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Research Article

TOPSIS Method Based on Entropy Measure for Solving Multiple-Attribute Group Decision-Making Problems with Spherical Fuzzy Soft Information

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A spherical fuzzy soft set (SFSS) is a generalized soft set model, which is more sensible, practical, and exact. Being a very natural generalization, introducing uncertainty measures of SFSSs seems to be very important. In this paper, the concept of entropy, similarity, and distance measures are defined for the SFSSs and also, a characterization of spherical fuzzy soft entropy is proposed. Further, the relationship between entropy and similarity measures as well as entropy and distance measures are discussed in detail. As an application, an algorithm is proposed based on the improved technique for order preference by similarity to an ideal solution (TOPSIS) and the proposed entropy measure of SFSSs, to solve the multiple attribute group decision-making problems. Finally, an illustrative example is used to prove the effectiveness of the recommended algorithm.

1. Introduction

In many real-life situations, individuals have to face different kinds of uncertainty problems, and that is unavoidable. Zadeh's fuzzy set theory [1] is a very effective tool to deal with data having uncertainty or vagueness. Since it is very useful and effective, this theory has started to be used in a variety of scientific fields and also achieved very good acceptance. Many generalizations of fuzzy set theory were developed by researchers such as intuitionistic fuzzy set theory [2], Pythagorean fuzzy set theory [3], picture fuzzy set theory [4], spherical fuzzy set theory [5], and so on. Most of the theories are effectively utilized in a lot of research areas of engineering, business, medicine, natural science, etc.

The soft set theory developed by Molodtsov [6, 7] is considered a general mathematical tool for dealing with uncertainty in a parametric manner. Again, Maji et al. [8] introduced fuzzy soft sets by combining the ideas of fuzzy sets and soft sets. The fuzzy soft set model is a more generalized concept and is successfully used in a lot of decision-making problems. Many researchers are interested in this new concept, and they developed more generalized versions of the fuzzy soft sets such as generalized fuzzy soft sets [9], group generalized fuzzy soft sets [10], intuitionistic fuzzy soft sets [11], Pythagorean fuzzy soft sets [12], picture fuzzy soft sets [13], interval-valued picture fuzzy soft sets [14], spherical fuzzy soft sets [15], etc.

The spherical fuzzy soft sets are a novel concept proposed recently by Perveen et al. [15]. It is a combined version of spherical fuzzy sets and soft sets. Since it is a generalization of all other existing soft set models, it is more realistic and accurate. Therefore, in certain cases, this new extension can be applied more effectively and precisely in decision-making problems, than the other soft set models that are already existing. Thus in the current situation, finding expressions for entropy measures, similarity measures, and distance measures of SFSSs and studying their properties have great importance.

The entropy measure helps find the measure of fuzziness of objects. An object having less entropy will be more precise. Several researchers have conducted more studies on entropy measures, similarity measures, and distance measures. For example, the concept of entropy between fuzzy sets was initiated by Zadeh [1]. Then, De Luca and Termini [16] introduced entropy by making use of no probabilistic concepts for obtaining the measure of uncertainty. Again, using the distance measure between the membership function of fuzzy sets and its nearest crisp set, Kaufmann [17] defined another entropy measure of fuzzy sets. Higashi and Klir [18] also defined it with the help of the distance between fuzzy sets and their complements. Fan et al. [19] studied the relationship between fuzzy entropy and Hamming distance. Liu [20] defined the axiomatic definitions of entropy, similarity measure, and distance measure of fuzzy sets. He also discussed some basic relations between them. The connection between the similarity measure and entropy of intuitionistic fuzzy sets was investigated by Li et al. [21], and also they were given sufficient conditions to transform the entropy to the similarity measure of intuitionistic fuzzy sets and vice versa. In the case of soft sets and fuzzy soft sets, Majumdar and Samanta [22, 23] introduced the uncertainty measures. They presented certain similarity measures of soft sets as well as fuzzy soft sets. Based on fuzzy equivalence, a new category of similarity measures and entropies of fuzzy soft sets were proposed by Liu et al. [24]. Jiang et al. [25] defined a novel distance measure and entropy measure of intuitionistic fuzzy soft sets and proposed an expression to transform the structure of entropy of intuitionistic fuzzy soft sets to the interval-valued fuzzy soft sets. Later, Athira et al. [26] developed a characterization of the entropy of Pythagorean fuzzy soft sets, and also they proposed the expressions for Hamming distance and Euclidean distance of Pythagorean fuzzy soft sets. Also, a similarity measure of SFSSs is developed by Perveen et al. [27] and applied to a medical diagnosis problem. Recently, masum Raj et al. [28] defined cosine similarity, distance, and entropy measures for fuzzy soft matrices.

In modern society, multiple attribute group decision-making (MAGDM) problems are very common, which helps select an optimal solution from a finite set of available alternatives. The technique for order preference by similarity to an ideal solution (TOPSIS) was introduced by Hwang and Yoon [29]. It is a method of multicriteria decision analysis. To solve the problems involving ranking alternatives, the TOPSIS method is very effective. Chen et al. [30] proposed an extended version of the TOPSIS method to solve a supplier selection problem in a fuzzy environment. Since this method is very flexible, the TOPSIS method is combined with different generalizations of fuzzy sets as well as fuzzy soft sets to deal with MAGDM problems, as we can see in [31–33]. In recent times, a MAGDM method has been used by Qi Han et al. [34] based on Pythagorean fuzzy soft entropy and improved TOPSIS method.

In this paper, we introduce and explore the novel concepts of entropy measure of SFSSs. This marks the first instance in the literature where explicit formulas for these concepts are defined and presented. Here, an axiomatic definition of the entropy measure of SFSSs is defined, and the relationship between entropy and similarity measure as well as the relationship between entropy and distance measure are discussed. Our contribution enhances the existing body of knowledge in the realm of spherical fuzzy soft environments by laying the groundwork for comprehending the SFSS entropy measure concept through our newly introduced mathematical formulations This paper is structured as follows: in Section 2, some fundamental definitions regarding soft sets and fuzzy soft sets are recalled in Section 3, the axiomatic definition of the entropy measure of SFSSs is defined, and some properties are discussed. In Section 4, the concept of similarity measure of SFSSs is defined and the relationships between entropy and similarity measure of SFSSs are proposed. Section 5 deals with the relationships between entropy and distance measures of SFSSs. In Section 6, an algorithm based on the MAGDM method is presented based on the improved TOPSIS method and the newly proposed entropy measure as an application and illustrated with a numerical example. In Section 7, to prove the accuracy of the suggested algorithm, a comparison with an existing algorithm is made. Finally, conclusions are drawn in Section 8.

2. Preliminaries

In this section, we include some basic definitions and concepts, which are very helpful for further discussions.All over this paper, denote Z as the universal set and E as the parameter set which is related to the objects in Z and $A \subseteq E$.

Definition 1 [6]. Let $\Upsilon\colon A \longrightarrow P(Z)$ be a mapping from the parameter subset A into $P(Z)$, the power set of Z. Then, the pair $\langle \Upsilon, A\rangle$ is said to be a soft set over Z.

Let $\langle \Upsilon, A\rangle$ and $\langle \Omega, B\rangle$ be two soft sets over Z, where $A, B \subseteq E$. $\langle \Upsilon, A\rangle$ is said to be a soft subset of $\langle \Omega, B\rangle$, if it satisfies the following two conditions:

(1) $A \subseteq B$

(2) $\Upsilon(a) \subseteq \Omega(a)$, $\forall a \in A$.

Definition 2 [8]. Let $\Upsilon\colon A \longrightarrow \mathscr{F}(Z)$ be a mapping, where $\mathscr{F}(Z)$ be the set of all fuzzy subsets over Z. Then, the pair $\langle \Upsilon, A\rangle$ is called a fuzzy soft over Z. That is, for each $a \in A$, $\Upsilon(a)$ is a fuzzy subset over Z.

Definition 3 [5]. An object of the form $\mathscr{S} = \{\zeta, \mu_{\mathscr{S}}(\zeta), \eta_{\mathscr{S}}(\zeta), \vartheta_{\mathscr{S}}(\zeta) \mid \zeta \in Z\}$ is said to be a spherical fuzzy set (SFS), where $\mu_{\mathscr{S}}(\zeta), \eta_{\mathscr{S}}(\zeta), \vartheta_{\mathscr{S}}(\zeta)$ are the functions from Z to [0,1], denote the degree of positive, neutral, and negative membership of $\zeta \in Z$, respectively, with the condition, $\mu^2_{\mathscr{S}}(\zeta) + \eta^2_{\mathscr{S}}(\zeta) + \vartheta^2_{\mathscr{S}}(\zeta) \le 1$, $\forall \zeta \in Z$.

Definition 4 [15]. Let $\mathrm{SFS}(Z)$ be the set of all spherical fuzzy sets over Z. A pair $\langle \Upsilon, A\rangle$ is said to be a spherical fuzzy soft set (SFSS) over Z, where Υ is a mapping given by $\Upsilon\colon A \longrightarrow \mathrm{SFS}(Z)$.

For each $a \in A, \Upsilon(a)$ is a spherical fuzzy set such that

$$\Upsilon(a) = \left\{\left(\zeta, \mu_{\Upsilon(a)}(\zeta), \eta_{\Upsilon(a)}(\zeta), \vartheta_{\Upsilon(a)}(\zeta)\right) \quad \zeta \in Z\right\}, \tag{1}$$

where $\mu_{\Upsilon(a)}(\zeta), \eta_{\Upsilon(a)}(\zeta), \vartheta_{\Upsilon(a)}(\zeta) \in [0,1]$ are the membership degrees which are explained in Definition 3, with the same condition.

Example 1. Let $Z = \{\zeta_1, \zeta_2, \zeta_3\}$ be the set of three motor bikes under consideration and let $A = \{a_1, a_2, a_3, a_4\}$ be the set of parameters, a_1 = Good looking, a_2 = Cheap, a_3 = Quality of parts, and a_4 = Ride quality. Then, the SFSS $\langle \Upsilon, A\rangle$ describes the "user-friendly motor bikes" as given as follows:

$$\langle \Upsilon, A\rangle = \begin{array}{ccccc} & a_1 & a_2 & a_3 & a_4, \\ \zeta_1 & (0.4, 0.1, 0.3) & (0.6, 0.3, 0.1) & (0.7, 0.2, 0.3) & (0.7, 0.4, 0.3), \\ \zeta_2 & (0.5, 0.2, 0.6) & (0.6, 0.2, 0.1) & (0.9, 0.1, 0.0) & (0.6, 0.0, 0.3), \\ \zeta_3 & (0.4, 0.2, 0.2) & (0.3, 0.1, 0.3) & (0.3, 0.2, 0.4) & (0.7, 0.0, 0.3). \end{array} \tag{2}$$

Definition 5 [15]. Let $\langle \Upsilon, A\rangle$ and $\langle \Omega, B\rangle$ be two SFSSs over Z, where $A, B \subseteq E$. $\langle \Upsilon, A\rangle$ is called the spherical fuzzy soft subset of $\langle \Omega, B\rangle$, denoted by $\langle \Upsilon, A\rangle \subseteq \langle \Omega, B\rangle$, if

(1) $A \subseteq B$ and

(2) $\forall a \in A, \Upsilon(a) \subseteq \Omega(a)$

3. Entropy Measure of SFSSs

Entropy measures the uncertainty associated with a system. Less entropy implies less uncertainty. Consequently, an element having less uncertainty or vagueness will carry more stable information. SFSSs being one of the most generalized versions of fuzzy soft sets, introducing entropy measures between SFSSs, are highly essential in both theoretical and practical scenarios. In this section, the definition of the entropy of SFSSs is put forward along with some results and illustrated examples.

Definition 6. A real valued function $\mathbf{E}$: SFSS$(Z) \longrightarrow [0,1]$ is called a spherical fuzzy soft entropy on SFSS(Z) if $\mathbf{E}$ satisfies the following four properties:

(**E**1) $\mathbf{E}(S) = 0$ if and only if S is a crisp soft set.

(**E**2) Let $S = \langle \Upsilon, E\rangle$ be a SFSS. $\mathbf{E}(S) = 1$ if and only if $\mu_{\Upsilon(e)}(\zeta) = \vartheta_{\Upsilon(e)}(\zeta)$, $\forall e \in E$ and $\zeta \in Z$.

(**E**3) $\mathbf{E}(S) = \mathbf{E}(S^c)$, $S \in$ SFSS(Z).

(**E**4) Let $S = \langle \Upsilon, E\rangle$ and $S' = \langle \Omega, E\rangle$ be two SFSSs. $\mathbf{E}(S) \le \mathbf{E}(S')$, if S is less fuzzy than S'. That is, $\mu_{\Upsilon(e)}(\zeta) \le \mu_{\Omega(e)}(\zeta)$, $\eta_{\Upsilon(e)}(\zeta) \le \eta_{\Omega(e)}(\zeta)$ and $\vartheta_{\Upsilon(e)}(\zeta) \ge \vartheta_{\Omega(e)}(\zeta)$ for $\mu_{\Omega(e)}(\zeta) \le \vartheta_{\Omega(e)}(\zeta)$ and $\eta_{\Omega(e)}(\zeta) \le \vartheta_{\Omega(e)}(\zeta)$, or $\mu_{\Upsilon(e)}(\zeta) \ge \mu_{\Omega(e)}(\zeta)$, $\eta_{\Upsilon(e)}(\zeta) \ge \eta_{\Omega(e)}(\zeta)$ and $\vartheta_{\Upsilon(e)}(\zeta) \le \vartheta_{\Omega(e)}(\zeta)$ for $\mu_{\Omega(e)}(\zeta) \ge \vartheta_{\Omega(e)}(\zeta)$ and $\eta_{\Omega(e)}(\zeta) \ge \vartheta_{\Omega(e)}(\zeta)$.

Now, a function Ψ is defined as Ψ: $D \longrightarrow [0,1]$ to get an expression for entropy, where $D = \{(x, y, z) \in [0,1] \times [0,1] \times [0,1]: x^2 + y^2 + z^2 \le 1\}$. Also, Ψ satisfies the conditions given as follows:

(1) $\Psi(x, y, z) = 1$ if and only if $(x, y, z) = (1, 0, 0)$ or $(0, 0, 1)$.

(2) $\Psi(x, y, z) = 0$ if and only if $x = z$.

(3) $\Psi(x, y, z) = \Psi(z, y, x)$.

(4) If $x \le x'$, $y \le y'$ and $z \ge z'$ for $x' \le z'$ and $y' \le z'$, or $x \ge x'$, $y \ge y'$ and $z \le z'$ for $x' \ge z'$ and $y' \ge z'$, then $\Psi(x, y, z) \ge \Psi(x', y', z')$.

Now, Theorem 7 provides an expression to define different spherical fuzzy soft entropies using the function Ψ that was defined above.

Theorem 7. *Let* $\mathbf{E}$ *be a function defined from SFSS(Z) to [0,1] and let* $S = \langle \Upsilon, E\rangle$ *be a SFSS. If*

$$E(S) = \frac{1}{mn}\sum_{j=1}^{n}\sum_{i=1}^{m}\left(1 - \Psi\left(\mu_{\Upsilon(e_j)}(\zeta_i), \eta_{\Upsilon(e_j)}(\zeta_i), \vartheta_{\Upsilon(e_j)}(\zeta_i)\right)\right), \tag{3}$$

where Ψ *is the function that satisfies conditions (1) to (4) that were defined above, then* $\mathbf{E}$ *is an entropy measure of SFSS.*

Proof

(1) $\mathbf{E}(S) = 0$

$\Leftrightarrow \sum_{j=1}^{n}\sum_{i=1}^{m}(1 - \Psi(\mu_{\Upsilon(e_j)}(\zeta_i), \eta_{\Upsilon(e_j)}(\zeta_i), \vartheta_{\Upsilon(e_j)}(\zeta_i))) = 0$

$\Leftrightarrow \Psi(\mu_{\Upsilon(e_j)}(\zeta_i), \eta_{\Upsilon(e_j)}(\zeta_i), \vartheta_{\Upsilon(e_j)}(\zeta_i)) = 1$

$\Leftrightarrow$ either $\mu_{\Upsilon(e_j)}(\zeta_i) = 1$, $\eta_{\Upsilon(e_j)}(\zeta_i) = 0$, $\vartheta_{\Upsilon(e_j)}(\zeta_i) = 0$ or $\mu_{\Upsilon(e_j)}(\zeta_i) = 0$, $\eta_{\Upsilon(e_j)}(\zeta_i) = 0$, $\vartheta_{\Upsilon(e_j)}(\zeta_i) = 1$, $\forall e_j \in E$ and $\zeta_i \in Z$.

$\Leftrightarrow S$ is a crisp soft set.

(2) $\mathbf{E}(S) = 1$

$\Leftrightarrow \sum_{j=1}^{n}\sum_{i=1}^{m}(1 - \Psi(\mu_{\Upsilon(e_j)}(\zeta_i), \eta_{\Upsilon(e_j)}(\zeta_i), \vartheta_{\Upsilon(e_j)}(\zeta_i))) = mn$

$\Leftrightarrow \Psi(\mu_{\Upsilon(e_j)}(\zeta_i), \eta_{\Upsilon(e_j)}(\zeta_i), \vartheta_{\Upsilon(e_j)}(\zeta_i)) = 0, \quad \forall e_j \in E$ and $\zeta_i \in Z$

$\Leftrightarrow \mu_{\Upsilon(e_j)}(\zeta_i) = \vartheta_{\Upsilon(e_j)}(\zeta_i)$, $\forall e_j \in E$ and $\zeta_i \in Z$.

(3) $\mathbf{E}(S) = 1/mn\sum_{j=1}^{n}\sum_{i=1}^{m}(1 - \Psi(\mu_{\Upsilon} \ (e_j)(\zeta_i), \eta_{\Upsilon(e_j)}(\zeta_i), \vartheta_{\Upsilon(e_j)}(\zeta_i)))$ and

$\mathbf{E}(S^c) = 1/mn\sum_{j=1}^{n}\sum_{i=1}^{m}(1 - \Psi(\vartheta_{\Upsilon(e_j)}(\zeta_i), \eta_{\Upsilon(e_j)}(\zeta_i), \mu_{\Upsilon(e_j)}(\zeta_i)))$

Since, $\Psi(\mu_{\Upsilon(e_j)}(\zeta_i), \eta_{\Upsilon(e_j)}(\zeta_i), \vartheta_{\Upsilon(e_j)}(\zeta_i)) = \Psi(\vartheta_{\Upsilon(e_j)}(\zeta_i), \eta_{\Upsilon(e_j)}(\zeta_i), \mu_{\Upsilon(e_j)}(\zeta_i))$, $\mathbf{E}(S) = \mathbf{E}(S^c)$

(4) If $\mu_{\Upsilon(e_j)}(\zeta_i) \le \mu_{\Omega(e_j)}(\zeta_i)$, $\eta_{\Upsilon(e_j)}(\zeta_i) \le \eta_{\Omega(e_j)}(\zeta_i)$ and $\vartheta_{\Upsilon(e_j)}(\zeta_i) \ge \vartheta_{\Omega(e_j)}(\zeta_i)$ for $\mu_{\Omega(e_j)}(\zeta_i) \le \vartheta_{\Omega(e_j)}(\zeta_i)$ and $\eta_{\Omega(e_j)}(\zeta_i) \le \vartheta_{\Omega(e_j)}(\zeta_i)$.

That is, $\mu_{\Upsilon(e_j)}(\zeta_i) \le \mu_{\Omega(e_j)}(\zeta_i) \le \vartheta_{\Omega(e_j)}(\zeta_i) \le \vartheta_{\Upsilon(e_j)}(\zeta_i)$ and $\eta_{\Upsilon(e_j)}(\zeta_i) \le \eta_{\Omega(e_j)}(\zeta_i) \le \vartheta_{\Omega(e_j)}(\zeta_i) \le \vartheta_{\Upsilon(e_j)}(\zeta_i)$.

Thus, $\Psi(\mu_{\Upsilon(e_j)}(\zeta_i), \eta_{\Upsilon(e_j)}(\zeta_i), \vartheta_{\Upsilon(e_j)}(\zeta_i)) \ge \Psi(\mu_{\Omega(e_j)}(\zeta_i), \eta_{\Omega(e_j)}(\zeta_i), \vartheta_{\Omega(e_j)}(\zeta_i)) \Rightarrow 1\text{-}\Psi(\mu_{\Upsilon(e_j)}(\zeta_i), \eta_{\Upsilon(e_j)}(\zeta_i), \vartheta_{\Upsilon(e_j)}(\zeta_i)) \le 1\text{-}\Psi(\mu_{\Omega(e_j)}(\zeta_i), \eta_{\Omega(e_j)}(\zeta_i), \vartheta_{\Omega(e_j)}(\zeta_i)), \Rightarrow \mathbf{E}(S) \le \mathbf{E}(S')$

The proof is similar if $\mu_{\Upsilon(e_j)}(\zeta_i) \le \mu_{\Omega(e_j)}(\zeta_i)$, $\eta_{\Upsilon(e_j)}(\zeta_i) \le \eta_{\Omega(e_j)}(\zeta_i)$ and $\vartheta_{\Upsilon(e_j)}(\zeta_i) \ge \vartheta_{\Omega(e_j)}(\zeta_i)$ for $\mu_{\Omega(e_j)}(\zeta_i) \le \vartheta_{\Omega(e_j)}(\zeta_i)$ and $\eta_{\Omega(e_j)}(\zeta_i) \le \vartheta_{\Omega(e_j)}(\zeta_i)$

Therefore, $\mathbf{E}$ is an entropy measure of SFSS. □

Example 2. Consider,

$$\mathbf{E}^1(S) = \frac{1}{n}\sum_{j=1}^{n}\epsilon(\Upsilon, e_j), \tag{4}$$

where $\epsilon(\Upsilon, e_j)$ is the spherical fuzzy entropy given by

$$\epsilon(\Upsilon, e_j) = \begin{cases} \dfrac{1}{m}\displaystyle\sum_{i=1}^{m} 1 - \left[\left(1 - \eta^2_{\Upsilon(e_j)}(\zeta_i)\right)\left|\mu^2_{\Upsilon(e_j)}(\zeta_i) - \vartheta^2_{\Upsilon(e_j)}(\zeta_i)\right|\right], & \eta_{\Upsilon(e_j)}(\zeta_i)) \ne 1, \\ \dfrac{1}{m}\displaystyle\sum_{i=1}^{m} 1 - \left[\left(\eta^2_{\Upsilon(e_j)}(\zeta_i)\right)\left|\mu^2_{\Upsilon(e_j)}(\zeta_i) - \vartheta^2_{\Upsilon(e_j)}(\zeta_i)\right|\right], & \eta_{\Upsilon(e_j)}(\zeta_i)) = 1. \end{cases} \tag{5}$$

That is,

$$\mathbf{E}^1(S) = \begin{cases} \dfrac{1}{mn}\displaystyle\sum_{j=1}^{n}\sum_{i=1}^{m} 1 - \left[\left(1 - \eta^2_{\Upsilon(e_j)}(\zeta_i)\right)\left|\mu^2_{\Upsilon(e_j)}(\zeta_i) - \vartheta^2_{\Upsilon(e_j)}(\zeta_i)\right|\right], & \eta_{\Upsilon(e_j)}(\zeta_i)) \ne 1, \\ \dfrac{1}{mn}\displaystyle\sum_{j=1}^{n}\sum_{i=1}^{m} 1 - \left[\left(\eta^2_{\Upsilon(e_j)}(\zeta_i)\right)\left|\mu^2_{\Upsilon(e_j)}(\zeta_i) - \vartheta^2_{\Upsilon(e_j)}(\zeta_i)\right|\right], & \eta_{\Upsilon(e_j)}(\zeta_i)) = 1. \end{cases} \tag{6}$$

To prove $\mathbf{E}^1(S)$, defined above is an entropy measure of SFSS, it is enough to prove that

$$\Psi\left(\mu_{\Upsilon(e)}(\zeta), \eta_{\Upsilon(e)}(\zeta), \vartheta_{\Upsilon(e)}(\zeta)\right) = \begin{cases} \left(1 - \eta^2_{\Upsilon(e)}(\zeta)\right)\left|\mu^2_{\Upsilon(e)}(\zeta) - \vartheta^2_{\Upsilon(e)}(\zeta)\right|, & \eta_{\Upsilon(e)}(\zeta)) \ne 1, \\ \left(\eta^2_{\Upsilon(e)}(\zeta)\right)\left|\mu^2_{\Upsilon(e)}(\zeta) - \vartheta^2_{\Upsilon(e)}(\zeta)\right|, & \eta_{\Upsilon(e)}(\zeta)) = 1, \end{cases} \tag{7}$$

is a function from $D = \{(\mu_{\Upsilon(e)}(\zeta), \eta_{\Upsilon(e)}(\zeta), \vartheta_{\Upsilon(e)}(\zeta)) \in [0,1] \times [0,1] \times [0,1]: \mu^2_{\Upsilon(e)}(\zeta) + \eta^2_{\Upsilon(e)}(\zeta) + \vartheta^2_{\Upsilon(e)}(\zeta) \le 1\}$ to $[0, 1]$, satisfy the four conditions of Ψ defined above.

Consider

Case 8. If $\eta_{\Upsilon(e)}(\zeta) \ne 1$

(1) $\Psi(\mu_{\Upsilon(e)}(\zeta), \eta_{\Upsilon(e)}(\zeta), \vartheta_{\Upsilon(e)}(\zeta)) = 1$

$\Leftrightarrow (1 - \eta^2_{F(e)}(\zeta))|\mu^2_{F(e)}(\zeta) - \vartheta^2_{F(e)}(\zeta)| = 1$

$\Leftrightarrow \mu_{\Upsilon(e)}(\zeta) = 1, \eta_{\Upsilon(e)}(\zeta) = 0, \vartheta_{\Upsilon(e)}(\zeta) = 0$ or $\mu_{\Upsilon(e)}(\zeta) = 0, \eta_{\Upsilon(e)}(\zeta) = 0, \vartheta_{\Upsilon(e)}(\zeta) = 1$

(2) $\Psi(\mu_{\Upsilon(e)}(\zeta), \eta_{\Upsilon(e)}(\zeta), \vartheta_{\Upsilon(e)}(\zeta)) = 0$

$\Leftrightarrow (1 - \eta^2_{\Upsilon(e)}(\zeta))|\mu^2_{\Upsilon(e)}(\zeta) - \vartheta^2_{\Upsilon(e)}(\zeta)| = 0$

$\Leftrightarrow \mu^2_{\Upsilon(e)}(\zeta) = \vartheta^2_{\Upsilon(e)}(\zeta)$, because $\eta_{\Upsilon(e)}(\zeta) \ne 1$

$\Leftrightarrow \mu_{\Upsilon(e)}(\zeta) = \vartheta_{\Upsilon(e)}(\zeta)$

(3) $\Psi(\mu_{\Upsilon(e)}(\zeta), \eta_{\Upsilon(e)}(\zeta), \vartheta_{\Upsilon(e)}(\zeta)) = (1 - \eta^2_{\Upsilon(e)}(\zeta))|\mu^2_{\Upsilon(e)}(\zeta) - \vartheta^2_{\Upsilon(e)}(\zeta)|$

$= (1 - \eta^2_{\Upsilon(e)}(\zeta))|\vartheta^2_{\Upsilon(e)}(\zeta) - \mu^2_{\Upsilon(e)}(\zeta)|$

$= \Psi(\vartheta_{\Upsilon(e)}(\zeta), \eta_{\Upsilon(e)}(\zeta), \mu_{\Upsilon(e)}(\zeta))$

(4) Let $S = \langle\Upsilon, E\rangle$, $S' = \langle\Omega, E\rangle$

If $\mu_{\Upsilon(e)}(\zeta) \le \mu_{\Omega(e)}(\zeta)$, $\eta_{\Upsilon(e)}(\zeta) \le \eta_{\Omega(e)}(\zeta)$ and $\vartheta_{\Upsilon(e)}(\zeta) \ge \vartheta_{\Omega(e)}(\zeta)$ for $\mu_{\Omega(e)}(\zeta) \le \vartheta_{\Omega(e)}(\zeta)$ and $\eta_{\Omega(e)}(\zeta) \le \vartheta_{\Omega(e)}(\zeta)$

That is, $\mu_{\Upsilon(e)}(\zeta) \le \mu_{\Omega(e)}(\zeta) \le \vartheta_{\Omega(e)}(\zeta) \le \vartheta_{\Upsilon(e)}(\zeta)$ and

$\eta_{\Upsilon(e)}(\zeta) \le \eta_{\Omega(e)}(\zeta) \le \vartheta_{\Omega}(e)(\zeta) \le \vartheta_{\Upsilon(e)}(\zeta)$

$\Rightarrow |\mu^2_{\Upsilon(e)}(\zeta) - \vartheta^2_{\Upsilon(e)}(\zeta)| \ge |\mu^2_{\Omega(e)}(\zeta) - \vartheta^2_{\Omega(e)}(\zeta)|$ and

$\eta_{\Upsilon(e)}(\zeta) \le \eta_{\Omega(e)}(\zeta) \Rightarrow \eta^2_{\Upsilon(e)}(\zeta) \le \eta^2_{\Omega(e)}(\zeta) \Rightarrow 1 - \eta^2_{\Upsilon(e)}(\zeta) \ge 1 - \eta^2_{\Omega(e)}(\zeta)$

$\Rightarrow (1 - \eta^2_{\Upsilon(e)}(\zeta))|\mu^2_{\Upsilon(e)}(\zeta) - \vartheta^2_{\Upsilon(e)}(\zeta)| \ge (1 - \eta^2_{\Omega(e)}(\zeta))\,|\mu^2_{\Omega(e)}(\zeta) - \vartheta^2_{\Omega(e)}(\zeta)|$

$\Rightarrow \Psi(\mu_{\Upsilon(e)}(\zeta), \eta_{\Upsilon(e)}(\zeta), \vartheta_{\Upsilon(e)}(\zeta)) \ge \Psi(\mu_{\Omega(e)}(\zeta), \eta_{\Omega(e)}(\zeta), \vartheta_{\Omega(e)}(\zeta))$

Case 9. If $\eta_{\Upsilon(e)}(\zeta) = 1$, proof is direct.

Example 3. Consider the SFSSs $S_1 = \langle \Upsilon, E\rangle$ and $S_2 = \langle \Omega, E\rangle$ over the universe $Z = \{\zeta_1, \zeta_2\}$ with parameter set $E = \{e_1, e_2\}$ is given as follows:

$$\begin{array}{llll} & & e_1 & e_2 \\ S_1 = \langle \Upsilon, E\rangle = & \zeta_1 & (0.5, 0.2, 0.3) & (0.7, 0.3, 0.3) \\ & \zeta_2 & (0.6, 0.4, 0.3) & (0.5, 0.2, 0.1), \\ & & e_1 & e_2 \\ S_2 = \langle \Omega, E\rangle = & \zeta_1 & (0.8, 0.1, 0.1) & (0.2, 0.1, 0.9) \\ & \zeta_2 & (0.7, 0.1, 0.2) & (0.9, 0.1, 0.1), \end{array} \tag{8}$$

Then, we get $\mathbf{E}^1(S_1) = 0.7563$ and $\mathbf{E}^1(S_2) = 0.3441$

By examining these two examples, the soundness of our proposed entropy measure for SFSS becomes evident. Specifically, when analyzing the SFSSs S_1 and S_2, it becomes apparent that S_1 exhibits a greater degree of fuzziness compared to S_2. This distinction arises from the fact that the disparity between the positive and negative membership degrees in S_1 is less pronounced than that observed in S_2.

4. Entropy and Similarity Measure of SFSSs

A similarity measure is an essential tool for measuring similarities between two or more objects. Objects having high similarity measures are very close to each other. Thus, they have almost similar characteristics. In this section, we define the similarity measure between SFSSs and propose the relationships between entropy and similarity measures of SFSSs.

Definition 10. A mapping $\mathbf{S}$: $\mathrm{SFSS}(Z) \times \mathrm{SFSS}(Z) \longrightarrow [0,1]$ is said to be a similarity measure between spherical fuzzy soft sets $S_1 = \langle \Upsilon, E\rangle$ and $S_2 = \langle \Omega, E\rangle$, denoted by $\mathbf{S}(S_1, S_2)$, if it satisfies the conditions listed as follows:

(**S1**) $\mathbf{S}(S_1, S_1^c) = 0$ if and only if S_1 is a crisp soft set.

(**S2**) $\mathbf{S}(S_1, S_2) = \mathbf{S}(S_2, S_1)$

(**S3**) $\mathbf{S}(S_1, S_2) = 1$ if and only if $S_1 = S_2$;

(**S4**) Let $S_3 = \langle \Pi, E\rangle$ be a spherical fuzzy soft set, if $S_1 \subseteq S_2 \subseteq S_3$, then $\mathbf{S}(S_1, S_3) \le \mathbf{S}(S_1, S_2)$ and $\mathbf{S}(S_1, S_3) \le \mathbf{S}(S_2, S_3)$.

Example 4. Let $Z = \{\zeta_1, \zeta_2, \ldots, \zeta_m\}$ be the universal set with $E = \{e_1, e_2, \ldots, e_n\}$ be the set of parameters. Let $S_1 = \langle \Upsilon, E\rangle$ and $S_2 = \langle \Omega, E\rangle$ be two SFSSs. Then,

$$\mathbf{S}^1(S_1, S_2) = 1 - \frac{1}{mn}\sum_{j=1}^{n}\sum_{i=1}^{m}\max\left\{\left|\mu^2_{\Upsilon(e_j)}(\zeta_i) - \mu^2_{\Omega(e_j)}(\zeta_i)\right|, \left|\eta^2_{\Upsilon(e_j)}(\zeta_i) - \eta^2_{\Omega(e_j)}(\zeta_i)\right|, \left|\vartheta^2_{\Upsilon(e_j)}(\zeta_i) - \vartheta^2_{\Omega(e_j)}(\zeta_i)\right|\right\}, \tag{9}$$

is a similarity measure of SFSS.

Example 5. Consider the SFSSs $S_1 = \langle \Upsilon, E\rangle$ and $S_2 = \langle \Omega, E\rangle$ in Example 3. Then, we get $\mathbf{S}^1(S_1, S_2) = 0.545$

4.1. Relationship between Entropy and Similarity Measures of SFSSs. In this subsection, we present a method for transforming the entropy measure into a similarity measure. This method holds significance in the realm of spherical fuzzy soft sets (SFSS) as it facilitates the derivation of novel similarity measures. The importance lies in the potential enhancement of our ability to discern and analyze relationships within SFSSs. By obtaining new similarity measures, we gain a more nuanced understanding of the structural nuances and interdependencies within complex data sets characterized by fuzzy and soft attributes. This, in turn, contributes to the refinement of decision-making processes and pattern recognition in fields where SFSSs find application, ultimately advancing the utility and effectiveness of spherical fuzzy soft set theory in diverse domains.

4.1.1. Method to Transform the Entropy Measure into the Similarity Measure of SFSSs. Proposing a method to transform the entropy measure into a similarity measure for SFSSs is pivotal in refining our understanding of relationships within fuzzy soft sets. This method not only establishes novel similarity measures but also contributes to the broader utility of SFSS theory in diverse analytical and decision-making contexts.

Based on the definitions of entropy measure and similarity measure of SFSSs, the following methods are proposed to transform an entropy measure into a similarity measure between SFSSs.

Let $\langle \Upsilon, E\rangle$, $\langle \Omega, E\rangle \in \mathrm{SFSS}(Z)$, define another SFSS denoted by $\varphi(\Upsilon\Omega, E)\forall \zeta \in Z, e \in E$ as follows:

$$\mu_{\varphi(\Upsilon\Omega,E)(e)}(\zeta)=\sqrt{\frac{1-\left[\left|\mu^2_{\Upsilon(e)}(\zeta)-\mu^2_{\Omega(e)}(\zeta)\right|\vee\left|\eta^2_{\Upsilon(e)}(\zeta)-\eta^2_{\Omega(e)}(\zeta)\right|\vee\left|\vartheta^2_{\Upsilon(e)}(\zeta)-\vartheta^2_{\Omega(e)}(\zeta)\right|\right]^2}{3}},$$
$$\eta_{\varphi(\Upsilon\Omega,E)(e)}(\zeta)=\sqrt{\frac{1-\left[\left|\mu^2_{\Upsilon(e)}(\zeta)-\mu^2_{\Omega(e)}(\zeta)\right|\vee\left|\eta^2_{\Upsilon(e)}(\zeta)-\eta^2_{\Omega(e)}(\zeta)\right|\vee\left|\vartheta^2_{\Upsilon(e)}(\zeta)-\vartheta^2_{\Omega(e)}(\zeta)\right|\right]}{3}}, \tag{10}$$
$$\vartheta_{\varphi(\Upsilon\Omega,E)(e)}(\zeta)=\sqrt{\frac{1+\left[\left|\mu^2_{\Upsilon(e)}(\zeta)-\mu^2_{\Omega(e)}(\zeta)\right|\vee\left|\eta^2_{\Upsilon(e)}(\zeta)-\eta^2_{\Omega(e)}(\zeta)\right|\vee\left|\vartheta^2_{\Upsilon(e)}(\zeta)-\vartheta^2_{\Omega(e)}(\zeta)\right|\right]}{3}}.$$

Theorem 11. *Let* $\mathbf{E}$ *be an entropy measure of SFSSs, then* $\mathbf{E}_\varphi$: $SFSS(Z)\times SFSS(Z)\longrightarrow[0,1]$, $(\langle\Upsilon,E\rangle,\langle\Omega,E\rangle)\longrightarrow\mathbf{E}_\varphi(\langle\Upsilon,E\rangle,\langle\Omega,E\rangle)=\mathbf{E}(\varphi(\Upsilon\Omega,E))$ *is a similarity measure of SFSSs.*

Proof. Here, $\mathbf{E}_\varphi(\langle\Upsilon,E\rangle,\langle\Omega,E\rangle)$ satisfies the four conditions (**S**1–**S**4) as follows:

(**S**1) Proof is direct.

(**S**2) Proof is direct.

(**S**3) $\mathbf{E}_\varphi(\langle\Upsilon,E\rangle,\langle\Omega,E\rangle)=1$
$\Leftrightarrow\mathbf{E}(\varphi(\Upsilon\Omega,E))=1\Leftrightarrow\mu_{\varphi(\Upsilon\Omega,E)(e)}(\zeta)=\vartheta_{\varphi(\Upsilon\Omega,E)(e)}(\zeta)$
$\Leftrightarrow|\mu^2_{\Upsilon(e)}(\zeta)-\mu^2_{\Omega(e)}(\zeta)|\vee|\eta^2_{\Upsilon(e)}(\zeta)-\eta^2_{\Omega(e)}(\zeta)|\vee|\vartheta^2_{\Upsilon(e)}(\zeta)-\vartheta^2_{\Omega(e)}(\zeta)|=0$
$\Leftrightarrow\mu^2_{\Upsilon(e)}(\zeta)=\mu^2_{\Omega(e)}(\zeta),\eta^2_{\Upsilon(e)}(\zeta)=\eta^2_{\Omega(e)}(\zeta)$ and $\vartheta^2_{\Upsilon(e)}(\zeta)=\vartheta^2_{\Omega(e)}(\zeta)$
$\Leftrightarrow\mu_{\Upsilon(e)}(\zeta)=\mu_{\Omega(e)}(\zeta),\eta_{\Upsilon(e)}(\zeta)=\eta_{\Omega(e)}(\zeta)$ and $\vartheta_{\Upsilon(e)}(\zeta)=\vartheta_{\Omega(e)}(\zeta)\forall\zeta\in Z,e\in E$
$\Leftrightarrow\langle\Upsilon,E\rangle=\langle\Omega,E\rangle$

(**S**4) If $\langle\Upsilon,E\rangle\subseteq\langle\Omega,E\rangle\subseteq\langle\Pi,E\rangle$, then $\forall\zeta\in Z$ and $e\in E$, $\mu_{\Upsilon(e)}(\zeta)\le\mu_{\Omega(e)}(\zeta)\le\mu_{\Pi(e)}(\zeta)$, $\eta_{\Upsilon(e)}(\zeta)\le\eta_{\Omega(e)}(\zeta)\le\eta_{\Pi}(e)(\zeta)$ and $\vartheta_{\Upsilon(e)}(\zeta)\ge\vartheta_{\Omega(e)}(\zeta)\ge\vartheta_{\Pi(e)}(\zeta)$
$\Rightarrow|\mu^2_{\Upsilon(e)}(\zeta)-\mu^2_{\Omega(e)}(\zeta)|\le|\mu^2_{\Upsilon(e)}(\zeta)-\mu^2_{\Pi(e)}(\zeta)|$, $|\eta^2_{\Upsilon(e)}(\zeta)-\eta^2_{\Omega(e)}(\zeta)|\le|\eta^2_{\Upsilon(e)}(\zeta)-\eta^2_{\Pi(e)}(\zeta)|$ and $|\vartheta^2_{\Upsilon(e)}(\zeta)-\vartheta^2_{\Omega(e)}(\zeta)|\le|\vartheta^2_{\Upsilon(e)}(\zeta)-\vartheta^2_{\Pi(e)}(\zeta)|$
$\Rightarrow\mu_{\varphi(\Upsilon\Pi,E)}(\zeta)\le\mu_{\varphi(\Upsilon\Omega,E)}(\zeta)\le1/\sqrt{3}\le\vartheta_{\varphi(\Upsilon\Omega,E)}(\zeta)\le\vartheta_{\varphi(\Upsilon\Pi,E)}(\zeta)$ and
$\eta_{\varphi(\Upsilon\Pi,E)}(\zeta)\le\eta_{\varphi(\Upsilon\Omega,E)}(\zeta)\le1/\sqrt{3}\le\vartheta_{\varphi(\Upsilon\Omega,E)}(\zeta)\le\vartheta_{\varphi(\Upsilon\Pi,E)}(\zeta)$
$\Rightarrow\varphi(\Omega\Pi,E)$ is less fuzzy than $\varphi(\Upsilon\Omega,E)$
$\mathbf{E}_\varphi(\langle\Upsilon,E\rangle,\langle\Pi,E\rangle)\le\mathbf{E}_\varphi(\langle\Upsilon,E\rangle,\langle\Omega,E\rangle)$

Therefore, $\mathbf{E}_\varphi(\langle\Upsilon,E\rangle,\langle\Omega,E\rangle)$ is a similarity measure of SFSSs. □

Example 6. Consider $\langle\Upsilon,E\rangle,\langle\Omega,E\rangle\in SFSS(Z)$, and $\mathbf{E}^1$ is the entropy measure defined as equation (4), assume that $\eta_{\varphi(\Upsilon\Omega,E)(e)}(\zeta)\neq1$, then

$$\mathbf{E}^1_\varphi(\langle\Upsilon,E\rangle,\langle\Omega,E\rangle)=\mathbf{E}^1(\varphi(\Upsilon\Omega,E))=\frac{1}{mn}\sum_{j=1}^{n}\sum_{i=1}^{m}\left[1-\frac{1}{3}(1+X)^2(X)\right], \tag{11}$$

where $X=|\mu^2_{\Upsilon(e_j)}(\zeta_i)-\mu^2_{\Omega(e_j)}(\zeta_i)|\vee|\eta^2_{\Upsilon(e_j)}(\zeta_i)-\eta^2_{\Omega(e_j)}(\zeta_i)|\vee|\vartheta^2_{\Upsilon(e_j)}(\zeta_i)-\vartheta^2_{\Omega(e_j)}(\zeta_i)|$ is a similarity measure between SFSSs $\langle\Upsilon,E\rangle$ and $\langle\Omega,E\rangle$.

Example 7. Consider the SFSSs $\langle\Upsilon,E\rangle$ and $\langle\Omega,E\rangle$ in Example 3. Then, we get $\mathbf{E}^1_\varphi(\langle\Upsilon,E\rangle,\langle\Omega,E\rangle)=0.6295$

Corollary 12. *Let* $\mathbf{E}$ *be an entropy measure of SFSSs, then* $\mathbf{E}^c_\varphi$: $SFSS(Z)\times SFSS(Z)\longrightarrow[0,1]$, $(\langle\Upsilon,E\rangle,\langle\Omega,E\rangle)\longrightarrow\mathbf{E}^c_\varphi(\langle\Upsilon,E\rangle,\langle\Omega,E\rangle)=\mathbf{E}((\varphi(\Upsilon\Omega,E))^c)$ *is a similarity measure of SFSSs.*

5. Entropy and Distance Measure of SFSSs

We can define distance measures from the entropy measure, a crucial aspect in the context of spherical fuzzy soft sets (SFSS). This enhancement in our ability to accurately quantify dissimilarity holds particular significance in decision-making and pattern recognition. In many cases, similarity measures are inherently connected to distance measures, underscoring the importance of introducing distance measures between SFSSs. In this regard, we not only define distance measures for SFSSs but also delve into the relationships between entropy measures and distance measures. This comprehensive exploration is complemented by illustrative examples, shedding light on how these measures interplay in practical scenarios. The resulting insights contribute to a more nuanced understanding of the structural variations within SFSSs, thereby fortifying the applicability and versatility of spherical fuzzy soft set theory across diverse domains, including data analysis and decision support systems.

Definition 13. A mapping $\mathbf{d}$: $\mathrm{SFSS}(Z)\times\mathrm{SFSS}(Z)\longrightarrow[0,1]$ is said to be a distance measure between spherical fuzzy soft sets $S_1 = \langle \Upsilon, E\rangle$ and $S_2 = \langle \Omega, E\rangle$, denoted by $\mathbf{d}(S_1, S_2)$, if it satisfies the conditions listed as follows:

($\mathbf{d}$1) $\mathbf{d}(S_1, S_1^c) = 1$ if and only if S_1 is a crisp soft set.

($\mathbf{d}$2) $\mathbf{d}(S_1, S_2) = \mathbf{d}(S_2, S_1)$

($\mathbf{d}$3) $\mathbf{d}(S_1, S_2) = 0$ if and only if $S_1 = S_2$;

($\mathbf{d}$4) Let $S_3 = \langle \Pi, E\rangle$ be a spherical fuzzy soft set, if $S_1 \subseteq S_2 \subseteq S_3$, then $\mathbf{d}(S_1, S_3) \geq \mathbf{d}(S_1, S_2)$ and $\mathbf{d}(S_1, S_3) \geq \mathbf{d}(S_2, S_3)$.

Example 8. Consider two SFSSs $S_1 = \langle \Upsilon, E\rangle$ and $S_2 = \langle \Omega, E\rangle$ defined over the universal set $Z = \{\zeta_1, \zeta_2, \ldots, \zeta_m\}$ with the parameter set $E = \{e_1, e_2, \ldots, e_n\}$, then the normalized Hamming distance and the normalized Euclidean distance between SFSSs S_1 and S_2 denoted by $\mathbf{l}(S_1, S_2)$ and $\mathbf{q}(S_1, S_2)$, respectively, are defined as follows:

$$\mathbf{l}(S_1, S_2) = \frac{1}{2mn}\sum_{j=1}^{n}\sum_{i=1}^{m}\left[\mu^2_{\Upsilon(e_j)}(\zeta_i) - \mu^2_{\Omega(e_j)}(\zeta_i) + \left|\eta^2_{\Upsilon(e_j)}(\zeta_i) - \eta^2_{\Omega(e_j)}(\zeta_i)\right| + \left|\vartheta^2_{\Upsilon(e_j)}(\zeta_i) - \vartheta^2_{\Omega(e_j)}(\zeta_i)\right|\right],$$

$$\mathbf{q}(S_1, S_2) = \sqrt{\frac{1}{2mn}\sum_{j=1}^{n}\sum_{i=1}^{m}\left[\left(\mu^2_{\Upsilon(e_j)}(\zeta_i) - \mu^2_{\Omega(e_j)}(\zeta_i)\right)^2 + \left(\eta^2_{\Upsilon(e_j)}(\zeta_i) - \eta^2_{\Omega(e_j)}(\zeta_i)\right)^2 + \left(\vartheta^2_{\Upsilon(e_j)}(\zeta_i) - \vartheta^2_{\Omega(e_j)}(\zeta_i)\right)^2\right]}. \tag{12}$$

Example 9. Consider the SFSSs $S_1 = \langle \Upsilon, E\rangle$ and $S_2 = \langle \Omega, E\rangle$ in Example 3. Then, we get $\mathbf{l}(S_1, S_2) = 0.3337$ and $\mathbf{q}(S_1, S_2) = 0.3942$.

5.1. Relationship between Entropy and Distance Measures of SFSSs. Let $\langle \Upsilon, E\rangle$ and $\langle \Omega, E\rangle \in$ SFSS(Z). By considering the SFSSs $\varphi(\Upsilon\Omega, E)$, $\langle P_\Upsilon, E\rangle$, $\langle Q_\Upsilon, E\rangle$, already defined in Section 4, here some methods are proposed to transform entropy measure into distance measure and vice versa.

5.1.1. Method to Transform the Entropy Measure into the Distance Measure of SFSSs. Introducing a method to transform the entropy measure into the distance measure of SFSSs holds significant implications for enhancing dissimilarity quantification within fuzzy soft sets. This approach not only defines distance measures for SFSSs but also elucidates their relationships with entropy measures, providing valuable insights into the structural nuances of these sets.

Theorem 14. *Let* $\mathbf{E}$ *be an entropy measure of SFSSs, then* $\mathbf{E}_\varphi : SFSS(Z) \times SFSS(Z) \longrightarrow [0,1]$, $(\langle \Upsilon, E\rangle, \langle \Omega, E\rangle) \longrightarrow \mathbf{E}_\varphi(\langle \Upsilon, E\rangle, \langle \Omega, E\rangle) = 1\text{-}\mathbf{E}(\varphi(\Upsilon\Omega, E))$ *is a distance measure of SFSSs.*

Proof. The proof is similar to the proof of Theorem 11. □

Example 10. Consider $\langle \Upsilon, E\rangle, \langle \Omega, E\rangle \in$ SFSS(Z), and $\mathbf{E}^1$ is the entropy measure defined as equation (4), assume that $\eta_{\varphi(\Upsilon\Omega,E)(e)}(\zeta) \neq 1$, then

$$\acute{\mathbf{E}}'^{1}_{\varphi}(\langle \Upsilon, E\rangle, \langle \Omega, E\rangle) = 1 - \mathbf{E}^1(\varphi(\Upsilon\Omega, E)) = 1 - \frac{1}{mn}\sum_{j=1}^{n}\sum_{i=1}^{m}\left[1 - \frac{1}{3}(1+X)^2(X)\right], \tag{13}$$

where $X = |\mu^2_{\Upsilon(e_j)}(\zeta_i) - \mu^2_{\Omega(e_j)}(\zeta_i)| \vee |\eta^2_{\Upsilon(e_j)}(\zeta_i) - \eta^2_{\Omega(e_j)}(\zeta_i)| \vee |\vartheta^2_{\Upsilon(e_j)}(\zeta_i) - \vartheta^2_{\Omega(e_j)}(\zeta_i)|$, is a distance measure between SFSSS $\langle \Upsilon, E\rangle$ and $\langle \Omega, E\rangle$.

Example 11. Consider the SFSSs $\langle \Upsilon, E\rangle$ and $\langle \Omega, E\rangle$ in Example 3. Then, we get $\acute{\mathbf{E}}_\varphi(\langle \Upsilon, E\rangle, \langle \Omega, E\rangle) = 0.3705$

Corollary 15. *Let* $\mathbf{E}$ *be an entropy measure of SFSSs, then* $\acute{\mathbf{E}}_\varphi$: $SFSS(Z) \times SFSS(Z) \longrightarrow [0,1]$, $(\langle \Upsilon, E\rangle, \langle \Omega, E\rangle) \longrightarrow \acute{\mathbf{E}}_\varphi(\langle \Upsilon, E\rangle, \langle \Omega, E\rangle) = 1 - \mathbf{E}((\varphi(\Upsilon\Omega, E))^c)$ *is a distance measure of SFSSs.*

6. Application of the Entropy Measure of Spherical Fuzzy Soft Set Model Based on Multiple-Attribute Group Decision-Making Method

Using the MAGDM method based on the improved TOPSIS method and the novel spherical fuzzy soft entropy, we propose an algorithm for SFSSs. This new algorithm helps find the optimal object by ranking all objects in our universe.

For that, we first discuss some fundamental concepts as follows.

Consider $Z=\{\zeta_i; i=1,2,\ldots,m\}$ as the set of universe with $E=\{e_j; j=1,2,\ldots,n\}$ as the set of parameters. Suppose that, there are p decision-makers $D_k, k=1,2,\ldots,p$ with $w_k\in[0,1]$ as their weight vector with $\sum_{k=1}^{p} w_k=1$.

Definition 16. Let $S_k=\langle \Upsilon_k, E\rangle=(\mu_{\Upsilon_k(e_j)}(\zeta_i), \eta_{\Upsilon_k(e_j)}(\zeta_i), \vartheta_{\Upsilon_k(e_j)}(\zeta_i))$ be the SFSSs corresponding to each decision-makers $D_k, k=1,2,\ldots,p$. Then, the overall weighted average spherical fuzzy soft set is given by

$$S(S_1,S_2,\ldots,S_p)=\left(\sum_{k=1}^{p} w_k\mu_{\Upsilon_k(e_j)}(\zeta_i), \sum_{k=1}^{p} w_k\eta_{\Upsilon_k(e_j)}(\zeta_i), \sum_{k=1}^{p} w_k\vartheta_{\Upsilon_k(e_j)}(\zeta_i)\right), \tag{14}$$

Entropy is the measure of fuzziness. In the case of parameters, a parameter with smaller entropy decreases uncertainty and evaluation becomes more accurate. Thus, more importance can be given to that parameter. Suppose that, in multiple attribute decision-making problems, decision-makers give the subjective weight vectors $\xi=\{\xi_1,\xi_2,\ldots,\xi_n\}$. Then, the objective weight and integrated weight of parameters are defined as follows:

Definition 17. For each $e_j\in E$, $j=1,2,\ldots,n$, the objective parameter weight ρ_j is

$$\rho_j=\frac{1-\epsilon(\Upsilon,e_j)}{\sum_{j=1}^{n}\left(1-\epsilon(\Upsilon,e_j)\right)}, \tag{15}$$

where, $\epsilon(\Upsilon,e_j)$ is the spherical fuzzy entropy given in (5)

The integrated parameter weight vector γ_j for each $e_j, j=1,2,\ldots,n$ is

$$\gamma_j=\frac{\rho_j\xi_j}{\sum_{j=1}^{n}\rho_j\xi_j}. \tag{16}$$

Now, we are proposing a new algorithm for SFSSs based on the multiple-attribute decision-making method as follows.

6.1. Algorithm Based on Multiple-Attribute Group Decision-Making Method. Let $Z=\{\zeta_1,\zeta_2,\ldots,\zeta_m\}$ be the universal set and $E=\{e_1,e_2,\ldots,e_n\}$ be the parameter set. Let $D_k, k=1,2,\ldots,p$ be the "p" decision-makers. Decision-makers can choose parameters that are very much familiar to them. It is not necessary to give evaluation values to all the parameters because all the parameters are independent. Express evaluation of each decision-maker as SFSSs $S_k=\langle\Upsilon_k,E\rangle, k=1,2,\ldots,p$. Let $\xi=\{\xi_1,\xi_2,\ldots,\xi_n\}$ be the subjective weights of each of the parameters.

Step 1: Obtain the overall weighted average spherical fuzzy soft set $S(S_1,S_2,\ldots,S_p)$ using (14), where $W_k\in[0,1], k=1,2,\ldots,p$ is the weight vector corresponding to each decision-maker "D_k," $k=1,2,\ldots,p$ and express it in a tabular form as Table 1

Step 2: For each $e_j, j=1,2,\ldots n$, calculate $\epsilon(\Upsilon,e_j)$ on SFSSs using (5)

Step 3: Compute objective weight vector "ρ_j" and integrated weight vector "γ_j" corresponding to each parameter using (15) and (16)

Step 4: Obtain the +ve ideal solution R^+ and -ve ideal solution R^-, where

$$\begin{aligned} R^+ &=\{(\mu_1^+,\eta_1^+,\vartheta_1^+),(\mu_2^+,\eta_2^+,\vartheta_2^+),\ldots\ldots,(\mu_n^+,\eta_n^+,\vartheta_n^+)\},\\ R^- &=\{(\mu_1^-,\eta_1^-,\vartheta_1^-),(\mu_2^-,\eta_2^-,\vartheta_2^-),\ldots\ldots,(\mu_n^-,\eta_n^-,\vartheta_n^-)\}. \end{aligned} \tag{17}$$

If parameters are in benefit category: where $\mu_j^+=\max_{1\le i\le m}\{\mu_{ij}\}$, $\eta_j^+=\min_{1\le i\le m}\{\eta_{ij}\}$, $\vartheta_j^+=\min_{1\le i\le m}\{\vartheta_{ij}\}$, and $\mu_j^-=\min_{1\le i\le m}\{\mu_{ij}\}$, $\eta_j^-=\min_{1\le i\le m}\{\eta_{ij}\}$, $\vartheta_j^-=\max_{1\le i\le m}\{\vartheta_{ij}\}$.

If parameters are in cost category: where, $\mu_j^+=\min_{1\le i\le m}\{\mu_{ij}\}$, $\eta_j^+=\min_{1\le i\le m}\{\eta_{ij}\}$, $\vartheta_j^+=\max_{1\le i\le m}\{\vartheta_{ij}\}$, and $\mu_j^-=\max_{1\le i\le m}\{\mu_{ij}\}$, $\eta_j^-=\min_{1\le i\le m}\{\eta_{ij}\}$, $\vartheta_j^-=\min_{1\le i\le m}\{\vartheta_{ij}\}$.

Step 5: Calculate the weighted spherical fuzzy distances $d(t_i,R^+)$ and $d(t_i,R^-)$, where

$$\begin{aligned} d(\zeta_i,R^+) &=\frac{1}{2}\sum_{j=1}^{n}\gamma_j\left[\left|\mu_{ij}^2-(\mu_j^+)^2\right|+\left|\eta_{ij}^2-(\eta_j^+)^2\right|+\left|\vartheta_{ij}^2-(\vartheta_j^+)^2\right|\right],\\ d(\zeta_i,R^-) &=\frac{1}{2}\sum_{j=1}^{n}\gamma_j\left[\left|\mu_{ij}^2-(\mu_j^-)^2\right|+\left|\eta_{ij}^2-(\eta_j^-)^2\right|+\left|\vartheta_{ij}^2-(\vartheta_j^-)^2\right|\right]. \end{aligned} \tag{18}$$

Step 6: Obtain the relative closeness coefficient "C_i" defined as

$$C_i=\frac{d(\zeta_i,R^-)}{d(\zeta_i,R^-)+d(\zeta_i,R^+)}. \tag{19}$$

Step 7: Based on the relative closeness coefficient "C_i," rank each $\zeta_i \in Z, i = 1, 2, \ldots, m$ from larger to smaller. Choose the highest rank element as the optimal solution.

Example 12. A global automaker group decided to start their production unit in Kerala. The following factors are considered as the parameters:

e_1: Good site characteristics

e_2: Availability of utilities

e_3: Lower disaster risk

e_4: Good business climate

e_5: Proximity to original equipment makers

Table 1: Tabular form of SFSS $S(S_1, S_2, \ldots, S_p)$.

	e_1	e_2	$\ldots$	e_n
t_1	$(\mu_{11}, \eta_{11}, \vartheta_{11})$	$(\mu_{12}, \eta_{12}, \vartheta_{12})$	$\ldots$	$(\mu_{1n}, \eta_{1n}, \vartheta_{1n})$
t_2	$(\mu_{21}, \eta_{21}, \vartheta_{21})$	$(\mu_{22}, \eta_{22}, \vartheta_{22})$	$\ldots$	$(\mu_{2n}, \eta_{2n}, \vartheta_{2n})$
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$
t_m	$(\mu_{m1}, \eta_{m1}, \vartheta_{m1})$	$(\mu_{m2}, \eta_{m2}, \vartheta_{m2})$	$\ldots$	$(\mu_{mn}, \eta_{mn}, \vartheta_{mn})$

After a deep searching process, 4 places $(\zeta_1, \zeta_2, \zeta_3, \zeta_4)$ are shortlisted to start their production unit. Three decision-makers/experts (D_1, D_2, D_3) are selected to take a better decision. Among the three decision-makers, D_1 is expertized in the case of parameters $E_1 = \{e_1, e_2, e_3\}$, D_2 is expertized in $E_2 = \{e_1, e_4, e_5\}$, and D_3 is expertized in $E_3 = \{e_2, e_3, e_5\}$. The SFSSs $S_k = \langle F_k, E_k\rangle$, $k = 1, 2, 3$ corresponding to the evaluation of the three decision-makers are given as follows:

$$
\begin{array}{rcccc}
 & & e_1 & e_2 & e_3 \\
 & \zeta_1 & (0.5, 0.1, 0.3) & (0.8, 0.1, 0.1) & (0.7, 0.2, 0.3) \\
S_1 = \langle \Upsilon_1, E_1\rangle = & \zeta_2 & (0.4, 0.2, 0.3) & (0.7, 0.2, 0.1) & (0.9, 0.1, 0.0) \\
 & \zeta_3 & (0.8, 0.1, 0.2) & (0.7, 0.1, 0.3) & (0.6, 0.2, 0.1) \\
 & \zeta_4 & (0.4, 0.1, 0.3) & (0.8, 0.1, 0.1) & (0.9, 0.1, 0.1), \\
 & & e_1 & e_4 & e_5 \\
 & \zeta_1 & (0.9, 0.1, 0.1) & (0.4, 0.1, 0.2) & (0.6, 0.3, 0.2) \\
S_2 = \langle \Upsilon_2, E_2\rangle = & \zeta_2 & (0.4, 0.1, 0.3) & (0.8, 0.1, 0.0) & (0.6, 0.1, 0.0) \\
 & \zeta_3 & (0.9, 0.1, 0.0) & (0.2, 0.3, 0.5) & (0.8, 0.2, 0.2) \\
 & \zeta_4 & (0.7, 0.1, 0.4) & (0.5, 0.2, 0.3) & (0.6, 0.1, 0.1), \\
 & & e_2 & e_3 & e_5 \\
 & \zeta_1 & (0.7, 0.2, 0.1) & (0.8, 0.1, 0.2) & (0.4, 0.3, 0.3) \\
S_3 = \langle \Upsilon_3, E_3\rangle = & \zeta_2 & (0.5, 0.2, 0.1) & (0.7, 0.1, 0.2) & (0.9, 0.1, 0.1) \\
 & \zeta_3 & (0.3, 0.2, 0.5) & (0.6, 0.2, 0.4) & (0.5, 0.2, 0.1) \\
 & \zeta_4 & (0.8, 0.0, 0.3) & (0.3, 0.1, 0.5) & (0.8, 0.3, 0.0),
\end{array}
\tag{20}
$$

Here, we are considering the weight vectors of the three experts are equal. Therefore, using (14), we get $S(S_1, S_2, S_3) = \langle \Upsilon, E\rangle$ and expressed in a tabular form in Table 2.

Using (5), compute the spherical fuzzy entropy $\epsilon(\Upsilon, e_j)$. That is,

$$
\begin{aligned}
\varepsilon(\Upsilon, e_1) &= 0.65,\\
\varepsilon(\Upsilon, e_2) &= 0.61,\\
\varepsilon(\Upsilon, e_3) &= 0.58,\\
\varepsilon(\Upsilon, e_4) &= 0.73, \text{and}\\
\varepsilon(\Upsilon, e_5) &= 0.607.
\end{aligned}
\tag{21}
$$

After a careful examination of each of the parameters e_j, $j = 1, 2, .., 5$, decision-makers assigned their subjective weights $\xi = \{0.23, 0.15, 0.20, 0.25, 0.17\}$. Using (15) and (16), obtain the objective weight vectors "ρ_j" and integrated weight vectors "γ_j" of each parameter and which are shown in Table 3.

Next, calculate the +ve ideal solution R^+ and −ve ideal solution R^- as follows:

$$
\begin{aligned}
R^+ &= \{(0.85, 0.10, 0.10), (0.80, 0.05, 0.10), (0.80, 0.10, 0.10), (0.80, 0.10, 0.00), (0.75, 0.10, 0.05)\},\\
R^- &= \{(0.40, 0.10, 0.35), (0.50, 0.05, 0.40), (0.60, 0.10, 0.30), (0.20, 0.10, 0.50), (0.50, 0.10, 0.25)\}.
\end{aligned}
\tag{22}
$$

Table 2: Tabular form of spherical fuzzy soft weighted average $S(S_1, S_2, S_3)$.

	e_1	e_2	e_3	e_4	e_5
ζ_1	(0.70, 0.10, 0.20)	(0.75, 0.15, 0.10)	(0.75, 0.15, 0.25)	(0.40, 0.10, 0.20)	(0.50, 0.30, 0.25)
ζ_2	(0.40, 0.15, 0.30)	(0.60, 0.20, 0.10)	(0.80, 0.10, 0.10)	(0.80, 0.10, 0.00)	(0.75, 0.10, 0.05)
ζ_3	(0.85, 0.10, 0.10)	(0.50, 0.15, 0.40)	(0.60, 0.20, 0.25)	(0.20, 0.30, 0.50)	(0.65, 0.20, 0.15)
ζ_4	(0.55, 0.10, 0.35)	(0.80, 0.05, 0.20)	(0.60, 0.10, 0.30)	(0.50, 0.20, 0.30)	(0.70, 0.20, 0.05)

Table 3: The weights of parameters.

	ξ	ρ	γ
e_1	0.23	0.192	0.2256
e_2	0.15	0.214	0.1639
e_3	0.20	0.231	0.2358
e_4	0.25	0.148	0.1889
e_5	0.17	0.214	0.1858

Then, compute the weighted spherical fuzzy distances $d(\zeta_i, R^+)$ and $d(\zeta_i, R^-)$ for each ζ_i, $i = 1, 2, 3, 4$. That is,

$$\begin{aligned} d(\zeta_1, R^+) &= 0.1456, \\ d(\zeta_1, R^-) &= 0.1565, \\ d(\zeta_2, R^+) &= 0.0999, \\ d(\zeta_2, R^-) &= 0.1869, \\ d(\zeta_3, R^+) &= 0.1988, \\ d(\zeta_3, R^-) &= 0.1204, \\ d(\zeta_4, R^+) &= 0.1623, \text{ and} \\ d(\zeta_4, R^-) &= 0.1264. \end{aligned} \tag{23}$$

Finally, find the relative closeness coefficients C_i, $i = 1, 2, 3, 4$, as follows:

$$\begin{aligned} C_1 &= 0.5180, \\ C_2 &= 0.6517, \\ C_3 &= 0.3772, \\ C_4 &= 0.4378. \end{aligned} \tag{24}$$

Thus, we can rank the places as $\zeta_2 > \zeta_1 > \zeta_4 > \zeta_3$. This implies that "$\zeta_4$" is the optimal solution. That is, the automaker group can choose place "ζ_4" to start their production unit in Kerala.

7. Comparison Analysis

This section seeks to compare the proposed algorithm with the already existing algorithm for SFSSs to prove the validity, reliability, and dependability of the novel algorithm. Here, we are considering the algorithm for SFSSs based on the group decision-making method and the extension of the TOPSIS (Technique of Order Preference by Similarity to an Ideal Solution) approach developed by Garg et al. [35]. While examining Example 12 using this approach, we get $\widehat{\mathbf{C}_1} = 0.5041$, $\widehat{\mathbf{C}_2} = 0.6434$, $\widehat{\mathbf{C}_3} = 0.4059$, and $\widehat{\mathbf{C}_4} = 0.4372$. That is, the order relation between the alternatives is given by $\zeta_2 > \zeta_1 > \zeta_4 > \zeta_3$.

It can be seen that the order relation and the optimal solution for the previously known method are the same as the proposed algorithm based on the multiple-attribute group decision-making method.

The following is a succinct summary of the benefits of the work illustrated in earlier sections:

(i) The entropy, distance, and similarity measures of SFSSs have a lot of applications in real-life situations including pattern recognition, group decision-making, image processing, medical diagnosis, etc.

(ii) SFSS is one of the perfect generalized forms of fuzzy soft sets and it is certainly the more realistic, useful, and accurate.

(iii) Introducing the entropy, distance, and similarity measures to SFSS appears to be crucial in both theoretical and practical contexts.

(iv) The proposed approach is more consistent and reliable for dealing with SFSS multi-attribute group decision-making problems.

8. Conclusions

In this paper, we introduced an axiomatic definition of the entropy measure of SFSSs and proposed a characterization of spherical fuzzy soft entropy. Also, an expression for calculating the entropy measure of SFSSs has been put forward. Again, the notions of similarity measure and distance measure of SFSSs are defined, and the relationships between them with the entropy measure of SFSSs are studied in detail, which includes the transformations between entropy and similarity measure of SFSSs as well as the transformations between entropy and distance measure of SFSSs. For the application, we generalized the TOPSIS method to cope with MAGDM problems for SFSSs. The newly proposed spherical fuzzy soft entropy measure is used to obtain the weights of the parameters. Finally, an algorithm based on the MAGDM method is presented and applied in a numerical example to show the usefulness of the proposed algorithm.

More applications can be found out to solve the decision-making problems of SFSSs, and also, the algebraic and topological structures can be introduced for SFSSs as future work.

Data Availability

No data were used to support the findings of this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

The authors Fathima Perveen P. A., Sunil Jacob John, and Baiju T. confirm sole responsibility for the following: study conception and design, data collection, analysis and interpretation of results, and manuscript preparation.

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