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Research Article

Time-Leveled Hypersoft Matrix, Level Cuts, Operators, and COVID-19 Collective Patient Health State Ranking Model

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This article is the first step to formulate such higher dimensional mathematical structures in the extended fuzzy set theory that includes time as a fundamental source of variation. To deal with such higher dimensional information, some modern data processing structures had to be built. Classical matrices (connecting equations and variables through rows and columns) are a limited approach to organizing higher dimensional data, composed of scattered information in numerous forms and vague appearances that differ on time levels. To extend the approach of organizing and classifying the higher dimensional information in terms of specific time levels, this unique plithogenic crisp time-leveled hypersoft-matrix (PCTLHS matrix) model is introduced. This hypersoft matrix has multiple parallel layers that describe parallel universes/realities/information on some specific time levels as a combined view of events. Furthermore, a specific kind of view of the matrix is described as a top view. According to this view, *i*-level cuts, sublevel cuts, and sub-sublevel cuts are introduced. These level cuts sort the clusters of information initially, subject-wise then attribute-wise, and finally time-wise. These level cuts are such matrix layers that focus on one required piece of information while allowing the variation of others, which is like viewing higher dimensional images in lower dimensions as a single layer of the PCTLHS matrix. In addition, some local aggregation operators are designed to unify i-level cuts. These local operators serve the purpose of unifying the material bodies of the universe. This means that all elements of the universe are fused and represented as a single body of matter, reflecting multiple attributes on different time planes. This is how the concept of a unified global matter (something like dark matter) is visualized. Finally, to describe the model in detail, a numerical example is constructed to organize and classify the states of patients with COVID-19.

1. Introduction

The discipline of modeling and decision-making in an uncertain and ambiguous environment is an incredible endeavor for the human mind. To optimize the field of modeling and decision-making in an uncertain and ambiguous universe, Zadeh [1] developed the fuzzy set. The author extended the crisp state (either yes or no) of the human mind by introducing fuzziness into mathematical structures, i.e., partial yes and partial no represented by some membership and nonmembership in the area of decision-making. Later, in 1986, Atanassov [2] extended this state of vagueness further by introducing intuition, or hesitation, into decision-making structures, which were called intuitionistic fuzzy set theory. This means adding some layers of hesitation with the layers of partial yes and partial no. These three states of the human mind were represented by the degree of membership, the degree of nonmembership, and the degree of hesitation. In addition, Atanassov [3] introduced an interval-valued fuzzy set (IVFS) in 1999, which is somewhat extended form of IFS (hesitating partial yes and partial no values packed in interval units). However, these extended set theories were unable to deal with uncertainty. To deal with uncertainty, Smarandache [4] introduced neutrosophy by generalizing hesitation as an independent indeterminate neutral factor and mentioning that the neutrosophic set is a generalization of the intuitionistic fuzzy set, the inconsistent intuitionistic fuzzy set, and the Pythagorean fuzzy set, and some other applications of the neutrosophic set were discussed by Smarandache and coauthors [5, 6]. Later, Molodtsov formulated a soft set in 1999 [7–10], in which the author further extended this extended fuzzy theory by considering multiple attributes parameterized by multiple subjects. Later, in 2018 [11], Smarandache further expanded this soft set to a hypersoft set and a plithogenic hypersoft set by splitting the attributes into various levels of attributes called subattributes. The author presented the hypersoft set as a set containing many subjects (matter bodies) parametrized by a combination of attributes/subattributes. In the case of the hypersoft set, the observer experiences an outside perception of the information that can be displayed in any extended fuzzy environment such as fuzzy, intuitionistic, and neutrosophic. The plithogenic hypersoft set is a set whose elements are structured by one or more attributes, and each attribute can have many values such that each attribute value has a corresponding degree of membership of an element x (subject) to the set P, with respect to each separately given criterion. It is an extended version of the hypersoft set and more widely applicable since the observer can intrinsically see the state of the element x (subject) by looking at each attribute separately. Smarandache introduced plithogeny [12–14] and expanded the vision of an ambiguous, uncertain universe. The author raised some open problems in this extended fuzzy set theory such as designing multicriterion decision-making techniques and construction of operators. Rana et al. [15] addressed these open problems and expanded the dimension of the plithogenic hypersoft set [11] by introducing and representing these PFWHSS in a matrix form called the plithogenic fuzzy whole hypersoft matrix. In addition, some local aggregation operators have been developed for the plithogenic fuzzy hypersoft set (PFHSS). This matrix was developed for a specific combination of attributes and subattributes, which was a limited structure designed for a single combination of attributes and subattributes, and a general model was required to be formulated. Therefore, later, Rana et al. [16] generalized the plithogenic whole hypersoft matrix to an extended form of the matrix called the plithogenic subjective hyper-supersoft matrix. It is a generalized form and a matrix superior to the previously developed matrix. It has a greater capacity to express the variations of connected attributive levels as well. These attributive levels are represented in the form of matrix layers. The application of this matrix is provided in the form of a new ranking model called the plithogenic subjective local-global universal ranking model. Some other researchers also addressed this extended fuzzy set theory. Saeed et al. [17] discussed the prognosis of allergy-based diseases by using Pythagorean fuzzy hypersoft and recommended medication. Muhammad et al. and Zulqarna et al. [18–21] described a correlation coefficient-based decision-making approach and discussed some of its properties for interval-valued neutrosophic hypersoft sets. The authors also outlined some basic operations and discussed their properties for interval-valued neutrosophic hypersoft sets. Furthermore, generalized aggregation operators have been established for neutrosophic hypersoft sets, and robust aggregation operators have been established for intuitionistic fuzzy hypersoft sets. Yolcu and Ozturk [22] described fuzzy hypersoft sets and discussed their application as a decision-making model. Siddique et al. [23] established a multicriterion decision-making approach for aggregation operators by using Pythagorean fuzzy hypersoft sets. Abdel-Basset et al. [24] designed an integrated plithogenic MCDM approach for the financial performance evaluation of manufacturing industries. Grida et al. [25] evaluated the impact of COVID-19 prevention policies on aspects of the supply chain under uncertainty. Akram et al. stated that after all these developments, there is still a vacuum in this extended fuzzy set theory. As we know, the source of the variation of the universe as information, data, and realities is time. It is a grim need to discuss and include this source of variation in the extended fuzzy set theory, and it is essential to design such mathematical higher dimensional structures that include time as a variation unit.

This recent paper is a first step toward designing such higher dimensional mathematical structures that include time as a fundamental source of variation. It further provides an upgraded and broadened plithogenic universal model by introducing a time-leveled variation in a hypersoft matrix, which deals with data and information as a magnified angle of vision. As we know, most of the variations in this universe are time-dependent like weather graphs, stock exchange, and website ratings, therefore, it is of tremendous help if we use a plithogenic hypersoft matrix to cope with scattered time-varying pieces of information of the plithogenic universe in crisp and other environments (i.e., fuzzy and neutrosophic environments). Initially, a three-dimensional broadened view of the PCHS-Matrix is portrayed to represent the plithogenic crisp time-leveled hypersoft set. This PCTLHS matrix is a rank-three tensor that shows three types of variations. It contains several matrix layers, whereas each layer is a tensor of rank two (i.e., a case of the ordinary matrix) expressed in the crisp environment. Furthermore, this PCTLHS matrix represents time-dependent multiple parallel universes/parallel realities/information. For example, if we are organizing the information of COVID-19 patients (subjects) admitted to a certain hospital, the record of their symptoms (attributes) at some time levels can be organized as a PCTLHS matrix. By using this connected matrix expression, we can see and classify all information immediately; i.e., the information from a group of patients is assigned to a combination of attributes and observed at different time levels. The three types of PCTLHS matrix variation indices would describe patients (subjects), their symptoms (attributes), and the time-leveled states of their symptoms as time-based subattributes, whereas one kind of level cuts (i-level cuts) related to the top view of the

PCTLHS matrix focuses on the subject (patient) separately and would display their attributes (symptoms) at several time levels as matrix layers.

In this way, these level cuts can focus on required information while displaying the variation of other information as a single matrix layer of the hypersoft matrix. Furthermore, these level cuts further cleave into sublevel cuts by splitting the matrix layer at one of the two remaining variation indices, whereas these sublevel cuts do offer the display of the previous lower dimension in the further lower dimension and enable us to sneak in an inside view of the expanded universe; i.e., after focusing on a subject explicitly through an i-level cut (single layer of the layered matrix), our next focus is on the attribute (a specific symptom) of that subject (patient) through the sublevel cut (row or a column of the one layer of the multiple layered matrix). One may call the expanded view of the matrix an implicit view of the universe. In the next stage of these sub-level cuts, sub-sublevel cuts are constructed by splitting the sublevel cuts (row or column of a specific layer of the PCTLHS matrix) at the third variation index of the matrix, which means after focusing on the subject (patient) and attribute (symptom) through the level cut and sublevel cut, our next focus is on the specific time level of information. After applying all splits to all indices, outcomes would be reflected as singletons. These level cuts, sublevel cuts, and sub-sublevel cuts are displaying their traits as zoom-in and zoom-out functions to provide the interior and exterior view of these time-bounded events, which can be argued as a contraction of the expanded higher dimensional universe. The sub-sub-level cuts provide a contract picture of the smallest part of a single or multiple universe. In this way, the expanded universe of matrix layers could be contracted to a single point. Similarly, by reversing the process, one can expand the same singleton into higher dimensions of rows, columns, matrices, matrix layers, and clusters of matrix layers. The matrix expression is the more appropriate expression to represent multidimensional data compared to the classic set expression. The question now arises as to why a collective state ranking would be preferable to an individual state ranking. The answer is precise and obvious since the collective state ranking sorts attributes of a group of people (subjects) as compared to an individual state ranking in which attributes of a person (subject) are categorized. By using the collective state ranking, one can categorize attributes on a broader spectrum such as attributes (health states) of a group of patients that can be distinguished, and hence, remedy for the most dominant attribute can be identified. Then, the cure for that attribute, which would be cough syrup, would be introduced to the market and manufactured on a large scale. Therefore, by considering a collective state ranking, any product associated with the dominant attribute may be imported or manufactured on a larger scale, which is not possible when considering an individual state ranking.

In the final stage, plithogenic local aggregation operators are developed and utilized to elaborate the activity of these several types of level cuts based on variation indices. These local operators serve the purpose of the unification of matter bodies in the universe, which means all their attributes are observed as they are reflected from a single entity. In this way, attributes are focused and subjects (matter bodies) are merged. This means that all elements of the universe are fused and represented as a single body of matter reflecting multiple attributes at different time levels. These i-level cuts unify the subjects using the aggregation operators, which help introduce the concept of a unified global matter that appears like dark matter that can have attributes but not individual bodies of matter. These operators are named plithogenic disjunction, plithogenic conjunction, and plithogenic averaging operators. For further precise applications of the model, a numerical example is derived for the organization and classification of COVID-19 data or information at two distinct time levels.

This article is organized into seven basic sections. After the introduction (Section 1), Section 2 summarizes some related preliminaries. Section 3 introduces some fundamental new concepts and definitions of the PTCHS-Matrix. Section 4 gives a mathematical description of the top-to-bottom view of the PLCHS-Matrix, and its splits major structures (connected matrix layers) into smaller structures such as i-level cuts, sublevel cuts, and sub-sublevel cuts. Section 5 describes the application of these split structures as the health state model for patients withCOVID-19 by using PCTLHS-Matrix. Section 6 introduces the set-based operations for the unification of i-level cuts, sublevel cuts, and sub-sublevel cuts of the PCTLHS matrix. Section 7 summarizes some conclusions and discussions on future research.

2. Preliminaries

This section summarizes some basic definitions of soft sets, hypersoft sets, crisp hypersoft sets, plithogenic hypersoft sets, and plithogenic crisp hypersoft Sets. These definitions would help expand the theory of plithogenesis.

Definition 1 (see [7]). (Soft set) Let U be the initial Universe of discourse and E be a set of parameters or attributes with respect to U, let $P(U)$ denote the power set of U, and $A \subseteq E$ is a set of attributes. Then, the pair (F, A) where $F\colon A \longrightarrow P(U)$ is called soft set over U. In other words, a soft set (F, A) over U is a parameterized family of subsets of U. For $e \in A$, $F(e)$ may be considered as a set of e elements or e approximate elements:

$$(F, A) = \{(F(e) \in P(U))\colon e \in E, F(e) = \varphi \quad \text{if } e \notin A\}. \quad (1)$$

Definition 2 (see [11]). (Hypersoft set)Let U be the initial universe of discourse and $P(U)$ be the power set of U.

Let $a_1, a_2, \ldots, a_n$ for $n \geq 1$ be n distinct attributes, whose corresponding attributes values are, respectively, the sets $A_1, A_2, \ldots, A_n$ with $A_i \cap A_j = \varphi$ for $i \neq j$ and $i, j \in \{1, 2, \ldots, n\}$.

Then, the pair $(F, A_1 \times A \times \cdots \times A_n)$ where

$$F\colon A_1 \times A \times \cdots \times A_n \longrightarrow P(U), \tag{2}$$

is called a hypersoft set over$U$.

Definition 3 (see [11]). (Plithogenic crisp hypersoft set) Let U_c be the initial crisp universe of discourse and $P(U_c)$ be the power set of U. Let $a_1, a_2, \ldots, a_n$ for $n \geq 1$ be n distinct attributes, whose corresponding attributes values are, respectively, the sets $A_1, A_2, \ldots, A_n$ with $A_i \cap A_j = \varphi$ for $i \neq j$ and $i, j \in \{1, 2, \ldots, n\}$. Then $\{F_c, A_1 \times A_2 \times \cdots \times A_n\}$ is called plithogenic crisp hypersoft set over U_c where $F_c\colon A_1 \times A \times \cdots \times A_n \longrightarrow P(U_c)$.

Definition 4 (see [25, 26]). (Supermatrices) Square or rectangular arrangements of numbers in rows and columns are matrices, and we call them simple matrices, while the supermatrix is the one whose elements are themselves matrices with elements that can be either scalars or other matrices.

$$a = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \tag{3}$$

where

$$\begin{aligned} a_{11} &= \begin{bmatrix} 2 & -4 \\ 0 & 1 \end{bmatrix}, \\ a_{12} &= \begin{bmatrix} 0 & 40 \\ 21 & -12 \end{bmatrix}, \\ a_{21} &= \begin{bmatrix} 3 & -1 \\ 5 & 7 \\ -2 & 9 \end{bmatrix}, \\ a_{22} &= \begin{bmatrix} 4 & 12 \\ -17 & 6 \\ 3 & 7 \end{bmatrix}, \end{aligned} \tag{4}$$

where a is a supermatrix.

Note: the elements of supermatrices are called submatrices; i.e., $a_{11}, a_{12}, a_{21}, a_{22}$ are submatrices of the supermatrix a. In this example, the order of the supermatrix a is 2×2, and the order of submatrices a_{11} is 2×2,a_{12} is 2×2, anda_{21} is 3×2, and the order of the submatrix a_{22} is 3×2, and we can see that the order of the supermatrix does not tell us about the order of its submatrices.

Definition 5 (see [27]). (Hypermatrices) For $n_1, \ldots, n_d \in N$, a function $f\colon (n_1) \times \cdots \times (n_d) \longrightarrow F$ is a hypermatrix, also called an order-d hypermatrix or d-hypermatrix. We often just write $a_{k_1 \cdots k_d}$ to denote the value $f(k_1 \cdots k_d)$ of f at $(k_1 \cdots k_d)$ and think of f (renamed as A) as specified by a d-dimensional table of values, writing $A = [a_{k_1 \cdots k_d}]_{k_1 \cdots k_d}^{n_1, \ldots, n_d}$.

A 3-hypermatrix may be conveniently written down on a (2-dimensional) piece of paper as a list of usual matrices, called slices. For example, we obtain

$$A = \left[a_{ijk}\right] = \begin{bmatrix} a_{111} & a_{121} & a_{131} & \cdot & a_{112} & a_{122} & a_{132} \\ a_{211} & a_{221} & a_{231} & \cdot & a_{212} & a_{222} & a_{232} \\ a_{311} & a_{321} & a_{331} & \cdot & a_{312} & a_{322} & a_{332} \end{bmatrix}. \tag{5}$$

3. Plithogenic Crisp Time-Leveled Hypersoft Matrix

3.1. (PCTLHS Matrix). Let $U_C(X)$ be the crisp universe of discourse and $P(U_C)$ be the power set ofU_C. A_j^k is a combination of attributes and subattributes (time-leveled attributes) for some $j = 1, 2, 3, \ldots, N$ represents N number of attributes, $k = 1, 2, 3, \ldots, L$ represents L number of time levels and x_i $i = 1, 2, 3, \ldots,$ and M represents the M number of subjects under consideration. Then, plithogenic crisp time-leveled hypersoft matrix (PCTLHS matrix) is a mapping from the cross product of attributes/time-leveled attributes on the power set of the universe $P(U_C)$, represented in the matrix form. This mapping with its matrix form is described in equations as follows:

$$F\colon A_1^k \times A_2^k \times A_3^k \times \cdots \times A_N^k \longrightarrow P(U_C), \tag{6}$$

$$F = \left[\mu_{A_j^k}(x_i)\right], \tag{7}$$

where $\mu_{A_j^k}(x_i) \in \{0.\ 1\}$ are crisp memberships for a given x_i subject regarding each given A_j^k attribute/time-leveled attributes.

In simple words, a plithogenic crisp hypersoft set, represented in the matrix form, is called plithogenic crisp time-lined hypersoft matrix (PCTLHS matrix).

This matrix has three possible expansions associated with its three-dimensional views, which are described in crisp environments.

3.2. Three-Dimensional Views and Level Cuts of the PCTLHS Matrix. As we know, all ordinary matrices in the real vector space are rank 2 tensors. Similarly, as an extended matrix version, the PCTLHS matrix with its three variation indices is a rank 3 tensor. The PCTLHS matrix contains layers of ordinary matrices called matrix layers or level cuts.

For a detailed description, we consider the example of the PCTLHS matrix $A = [A_{ijk}]$. The index i refers to variations of rows used to represent the subjects under consideration, j specifies a variation of columns used to represent attributes of subjects, and k provides variations of layers of rows and columns that would be used to represent the attributes on specific time levels (varying matrix layers as clusters of rows and columns). Similarly, $[A_{jki}]$ is interpreted as the index, and j offers a variation of rows, k gives a variation of columns, and i offers variation of clusters of rows and columns.

The PCTLHS matrix contains layers of ordinary matrices, termed matrix layers or level cuts of the PCTLHS matrix

The level cuts, sublevel cuts, and sub-sublevel cuts would be defined by specifying variation indices i, j, k for their positive integer values.

3.2.1. Level Cuts. Level cuts are submatrices (first-level splits) of the PCTLHS matrix that can be further described as parallel matrix layers. The PCTLHS matrix is generated by uniting these matrix layers. These level cuts of the PCTLHS matrix are obtained by assigning a specific integer value to the first variation index at a time. According to three types of view of the PCTLHS matrix, level cuts are distributed in three categories, i.e.,

i-level cuts
j-level cuts
k-level cuts

Note: in this article, only the first type of level cuts are formulated and explored, whereas the other two types would be discussed in the upcoming version.

3.2.2. Sublevel Cuts. Sublevel cuts are perceived as level Cuts of level cuts (second splits applied over first splits) of the PCTLHS matrix. These sublevel cuts are columns or rows of the submatrix (the matrix obtained after the first split). The sublevel cuts are obtained by assigning a specific integer value to one of the two variation indices of a parallel layer (submatrix) of the PCHS matrix.

3.2.3. Sub-Sublevel Cuts. Sub-sublevel cuts are obtained by assigning a specific integer value to the variation index of sublevel cuts (the third-level split over the second split). The sub-sublevel cut is one specific element (point) of the sublevel cut (column or row). These level cuts, sublevel cuts, and sub-sublevel cuts are images of the higher dimensional universe in lower dimensions and can be used as tools for getting images and transformations. The detailed classification of these level cuts, sublevel cuts, and sub-sublevel cuts is described below.

The utilization of these cuts is that one can contract the expanded dimension of the PCTLHS matrix to a matrix, then to a row or column matrix, and then further to a single point. Similarly, the reverse procedure would provide an expansion of the universe.

3.2.4. Variation Indices of the PCTLHS Matrix. Three types of variation indices are used to represent a general element ($\mu_{A_j^k}(x_i)$) of the PCHS matrix. The first variation index i (associated with subjects) represents M rows of an $M \times N$ submatrix which is a single layer of the $M \times N \times L$ PCHS matrix. The second variation j (used to specify attributes) represents N columns of this submatrix. A third variation index k (represent attributive levels) represents L layers or L level cuts of the $M \times N \times L$ PCHS matrix. The index-based views, level cuts, sublevel cuts, and Sub-sublevel cuts are categorized into the following types.

4. *i*-Level Cuts, Sublevel Cuts, and Sub-Sublevel Cuts of the PCTLHS Matrix

The top-to-bottom view of the PCTLHS matrix consists of top to the bottom layer of the matrix. These layers are formulated by specifying the variation index i. Equation (8a) describes the top-to-bottom view of the PCTLHS matrix:

$$A = \begin{bmatrix} \mu_{A_1^1}(x_i) & \mu_{A_1^2}(x_i) & \cdots & \mu_{A_1^L}(x_i) \\ \mu_{A_2^1}(x_i) & \mu_{A_2^2}(x_i) & \cdots & \mu_{A_2^L}(x_i) \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \mu_{A_N^1}(x_i) & \mu_{A_N^2}(x_i) & \cdots & \mu_{A_N^L}(x_i) \end{bmatrix}. \tag{8a}$$

The top-down view of the PCTLHS matrix in an expanded form is described as follows:

$$A = \begin{bmatrix} \begin{bmatrix} \mu_{A_1^1}(x_1) & \mu_{A_1^2}(x_1) & \cdots & \mu_{A_1^L}(x_1) \\ \mu_{A_2^1}(x_1) & \mu_{A_2^2}(x_1) & \cdots & \mu_{A_2^L}(x_1) \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \mu_{A_N^1}(x_1) & \mu_{A_N^2}(x_1) & \cdots & \mu_{A_N^L}(x_1) \end{bmatrix} \\ \begin{bmatrix} \mu_{A_1^1}(x_2) & \mu_{A_1^2}(x_2) & \cdots & \mu_{A_1^L}(x_2) \\ \mu_{A_2^1}(x_2) & \mu_{A_2^2}(x_2) & \cdots & \mu_{A_2^L}(x_2) \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \mu_{A_N^1}(x_2) & \mu_{A_N^2}(x_2) & \cdots & \mu_{A_N^L}(x_2) \end{bmatrix} \\ \cdot \\ \cdot \\ \begin{bmatrix} \mu_{A_1^1}(x_M) & \mu_{A_1^2}(x_M) & \cdots & \mu_{A_1^L}(x_M) \\ \mu_{A_2^1}(x_M) & \mu_{A_2^2}(x_M) & \cdots & \mu_{A_2^L}(x_M) \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \mu_{A_N^1}(x_M) & \mu_{A_N^2}(x_M) & \cdots & \mu_{A_N^L}(x_M) \end{bmatrix} \end{bmatrix}. \tag{8b}$$

4.1. i-Level Cuts $A^{[i]}$. i-level cuts expressed as $A^{[i]}$ are the top-to-bottom split layers of the PCTLHS matrix. Figure 1 depicts i-level cuts of the PCHTLS matrix. The observer can sort the clusters of information subject-wise, using this top-down view of the PCTLHS matrix. For each specific value of i, an $N \times L$ submatrix is obtained termed as i-level cut. These i-level cuts are M numbers of top-to-bottom matrix layers. Each layer is an $N \times L$ submatrix of the $M \times N \times L$ PCTLH matrix. The top-to-bottom layers of equation (9) represent i-level cuts.

Furthermore, these i-level cuts focus on each subject (patient) separately and display variations of their attributes (symptoms) at several time levels.

$A = [\mu_{A_j^k}(x_i)]$ is a PCTLHS matrix.

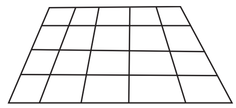

Figure 1: Level cut.

For some given $i = 1, 2, \ldots, M$, $j = 1, 2, \ldots, N$ and $k = 1, 2, 3, \ldots, L$.

The i-level cuts of the PCTLHS matrix, constructed by splitting the index i, are called i-leveled splits or i-level cuts.

i-level cuts $A^{[i]}$ of the PCTLHS matrix (equations (8a) and (8b)) for fixed $i = 1, 2, \ldots, M$ are given as follows:

$$A^{[1]} = \begin{bmatrix} \mu_{A_1^1}(x_1) & \mu_{A_2^1}(x_1) & \cdot\cdot\cdot & \mu_{A_N^1}(x_1) \\ \mu_{A_1^2}(x_1) & \mu_{A_2^2}(x_1) & \cdot\cdot\cdot & \mu_{A_N^2}(x_1) \\ \cdot & \cdot & \cdot\cdot\cdot & \cdot \\ \cdot & \cdot & \cdot\cdot\cdot & \cdot \\ \cdot & \cdot & \cdot\cdot\cdot & \cdot \\ \mu_{A_1^L}(x_1) & \mu_{A_2^L}(x_1) & \cdot\cdot\cdot & \mu_{A_N^L}(x_1) \end{bmatrix}, \tag{9}$$

$$A^{[2]} = \begin{bmatrix} \mu_{A_1^1}(x_2) & \mu_{A_2^1}(x_2) & \cdot\cdot\cdot & \mu_{A_N^1}(x_2) \\ \mu_{A_1^2}(x_2) & \mu_{A_2^2}(x_2) & \cdot\cdot\cdot & \mu_{A_N^2}(x_2) \\ \cdot & \cdot & \cdot\cdot\cdot & \cdot \\ \cdot & \cdot & \cdot\cdot\cdot & \cdot \\ \cdot & \cdot & \cdot\cdot\cdot & \cdot \\ \mu_{A_1^L}(x_2) & \mu_{A_2^L}(x_2) & \cdot\cdot\cdot & \mu_{A_N^L}(x_2) \end{bmatrix}, \tag{10}$$

$$A^{[M]} = \begin{bmatrix} \mu_{A_1^1}(x_M) & \mu_{A_2^1}(x_M) & \cdot\cdot\cdot & \mu_{A_N^1}(x_M) \\ \mu_{A_1^2}(x_M) & \mu_{A_2^2}(x_M) & \cdot\cdot\cdot & \mu_{A_N^2}(x_M) \\ \cdot & \cdot & \cdot\cdot\cdot & \cdot \\ \cdot & \cdot & \cdot\cdot\cdot & \cdot \\ \cdot & \cdot & \cdot\cdot\cdot & \cdot \\ \mu_{A_1^L}(x_M) & \mu_{A_2^L}(x_M) & \cdot\cdot\cdot & \mu_{A_N^L}(x_M) \end{bmatrix}. \tag{11}$$

These i-level slices are submatrices of the PCTLHS matrix, which can be further described as parallel layers of the matrix. The PCTLHS matrix is created by uniting these matrix layers. It is observed that i-level cuts can focus and classify subjects (matter bodies) in scattered clusters of information. For example, the first i-level cut $A^{[1]}$ focuses on the first subject x_1 and displays the N-number of attributes at L-number of time levels. Similarly, the second i-level cut $A^{[2]}$ focuses on the second subject x_2 and $A^{[M]}$ focuses on Mth subject x_M.

4.2. i_j-Sublevel Cuts. i_j-sublevel cuts are constructed by first specifying $i = m$ where m is any positive integer from 1 to M, then further specifying $j = n$ such that n is a positive integer from 1 to N, and then varying $k = 1, 2, \ldots, L$ as given in equation (12). The observer can sort the clusters of information first subject-wise and then attribute-wise, respectively, by using this top-down view of the PCTLHS matrix These i_j-sublevel cuts are the column-wise split of each top-to-bottom layer of the matrix:

$$A^{[m_n]} = \begin{bmatrix} \mu_{A_n^1}(x_m) \\ \mu_{A_n^2}(x_m) \\ \cdot \\ \cdot \\ \cdot \\ \mu_{A_n^L}(x_m) \end{bmatrix}. \tag{12}$$

It is observed that after classifying information subject-wise through i-level cuts, one would further distinguish information attribute-wise by using i_j-sublevel cuts. For example, for a fixed $i = 1$ and $j = 1$, a sublevel cut is a column matrix, displaying the first subject's information for its first attribute at k time levels where $k = 1, 2, \ldots, L$.

4.3. i_k-Sublevel Cuts. i_k-sublevel cuts of i-level cut $A^{[i]}$ are obtained by first specifying the index $i = m$, then further specifying the index k, and varying only j. These sub-level cuts are rows of an $N \times L$ submatrix as shown in equation:

$$A^{[m_l]} = \left[\mu_{A_1^l}(x_m) \;\; \mu_{A_2^l}(x_m) \;\; \cdot\cdot\cdot \;\; \mu_{A_N^l}(x_m) \right]. \tag{13}$$

4.4. i_{k_j}-Sub-Sublevel Cuts. i_{k_j}-sub-sublevel cuts $A^{[i_{k_j}]}$ are constructed by first specifying i and then further specifying both j and k. The mathematical expression of these sub-sublevel cuts is given as follows:

$$A^{\left[i_{k_j}\right]} = A^{\left[m_{l_n}\right]} = \left[\mu_{A_n^l}(x_m)\right]. \tag{14}$$

Note: it is obvious that $A^{[i_{j_k}]} = [\mu_{A_n^l}(x_m)]$ represents a single element of the expanded matrix and serves as a zoom-in view of the matrix. For example, $A^{[1_{1_1}]}$ is an i_{k_j}-sub-sublevel cut that focuses on the information of the first subject for its first attribute at the first time level.

i_{k_j} sub-sublevel cut: for a given i-level cut by specifying j, one can obtain the i_{k_j}-sub-sublevel cut.

It is obvious that by following the abovementioned split procedure, one would zoom into the given PCTLHS matrix. The level cutbehaves like a zoom into the first layer (matrix) of the hypermatrix, then the sub level cut reaches the the column or row of matrix, and the sub sub level cut would be an element of the column or row. Similarly, the reverse process can serve as a zoom-out function. In this way, one can approach the smallest unit of the extended universe, which is an explicit view of the event as an element of the PCHS matrix. Similarly, the matrix itself serves as an implicit view of the event.

5. Application

5.1. Example 5.1. Let $U_{PC} = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ is a group of six patients who visited the hospital with symptoms of COVID-19. They were examined by a doctor, who examined three of them under four basic symptoms. These symptoms were observed in their two different visits to the doctor. These two visits at two distinct times are considered two-time levels. In both these times, the symptoms (time-lined subattributes) of these patients were observed and recorded during the first and second meetings with the doctor. The three patients under observation are considered subjects. The information/data of their health condition associated with both visits are organized in a PCTLHS matrix. The clinical observations of the visits are expressed in the crisp environment and analyzed by using the plithogenic crisp hypersoft matrix.

Consider the set of the first three patients who are under observation.

Let the four attributes be $A_j^k; j = 1,2,3,4$ observed at two-time levels $k = 1,2$ which are described as follows.

Fever with numeric values, $k = 1,2$ representing first and second-time levels

A_1^1 = State of fever at the first visit
A_1^2 = State of fever at the second visit
A_2^k = Dry cough, with numeric values, $k = 1,2$
A_2^1 = Condition of cough at the first visit
A_2^2 = Condition of cough at the second visit
A_3^k = Breathing difficulty with numeric values, $k = 1,2$
A_3^1 = Breathing difficulty level at the first visit
A_3^2 = Breathing difficulty level at the first visit
A_4^k = Sickness record, with numeric values $k = 1,2$
A_4^1 = Sickness state at the first visit
A_4^2 = Sickness state at the second visit

Now, in the next two subsections, this information consisted of symptoms (attributes) of patients (subjects) observed at two levels of time. Two forms of expressions are formulated to organize information. One way of representation is the set expression, i.e., the PCTLHS set, and the other way of representation is the matrix expression, which is a connected matrix or a hypermatrix termed the PCTLHS matrix.

5.1.1. PCTLHS Set Representation. Let the function A reflects given attributes/time-leveled attributes described as follows:

$$\begin{aligned} \mathbf{A}: \quad & A_1^k \times A_2^k \times A_3^k \times A_4^k \longrightarrow P(U_C), \\ \text{S.t.} \quad & \mathbf{A}\left(A_1^1, A_2^1, A_3^1, A_4^1\right) = \{x_1, x_2, x_3\}, \\ & \mathbf{A}\left(A_1^2, A_2^2, A_3^2, A_4^2\right) = \{x_1, x_2, x_3\}. \end{aligned} \tag{15}$$

$A_1^1, A_2^1, A_3^1, A_4^1$ is a combination of attributes at the first visit level (α-combination). $A_1^2, A_2^2, A_3^2, A_4^2$ is a combination of attributes at the second visit level (β-combination).

Individual crisp memberships are assigned to $A = \{x_1, x_2, x_3\}$ according to the opinion of the doctor, and then, information is represented as the PCTLHS set by using crisp memberships; that is, if a given symptom is present in the patient, the assigned membership is one, and if it is not present, the membership is zero. The formal notation of the PCTLHS set A is described as follows:

$$\mathbf{A} = \left\{x_1\left(\mu_{A_j^1}(x_1)\right), x_2\left(\mu_{A_j^1}(x_2)\right), x_3\left(\mu_{A_j^1}(x_3)\right)\right\}, \tag{16}$$

where $\mu_{A_j^k}(x_i)$ represents crisp memberships assigned to three subjects (patients), $i = 1,2,3$ according to four different attributes (symptoms) $j = 1,2,3,4$ that are observed at two distinct time levels $k = 1,2$, (these plithogenic crisp memberships reflect whether the A_j^k attribute is present ($\mu_{A_j^k}(x_i) = 1$) in the x_i subject or not present ($\mu_{A_j^k}(x_i) = 0$).

The information of the first visit of patients x_i associated with four symptoms as α-combination of attributes is organized as a PCTLHS set:

$$A(\alpha) = A\left(A_1^1, A_2^1, A_3^1, A_4^1\right) = \left\{\begin{array}{l} x_1(1,0,1,1), \\ x_2(1,1,1,1), \\ x_3(1,0,0,1). \end{array}\right\} \tag{17}$$

It is clear that the first-level layer of the PCTLHS matrix is generated by this $A(\alpha)$.

Now, regarding the second observation of patients as β -combination of attributes, information is portrayed as a PCTLHS set:

$$A(\beta) = A\left(A_1^1, A_2^1, A_3^1, A_4^1\right) = \left\{\begin{array}{l} x_1(0,0,0,0), \\ x_2(0,1,0,0), \\ x_3(1,0,1,0). \end{array}\right\} \tag{18}$$

$A(\beta)$ generates the second-level layer of the matrix.

5.1.2. PCTLHS Matrix Representation. Let A is the matrix of representation for both PCTLHS Sets. Here, rows of this matrix represent x_1, x_2, x_3 (physical bodies or subjects) and columns represent (the nonphysical aspect of subjects as symptoms) attributes $A_1^k, A_2^k, A_3^k, A_4^k$.

This whole information of two sets $A(\alpha)$ and $A(\beta)$ is organized as two connected layers of PCTLHS matrix A described as follows:

$$F = \begin{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{bmatrix}. \tag{19}$$

This PCTLHS matrix consists of two layers. The first layer is interpreted as the first observation, representing the state of health of three patients. By observing the matrix given in equation (19), one can clearly see that, on the first visit, the patient x_1 is suffering from fever with no dry cough but feeling suffocation and nausea. The patient x_2 has a fever with dry cough, suffocation, and nausea. The patient x_3 is suffering from fever with dry cough, no breathing difficulty, and feeling nausea. However, we can observe from matrix equation (19) that, on the second visit, all symptoms are cleared in the x_1 patient, while the x_2 patient is having breathing difficulty, and hence, the x_2 patient is suffering from fever and breathing difficulty both. In this way, one can see and classify all information at a glance. Therefore, it is obvious that the matrix expression is the most appropriate expression to represent multidimensional data compared to the classical set expression.

$$A = \left[\mu_{A_j^k}(x_i)\right], \quad i = 1, 2, 3, j = 1, 2, 3, 4, \text{and } k = 1, 2. \tag{20}$$

The given PCTLHS matrix **A** is a rank-three tensor. The order of **A** is $(i \times j \times k) = 3 \times 4 \times 2$.

The top-to-bottom view of this PCTLHS Matrix consists of three parallel layers of ordinary matrices, and the order of each matrix is 2×4. These top-to-bottom layers in the separated form are i-level cuts.

5.2. *i-Level Cuts $A^{[i]}$ of the PCTLHS-Matrix (Subjectwise Level Cuts)*

$$A = \begin{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{bmatrix}. \tag{21}$$

Equation (21) represents a top-to-bottom view of the PCTLHS matrix, which is used to formulate i-level cuts $A^{[i]}$ (subjectwise level cuts) of the PCTLHS matrix.

We split the matrix at $i = 1, 2, 3$. These three integer levels (i splits) are three i-level cuts known as subjectwise level splits. These level cuts focus on subjects initially as follows:

$$A^{[1]} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{22}$$

$A^{[1]}$ reflects the attributive state of the first patient x_1 observed in the two given time levels.

$$A^{[2]} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}. \tag{23}$$

$A^{[2]}$ reflects the attributive state of the second patientx_2 observed in the two given time levels.

$$A^{[3]} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}. \tag{24}$$

$A^{[3]}$ reflects the attributive state of the third patient x_3 observed in the two given time levels.

This matrix form represents three top-to-bottom layers, and each layer is a matrix of order 2×4. These layers are obtained by splitting the given matrix on the index $i = 1, i = 2$, $i = 3$. Each i-level cut represents states of four attributes (symptoms) associated with a specific subject (patient) for the first and second examination levels. These three i-level cuts focus on the patient first and then describe their health state at time levels. These levels are utilized to organize the information for each patient separately. For example, the first layer (i-level cut) of the above matrix reflects the state of the patient x_1. From the first layer, it can be seen that they have all three symptoms except for cough on the first visit. During the second visit, all symptoms disappeared. Similarly, the second patient x_2 has all symptoms in the initial stage, and only one (cough) is left in the next stage. A description for the third patient can be given similarly by observing the third-level layer.

5.2.1. *i_j-Sublevel Cuts.*

For specified $i = 1$ and then further specify each $j = 1, 2, 3, 4$, respectively, i_j-sublevel cuts $A^{[i_j]}$ of the i-level cut $A^{[1]}$ are constructed as follows:

$$A^{[1_1]} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad A^{[1_2]} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad A^{[1_3]} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad A^{[1_4]} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \tag{25}$$

$A^{[1_1]}$represents the first symptom of the first patient in two given time levels, which means after focusing on the patient, the next focus is on the attribute. Similarly, one can describe other two level cuts $A^{[1_2]}$, $A^{[1_3]}$ and i_j-sublevel cuts.

For fixed $i = 2$ and then further specify each $j = 1, 2, 3, 4$, i_j-sublevel cuts of i-level cut $A^{[2]}$ are constructed as follows:

$$A^{[2_1]} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad A^{[2_2]} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad A^{[2_3]} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad A^{[2_4]} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \tag{26}$$

For fixed $i = 3$ and $j = 1, 2, 3, 4$, i_j-sublevel cuts of i-level cut $A^{[3]}$ are as follows:

$$A^{[3_1]} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad A^{[3_2]} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad A^{[3_3]} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad A^{[3_4]} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \tag{27}$$

5.2.2. *i_{j_k}-Sub-Sublevel Cut.*

i_{j_k}-sub-sublevel cuts are constructed as described. By using a level cut, one would observe the patient first $i = m$, $j = n$ and $k = l$, then at the next level by using a sublevel cut, one would observe his symptom (attribute) at several given time levels. And by using sub-sub-level cut, one would observe its symptom at a certain time level. The sub-sublevel cut is the smallest unit of the matrix, which is the single element. The general form is described as follows:

$$A^{\left[i_{j_k}\right]} = A^{\left[m_{n_l}\right]} = \left[\mu_{A_n^l}\left(x_m\right)\right]. \tag{28}$$

i_{j_k} *sub*-sublevel cuts of A are as follows:

$$\begin{aligned} A^{\left[1_{1_1}\right]} &= [1], \\ A^{\left[1_{1_2}\right]} &= [0], \\ A^{\left[1_{2_1}\right]} &= [0], \\ A^{\left[1_{2_2}\right]} &= [0], \\ A^{\left[1_{3_1}\right]} &= [1], \\ A^{\left[1_{3_2}\right]} &= [0], \\ A^{\left[1_{4_1}\right]} &= [1], \\ A^{\left[1_{4_2}\right]} &= [0], \\ A^{\left[2_{1_1}\right]} &= [1], \\ A^{\left[2_{1_2}\right]} &= [0], \\ A^{\left[2_{2_1}\right]} &= [1], \\ A^{\left[2_{2_2}\right]} &= [1], \\ A^{\left[2_{3_1}\right]} &= [1], \\ A^{\left[2_{3_2}\right]} &= [0] \\ A^{\left[2_{4_1}\right]} &= [1], \\ A^{\left[2_{4_2}\right]} &= [0], \\ A^{\left[3_{1_1}\right]} &= [1], \\ A^{\left[3_{1_2}\right]} &= [1], \\ A^{\left[3_{2_1}\right]} &= [1], \\ A^{\left[3_{2_2}\right]} &= [0], \\ A^{\left[3_{3_1}\right]} &= [0], \\ A^{\left[3_{3_2}\right]} &= [1], \\ A^{\left[3_{4_1}\right]} &= [1], \\ A^{\left[3_{4_2}\right]} &= [0]. \end{aligned} \tag{29}$$

5.2.3. i_k-Sublevel Cuts. For given i-level cuts, further specifying k and varying only j, one can obtain i_k-sublevel cuts. These level cuts are rows of an $N \times L$ submatrix.

Specifying $i = mk = l$ and varying $j = 1, 2, \ldots, N i_k$-sublevel cuts for a fixed i-level cut are constructed as follows:

$$A^{[i_k]} = A^{[m_l]} = \left[\, \mu_{A_1^l}\left(x_m\right) \;\; \mu_{A_2^l}\left(x_m\right) \;\; \cdot \;\; \cdot \;\; \cdot \;\; \mu_{A_N^l}\left(x_m\right) \right]. \tag{30}$$

i_k-sublevel cuts of i-level cut $A^{[i]}$ are as follows:

$$\begin{aligned} A^{[1_1]} &= \left[\, 1 \;\; 0 \;\; 1 \;\; 1 \,\right], \\ A^{[1_2]} &= \left[\, 0 \;\; 0 \;\; 0 \;\; 0 \,\right]. \end{aligned} \tag{31}$$

For fixed $i = 2$ and $k = 1, 2, i_k$-sublevel cuts of i-level cut $A^{[2]}$ are as follows:

$$\begin{aligned} A^{[2_1]} &= \left[\, 1 \;\; 1 \;\; 1 \;\; 1 \,\right], \\ A^{[2_2]} &= \left[\, 0 \;\; 1 \;\; 0 \;\; 0 \,\right]. \end{aligned} \tag{32}$$

For fixed $i = 3$ and $k = 1, 2, i_k$-sublevel cuts of i-level cut $A^{[3]}$ are as follows:

$$\begin{aligned} A^{[3_1]} &= \left[\, 1 \;\; 1 \;\; 0 \;\; 1 \,\right], \\ A^{[3_2]} &= \left[\, 0 \;\; 1 \;\; 1 \;\; 0 \,\right]. \end{aligned} \tag{33}$$

5.2.4. i_{k_j}-Sub-Sublevel Cut. i_{k_j} sub-sublevel cuts are constructed by specifying $i = m$. Then, by further specifying $j = n$ and $k = l$, the general form is described as follows:

$$A^{\left[i_{k_j}\right]} = A^{\left[m_{l_n}\right]} = \left[\mu_{A_n^l}\left(x_m\right)\right]. \tag{34}$$

i_{j_k} sub-sublevel cuts of A are as follows:

$$\begin{aligned} &A^{\left[1_{1_1}\right]} = [1],\ A^{\left[1_{1_2}\right]} = [0],\ A^{\left[1_{1_3}\right]} = [1],\ A^{\left[1_{1_4}\right]} = [1], \\ &A^{\left[1_{2_1}\right]} = [0], A^{\left[1_{2_2}\right]} = [0], A^{\left[1_{2_3}\right]} = [0], A^{\left[1_{2_4}\right]} = [0], \\ &A^{\left[2_{1_1}\right]} = [1], A^{\left[2_{1_2}\right]} = [1], A^{\left[2_{1_3}\right]} = [1], A^{\left[2_{1_4}\right]} = [1], \\ &A^{\left[2_{2_1}\right]} = [0], A^{\left[2_{2_2}\right]} = [1], A^{\left[2_{2_3}\right]} = [0], A^{\left[2_{2_4}\right]} = [0], \\ &A^{\left[3_{1_1}\right]} = [1], A^{\left[3_{1_2}\right]} = [1], A^{\left[3_{1_3}\right]} = [0], A^{\left[3_{1_4}\right]} = [1], \\ &A^{\left[3_{2_1}\right]} = [0], A^{\left[3_{2_2}\right]} = [1], A^{\left[3_{2_3}\right]} = [1], A^{\left[3_{2_4}\right]} = [0]. \end{aligned} \tag{35}$$

Example 1

$$B = \begin{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \end{bmatrix}. \tag{36}$$

The matrix in equation (36) is a $3 \times 3 \times 2$ PCTLHS matrix with three subjects, three attributes, and two time levels.

$i = 1, 2, 3$

$j = 1, 2, 3$

$k = 1, 2$

A top-to-bottom view of the PCTLHS matrix with three i -level cuts is given in equation (37). Every level cut is a 2×3 matrix.

$$B = \begin{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \end{bmatrix}. \tag{37}$$

$i -$ level cuts of B are given as follows:

$$B^{[1]} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad B^{[2]} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}, \quad B^{[3]} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}. \tag{38}$$

The local aggregation operators for i-level cuts are the union, intersection, average, and compliment operators described as follows.

5.2.5. Union of $i -$ Level Cuts. The union of $A^{[i]}$ is defined as follows:

$$\bigcup\left(A^{[i]}\right) = \underset{i}{\mathrm{Max}}\left(\mu_{A_j^k}(x_i)\right) = \left[\Omega_{A_j^k}(X)\right], \tag{39}$$

$[\Omega_{A_j^k}(X)]$ is an accumulated layer (submatrix) of the highest memberships considered as the top-level layer.

Example 2. For matrix **A** given in Ex-1, $\bigcup(A^{[i]}))$ is given as follows:

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \bigcup \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \bigcup \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}. \tag{40}$$

The cumulative attributive state of all three subjects reflects the highest level of each symptom among the given group of patients.

5.2.6. Intersection of $i -$ Level Cuts. The intersection of $A^{[i]}$ defined as follows:

$$\cap\left(A^{[i]}\right) = \underset{i}{\mathrm{Min}}\left(\mu_{A_j^k}(x_i)\right) = \left[\Omega_{A_j^k}(x)\right]. \tag{41}$$

$[\mu_{A_j^k}(x)]$ is the accumulated lowest membership as a bottom layer that represents the lowest state of each symptom of the group of three patients.

Example 3. For Matrix A in Ex-1, $\cap(A^{[i]})$ is given as follows:

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cap \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \cap \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{42}$$

This is the accumulated lowest attributive state of all three subjects as a group.

5.2.7. Average of $i -$ Level Cuts. The average memberships as an interior layer are defined as follows:

$$\Gamma\left(A^{[i]}\right) = \sum_{i=1}^{M} \frac{\left(\mu_{A_j^k}(x_i)\right)}{M} = \left[\Omega_{A_j^k}(x)\right] = \begin{bmatrix} 1, & \text{if } \sum_{i=1}^{M} \frac{\left(\mu_{A_j^k}(x_i)\right)}{M} \geq 0.5, \\ 0, & \text{if } \sum_{i=1}^{M} \frac{\left(\mu_{A_j^k}(x_i)\right)}{M} < 0.5, \end{bmatrix} \tag{43}$$

$[\Omega_{A_j^k}(x)]$ is the accumulated layer of average memberships considered as the interior-level layer. This average operator presents the neutral or average state of the universe.

Example 4. For matrix A given in Ex-1, $\Gamma(A^{[i]}))$ is

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Gamma \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \Gamma \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{44}$$

This is the accumulative attributive state of all three subjects as they are viewed as a single entity, reflecting the average level of each symptom.

5.2.8. Complement of $i-$Level Cuts. The complement of each membership of the i-level cut is defined as follows:

$$C\left(A^{[i]}\right) = \left[1 - \mu_{A_j^k}\left(x_i\right)\right]. \tag{45}$$

Example 5. Complement of i-level cuts of A is given as follows:

$$\begin{aligned} C\left(A^{[1]}\right) &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}, \\ C\left(A^{[2]}\right) &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}, \\ C\left(A^{[3]}\right) &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}. \end{aligned} \tag{46}$$

These complements can be further accumulated through previously defined aggregation operators. These complement operators reflect the inverted state of the merged universe.

6. Conclusion

6.1. Conclusion and Discussion. This novice model would help improve and expand the field of decision-making and artificial intelligence. Using the PCTLHS matrix, one would be able to represent an extensive, indeterminate plithogenic universe. Some important facts of this model are discussed as follows:

(1) The PCTLHS matrix provides multidimensional views of the universe (subjects versus attributes and time-lined attributes), whereas in this first version, only one of the three possible views is described as a model. In this view, the subjects are initially focused by the observer, and the next focus would be attributes or time levels depending on the interest of the observer or decision-maker.

(2) One can classify and analyze the universe explicitly and implicitly through level cuts, sublevel cuts, and sub-sublevel cuts. In this way, one could be able to enhance his approach to knowing the universe (all information) in a more dynamic and modern form.

(3) By choosing specific i, j, or k level cuts, one can make his priority choices by subjects, attributes, or time levels, and by specifying further sublevel cuts, one can further specify his next level of selection by subject, attribute, or time levels. Through the next level of selection, i.e., using sub-sublevel cuts, one can approach a final selection of the subject, attribute, or time level as per requirements.

(4) The PCTLHS matrix provides the broader exterior and interior views of the matrix by displaying all possible events (realities) together. Therefore, we expect that this PCTLHS Matrix will deal with multiple information networks under one command and one control system.

(5) The level cuts of the PCTLHS matrix present an explicit event or reality at a single instance of time. Therefore, one would be able to observe the state of an event (health state of patient) from moment to moment. This will improve his understanding of each event and every state according to its time level during that state.

(6) Variation index-based level cuts provide the view of reality or events from multiple angles of vision. This will improve the mathematical visualization of facts/events/information.

(7) One can analyze the universe by choosing the best possible reality from several possible realities using level cuts and operators. This fact would be helpful in the development of artificial intelligence programs.

(8) This PCTLHS matrix is very close to the functions of the human mind. Therefore, it acts as a pioneer in the modeling of the human mind. If this application can be opted by some computer specialists/software developers, it has every potential not only to improve artificial intelligence (AI) systems but also has great promises in the field of real intelligence (RI).

(9) The disjunction operator, i.e., the max operator, provides the optimist view of reality.

(10) The conjunction operator, i.e., the min operator, offers the pessimist perception of the event or reality

(11) The averaging operator offers the neutral states of the unified universe, which means that all the elements of the universe are considered as a single entity and only its neutral states are focused

(12) The complement operator presents the inverted reflection of the event or reality.

(13) The local operators designed for i-level cuts serve the purpose of unifying the matter bodies of the universe. This means that all the elements of the universe are merged and presented as a single matter body reflecting several attributes in distinct time levels. In this way, the concept of the unified global matter (something like dark matter) is visualized.

(14) These local operators serve the purpose of unification of matter bodies of the universe. This means that all the elements of the universe are merged and presented as a single matter body reflecting several attributes in distinct time levels. This is how the concept of the unified global matter (something like dark matter) arises. In this way, the concept of the unified global matter (something like dark matter) emerges.

6.2. Comparisons of Former Fuzzy Extensions and Models. This section contains a brief comparison of previous and current fuzzy extensions and models.

The soft set is an advanced and extended version of the fuzzy set because it manages many attributes simultaneously regardless of the fuzzy set, which only handles one attribute at a time.

The hypersoft set is an enhanced extension of a soft set because it can accommodate multidimensional information by managing many attributes and their various values simultaneously.

The plithogenic hypersoft set is a more advanced and applicable version than the hypersoft set, soft set, and fuzzy set. It is a higher dimensional version that manages detailed information. The observer can intrinsically see the state of the element x (subject) by looking at each attribute separately. In simple terms, the plithogenic hypersoft set manages multiple attributes and their values (subattributes) simultaneously and beyond by observing each attribute separately.

The plithogenic fuzzy whole hypersoft set/matrix (PFWHS set/matrix) is a more appropriate choice than the previously mentioned extensions as it indicates the states of subjects (attributes/subattributes) at the individual level for each attribute/subattribute (as in the case of the plithogenic hypersoft set) and also at the unified or combined level for all attributes together as a whole (the case of the hypersoft set). Therefore, it is an extended hybrid version of the hypersoft set and the plithogenic hypersoft set. By using the PFWHS set/matrix, one can observe a more transparent inner perception (case of a single state representation) or outer view (case of a combined state representation) of information/facts/events.

The plithogenic subjective hyper-supersoft matrix (PSHSS matrix) is a generalized advanced form of the PFWHS matrix and is more applicable as than previously developed models, as it has a greater capacity to express and manage various connected attributes/subattributes separately and as a whole by considering connected attribute/subattribute levels.

The plithogenic time-leveled hypersoft matrix (PTLHS matrix) is a special case of the previous generalized form (PSHSS matrix) that manages time-based connected attributes. It is more suitable than other extended fuzzy sets mentioned (soft set, hypersoft set, plithogenic hypersoft set, PFWHS set/matrix, and PSHSS matrix) for the following valid reasons:

(1) The plithogenic time-leveled hypersoft matrix includes time as the fundamental source of variation. As most of the variations in this universe are time-dependent like weather graphs, stock exchange, and website ratings, therefore, it is of tremendous help if this PCTLHS matrix is used to cope with the scattered time-varying piece of information.

(2) It deals with several attributes interiorly such that each attribute has many values (subattributes). These subattributes are varying with the flow of time. By using the PCTLHS matrix, one can organize and classify multidimensional information into the shape of connected matrix layers as hypermatrices.

(3) The matrix expression is the most appropriate expression to represent multidimensional data compared to the classic set expression.

(4) This PTLHS Matrix allows the viewer to see information down to its innermost level through level cuts, sublevel cuts, and sub-sublevel cuts.

(5) In addition, it offers a broader view of multidimensional information by viewing the entire universe as a hypersoft time-leveled matrix. Therefore, the observer can see and analyze the whole universe externally at a single glance.

(6) Level cuts can focus on one required piece of information that is displayed in the form of a single matrix layer of the PTLHS matrix, while the other information can vary with the flow of time being displayed as other matrix layers.

(7) Sublevel cuts can focus on required information that is displayed as a single column or row of the given layer (submatrix) of the PTLHS matrix.

(8) Sub-sublevel cuts can focus on required information that is displayed as a single element of the submatrix of the PTLHS matrix.

(9) Sublevel cuts offer the representation of the previous lower dimension in the further lower dimension and enable us to sneak in an inside view of the expanded universe; i.e., after explicitly focusing on a subject through an i-level cut (single level of the layered matrix), our next focus is on that subject's (patient's) attribute (a particular symptom) through the sublevel cut (row or column of one layer of the multilayered matrix).

(10) It also offers the unification of the information by applying aggregation operators, and in this way, all the extended information of the universe that is represented as a matrix having multiple layers can be transformed into a single layer of the matrix.

6.3. Future Research. Now, let us list some of the open problems that might be addressed in future research.

(i) In this article, we have portrayed the plithognic hypersoft matrix in a crisp environment. One can extend this model in other environments such as fuzzy, intuitionistic, and neutrosophic, or any mixed or combined environment; i.e., containing several environments would provide the variation of fuzziness levels of reflected events.

(ii) Moreover, some other kinds of local operators can be provided for the unification purpose of littered data according to the requirement of the concerned bodies.

(iii) The operations and properties of these hypersoft matrices need to be explored.

Data Availability

No data were used to support the findings of this study as it is a theoretical model.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

All authors reviewed the results and approved the final version of the manuscript for publication. Shazia Rana is responsible for the study, conception, drafting, design, interpretation of the work, and review of the results for the final version of the manuscript. Mohammad Saeed is responsible for the analysis of the research work and its critical revision for key intellectuals' content agreement to be responsible for all aspects of the work to ensure that issues related to the accuracy or integrity of any part of the work are properly investigated and resolved. Badria Almaz Ali Yousif is responsible for analyzing the work and critically reviewing it for important intellectual content, reviewing the results, and approving the final version of the manuscript. Florentin Smarandache is responsible for analyzing and critically revising the manuscript for the acquisition of important intellectual content and the agreement to all aspects of the work, to ensure that issues related to the accuracy or integrity of any part of the work are adequately investigated and resolved. Hamiden Abd-ul-Wahid Khalifa is responsible for critically revising the manuscript for important intellectual content, reviewing the results, and approving the final version of the manuscript.

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