dat, V. F. Yu, and S. Y. Chou, "An improved ranking method for fuzzy numbers using left and right indices," in *Proceedings of the 2nd International Conference on Computer Design and Engineering (IPCSIT '12)*, vol. 49, pp. 89–94, 2012.
- [23] B. Farhadinia, "Ranking fuzzy numbers on lexicographical ordering," *International Journal of Applied Mathematics and Computer Sciences*, vol. 5, no. 4, pp. 248–251, 2009.
- [24] N. Parandin, Araghi, and M. A. Fariborzi, "Ranking of fuzzy numbers by distance method," *Journal of Applied Mathematics*, vol. 5, no. 19, pp. 47–55, 2008.
- [25] N. R. Shankar and P. P. B. Rao, "Ranking fuzzy numbers with a distance method using circumcenter of centroids and an index of modality," *Advances in Fuzzy Systems*, vol. 2011, Article ID 178308, 7 pages, 2011.

- [26] R. Saneifard and R. Ezzati, "A new approach for ranking fuzzy numbers with continuous weighted quasi-arithmetic means," *Mathematical Sciences*, vol. 4, no. 2, pp. 143–158, 2010.
- [27] R. Biswas, "Fuzzy numbers redefined," *Information*, vol. 15, no. 4, pp. 1369–1380, 2012.



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