A Novel Smart Method for State Estimation in a Smart Grid Using Smart Meter Data

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Smart grids have brought new possibilities in power grid operations for control and monitoring. For this purpose, state estimation is considered as one of the effective techniques in the monitoring and analysis of smart grids. State estimation uses a processing algorithm based on data from smart meters. The major challenge for state estimation is to take into account this large volume of measurement data. In this article, a novel smart distribution network state estimation algorithm has been proposed. The proposed method is a combined high-gain state estimation algorithm named adaptive extended Kalman filter (AEKF) using extended Kalman filter (EKF) and unscented Kalman filter (UKF) in order to achieve better intelligent utility grid state estimation accuracy. The performance index and the error are indicators used to evaluate the accuracy of the estimation models in this article. An IEEE 37-node test network is used to implement the state estimation models. The state variables considered in this article are the voltage module at the measurement nodes. The results obtained show that the proposed hybrid algorithm has better performance compared to single state estimation methods such as the extended Kalman filter, the unscented Kalman filter, and the weighted least squares (WLS) method.

1. Introduction

The power network has undergone technological changes in production, transmission and distribution in recent years. Additionally, new technologies are expected to revolutionize the use of energy through the adoption of various demand management programs and methods. These changes make it possible to migrate to the smart grid with the installation of smart meters for measuring consumption. The principle of the smart grid is based on data analysis to better manage energy systems [1, 2]. These new technological requirements in distribution networks have led some countries to develop their research towards smart power networks [3]. Smart meters monitor energy usage in real time by collecting and reporting data to utility companies. The accuracy of smart meters deteriorates over time, making it necessary for power companies to verify their performance [4, 5].

The smart meters collect electrical data over an interval of 15 minutes or 1 hour. The deployment of smart meters around the world can benefit energy suppliers, network operators, and consumers with its ability to collect detailed information to enable real-time monitoring of electrical energy consumption [6, 7]. To this end, the management of these data from smart meters requires operational calculation techniques to understand the behavior of the network and consumers [8, 9]. In addition, state estimation is one of the most effective techniques in power network management. State estimation is a method that reduces the complexity in the control of power networks by increasing the state parameters of the system, thus contributing to the...
efficient management of electrical energy [10, 11]. Moreover, state estimation techniques can be used for modeling, optimization, bad data detection and power resilience analysis to improve network monitoring capabilities in real-time control [12]. Thus, in power distribution networks with multiple measurements, state estimation is used to reduce errors and tune the measurements. Despite the increase in the quantity of measurement results in the state estimation, a large part of these data transmitted to the control center reduces the processing speed, thus causing disturbances in the online control of the network. State estimation makes it possible to follow the dynamics of electrical networks by considering real measurement data from smart meters [13].

The network state vector is made up of electrical parameters such as bus voltage or branch currents. Most often, the bus voltage is used as the state vector. To this end, the inputs of such as bus voltage or branch currents. Most often, the bus voltage is used as the state vector. To this end, the inputs of these data transmitted to the control center.

Several works on smart distribution grid state estimation have been done. System state variables come from smart meters [15, 16]. These estimation algorithms are mostly based on Kalman filters, neural network filters, and the weighted least squares method [17]. In [17], a state estimation of the distribution network using artificial neural networks has been proposed. The proposed method uses load profiles and network data to train two neural networks: one for active power and the other for reactive power. A 9 bus section of the United Kingdom distribution network was used to demonstrate the effectiveness of the state estimation model using artificial neural networks. Matlab software was used for training and testing neural networks and applying state estimation. The simulation results show that the maximum convergence time for the state estimation algorithm among all 88 samples is 0.7 s.

In addition, the author [18] studied an algorithm for estimating the measurement error of smart meters. This algorithm uses the extended Kalman filter method and the recursive least memory least squares method. In this article, a modified estimation model is obtained by the selection of the estimation stage that corresponds to the real operating conditions and filters out the abnormal estimation values related to the line loss characteristics. The model is applied on the experimental data obtained from the simulation system in order to obtain the precision of the estimation of the measurement error. Thus, the mean absolute percentage error (MAPE) obtained respectively for each of these methods is 3.71 and 0.77. The author [19] proposed a distance estimation method based on the neural network filter and the recursive least squares method. In this work, an error estimation model of the smart meter with a loss noise filter is built with a neural network. The effectiveness of this model is verified through simulations on the distribution network. The results obtained show that the measurement error estimation range of smart meters is between 2 and 3%.

The author [20] presented a two-layer estimation technique based on smart meters. This technique makes it possible to integrate the control of medium-voltage and low-voltage distribution networks. This technique is demonstrated on the IEEE 13-node and 906-node distribution networks. From this state estimation model, the MAPE is between 0.035 and 0.038%. Similarly, the work in [21] has made it possible to adopt a method for continuous estimation of current transformer and power transformer errors using an adhesion cloud and dynamic time warping. Moreover, a new mixed semi-trapezoidal adhesion cloud generator is built to take into account the randomness and fuzziness of the different error variation factors. Current and voltage transformer errors can be estimated using the similarity between error time series and the influence factor quantified using a modified dynamic time warping algorithm. The proposed technique is verified by test data from the 110 kV substation metering equipment. The normalized root-mean-square error (NRMSE) obtained by the mixed semi-trapezoidal generator method and the modified dynamic time warping algorithm is 13.85%.

The authors in [22] proposed a method to estimate the admittance matrix for electrical distribution networks using voltage, and active and reactive power data. This work makes a derivative of the linear relationship between the admittance matrix and the measurements based on the power flow equations. The IEEE 33-node network was used to verify the efficiency and accuracy of the proposed method. The results obtained from this model give an estimation error of 1%.

The author [23] made a review on state estimation techniques in the intelligent distribution network. This work studies the subjects of state estimation of distribution networks in occurrence the application of pseudo-measurements, the placement of measuring instruments, the problems of network topology, the impact of the integration of renewable energies, and cyber security. The author [24] described a method for online smart meter error calibration using smart meter data. This method is based on the combination of K-means theory and regularization theory in order to evaluate the errors of smart meters. The relative error of the proposed algorithm is 11.27%. The interval of difference between the estimated error and the real error is between 0.23% and 0.31%. In addition, the author [4] conducted research on the influence of temperature and humidity on smart meter errors based on regression analysis. The mean square error (MSE) of the value predicted by the linear regression is $8.076 \times 10^{-6}$.

The work in [25] proposed a decentralized robust state estimation method for AC/DC distribution systems using multiple data sources. In order to improve the accuracy of the estimation, a deep neural network based on smart meter data is used to extract statistical information from the system and allow the derivation of nodal power injections.

The author [26] presented a state estimation using the extended modified Kalman filter in which the states are estimated directly from the compressed data without reconstruction procedure. The IEEE 33-node network with two decentralized productions is used to verify the performance of the proposed method. The results show that the states of the test network are accurately estimated with only 50% of the measurements. In [27], the states were estimated using the dynamic power flow model and an extended Kalman filter, and its performance was compared with the least squares method. Similarly, the author [28] made a...
comparison between the least squares method and the Kalman filter using PMUs as a measurement vector.

The authors in [29] described a state estimation in electric power networks using the extended Kalman filter (EKF) and Holt’s method in order to linearize the processing model and calculate performance indicators. IEEE 14-node and 30-node networks were used to implement these algorithms; moreover, Matlab was used for the simulation of the method. This author through his results shows that the estimated values adequately follow the real values. In [30], a state estimation method using the unscented Kalman filter (UKF) has been proposed. The algorithm was implemented on an IEEE 18-node network in order to observe the estimation performance during the simulation. This technique based on the unscented Kalman filter (UKF) makes it possible to linearize the dynamic state variables such as the voltage in order to obtain a better performance compared to the extended Kalman filter (EKF).

In [31], smart meters are used for state estimation of medium-voltage distribution networks. The estimation algorithm was developed and tested using smart meter measurements from the 11 kV residential distribution network in England. The estimation produces the amplitudes and angles of the voltages on each node. Simulation results show that smart meter measurements can be used for state estimation to extend the observability of the distribution network.

The author [32] proposed a state estimation algorithm for the inclusive monitoring of smart grids. The proposed method is a hybrid technique using weighted least squares (WLS) and firefly algorithm (FA) to achieve a more reliable and accurate state estimation of the power grid. The firefly algorithm is equipped with new optimization operators that make it possible to solve multiple problems using multiple subdivision components. In order to improve the overall search ability of the algorithm, a new two-phase modification method has been implemented. This novel hybrid method can effectively estimate the voltage angle using the weighted least squares method.

In [33], a discussion on relevant experiences on distribution system state estimation (DSSE) for radial distribution networks is presented. In this article, the authors elaborated the connections between state estimation implementation and practical variables, such as switch flows, line lengths, voltage regulation, and measurement areas.

Figure 1 shows the structure of a smart grid. Here, smart meters are deployed across the network to measure, synchronize, store, and retransmit data.

The state estimation algorithms can also be used for the state of charge estimation of lithium-ion batteries and power capacity estimation. In this way, the works in [34] proposed a co-estimation of load current and the state of charge for current sensor-free lithium-ion battery. This article aims to evaluate the need for installing the current sensor for the management of lithium-ion batteries. Moreover, a constrained optimization problem is obtained from the state observation using the proposed method, and it is resolved digitally in a moving horizon estimation algorithm to allow the online co-estimation of the state of charge and input current. Using load current can allow to mitigate the accuracy of lithium-ion battery state of charge estimation. This method can reduce the structural complexity and cost of future lithium utilization with better accuracy than the EKF. In same way, author [35] developed a lithium-ion polymer battery state of charge estimation based on adaptive unscented Kalman filter (UKF) and support vector machine (SVM). In this article, the proposed algorithm can effectively estimate the state of charge of battery considering the load current and voltage. The error coefficients obtained showed that the proposed method is better comparing to single extended Kalman filter, unscented Kalman filter, and weighted least square method.

Moreover, author [36] implemented a sensor-less state of charge estimation which combined a simple equivalent circuit model (ECM) with voltage filtering. However, the proposed filter is basic, and frequent relaxations are necessary to reduce the estimation error. With the aim to remedy such limits, author [37] refined the simple equivalent circuit model (ECM) based on an unknown input observer designed to estimate the lithium-ion battery current. Using moving horizon estimation (MHE) model, author [38] estimated the accurate state of charge of lithium-ion batteries. It can be observed that MHE model can
precisely estimate the lithium-ion battery state of charge with high accuracy.

Using the EKF method [39] and the UKF method [35], authors have also applied the lithium-ion battery state of charge estimation. Considering the impact of noise corruption, author [40] implemented an online model identification method based on adaptive forgetting recursive total least squares (AF-RTLS) to compensate the noise effect and attenuate the identification bias of model parameters. The results obtained show that the proposed method is superior in terms of accuracy compared to other methods. In [41], a fourth-order Butterworth method has been implemented to reduce the estimation error and remove the high-frequency measurement noise corruption. Moreover, author [42] developed a bias compensation method to remove the identification bias on model parameters caused by noises.

Author [43] combined the recursive least squares (RLS)-based online model identification with adaptive extended Kalman filter (AEKF) for the state of charge and power capacity joint estimation. Author [44] developed a hybrid pulse power characterization (HPPC) model known as ECM-based power capacity estimator. However, this model only considers voltage-limited instantaneous power with a primitive model.

In [45], an analytical model-based close-loop method was performed combined with the RLS-based identification for power capacity estimation. A total least squares (TLS) model has been developed in [46, 47] to provide solution to the error-in-variable (EIV) problem. In [48], the authors proposed a balancing current ratio (BCR)-based solution to realize satisfactory state of health (SoH) estimation performance for all series-connected cells with a pack. The proposed model gives an estimation error around 1.5% which is lower for 70% than the benchmarking models. Author [49] proposed a co-estimation framework for the state of charge, state of health, and state of function of lithium-ion batteries in electrical vehicles. This model is performed in real battery management system with better real-time performance and effective estimation accuracy. In same way, author [50] developed a framework using the reduced-order electrochemical model and the dual non-linear filters with the aim to maintain the accuracy of battery state estimation. The experimental verification showed that the proposed state of charge and state of health cosimulation model is superior to other models.

Author [51] focused on data science technologies for battery manufacturing management. In this article, the battery manufacturing line consists of numerous intermediate stages involving strongly coupled interdependency which determine the performance of manufactured battery. Author [52] proposed a novel tree boosting-based model to analyze and predict the variation of battery electrode properties considering parameters during the production stage. Other estimation methods such as dual extended Kalman filter [53], recursive least squares (RLS) [54], and recursive extended least squares (RELS) [55] have been used to solve this problem.

For smart distribution grid, during the state estimation process, bad data and erroneous measurements are extracted from the database. Additionally, system clarity must be maintained before and after the cleaning process. The major challenge is that the smart grid requires a large number of data processing devices including measurement equipment’s and control devices which would lead to the complexity of the smart grid. The use of an appropriate estimator can reduce the number of materials used in the system.

The work proposed in this article effectively solves the problems encountered in the estimation process by proposing a new estimation algorithm using the extended Kalman filter (EKF) and the unscented Kalman filter (UKF).

The rest of the article is organized as follows: Section 2 gives the material and the method, the extended Kalman filter (EKF), the unscented Kalman filter (UKF), and the weighted least square method (WLS) are explained in detail in order to build a hybrid model combining the EKF and UKF to obtain the AEKF algorithm. Section 3 presents the results and the discussion. These results will be obtained by implementing state estimation on IEEE 37-node network. The conclusion is given in Section 4 with some perspectives for the future.

2. Material and Method

2.1. Equipment

2.1.1. The IEEE 37-Node Network. The test network is an IEEE 37-node distribution network with smart meters on each node. These smart meters are used to measure data in real time in order to estimate the state of the distribution system. The IEEE 37 network has a 33 kV source substation, two transformers, and 37 buses at 4.8 KV, 32 lines, and 65 loads. Figure 2 gives the structure of the IEEE 37-node network modified with smart meters and a communication and monitoring system.

2.1.2. Matlab. State estimation is implemented in Matlab software. In this article, we use Matlab 2020b 64 bit. Matlab allows to realize the algorithm of the extended Kalman filter (EKF) and the unscented Kalman filter (UKF). The time interval for continuously reading data through the state estimation is 1 minute.

2.1.3. OpenDSS. OpenDSS (open distribution system simulator) is a software designed by Roger Dugan, allowing the simulation of electrical distribution networks [11, 56]. The simulation of the daily power flow is carried out using OpenDSS with a time interval of 1 minute to obtain the values measured by the smart meters. In this article, the real values of the OpenDSS simulation environment which are considered as the true values are subject to a state estimation algorithm.

2.1.4. Computer. The state estimation algorithm was implemented using a Dell computer with Windows 10 system, 3.5 GHz processor, 500 GB hard disk, and 8 GB RAM.
2.2. Method. The method is based on a hybrid algorithm combining the extended Kalman filter (EKF) and the unscented Kalman filter (UKF).

Figure 2 gives the structure of the hybrid state estimation algorithm using the extended Kalman filter and the unscented Kalman filter.

2.2.1. Extended Kalman Filter (EKF). The Kalman filter was developed by Rudolf E. Kalman; this algorithm makes it possible to generally estimate the states of a dynamic and linear system from a time series. In our work, we have developed a smart distribution network state estimation algorithm that uses data from smart meter measurements. For this purpose, we propose a novel dynamic algorithm, which estimates the states of the smart grid directly from the data of the smart meters. This method is an extended and modified Kalman filter for smart utility grid state estimation. Thus, the states are estimated from a reduced quantity of measurements transmitted by the smart meters. For the state estimation algorithm, the Kalman filter applies linearization to determine the network model. The advantage of this estimator is its ability to determine the dynamic model of the network using a more accurate method.

The proposed algorithm is implemented to estimate the amplitude of the voltages at the distribution network buses and to know the topology during the estimation period.

The purpose of the filter is to estimate the future value of a process that is stained with white noise. Figure 4 illustrates the stages of the Kalman filter for the state estimation of the distribution network.

(1) Initialization. This step initializes the algorithm from the initial values.

(2) Dynamic Modeling. In this step, how the state vector changes should be determined by the dynamic model of the network. In the extended Kalman filter algorithm, the dynamic model of the network is obtained from the first-order derivatives of the Taylor series and the general nonlinear relations of states and inputs. Although linearization is a simple and common method to determine the model of the network, the linearization error decreases the accuracy of the model. The proposed method is particular because it does not require the use of linearization to define a model of the network. The nonlinear stochastic state and measurement equations used to define the network model are as follows:
State equation:
\[ X_{k+1} = f(X_k, u_k) + w_k. \]  
(1)

Measurement equation:
\[ Y_{k+1} = h(X_k, u_k, w_k) + v_k, \]  
(2)

where \( X_{k+1} \) is the network state vector, \( Y_{k+1} \) is the measurement vector, \( u_k \) is the driving input function, \( w_k \) is the white noise of the process, and \( v_k \) is the white noise measurement.

The dynamic model of the distribution network is nonlinear; however, if the dynamic changes in the network are slow enough, the smart network can be considered in the quasi-stationary state. For this purpose, the extended Kalman filter will make it possible to determine the discrete linear approximation of the network model that can be presented by equation.

\[ X_{k+1} = F_k X_k + g_k + w_k, \]  
(3)

where \( F_k \) is the speed of transition between the states and \( g_k \) is the behavioral tendency of the states.

(3) Identification of Parameters. The identification of the parameters makes it possible to estimate the parameters \( F_k \) and \( g_k \) which are used in the step of prediction.

The values \( F_k \) and \( g_k \) can be obtained from the following equations:

\[ F_k = \alpha_k (1 + \beta_k) I, \]  
(4)

\[ g_k = (1 + \beta_k) (1 - \alpha_k) \hat{X}_k - \beta_k a_{k-1} + (1 - \beta_k) b_{k-1}, \]  
(5)

where \( \hat{X}_k \) is the predicted state vector and \( X_k \) is the true state vector.

Further, the state vector \( X_k \) for the \( N \)-bus distribution network is shown in the following equation:

\[ X_k = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_N \\ |V_1| \\ |V_2| \\ \vdots \\ |V_N| \end{bmatrix}, \]  
(8)

The parameters \( \alpha_k \) and \( \beta_k \) vary between 0 and 1.

The vectors \( a_k \) and \( b_k \) are obtained using the following equations:

\[ a_k = \alpha_k X_k^T + (1 - \alpha_k) \hat{X}_k, \]  
(6)

\[ b_k = \beta_k (a_k - a_{k-1}) + (1 - \beta_k) b_{k-1}, \]  
(7)

where \( \hat{X}_k \) is the predicted state vector and \( X_k \) is the true state vector.

Further, the state vector \( X_k \) for the \( N \)-bus distribution network is shown in the following equation:

\[ X_k = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \vdots \\ \phi_N \\ |V_1| \\ |V_2| \\ \vdots \\ |V_N| \end{bmatrix}, \]  
(8)

where \( |V_i| \) and \( \phi_i \) representing the magnitude and angle of the voltage at bus \( i \).
Apply the unscented transformation.

Calculation of state estimation and convariance.

Obtain the state prediction of the convariance.

Apply the unscented transformation.

Initialzation

End

Figure 5: UKF flowchart.

(4) Approximate estimation. This step provides an approximate estimation of the states including the state vector, the measurement error, and the performance index. The estimation error $\epsilon(k)$ at step $k$ is given in the following equation:

$$
\epsilon(k) = \frac{1}{N} \sum_{n=1}^{N} |\bar{X}_n(k) - X_n(k)|.
$$

(9)

The estimate of the error $\epsilon_r$ on the voltage amplitude is given by the following equation:

$$
\epsilon_r = \frac{1}{N} \sum_{k=1}^{N} |\bar{V}_k - V_k'|.
$$

(10)

The rough estimate of the performance index is as follows:

$$
H_k = \frac{\sum_{i=1}^{m} |\bar{Y}_k - Y_k'|}{\sum_{i=1}^{m} |Y_k - Y_k'|}.
$$

(11)

where $Y_k$ is the measurement vector, $Y_k'$ is the actual measure vector, and $\bar{Y}_k$ is the predicted measure vector.

Moreover, the covariance error is given by the following equation:

$$
P_k = F_k \bar{P}_k + \bar{P}_k F_k^T + Q_k,
$$

where $F_k'$ is the real speed of transition between the states and $Q_k$ is the covariance matrix.

(5) Calculation of the Kalman gain. The Kalman gain is calculated using the Jacobian matrix, which is given as follows:

$$
J_k = \frac{\partial h(X_k)}{\partial \bar{X}_k}.
$$

We therefore obtain the expression of the Kalman gain as follows:

$$
K_k = P_k (-J_k^T (\bar{X}_k (\bar{X}_k (-)))[J_k (\bar{X}_k (-)) P_k (-J_k^T (\bar{X}_k (-)) + R_k]^{-1},
$$

(14)

where $\bar{X}_k$ is the predicted state vector and $R_k$ is the uncertainty measure.

(6) State correction. This step makes it possible to make a correction of the states of the system by considering the model of the network as well as the white noise. The measurement vector considered makes it possible to obtain the estimated value of the amplitude of the voltage. By updating the measurement vector $Y_{k+1}$, the predicted state vector $\bar{X}_{k+1}$ can be corrected leading to a new vector $\bar{X}_{k+1}$ with a covariance error.

2.2.2. Unscented Kalman Filter (UKF). The unscented Kalman filter is an enhancement of the Kalman estimator to handle the nonlinear nature of dynamic state estimation. It is based on the theory of unscented transformation; this approach propagates a statistical distribution of the state via nonlinear equations to provide better results. The UKF does not require the linearization of the model and allows the calculation of Jacobian matrices in order to avoid the first feedback. Linearization can also cause errors in the transformation of means and covariance matrices when a random variable is transformed in the nonlinear equation. The UKF allows the direct propagation of means and covariance matrices through the nonlinear equations of the model. For this purpose, the UKF uses a quantity of weighted points based on the standard deviations (σ points) chosen in a deterministic way knowing that the properties of these points are in adequacy with those of the distribution before the transformation. The unscented transformation guarantees the same performance (mean and covariance) as the second-order Gaussian filter. Figure 5 gives the flowchart of the unscented Kalman filter.

Preliminarily, consider a sequence of points $x^{(i)} (i = 1, \ldots, 2n + 1)$ with $p = 2n + 1$. Each point associated with a weight $\omega^{(i)}$. Each point is propagated through a nonlinear function $g$ such that:

$$
y^{(i)} = g(x^{(i)}).
$$

(15)

According to the UKF, the average is then approximated by a weighted average over the transformed points.

$$
\bar{y}^{UKF} = \sum_{i=0}^{p} \omega^{(i)} y^{(i)},
$$

(16)

where $\sum_{i=0}^{p} \omega^{(i)} = 1$.

The covariance is calculated by the following equation:

$$
P^{UKF}_{y} = \sum_{i=0}^{p} \omega^{(i)} (y^{(i)} - \bar{y})(y^{(i)} - \bar{y})^T.
$$

(17)

The UKF algorithm is characterized by a discrete state model presented in the following equations:
\[ x_k = f(x_{k-1}, u_{k-1}, \omega_{k-1}), \quad (18) \]
\[ y_k = h(x_k, u_k, v_k), \quad (19) \]

where \( \omega \) is the processing noise of dimension \( q \) and the covariance matrix \( Q; \) \( v \) is the measurement noise of dimension \( r \) and the covariance matrix \( R; \) and \( x_k \) is the state of dimension \( n. \) It can be noted that equations (18) and (19) incorporate the noises in the terms \( f \) and \( h. \) If the noise terms are added to \( f \) and \( g, \) the covariance matrices of the processing noise \( Q_{k-1} \) and the measurement noise \( R_k \) must be added respectively in the expressions given by the matrices \( P_{x_k} \) and \( P_{y_k}. \)

An augmented state vector is introduced as follows:
\[
X_k^a = \begin{bmatrix} X_k \\ u_k \\ v_k \end{bmatrix}. \quad (20)
\]

The average of the augmented state vector is presented as follows:
\[
E(X_k^a) = \begin{bmatrix} X_k \\ 0^{q \times 1} \\ 0^{r \times 1} \end{bmatrix}, \quad (21)
\]

where \( 0^{q \times 1} \) is a column vector of zeros of dimension \( q. \)

The covariance matrix of the augmented state vector is given by the following equation:
\[
P_k^a = \begin{bmatrix} P_k & 0^{q \times r} & 0^{q \times r} \\ 0^{q \times r} & Q_k & P_v \\ 0^{q \times r} & P_v & R_k \end{bmatrix}. \quad (22)
\]

Let us initialize \( k = 0 \)
\[
\tilde{x}_0 = E(x_0),
\]
\[
E_0 = E\left((x_0 - \tilde{x}_0)(x_0 - \tilde{x}_0)^T\right),
\]
\[
E \left((x_0 - \tilde{x}_0)^T(x_0 - \tilde{x}_0)\right) = \begin{bmatrix} P_x & 0 \\ 0 & P_v \end{bmatrix}. \quad (23)
\]

Updating equations: the \( \sigma \) points are transformed through the following state equations:
\[
X_k^x = f(x_{k-1}^x, u_{k-1}, X_{k-1}^w). \quad (24)
\]

The prior estimate and the covariance are calculated in the following equations, respectively:
\[
\tilde{x}_{k-1} = \tilde{x}_{k-1}, \quad (25)
\]
\[
P_{x_k} = \sum_{i=0}^{2N} \omega_c^{(i)} (X_{ik-1}^x - \tilde{x}_k)(X_{ik-1}^x - \tilde{x}_k)^T, \quad (26)
\]

where the weights are defined by: \( \omega_m^{(0)} = (\lambda/(N + \lambda)) \)
\[
\omega_c^{(0)} = \frac{\lambda}{N + \lambda} + (1 - \alpha^2 + \beta), \quad (27)
\]
\[
\omega_m^{(i)} = \frac{\lambda}{2} (N + \lambda), \quad (27)
\]

where \( i = 1, \ldots, 2N \) and \( \beta \) is the positive weight parameter.

Updating measurement equations: \( \sigma \) points are transformed through measurement in equation:
\[
\tilde{y}_k = \tilde{y}_{k-1} = \sum_{i=1}^{2N} \omega_m^{(i)} \tilde{y}_{i,k-1}. \quad (28)
\]

The Kalman gain is given by the following equation:
\[
K_k = P_{x_k} (P_{y_k})^{-1}. \quad (30)
\]

Updating the estimated states gives the following equation:
\[
\tilde{x}_k = \tilde{x}_{k-1} + K_k (y_k - \tilde{y}_k). \quad (31)
\]

The corresponding covariance matrix is presented as follows:
\[ P_{x_k} = P_{x_k} - K_k P_{x_k} \beta_k^T. \]  

(32)

The Kalman gain can affect the estimated state values in different ways. Moreover, a remarkable characteristic of the UKF is that it allows to take into account the constraints on the states by making an appropriate conditioning of the \( \sigma \) points.

2.2.3. Weighted Least Square. Weighted least square (WLS) is an estimation technique that weights observations proportionally to the covariance error for that observation and thus solves the problem of nonconstant variance. Figure 6 shows the organization chart of weighted least squares method.

The model is presented in the following equation:

\[ Y_i = \beta_0 + \beta_1 X_1 + \epsilon_i, \]  

(33)

where \( i = 1, \ldots, n \) and \( \epsilon_i \sim N(0, \sigma^2/\omega_i) \) for constants \( \omega_1, \ldots, \omega_n \). WLS estimates \( \beta_0 \) and \( \beta_1 \) for quantity minimization.

\[ S_w(\hat{\beta}_0, \hat{\beta}_1) = \sum_{i=1}^{n} \omega_i (y_i - \beta_0 - \beta_1 x_i)^2. \]  

(34)

In the weighted sum of squares, the weights are inversely proportional to the corresponding variances; points with low variances will give larger weights and points with large variances will give smaller weights. The WLS estimates therefore become as the following equations:

\[ \hat{\beta}_0 = \bar{y}_w - \beta_1 \bar{x}_w, \]  

(35)

\[ \hat{\beta}_1 = \frac{\sum \omega_i (x_i - \bar{x}_w)(y_i - \bar{y}_w)}{\sum \omega_i (x_i - \bar{x}_w)^2}, \]  

(36)

where \( \bar{x}_w \) and \( \bar{y}_w \) are the weighted means.

3. Results and Discussion

Figures 7–10 show the behavior of the estimation values regarding the true voltage values using EKF, UKF, AEKF, and WLS, respectively, and their estimation errors. It can be seen that, in all the graphs, there is a tracking of the estimate signal towards the real signal.

Figure 7 shows a lower but not negligible performance, all due to the sensitivity of the EKF to sudden state changes. One option is to reduce the impact of the prediction step by increasing the corresponding matrix \( Q_k \).

On the other hand, Figure 8 also shows the outperformance of UKF compared to other observers, in a transient and stable state of the system. After a few first steps, the state vector is predicted quite close to the true value during steady state.

Figure 9 compares the same performance with that of an AEKF whose high-gain value has been set to \( \theta_{\text{max}} \). Figure 9 shows a poor performance after a few first samples, but the estimation improves during the simulation, this is explained by the fact that \( \theta \) increases only when the disturbance has an impact on the innovation. And we can also say that, the convergence speed of the AEKF is comparable to the
convergence of the high-gain observer but with a delay caused by the computation of the innovation. It is clear that AEKF is more sensitive than UKF and EKF to abrupt or sudden state changes. If not selected appropriately, the values of the Holt parameters $a_k$ and $b_k$ can affect the accuracy of the prediction step. The values considered in this article have delivered very good prediction results. Nevertheless, other values could be selected after some offline studies for a particular power grid. Changing the values of any of the parameters mentioned above does not affect the convergence of the observers.

Figure 10 shows an estimation of the voltage by the WLS method. We see that during the first stages of simulations, the method struggles to produce a suitable estimate, but over time, the method adapts to the measurements and provides a fairly good estimate in order to have an estimated value close to the real value.

Figure 11 presents a comparison of the various estimators and their estimation errors where one can easily evaluate the behavior of each one compared to the others.

In Figure 12, we can notice that, the index of performance of the AEKF is rather high; it is because the particularity of the observer AEKF functions like an EKF when the studied system is slightly disturbed and as a high gain when the system is heavily disturbed. Indeed, the behavior of our system being thus random and strongly disturbed, the behavior of the AEKF observer tends much more towards a behavior with a high gain. The performance index of the
AEKF is better than all the other filters because throughout the simulation the AEKF provides a constant and quite good estimate compared to the other filters.

4. Conclusion

In this article, a novel smart distribution grid state estimation method has been presented in order to solve the problems of processing measurement data from smart meters. The proposed model is a hybrid algorithm combining the extended Kalman filter (EKF) and the unscented Kalman filter (UKF) in order to obtain a new hybrid filter named adaptive extended Kalman filter (AEKF). This hybrid filter shows its effectiveness even more when using a large amount of measurement data. An IEEE 37-node network was used to test the performance of the proposed approach. Moreover, from the results obtained after simulation, the hybrid algorithm presents a better performance in
comparison with the single methods such as the extended Kalman filter (EKF), the unscented Kalman filter (UKF), and the weighted least squares (WLS) method. Considering the obtained results in terms of estimation accuracy and performance index, it can be seen that the proposed method outperforms the methods in literature, and it can be applied for all state estimation cases in smart grid applications. The limitation of this works is about the consideration of smart distribution grid changeable topologies. Future work will consist in developing a sequential estimation method that will consider changeable and random topologies as well as data from all Internet of things equipment related to smart distribution networks. Moreover, the future estimator will give us the state of physical sensor installed in distribution network.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare no conflicts of interest regarding this article.

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