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Research Article

The Characteristics of Circular Fermatean Fuzzy Sets and Multicriteria Decision-Making Based on the Fermatean Fuzzy t-Norm and t-Conorm

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When diverse decision makers are involved in the decision-making process, taking average of decision values might not reflect an accurate point of view. To overcome such a scenario, the circular Fermatean fuzzy (CFF) set, an advancement of the Fermatean fuzzy (FF) set, and the interval-valued Fermatean fuzzy set (IVFFS) are introduced in this current study. The proposed CFF set is a circle with a centre as association value (AV) and nonassociation value (NAV) with a radius at most equal to $\sqrt{2}$. It is built in such a way that it covers all the decision makers' opinion value through a circle. Due to its geometric structure, the CFF set resolves ambiguity and risk more accurately and effectively than FF and IVFF. FF t-norm and t-conorm are used to investigate the properties of CFF sets, subsequent to which the algebraic operations between them are defined. A couple of CFF distance measures between CFF numbers are introduced and used in the selection of an electric autorickshaw along with the CFF weighted averaging and geometric aggregation operators. The overview and comparison analysis of the generated reports exemplifies the viability and compatibility of the CFF set strategy for selecting the best choices.

1. Introduction

In 1965, Zadeh [1] pioneered the conception of the fuzzy set (FS) by employing AV that takes a value stretching from 0 to 1 instead of a characteristic function that concurs with the value of either 0 or 1 to address the consistency difficulties in real-life situations.

FS is capable of administering the AV. In reality, the NAV should be brought into consideration in many additional circumstances, and that is not essential that the amount of NAV remains the same as the AV reduced by one. As a consequence of this, Atanossav [2] proposed the intuitionistic fuzzy set (IFS) concept in 1986 that considers both AV and NAV into account, in which the NAV is not obtained from the AV and that culminates in the concept of hesitation degree with the assumption that the sum of AV, NAV, and hesitation degree is 1.

Despite the reality that IFS has been used in several areas, there were challenges. In particular, the IFS lacks the capacity to handle such information when a decision maker provides it, but the total amount of AV and NAV is higher than 1. With regard to this, Yager and Abbasov [3] put forwarded the Pythagorean fuzzy set (PFS) theory in 2013 by means of manipulating IFS's perspective. PFS provides the additional feature that the sum of squares of AV, NAV, and hesitation degree is 1, which extends the acceptance range.

The FF set proposed by Senapati and Yager [4] has comparatively extensive range of acceptance to IFS and PFS by taking AV and NAV with the condition that the sum of cubes of AV, NAV, and hesitation degree is 1.

The highly progressive advancement in FS is the circular IFS, which Atanassov [5] created in 2020. In contrast to IFS, a CIFS illustrates every component as a circle with a centre and radius. These sets are those in which every component of the universe has an AV and an NAV as the centre of a circle around them, and that circle has a radius in $[0, \sqrt{2}]$ such that the sum of the values of AV and NAV within this circle is at most equal to 1. A circular Pythagorean fuzzy set (CPFS) introduced by Bozyigit et al. [6] is described by a circle with a centre as AV and NAV, with the sum of their squares between 0 and 1 and radius in [0, 1]. Alsattar et al. [7] have used CPFS for decision-making.

As an extension of FFS and IVFS, the circular Fermatean fuzzy (CFF) set is introduced in this study. CFF sets are circles with a centre at the AV and NAV, with the sum of their cubes between 0 and 1 and of radius in $[0, \sqrt{2}]$. Compared to the FFS structure, a CFF set component is represented by a configurable circle with a radius of not more than $\sqrt{2}$ and a centre made up of AV and NAV. So, the CFF set structure is demonstrated in a two-dimensional space as a high-order ambiguous set where the components of a given finite universe of discourse take the AV and NAV enclosing in a circle. As an outcome, more informed decisions can be established by decision makers by assessing things in greater size and in willing regions.

Particular forms of t-norm and t-conorm have been studied by Deschrijver et al. [8] and Klement et al. [9]. The aggregation process facilitates the conversion of a list of items, all associated with the same set, into a single representation of that set. Grabisch et al. [10] have written about aggregation functions in the Encyclopedia of Mathematics and its Applications, while Beliakov et al. [11] introduced averaging operators for Atanassov's IFS. Xu and Yager [12], Wang and Liu [13], and Xu and Da [14] have studied geometric aggregation operators. Garg and Arora [15] and Xia et al. [16] have researched aggregation operators in Archimedean t-norm and t-conorm.

A structured approach for valuing options with competing criteria and selecting the most effective plan of action is known as multiple criteria decision analysis (MCDA). While MCDM shares similarities with cost-benefit analysis, it considers additional factors beyond just cost. MCDM is an approach that finds applications in a broad range of domains.

Kirisci [17] and Sahoo [18] introduced the SMs of the FF set. Atanassov and Evgeniy [19] introduced four distances of CIFS, Chen [20] has given evolved distance measures for CIFS, Hayat et al. [21] introduced group-based IFS AOs, and Yager [22] has discussed fuzzy measures.

Based on spherical fuzzy soft TOPSIS [23], IVFF TOPSIS [24], quasirung orthopair FS [25], 3, 4-quasirung FS [26], IVFF Dombi aggregation operators [27], FF Bonferroni mean [28], IVFFS [29], FF weighted averaging and geometric operators [30], IF Einstein hybrid AO [31], interval-valued picture fuzzy geometric Bonferroni mean AOs [32], logarithmic AOs [33], generalized IFAOs [21], q-rung orthopair fuzzy Choquet integral [34], and generalized IF AO [35] MCDM have been discussed.

The following are our objective and purposes:

(i) All currently available approaches employ the single averaged information, even when there are numerous decision makers. The opinion of the decision maker is converted to a circle using our recommended CFF set.

(ii) The CFF set is an effective tool to deal with uncertainty because of its special geometric structure. The CFF set identifies the range of opinions that a decision maker is allowed to have in order for the ranking of options remains unchanged.

(iii) The values can be aggregated easily by employing the CFF weighted averaging and geometric aggregation operator.

(iv) CFFCDM and CFFEDM are the effective tools to rank the alternatives, and they can be extended to other DM in the future work.

The following outcomes are illustrated in this study:

(i) The CFF sets are introduced, and their characteristics are investigated.

(ii) CFF aggregation operator and distance measures are defined using FF t-norm and t-conorm.

(iii) The suggested methods are applied in the selection of the best electric autorickshaw.

(iv) The sustainability and validity are examined through visualisation and comparison analysis with the existing methods.

Order of the remaining content is as follows: preliminaries are given in Section 2. In Section 3, the CFF set is introduced as well as the connections between them, and its characteristics are explored. Section 4 deals with CFF AOs and DMs. In Section 5, the proposed AOs and DMs are used for a MCDM on an electric autorickshaw. Section 6 winds up with a conclusion.

The acronyms used in the current research are listed as follows (see Table 1).

2. Preliminaries

This section conveys some of the essential concepts utilised in this study.

Definition 1 (see [4]). "A set $\mathscr{F} = \{\langle x, \alpha_F(x), \beta_F(x)\rangle : x \in X\}$ in the universe of discourse X is called Fermatean fuzzy (FF) set if $0 \leq (\alpha_F(x))^3 + (\beta_F(x))^3 \leq 1$, where $\alpha_F(x)\colon X \longrightarrow [0,1]$, $\beta_F(x)\colon X \longrightarrow [0, 1]$, and $\pi = \sqrt[3]{1 - (\alpha_F(x))^3 - (\beta_F(x))^3}$ are the degree of AV, NAV, and the degree of indeterminacy of x in F. The components of the FF set are taken as the FF number (FFN), and it is represented by $\mathscr{F} = (\alpha_F, \beta_F)$ whose complement is $\mathscr{F}^c = (\beta_F, \alpha_F)$."

The terminology of triangular norm and triangular conorm was originally put forward by Schweizer and Sklar

Table 1: List of acronyms.

Acronyms	Expansion
AV	Association value
NAV	Nonassociation value
FS	Fuzzy set
IFS	Intuitionistic fuzzy set
PFS	Pythagorean fuzzy set
IVFFS	Interval-valued Fermatean fuzzy set
FF	Fermatean fuzzy
FFN	Fermatean fuzzy number
CIFS	Circular intuitionistic fuzzy set
CPFS	Circular Pythagorean fuzzy set
CFF	Circular Fermatean fuzzy
CFFN	Circular Fermatean fuzzy number
MCDM	Multicriteria decision-making
DE	Decision experts
AO	Aggregation operator
DM	Distance measure
FFDM	Fermatean fuzzy decision matrix
NFFDM	Normalised Fermatean fuzzy decision matrix
NCFFDM	Normalised circular Fermatean fuzzy decision matrix
ACFFDM	Aggregated circular Fermatean fuzzy decision matrix
CFFWA	Circular Fermatean fuzzy weighted averaging aggregation operator
CFFWG	Circular Fermatean fuzzy weighted geometric aggregation operator
CFFCDM	Circular Fermatean fuzzy cosine distance measure
CFFEDM	Circular Fermatean fuzzy Euclidean distance measure
FFWA	Fermatean fuzzy weighted averaging aggregation operator
FFWG	Fermatean fuzzy weighted geometric aggregation operator
FFCDM	Fermatean fuzzy cosine distance measure
FFEDM	Fermatean fuzzy Euclidean distance measure

[36] by elaborating on Menger's [37] notion of the probabilistic metric spaces who originally introduced the terms t-norm and t-conorm. Such concepts fulfil the significant parts in decision-making and statistics.

Definition 2 (see [9, 36]). A t-norm is a function T: $[0,1] \times [0,1] \longrightarrow [0,1]$ that satisfies the following conditions:

(1) $T(a,1) = a$ for all $a \in [0,1]$
(2) $T(a,b) = T(b,a)$ for all $a,b \in [0,1]$
(3) $T(a,T(b,c)) = T(T(a,b),c)$ for all $a,b,c \in [0,1]$
(4) $T(a,b) \leq T(a',b')$ whenever $a \leq a'$ and $b \leq b'$ for all $a,b',b,b' \in [0,1]$

Definition 3 (see [9, 36]). A t-conorm is a function S: $[0,1] \times [0,1] \longrightarrow [0,1]$ that satisfies the following conditions:

(1) $S(a,0) = a$ for all $a \in [0,1]$
(2) $S(a,b) = S(b,a)$ for all $a,b \in [0,1]$
(3) $S(a,S(b,c)) = S(S(a,b),c)$ for all $a,b,c \in [0,1]$
(4) $S(a,b) \leq S(a',b')$ whenever $a \leq a'$ and $b \leq b'$ for all $a,a',b,b' \in [0,1]$

Definition 4 (see [9, 36]). A strictly decreasing function g: $[0,1] \longrightarrow [0,\infty]$ with $g(1) = 0$ is called the additive generator of a t-norm T if we have $T(x,y) = g^{-1}(g(x) + g(y))$ for all $(x,y) \in [0,1] \times [0,1]$.

Definition 5 (see [9, 36]). The additive generator of a dual t-conorm can therefore be found using the fuzzy complement notion.

(1) $N(0) = 1$ and $N(1) = 0$
(2) $N(\alpha) \geq N(\beta)$ whenever $\alpha \leq \beta$ for all $\alpha, \beta \in [0,1]$
(3) Continuity
(4) $N(N(\alpha)) = \alpha$ for all $\alpha \in [0,1]$

The function N: $[0,1] \longrightarrow [0,1]$ defined by $N(x) = (1 - x^k)^{1/k}$, where $k \in (0,\infty)$ is a fuzzy complement. When $k = 3$, N becomes the Fermatean fuzzy complement $N(x) = \sqrt[3]{1 - x^3}$. "T is an Archimedean t-norm if and only if $T(x,x) < x$ for all $x \in (0,1)$, and S is an Archimedean t-conorm if and only if $S(x,x) > x$ for all $x \in (0,1)$.

Definition 6 (see [9, 36]). In [0,1], let T be a t-norm and S be a t-conorm. If $T(x,y) = N(S(N(x),N(y)))$ and $S(x,y) = N(T(N(x),N(y)))$, then T and S are referred as dual with respect to the fuzzy complement N.

Remark 7 (see [9, 36]). Let T be a t-norm on [0, 1]. Then, the dual t-conorm S with respect to the Fermatean fuzzy complement N is as follows:

$$S(x,y)=\sqrt[3]{1-T^3}\left(\sqrt[3]{1-x^3},\sqrt[3]{1-y^3}\right). \quad (1)$$

3. Circular Fermatean Fuzzy Sets

This section addresses some of the basic features of CFF sets employed in this work.

Definition 8. A circular Fermatean fuzzy (CFF) set $c\mathscr{F}$ stated as $c\mathscr{F}=\{\langle\iota,\mu_{c\mathscr{F}}(\iota),\nu_{c\mathscr{F}}(\iota);\rho\rangle:\iota\in\mathscr{I}\}$ in the the space of discussion $\mathscr{I}$ is a circle with centre at AV and NAV $\mu_{c\mathscr{F}}(\iota),\nu_{c\mathscr{F}}(\iota):\mathscr{I}\longrightarrow[0,1]$ and of radius $\rho\in[0,\sqrt{2}]$ such that $0\le(\mu_{c\mathscr{F}}(\iota))^3+(\nu_{c\mathscr{F}}(\iota))^3\le 1$. $\pi_{c\mathscr{F}}(\iota)=\sqrt[3]{1-\mu_{c\mathscr{F}}(\iota))^3-(\nu_{c\mathscr{F}}(\iota))^3}$ is the value of the indeterminacy ι in $c\mathscr{F}$.

The component of the CFF set is called the circular Fermatean fuzzy number (CFFN), and it is indicated in the form of $c\mathscr{F}=(\mu_{c\mathscr{F}},\nu_{c\mathscr{F}};\rho_{c\mathscr{F}})$.

Remark 9. All FFS can be viewed as a CFF set because each FFS possesses the structure $c\mathscr{F}=\{\langle\iota,\mu_{c\mathscr{F}}(\iota),\nu_{c\mathscr{F}}(\iota);0\rangle\,\iota\in\mathscr{I}\}=\mathscr{F}$. That is, every FFS is a CFF set with radius 0.

Definition 10. Let $c\mathscr{F}=\{\langle\iota,\mu_{c\mathscr{F}}(\iota),\nu_{c\mathscr{F}}(\iota);\rho_{c\mathscr{F}}\rangle:\iota\in\mathscr{I}\}$ and $c\mathscr{G}=\{\langle\iota,\mu_{c\mathscr{G}}(\iota),\nu_{c\mathscr{G}}(\iota);\rho_{c\mathscr{G}}\rangle:\iota\in\mathscr{I}\}$ be two CFF sets in $\mathscr{I}$. Then,

(1) $c\mathscr{F}\subset c\mathscr{G}$ iff $\rho_{c\mathscr{F}}<\rho_{c\mathscr{G}}$ and $\mu_{c\mathscr{F}}(\iota)<\mu_{c\mathscr{G}}(\iota)$, $\nu_{c\mathscr{F}}(\iota)>\nu_{c\mathscr{G}}(\iota)\,\forall\iota\in\mathscr{I}$.
(2) $c\mathscr{F}=c\mathscr{G}$ iff $\rho_{c\mathscr{F}}=\rho_{c\mathscr{G}}$ and $\mu_{c\mathscr{F}}(\iota)=\mu_{c\mathscr{G}}(\iota)$, $\nu_{c\mathscr{F}}(\iota)=\nu_{c\mathscr{G}}(\iota)\,\forall\iota\in\mathscr{I}$.
(3) The complement $c\mathscr{F}^c=\{\langle\iota,\nu_{c\mathscr{F}}(\iota),\mu_{c\mathscr{F}}(\iota);\rho_{c\mathscr{F}}\rangle:\iota\in\mathscr{I}\}$.
(4) In the context of minimum and maximum, we define

$$\begin{aligned}
(i)\ c\mathscr{F}\bigcup_{\min}c\mathscr{G}&=\{\langle\iota,\max(\mu_{c\mathscr{F}}(\iota),\mu_{c\mathscr{G}}(\iota)),\min(\nu_{c\mathscr{F}}(\iota),\nu_{c\mathscr{G}}(\iota));\min(\rho_{c\mathscr{F}},\rho_{c\mathscr{G}})\rangle:\iota\in\mathscr{I}\},\\
(ii)\ c\mathscr{F}\bigcup_{\max}c\mathscr{G}&=\{\langle\iota,\max(\mu_{c\mathscr{F}}(\iota),\mu_{c\mathscr{G}}(\iota)),\min(\nu_{c\mathscr{F}}(\iota),\nu_{c\mathscr{G}}(\iota));\max(\rho_{c\mathscr{F}},\rho_{c\mathscr{G}})\rangle:\iota\in\mathscr{I}\},\\
(iii)\ c\mathscr{F}\bigcap_{\min}c\mathscr{G}&=\{\langle\iota,\min(\mu_{c\mathscr{F}}(\iota),\mu_{c\mathscr{G}}(\iota)),\max(\nu_{c\mathscr{F}}(\iota),\nu_{c\mathscr{G}}(\iota));\min(\rho_{c\mathscr{F}},\rho_{c\mathscr{G}})\rangle:\iota\in\mathscr{I}\},\\
(iv)\ c\mathscr{F}\bigcap_{\max}c\mathscr{G}&=\{\langle\iota,\min(\mu_{c\mathscr{F}}(\iota),\mu_{c\mathscr{G}}(\iota)),ma\iota(\nu_{c\mathscr{F}}(\iota),\nu_{c\mathscr{G}}(\iota));\max(\rho_{c\mathscr{F}},\rho_{c\mathscr{G}})\rangle:\iota\in\mathscr{I}\}.
\end{aligned} \quad (2)$$

(5) De Morgan's law is as follows:

$$\begin{aligned}
(i)\ \left(c\mathscr{F}\bigcup_{\min}c\mathscr{G}\right)^c&=c\mathscr{F}^c\bigcap_{\min}c\mathscr{G}^c,\\
(ii)\ \left(c\mathscr{F}\bigcup_{\max}c\mathscr{G}\right)^c&=c\mathscr{F}^c\bigcap_{\max}c\mathscr{G}^c,\\
(iii)\ \left(c\mathscr{F}\bigcap_{\min}c\mathscr{G}\right)^c&=c\mathscr{F}^c\bigcup_{\min}c\mathscr{G}^c,\\
(iv)\ \left(c\mathscr{F}\bigcap_{\max}c\mathscr{G}\right)^c&=c\mathscr{F}^c\bigcup_{\max}c\mathscr{G}^c.
\end{aligned} \quad (3)$$

Proposition 11. *Let* $\mathscr{F}_i=\left\{\left\langle\mu_{f_{i,1}},\nu_{f_{i,1}}\right\rangle,\left\langle\mu_{f_{i,2}},\nu_{f_{i,2}}\right\rangle,\ldots,\left\langle\mu_{f_{i,k_i}},\nu_{f_{i,k_i}}\right\rangle\right\}$ *be the collection of FFNs. Then,* $c\mathscr{F}_i=\{\langle\iota_i,\mu_{c\mathscr{F}}(\iota_i),\nu_{c\mathscr{F}}(\iota_i);\rho_c\mathscr{F}_i\rangle:\iota_i\in\mathscr{I}\}$ *are CFF sets, where*

$$\begin{aligned}
\langle\mu_{c\mathscr{F}}(\iota_i),\nu_{c\mathscr{F}}(\iota_i)\rangle&=\left\langle\sqrt[3]{\frac{\sum_{j=1}^{k_i}\mu^3_{f_{i,j}}}{k_i}},\sqrt[3]{\frac{\sum_{j=1}^{k_i}\nu^3_{f_{i,j}}}{k_i}}\right\rangle\ \text{and}\\
\rho_{c\mathscr{F}_i}&=\min\left\{\max_{1\le j\le k_i}\sqrt{\left(\mu_{c\mathscr{F}}(\iota_i)-\mu_{f_{i,j}}\right)^2+\left(\nu_{c\mathscr{F}}(\iota_i)-\nu_{f_{i,j}}\right)^2},\sqrt{2}\right\}.
\end{aligned} \quad (4)$$

Proof. Consider

$$0 \leq \mu_{c\mathscr{F}}^3(\iota_i) + \nu_{c\mathscr{F}}^3(\iota_i) = \frac{\sum_{j=1}^{k_i}\mu_{f_{i,j}}^3}{k_i} + \frac{\sum_{j=1}^{k_i}\nu_{f_{i,j}}^3}{k_i} = \frac{\sum_{j=1}^{k_i}\mu_{f_{i,j}}^3 + \sum_{j=1}^{k_i}\nu_{f_{i,j}}^3}{k_i} \leq \frac{\sum_{j=1}^{k_i}1}{k_i} = 1, \tag{5}$$

and meanwhile, $0 \leq \rho_{c\mathscr{F}_i} \leq \sqrt{2}, \forall i$. Consequently, $c\mathscr{F} = \left\{\left\langle \iota, \mu_{c\mathscr{F}_i}(\iota_i), \nu_{c\mathscr{F}}(\iota_i); \rho_{c\mathscr{F}_i}\right\rangle : \iota_i \in \mathscr{I}\right\}$ is a CFF set. □

Example 1. By Proposition 11, for the collection of FFNs $\mathscr{F}_1 = \{\langle 0.6,0.4\rangle, \langle 0.7,0.2\rangle, \langle 0.7,0.1\rangle, \langle 0.8,0.1\rangle\}$, $\mathscr{F}_2 = \{\langle 0.1,0.2\rangle, \langle 0.1,0.3\rangle, \langle 0.1,0.4\rangle, \langle 0.2,0.4\rangle\}$, and $\mathscr{F}_3 = \{\langle 0.33,0.96\rangle, \langle 0.3,0.92\rangle, \langle 0.25,0.89\rangle, \langle 0.4,0.7\rangle\}$, the corresponding CFF sets are $c\mathscr{F}_i = \{\langle 0.71,0.26; 0.19\rangle, \langle 0.14,0.34; 0.15\rangle \langle 0.33,0.88; 0.19\rangle\}$.

The transformation from FF sets to CFF sets, discussed in Example 1, is visualised in Figure 1.

Definition 12. $cf_1 = \left\langle \mu_{cf_1}, \nu_{cf_1}; \rho_{cf_1}\right\rangle$, $cf_2 = \left\langle \mu_{cf_2}, \nu_{cf_2}; \rho_{cf_2}\right\rangle$, and $cf = \left\langle \mu_{cf}, \nu_{cf}; \rho_{cf}\right\rangle$ be CFFNs and $\psi > 0$. Let us assume the continuous Archimedean t-norms m, a: $[0,1] \longrightarrow [0,\infty)$ are the additive generators for $T \& Q$, respectively. Also, the continuous Archimedean t-conorms n, b: $[0,1] \longrightarrow [0,\infty)$ taken as $n(x) = m(\sqrt[3]{1-x^3})$ and $b(x) = a(\sqrt[3]{1-x^3})$ are the additive generators for S and P, respectively:

$$\begin{aligned}
(i)\ cf_1 \oplus_Q cf_2 &= \left\langle S\left(\mu_{cf_1}, \mu_{cf_2}\right), T\left(\nu_{cf_1}, \nu_{cf_2}\right), Q\left(\rho_{cf_1}, \rho_{cf_2}\right)\right\rangle \\
&= \left\langle n^{-1}\left(n\left(\mu_{cf_1}\right) + n\left(\mu_{cf_2}\right)\right), m^{-1}\left(m\left(\nu_{cf_1}\right) + m\left(\nu_{cf_2}\right)\right), a^{-1}\left(a\left(\rho_{cf_1}\right) + a\left(\rho_{cf_2}\right)\right)\right\rangle, \\
(ii)\ cf_1 \oplus_P cf_2 &= \left\langle S\left(\mu_{cf_1}, \mu_{cf_2}\right), T\left(\nu_{cf_1}, \nu_{cf_2}\right), P\left(\rho_{cf_1}, \rho_{cf_2}\right)\right\rangle \\
&= \left\langle n^{-1}\left(n\left(\mu_{cf_1}\right) + n\left(\mu_{cf_2}\right)\right), m^{-1}\left(m\left(\nu_{cf_1}\right) + m\left(\nu_{cf_2}\right)\right), b^{-1}\left(b\left(\rho_{cf_1}\right) + b\left(\rho_{cf_2}\right)\right)\right\rangle, \\
(iii)\ cf_1 \otimes_Q cf_2 &= \left\langle T\left(\mu_{cf_1}, \mu_{cf_2}\right), S\left(\nu_{cf_1}, \nu_{cf_2}\right), Q\left(\rho_{cf_1}, \rho_{cf_2}\right)\right\rangle \\
&= \left\langle m^{-1}\left(m\left(\mu_{cf_1}\right) + m\left(\mu_{cf_2}\right)\right), n^{-1}\left(n\left(\nu_{cf_1}\right) + n\left(\nu_{cf_2}\right)\right), a^{-1}\left(a\left(\rho_{cf_1}\right) + a\left(\rho_{cf_2}\right)\right)\right\rangle, \\
(iv)\ cf_1 \otimes_P cf_2 &= \left\langle T\left(\mu_{cf_1}, \mu_{cf_2}\right), S\left(\nu_{cf_1}, \nu_{cf_2}\right), P\left(\rho_{cf_1}, \rho_{cf_2}\right)\right\rangle \\
&= \left\langle m^{-1}\left(m\left(\mu_{cf_1}\right) + m\left(\mu_{cf_2}\right)\right), n^{-1}\left(n\left(\nu_{cf_1}\right) + n\left(\nu_{cf_2}\right)\right), b^{-1}\left(b\left(\rho_{cf_1}\right) + b\left(\rho_{cf_2}\right)\right)\right\rangle, \\
(v)\ \psi_Q cf &= \left\langle n^{-1}\left(\psi n\left(\mu_{cf}\right)\right), m^{-1}\left(\psi m\left(\nu_{cf}\right)\right), a^{-1}\left(\psi a\left(\rho_{cf}\right)\right)\right\rangle, \\
(vi)\ \psi_P cf &= \left\langle n^{-1}\left(\psi n\left(\mu_{cf}\right)\right), m^{-1}\left(\psi m\left(\nu_{cf}\right)\right), b^{-1}\left(\psi b\left(\rho_{cf}\right)\right)\right\rangle, \\
(vii)\ cf^{\psi_Q} &= \left\langle m^{-1}\left(\psi m\left(\mu_{cf}\right)\right), n^{-1}\left(\psi n\left(\nu_{cf}\right)\right), a^{-1}\left(\psi a\left(\rho_{cf}\right)\right)\right\rangle, \\
(viii)\ cf^{\psi_P} &= \left\langle m^{-1}\left(\psi m\left(\mu_{cf}\right)\right), n^{-1}\left(\psi n\left(\nu_{cf}\right)\right), b^{-1}\left(\psi b\left(\rho_{cf}\right)\right)\right\rangle.
\end{aligned} \tag{6}$$

The preceding operators own the following features:

(i) $cf_1 \oplus_Q cf_2 = cf_2 \oplus_Q cf_1$
(ii) $cf_1 \otimes_Q cf_2 = cf_2 \otimes_Q cf_1$
(iii) $(cf_1 \oplus_Q cf_2) \oplus_Q cf_3 = cf_1 \oplus_Q (cf_2 \oplus_Q cf_3)$
(iv) $(cf_1 \otimes_Q cf_2) \otimes_Q cf_3 = cf_1 \otimes_Q (cf_2 \otimes_Q cf_3)$
(v) $\psi_Q (cf_1 \oplus_Q cf_2) = \psi_Q cf_1 \oplus_Q \psi_Q cf_2$
(vi) $(\psi_Q + \gamma_Q) cf = \psi_Q cf \oplus_Q \gamma_Q cf$
(vii) $(cf_1 \otimes_Q cf_2)^{\psi_Q} = cf_1^{\psi_Q} \otimes_Q cf_2^{\psi_Q}$
(viii) $cf^{\psi_Q} \otimes_Q cf^{\gamma_Q} = cf^{\psi_Q + \gamma_Q}$

Proof

(i) and (ii) are simple to prove.
(iii) We acquire

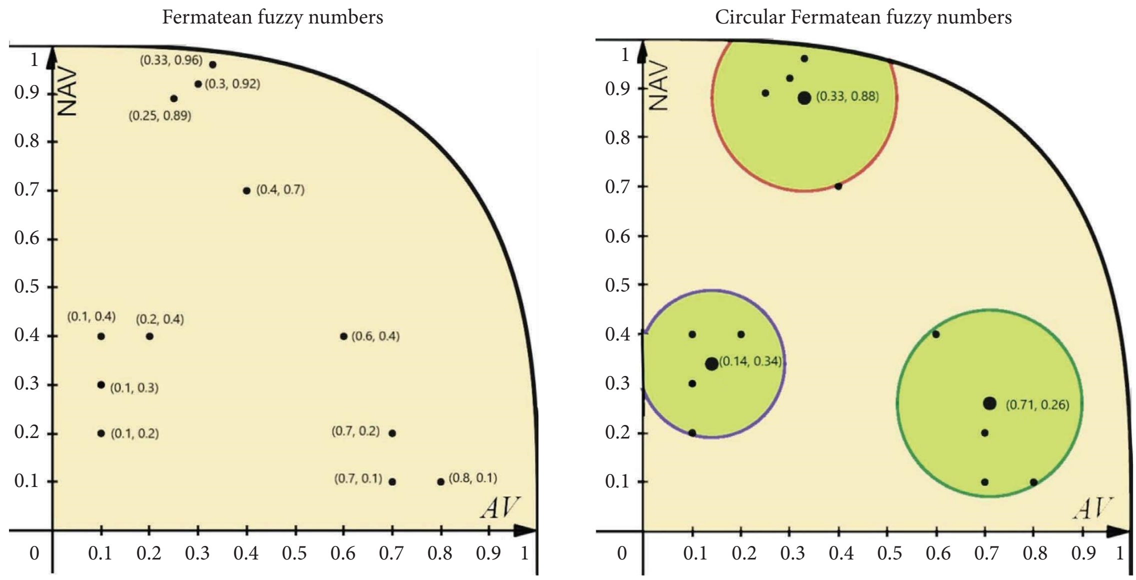

Figure 1: Representation of FFN and CFFN.

$$\begin{aligned}(cf_1 \oplus_Q cf_2) \oplus_Q cf_3 &= \left\langle n^{-1}\left(n(\mu_{cf_1}) + n(\mu_{cf_2})\right), m^{-1}\left(m(\nu_{cf_1}) + m(\nu_{cf_2})\right); a^{-1}\left(a(\rho_{cf_1}) + a(\rho_{cf_2})\right)\right\rangle \oplus_Q \left\langle \mu_{cf_3}, \nu_{cf_3}; \rho_{cf_3}\right\rangle \\
&= \left\langle n^{-1}\left(n(\mu_{cf_1}) + n(\mu_{cf_2}) + n(\mu_{cf_3})\right), m^{-1}\left(m(\nu_{cf_1}) + m(\nu_{cf_2}) + m(\nu_{cf_3})\right); a^{-1}\left(a(\rho_{cf_1}) + a(\rho_{cf_2}) + a(\rho_{cf_3})\right)\right\rangle \\
&= \left\langle n^{-1}\left(n(\mu_{cf_1}) + n\left(n^{-1}\left(n(\mu_{cf_2}) + n(\mu_{cf_3})\right)\right)\right), m^{-1}\left(m(\nu_{cf_1}) + m\left(m^{-1}\left(m(\nu_{cf_2}) + m(\nu_{cf_3})\right)\right)\right);\right. \\
&\quad \left. a^{-1}\left(a(\rho_{cf_1}) + a\left(a^{-1}\left(a(\rho_{cf_2}) + a(\rho_{cf_3})\right)\right)\right)\right\rangle \\
&= \left\langle \mu_{cf_1}, \nu_{cf_1}; \rho_{cf_1}\right\rangle \oplus_Q \left\langle n^{-1}\left(n(\mu_{cf_2}) + n(\mu_{cf_3})\right), m^{-1}\left(m(\nu_{cf_2}) + m(\nu_{cf_3})\right); a^{-1}\left(a(\rho_{cf_2}) + a(\rho_{cf_3})\right)\right\rangle \\
&= cf_1 \oplus_Q (cf_2 \oplus_Q cf_3).\end{aligned} \tag{7}$$

(iv) Consider

$$\begin{aligned}(cf_1 \otimes_Q cf_2) \otimes_Q cf_3 &= \left\langle m^{-1}\left(m(\mu_{cf_1}) + m(\mu_{cf_2})\right), n^{-1}\left(n(\nu_{cf_1}) + n(\nu_{cf_2})\right); a^{-1}\left(a(\rho_{cf_1}) + a(\rho_{cf_2})\right)\right\rangle \otimes_Q \left\langle \mu_{cf_3}, \nu_{cf_3}; \rho_{cf_3}\right\rangle \\
&= \left\langle m^{-1}\left(m(\mu_{cf_1}) + m(\mu_{cf_2}) + m(\mu_{cf_3})\right), n^{-1}\left(n(\nu_{cf_1}) + n(\nu_{cf_2}) + n(\nu_{cf_3})\right); a^{-1}\left(a(\rho_{cf_1}) + a(\rho_{cf_2}) + a(\rho_{cf_3})\right)\right\rangle \\
&= \left\langle m^{-1}\left(m(\mu_{cf_1}) + m\left(m^{-1}\left(m(\mu_{cf_2}) + m(\mu_{cf_3})\right)\right)\right), n^{-1}\left(n(\nu_{cf_1}) + n\left(n^{-1}\left(n(\nu_{cf_2}) + n(\nu_{cf_3})\right)\right)\right);\right. \\
&\quad \left. a^{-1}\left(a(\rho_{cf_1}) + a\left(a^{-1}\left(a(\rho_{cf_2}) + a(\rho_{cf_3})\right)\right)\right)\right\rangle \\
&= \left\langle \mu_{cf_1}, \nu_{cf_1}; \rho_{cf_1}\right\rangle \otimes_Q \left\langle m^{-1}\left(m(\mu_{cf_2}) + m(\mu_{cf_3})\right), n^{-1}\left(n(\nu_{cf_2}) + n(\nu_{cf_3})\right); a^{-1}\left(a(\rho_{cf_2}) + a(\rho_{cf_3})\right)\right\rangle \\
&= cf_1 \otimes_Q (cf_2 \otimes_Q cf_3).\end{aligned} \tag{8}$$

(v) We obtain

$$\begin{aligned}\psi_Q(cf_1 \oplus_Q cf_2) &= \psi\langle n^{-1}(n(\mu_{cf_1}) + n(\mu_{cf_2})), m^{-1}(m(\nu_{cf_1}) + m(\nu_{cf_2})); a^{-1}(a(\rho_{cf_1}) + a(\rho_{cf_2}))\rangle \\ &= \langle n^{-1}(\psi n(\mu_{cf_1}) + \psi n(\mu_{cf_2})), m^{-1}(\psi m(\nu_{cf_1}) + \psi m(\nu_{cf_2})); a^{-1}(\psi a(\rho_{cf_1}) + \psi a(\rho_{cf_2}))\rangle \\ &= \langle n^{-1}(n(n^{-1}(\psi n(\mu_{cf_1}))) + n(n^{-1}(\psi n(\mu_{cf_2})))), m^{-1}(m(m^{-1}(\psi m(\nu_{cf_1}))) + m(m^{-1}(\psi m(\nu_{cf_2})))); \\ &\quad a^{-1}(a(a^{-1}(\psi a(\rho_{cf_1}))) + a(a^{-1}(\psi a(\rho_{cf_2}))))\rangle \\ &= \langle n^{-1}(n(\mu_{\psi cf_1}) + n(\mu_{\psi cf_2})), m^{-1}(m(\nu_{\psi cf_1}) + m(\nu_{\psi cf_2})); a^{-1}(a(\rho_{\psi cf_1}) + a(\rho_{\psi cf_2}))\rangle \\ &= \psi_Q cf_1 \oplus_Q \psi_Q cf_2. \end{aligned} \tag{9}$$

(vi) It is clear that

$$\begin{aligned}(\psi_Q + \gamma_Q)cf &= \langle n^{-1}((\psi+\gamma)n(\mu_{cf})), m^{-1}((\psi+\gamma)m(\nu_{cf})); a^{-1}((\psi+\gamma)a(\rho_{cf}))\rangle \\ &= \langle n^{-1}(\psi n(\mu_{cf}) + \gamma n(\mu_{cf})), m^{-1}(\psi m(\nu_{cf}) + \gamma m(\nu_{cf})); a^{-1}((\psi a(\rho_{cf}) + \gamma a(\rho_{cf})))\rangle \\ &= \langle n^{-1}(n(n^{-1}(\psi n(\mu_{cf}))) + n(n^{-1}(\gamma n(\mu_{cf})))), m^{-1}(m(m^{-1}(\psi m(\nu_{cf}))) + m(m^{-1}(\gamma m(\nu_{cf})))); \\ &\quad a^{-1}(a(a^{-1}(\psi a(\rho_{cf}))) + a(a^{-1}(\gamma a(\rho_{cf}))))\rangle \\ &= \psi_Q cf \oplus_Q \gamma_Q cf. \end{aligned} \tag{10}$$

(vii) We have

$$\begin{aligned}(cf_1 \otimes_Q cf_2)^{\psi_Q} &= \langle m^{-1}(\psi m(\mu_{cf_1 \otimes_Q cf_2})), n^{-1}(\psi n(\nu_{cf_1 \otimes_Q cf_2})); a^{-1}(\psi a(\rho_{cf_1 \otimes_Q cf_2}))\rangle \\ &= \langle m^{-1}(\psi m(m^{-1}(m(\mu_{cf_1}) + m(\mu_{cf_2})))), n^{-1}(\psi n(n^{-1}(n(\nu_{cf_1}) + n(\nu_{cf_2})))); a^{-1}(\psi a(a^{-1}(a(\rho_{cf_1}) + a(\rho_{cf_2}))))\rangle \\ &= \langle m^{-1}(m(\mu_{cf_1^{\psi_Q}}) + m(\mu_{cf_2^{\psi_Q}})), n^{-1}(n(\nu_{cf_1^{\psi_Q}}) + n(\nu_{cf_2^{\psi_Q}})); a^{-1}(a(\rho_{cf_1^{\psi_Q}}) + a(\rho_{cf_2^{\psi_Q}}))\rangle \\ &= cf_1^{\psi_Q} \otimes_Q cf_2^{\psi_Q}. \end{aligned} \tag{11}$$

(viii) We have

$$\begin{aligned}cf^{\psi_Q+\gamma_Q} &= \langle m^{-1}((\psi+\gamma)m(\mu_{cf})), n^{-1}((\psi+\gamma)n(\nu_{cf})); a^{-1}((\psi+\gamma)a(\rho_{cf}))\rangle \\ &= \langle m^{-1}(\psi m(\mu_{cf}) + \gamma m(\mu_{cf})), n^{-1}(\psi n(\nu_{cf}) + \gamma n(\nu_{cf})); a^{-1}(\psi a(\rho_{cf}) + \gamma a(\rho_{cf}))\rangle \\ &= \langle m^{-1}(m(m^{-1}(\psi m(\mu_{cf}))) + m(m^{-1}(\gamma m(\mu_{cf})))), (n(n^{-1}(\psi n(\nu_{cf}))) + n(n^{-1}(\gamma m(\nu_{cf})))); \\ &\quad a^{-1}(a(a^{-1}(\psi a(\rho_{cf}))) + a(a^{-1}(\gamma a(\rho_{cf}))))\rangle \\ &= \langle m^{-1}(m(\mu_{cf^{\psi}}) + m(\mu_{cf^{\gamma}})), n^{-1}(n(\nu_{cf^{\psi}}) + m(\nu_{cf^{\gamma}})); a^{-1}(a(\rho_{cf^{\psi}}) + a(\rho_{cf^{\gamma}}))\rangle \\ &= cf^{\psi_Q} \otimes_Q cf^{\gamma_Q}. \end{aligned} \tag{12}$$

□

4. Circular Fermatean Fuzzy Aggregation Operator and Distance Measures

The aggregation's role in data consolidation, especially in decision-making circumstances, is to serve as an overview of the data before taking the final step. This section discusses the CFF weighted averaging and geometric aggregation operators. We also generate the CFF cosine and the Euclidean distance measure between CFFNs.

Definition 13. If $c\omega = (c\omega_1, c\omega_2, \ldots c\omega_\kappa)$ is the weight vector of the CFFNs, $cf_i = \langle \mu_{cf_i}, \nu_{cf_i}; \rho_{cf_i} \rangle, i = 1,2,3,..\kappa$ with $c\omega_i > 0$ and $\sum_{i=1}^{k} c\omega_i = 1$, then the circular Fermatean fuzzy weighted averaging aggregation operators according to Archimedean t-norm and t-conorm denoted by $\mathrm{ATS-CFFWA}_Q$ and $\mathrm{ATS-CFFWA}_P$ are defined as follows:

$$\begin{aligned} \mathrm{ATS-CFFWA}_Q(cf_1, cf_2, \ldots cf_\kappa) &= \bigoplus_{i=1}^{\kappa}{}_Q (c\omega_i cf_i), \\ \mathrm{ATS-CFFWA}_P(cf_1, cf_2, \ldots cf_\kappa) &= \bigoplus_{i=1}^{\kappa}{}_P (c\omega_i cf_i). \end{aligned} \tag{13}$$

Theorem 14. *The aggregated values $CFFWA_Q$ and $CFFWA_P$ of the collection of CFFNs $f_i = \langle \mu_{cf_i}, \nu_{cf_i}; \rho_{cf_i} \rangle, i = 1,2,3,..\kappa$ with $c\omega_i > 0$ and $\sum_{i=1}^{k} c\omega_i = 1$ are also CFFNs and are of the following form:*

$$\mathrm{ATS-CFFWA}_Q(cf_1, cf_2, \ldots cf_\kappa) = \left\langle n^{-1}\left(\sum_{i=1}^{\kappa} c\omega_i n\left(\mu_{cf_i}\right)\right), m^{-1}\left(\sum_{i=1}^{\kappa} c\omega_i m\left(\nu_{cf_i}\right)\right); a^{-1}\left(\sum_{i=1}^{\kappa} c\omega_i a\left(\rho_{cf_i}\right)\right)\right\rangle, \tag{14}$$

$$\mathrm{ATS-CFFWA}_P(cf_1, cf_2, \ldots cf_\kappa) = \left\langle n^{-1}\left(\sum_{i=1}^{\kappa} c\omega_i n\left(\mu_{cf_i}\right)\right), m^{-1}\left(\sum_{i=1}^{\kappa} c\omega_i m\left(\nu_{cf_i}\right)\right); b^{-1}\left(\sum_{i=1}^{\kappa} c\omega_i a\left(\rho_{cf_i}\right)\right)\right\rangle. \tag{15}$$

Proof. From Proposition 11, $\mathrm{ATS-CFFWA}_Q$ and $\mathrm{ATS-CFFWA}_P$ are CFFN*s*. We prove equation (14) through mathematical induction.

For $\kappa = 2$,

$$\begin{aligned} \bigoplus_{i=1}^{2}{}_Q (c\omega_i cf_i) &= c\omega_1 cf_1 \oplus_Q c\omega_2 cf_2 \\ &= \left\langle n^{-1}\left(c\omega_1 n\left(\mu_{cf_1}\right)\right), m^{-1}\left(c\omega_1 m\left(\nu_{cf_1}\right)\right); a^{-1}\left(c\omega_1 a\left(\rho_{cf_1}\right)\right)\right\rangle \oplus \left\langle n^{-1}\left(c\omega_2 n\left(\mu_{cf_2}\right)\right), m^{-1}\left(c\omega_2 m\left(\nu_{cf_2}\right)\right); a^{-1}\left(c\omega_2 a\left(\rho_{cf_2}\right)\right)\right\rangle \\ &= \left\langle n^{-1}\left(c\omega_1 n\left(\mu_{cf_1}\right) + c\omega_2 n\left(\mu_{cf_2}\right)\right), m^{-1}\left(c\omega_1 m\left(\nu_{cf_1}\right) + c\omega_2 m\left(\nu_{cf_2}\right)\right); a^{-1}\left(c\omega_1 a\left(\rho_{cf_1}\right) + c\omega_2 a\left(\rho_{cf_2}\right)\right)\right\rangle \\ &= \left\langle n^{-1}\left(\sum_{i=1}^{2} c\omega_i n\left(\mu_{cf_i}\right)\right), m^{-1}\left(\sum_{i=1}^{2} c\omega_i m\left(\nu_{cf_i}\right)\right); a^{-1}\left(\sum_{i=1}^{2} c\omega_i a\left(\rho_{cf_i}\right)\right)\right\rangle. \end{aligned} \tag{16}$$

Let us assume the result is true for $\kappa = n$

$$\bigoplus_{i=1}^{n}{}_Q (c\omega_i cf_i) = \left\langle n^{-1}\left(\sum_{i=1}^{n} c\omega_i n\left(\mu_{cf_i}\right)\right), m^{-1}\left(\sum_{i=1}^{n} c\omega_i m\left(\nu_{cf_i}\right)\right); a^{-1}\left(\sum_{i=1}^{n} c\omega_i a\left(\rho_{cf_i}\right)\right)\right\rangle. \tag{17}$$

Then, for $\kappa = n + 1$, we have the following expression:

$$\begin{aligned}
\bigoplus_{i=1}^{n+1}{}_Q (c\omega_i cf_i) &= \bigoplus_{i=1}^{n}{}_Q (c\omega_i cf_i) \oplus_Q \left(c\omega_{(n+1)} cf_{n+1}\right) \\
&= \left\langle n^{-1}\left(\sum_{i=1}^{n} c\omega_i n\left(\mu_{cf_i}\right)\right), m^{-1}\left(\sum_{i=1}^{n} c\omega_i m\left(\nu_{cf_i}\right)\right); \right. \\
&\quad \left. a^{-1}\left(\sum_{i=1}^{n} c\omega_i a\left(\rho_{cf_i}\right)\right)\right\rangle \oplus_Q \left\langle n^{-1}\left(c\omega_{(n+1)} n\left(\mu_{cf_{n+1}}\right)\right), m^{-1}\left(c\omega_{(n+1)} m\left(\nu_{cf_{n+1}}\right)\right); a^{-1}\left(c\omega_{(n+1)} a\left(\rho_{cf_{n+1}}\right)\right)\right\rangle \\
&= \left\langle n^{-1}\left(\sum_{i=1}^{n} c\omega_i n\left(\mu_{cf_i}\right) + c\omega_{(n+1)} n\left(\mu_{cf_{n+1}}\right)\right), m^{-1}\left(\sum_{i=1}^{n} c\omega_i m\left(\nu_{cf_i}\right) + c\omega_{(n+1)} m\left(\mu_{cf_{n+1}}\right)\right); \right. \\
&\quad \left. a^{-1}\left(\sum_{i=1}^{n} c\omega_i a\left(\rho_{cf_i}\right) + c\omega_{(n+1)} a\left(\rho_{cf_{n+1}}\right)\right)\right\rangle \\
&= \left\langle n^{-1}\left(\sum_{i=1}^{n+1} c\omega_i n\left(\mu_{cf_i}\right)\right), m^{-1}\left(\sum_{i=1}^{n+1} c\omega_i m\left(\nu_{cf_i}\right)\right); a^{-1}\left(\sum_{i=1}^{n+1} c\omega_i a\left(\rho_{cf_i}\right)\right)\right\rangle.
\end{aligned} \tag{18}$$

Thus, equation (14) is valid for $\kappa = n+1$. Hence, equation (14) holds for every κ. In the same way, we can prove equation (15). □

Definition 15. If $c\omega = (c\omega_1, c\omega_2, \ldots c\omega_\kappa)$ is the weight vector of the CFFNs $cf_i = \langle \mu_{cf_i}, \nu_{cf_i}; \rho_{cf_i} \rangle, i = 1,2,3,..\kappa$ with $c\omega_i > 0$ and $\sum_{i=1}^{k} c\omega_i = 1$, then circular Fermatean fuzzy weighted geometric aggregation operators according to Archimedean t-norm and t-conorm denoted by CFFWG_Q and CFFWG_P are defined as follows:

$$\begin{aligned}
\text{ATS} - \text{CFFWG}_Q (cf_1, cf_2, \ldots cf_\kappa) &= \bigotimes_{i=1}^{\kappa}{}_Q (c\omega_i cf_i), \\
\text{ATS} - \text{CFFWA}_P (cf_1, cf_2, \ldots cf_\kappa) &= \bigotimes_{i=1}^{\kappa}{}_P (c\omega_i cf_i).
\end{aligned} \tag{19}$$

Theorem 16. *The aggregated values $ATS - CFFWG_Q$ and $ATS - CFFWG_P$ of the collection of CFFNs $cf_i = \langle \mu_{cf_i}, \nu_{cf_i}; \rho_{cf_i} \rangle, i = 1,2,3,..\kappa$ with $c\omega_i > 0$ and $\sum_{i=1}^{k} c\omega_i = 1$ are also CFFNs and are of the following form:*

$$\text{ATS} - \text{CFFWG}_Q (cf_1, cf_2, \ldots cf_\kappa) = \left\langle m^{-1}\left(\sum_{i=1}^{k} c\omega_i m\left(\mu_{cf_i}\right)\right), n^{-1}\left(\sum_{i=1}^{k} c\omega_i n\left(\nu_{cf_i}\right)\right); a^{-1}\left(\sum_{i=1}^{k} c\omega_i a\left(\rho_{cf_i}\right)\right)\right\rangle, \tag{20}$$

$$\text{ATS} - \text{CFFWG}_P (cf_1, cf_2, \ldots cf_\kappa) = \left\langle m^{-1}\left(\sum_{i=1}^{k} c\omega_i m\left(\mu_{cf_i}\right)\right), n^{-1}\left(\sum_{i=1}^{k} c\omega_i n\left(\nu_{cf_i}\right)\right); b^{-1}\left(\sum_{i=1}^{k} c\omega_i b\left(\rho_{cf_i}\right)\right)\right\rangle. \tag{21}$$

Proof. The proof is similar to Theorem 14. □

Remark 17. If n, m, a, b: $[0,1] \longrightarrow [0,\infty)$ are defined as $m(x) = -\log x^3$, $n(x) = -\log(1-x^3)$, $a(x) = -\log x^3$, and $b(x) = -\log(1-x^3)$, then $m^{-1}(x) = \sqrt[3]{e^{-x}}$, $n^{-1}(x) = \sqrt[3]{1-e^{-x}}$, $a^{-1}(x) = \sqrt[3]{e^{-x}}$, and $b^{-1}(x) = \sqrt[3]{1-e^{-x}}$. From equations (14), (15), (20), and (21), we obtain the following expressions:

$$
\begin{aligned}
\mathrm{CFFWA}_Q\left(cf_1, cf_2, \ldots cf_\kappa\right) &= \left\langle \sqrt[3]{1-\prod_{i=1}^{\kappa}\left(1-\mu_{cf_i}^3\right)^{c\omega_i}}, \prod_{i=1}^{\kappa}\nu_{cf_i}^{c\omega_i}, \prod_{i=1}^{\kappa}\rho_{cf_i}^{c\omega_i}\right\rangle,\\
\mathrm{CFFWA}_P\left(cf_1, cf_2, \ldots cf_\kappa\right) &= \left\langle \sqrt[3]{1-\prod_{i=1}^{\kappa}\left(1-\mu_{cf_i}^3\right)^{c\omega_i}}, \prod_{i=1}^{\kappa}\nu_{cf_i}^{c\omega_i}, \sqrt[3]{1-\prod_{i=1}^{\kappa}\left(1-\rho_{cf_i}^3\right)^{c\omega_i}}\right\rangle,\\
\mathrm{CFFWG}_Q\left(cf_1, cf_2, \ldots cf_\kappa\right) &= \left\langle \prod_{i=1}^{k}\mu_{cf_i}^{c\omega_i}, \sqrt[3]{1-\prod_{i=1}^{\kappa}\left(1-\nu_{cf_i}^3\right)^{c\omega_i}}, \prod_{i=1}^{\kappa}\rho_{cf_i}^{c\omega_i}\right\rangle,\\
\mathrm{CFFWG}_P\left(cf_1, cf_2, \ldots cf_\kappa\right) &= \left\langle \prod_{i=1}^{k}\mu_{cf_i}^{c\omega_i}, \sqrt[3]{1-\prod_{i=1}^{\kappa}\left(1-\nu_{cf_i}^3\right)^{c\omega_i}}, \sqrt[3]{1-\prod_{i=1}^{\kappa}\left(1-\rho_{cf_i}^3\right)^{c\omega_i}}\right\rangle.
\end{aligned}
\tag{22}
$$

Definition 18. Let $c\mathcal{F} = \langle \mu_{c\mathcal{F}}, \nu_{c\mathcal{F}}; \rho_{cF}\rangle$ and $c\mathcal{G} = \langle \mu_{c\mathcal{G}}, \nu_{c\mathcal{G}}; \rho_{cG}\rangle$ be two CFFNs. The circular Fermatean fuzzy cosine distance measure (CFFCDM) and the circular Fermatean fuzzy Euclidean distance measure (CFFEDM) between $c\mathcal{F}$ and $c\mathcal{G}$ are defined as follows:

$$
\mathrm{CFFCDM}(c\mathcal{F}, c\mathcal{G}) = 1 - \frac{1}{2}\left(\frac{\mu_{c\mathcal{F}}^3\mu_{c\mathcal{G}}^3 + \nu_{c\mathcal{F}}^3\nu_{c\mathcal{G}}^3}{\sqrt[3]{\mu_{c\mathcal{F}}^6 + \nu_{c\mathcal{F}}^6}\sqrt[3]{\mu_{c\mathcal{G}}^6 + \nu_{c\mathcal{G}}^6}} + 1 - \frac{\left|\rho_{cF} - \rho_{cG}\right|}{\sqrt{2}}\right), \tag{23}
$$

$$
\mathrm{CFFEDM}(c\mathcal{F}, c\mathcal{G}) = \frac{1}{2}\left(\frac{\left|\rho_{cF} - \rho_{cG}\right|}{\sqrt{2}} + \sqrt{\frac{1}{2}\left[\left(\mu_{c\mathcal{F}}^3 - \nu_{c\mathcal{F}}^3\right)^2 + \left(\mu_{c\mathcal{G}}^3 - \nu_{c\mathcal{G}}^3\right)^2\right]}\right). \tag{24}
$$

Theorem 19. *If $c\mathcal{F} = \langle \mu_{c\mathcal{F}}, \nu_{c\mathcal{F}}; \rho_{cF}\rangle$ and $c\mathcal{G} = \langle \mu_{c\mathcal{G}}, \nu_{c\mathcal{G}}; \rho_{cG}\rangle$ are CFFNs, then all of the subsequent traits hold:*

(i) $0 \leq CFFCDM(c\mathcal{F}, c\mathcal{G}) \leq 1$

(ii) $CFFCDM(c\mathcal{F}, c\mathcal{G}) = CFFCDM(c\mathcal{G}, c\mathcal{F})$

(iii) $CFFCDM(c\mathcal{F}, c\mathcal{G}) = 0$ *iff* $c\mathcal{F} = c\mathcal{G}$

The above theorem is also true for CFFEDM.

5. Multicriteria Decision-Making Using Circular Fermatean Fuzzy Aggregation Operators and Distance Measures

We use our suggested AOs and DMs in MCDM in the phases that are as follows:

Step 1: The choices $\{cA_1, cA_2, \ldots, cA_m\}$ and essential criteria $\{cC_1, cC_2, \ldots, cC_n\}$ with weights $\{c\omega_1, c\omega_2, \ldots, c\omega_m\}$ that are nonnegative with all their sum 1 and decision experts $\{DE_1, DE_2, \ldots, DE_k\}$ who are representatives in associated sectors are identified for the establishment of a decision-making framework.

Step 2: Making reference to the scale shown in Table 2, the FF decision matrix (FFDM) in terms of FFNs executed up of FF linguistic terms pertaining to the DEs' perspectives is brought out.

Step 3: The cost criteria FFNs are replaced by their complement to get the normalised FFDM (NFFDM).

Step 4: With the advent of Proposition 11, a normalised CFFDM is generated from NFFDM.

Step 5: The values of the alternatives against each criteria are aggregated by using the formula stated in Remark 17.

Step 6: The distance measures established in Definition 18 are adopted for assessing how closure are the aggregated values to the ideal solution $\langle 1,0; 1\rangle$.

Step 7: The distances compiled in step 6 are sorted from minimal to high. The alternative with the closest distance gets first rank, followed by the choice with the next closest distance, and so on.

The MCDM flow is outlined in the flowchart shown in Figure 2.

5.1. Selection of Electric Autorickshaw in the Circular Fermatean Fuzzy Environment. Numerous everyday journeys are carried out on three-wheelers throughout Asia. The autorickshaw resembles a motorcycle in the front and has seating or cargo space for a group of people in the back. Rickshaws exist in a variety of forms, from fully enclosed boxes to more open versions topped with a simple shade canopy. They are descended from hand-pulled carts via a bicycle-based variation. They are often operated like taxis, with drivers who transport people and items from one location to another for a fee. They are designed for low speeds and urban situations. A conventional four-wheeler needs more power than a rickshaw does. Compared to an automobile, the electrification retrofits are quick and uncomplicated. In developing nations like India, three-wheeler autorickshaws play a significant role in the public transit

Table 2: FFNs for linguistic terms.

Linguistic terms	Acronym	FFN
Extremely good	EG	(1, 0)
Very very good	VVG	(0.95, 0.37)
Very good	VG	(0.85, 0.39)
Good	G	(0.76, 0.37)
Medium good	MG	(0.64, 0.49)
Medium	M	(0.56, 0.57)
Medium bad	MB	(0.43, 0.69)
Bad	B	(0.38, 0.77)
Very bad	VB	(0.28, 0.89)
Very very bad	VVB	(0.19, 0.99)
Extremely bad	EB	(0, 1)

system since they offer affordable and practical mobility. The use of electric autorickshaws will cut pollution greatly and be more affordable. In addition, compared to gas-powered cars, electric autorickshaws for public transportation significantly boost the country's economy by using power networks. The key feature of an E-autorickshaw is an electric motor, which together with batteries, replaces the internal combustion engine. Due to the swift progress of power electronics together with control technologies, electric vehicles (EVs) can presently exploit a wide variety of electric motor types [38]. One must primarily take into account performance, operating circumstances, efficiency, relative cost, and a number of other considerations when selecting the best motor for an E-autorickshaw. For electrically powered vehicles, such as three-wheeled autorickshaws, to cover short-range driving cycles, eminent-power density, less-weight, and inexpensive motors must be envisioned in terms of reliability and financial sustainability in the Indian environment. The central objective of this inquiry is to assess the diverse motors' functioning characteristics with the goal to decide which ones are most suited for the widespread use of battery-powered autorickshaws in India's transportation network. A three-wheeled autorickshaw powered by electricity costs approximately 1.12 lakhs to 2.80 lakhs in India with wider wheels and hydraulic suspension allowing for a maximum ground clearance of 220 mm. The seating capacity ranges from three passengers and a driver to five passengers and a driver. You can instantly track range, speed, position, and other statistics using the cloud-based NEMO (next-generation mobility) platform. Lithium-ion and lead-acid batteries are widely used [39, 40]. The motor has a range of 1.14 to 1.60 horsepower. It produces no emissions and makes no noise. Battery charging cycles might last anywhere from 4 to 10 hours. Six three-wheeled electric autorickshaws (cA_1, cA_2, cA_3, cA_4, cA_5, and cA_6) were reviewed by three experts (DE_1, DE_2, and DE_3) using five criteria, namely, cost (cC1), driving range (cC2), battery type and capacity (cC3), maintenance and charging time (cC4), and seating capacity (cC5) with a criterion weight vector $(0.2, 0.4, 0.1, 0.1, 0.2)$. The decision-making assistance of experts has been gathered and documented using FF phrases, as depicted in Table 3.

The FFDM, which is tabulated in Table 4, is constructed from Table 3 utilising the FFNs specified in Table 2.

By using the complement of the cost criteria, FFDM is normalised. Since cost of the vehicle (cC1) and charging time and maintenance (cC4) are nonbeneficial criteria, we swap out the AV and NAV to get the NFFDM tabulated in Table 5.

The NCFFDM in Table 6 is computed using Proposition 11 in Table 5. It enables us to estimate the decision makers' opinions regarding the alternative in comparison to the specific criteria as the region of a circle whose centre is at the AV and NAV.

Using the formula mentioned as follows, each alternative is aggregated with all its criteria along with a criterion weight, and the results are summarised in an aggregated circular Fermatean fuzzy matrix (ACFFDM) in Table 7:

$$
\begin{aligned}
\mathrm{CFFWA}_Q(\mathrm{cf}_1,\mathrm{cf}_2,\mathrm{cf}_3,\mathrm{cf}_4) &= \left\langle \sqrt[3]{1-\prod_{i=1}^{4}\left(1-\mu_{\mathrm{cf}_i}^3\right)^{c\omega_i}}, \prod_{i=1}^{4}\nu_{\mathrm{cf}_i}^{c\omega_i}, \prod_{i=1}^{4}\rho_{\mathrm{cf}_i}^{c\omega_i} \right\rangle,\\
\mathrm{CFFWA}_P(\mathrm{cf}_1,\mathrm{cf}_2,\mathrm{cf}_3,\mathrm{cf}_4) &= \left\langle \sqrt[3]{1-\prod_{i=1}^{4}\left(1-\mu_{\mathrm{cf}_i}^3\right)^{c\omega_i}}, \prod_{i=1}^{4}\nu_{\mathrm{cf}_i}^{c\omega_i}, \sqrt[3]{1-\prod_{i=1}^{4}\left(1-\rho_{\mathrm{cf}_i}^3\right)^{c\omega_i}} \right\rangle,\\
\mathrm{CFFWG}_Q(\mathrm{cf}_1,\mathrm{cf}_2,\mathrm{cf}_3,\mathrm{cf}_4) &= \left\langle \prod_{i=1}^{4}\mu_{\mathrm{cf}_i}^{c\omega_i}, \sqrt[3]{1-\prod_{i=1}^{4}\left(1-\nu_{\mathrm{cf}_i}^3\right)^{c\omega_i}}, \prod_{i=1}^{4}\rho_{\mathrm{cf}_i}^{c\omega_i} \right\rangle,\\
\mathrm{CFFWG}_P(\mathrm{cf}_1,\mathrm{cf}_2,\mathrm{cf}_3,\mathrm{cf}_4) &= \left\langle \prod_{i=1}^{4}\mu_{\mathrm{cf}_i}^{c\omega_i}, \sqrt[3]{1-\prod_{i=1}^{4}\left(1-\nu_{\mathrm{cf}_i}^3\right)^{c\omega_i}}, \sqrt[3]{1-\prod_{i=1}^{4}\left(1-\rho_{\mathrm{cf}_i}^3\right)^{c\omega_i}} \right\rangle.
\end{aligned}
\tag{25}
$$

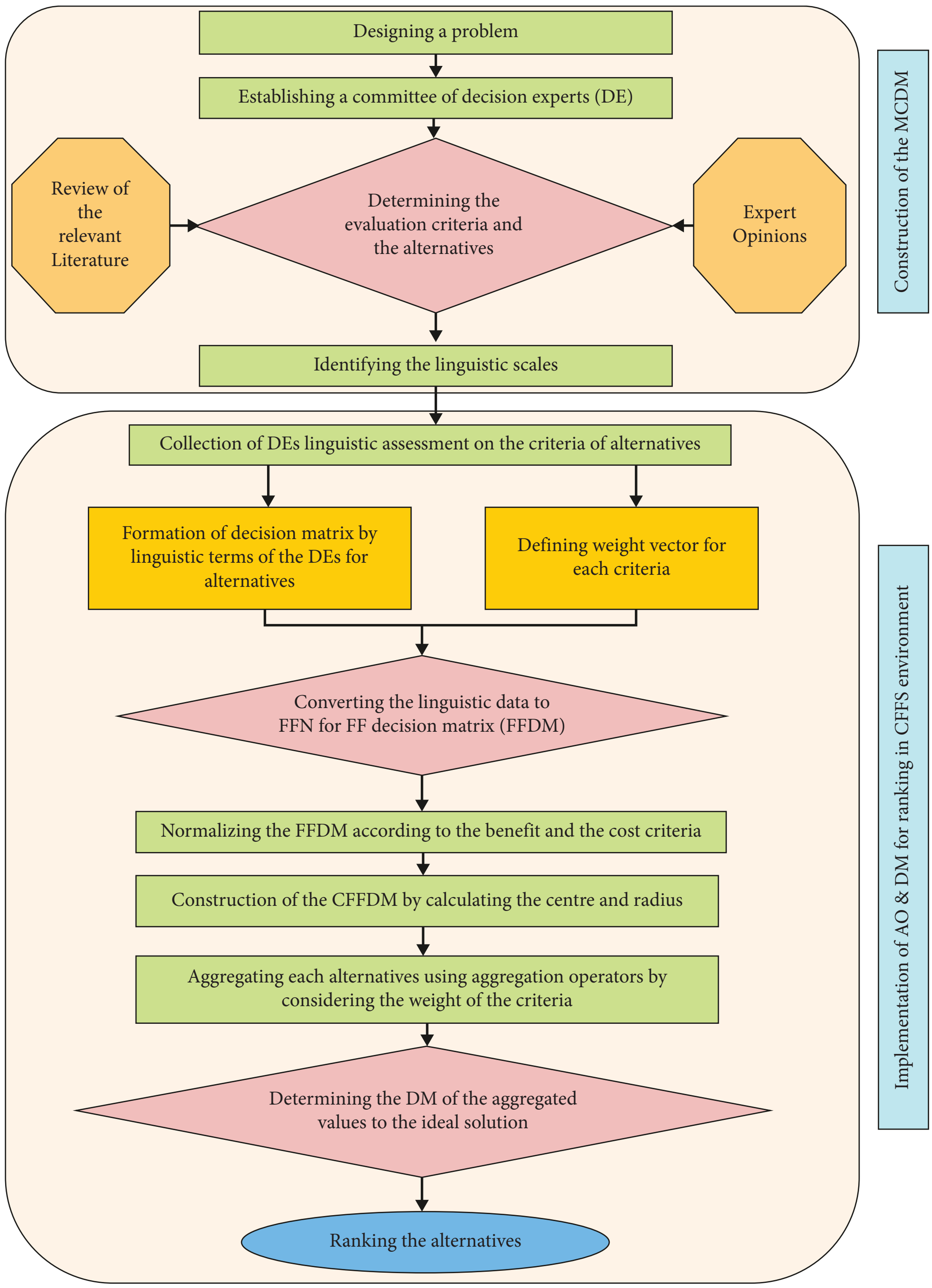

Figure 2: Flowchart of the MCDM.

CFFCDM and CFFEDM between the aggregated values and the ideal solution $\langle 1,0;1\rangle$ are obtained by formulas stated in equations (23) and (24) and are tabulated in Table 8. The options are also graded according to which of the aggregated value is closest to the ideal solution. To ensure the consistency of the proposed set, we compare the results with the existing methods FFWA, FFWG, FFWPA, and FFWPG via FFCDM and FFEDM. The ranks by using various methods are tabulated in Table 9.

Except for CFFEDM via CFFWA$_Q$, all the remaining aggregated values corresponding to both distance measures indicate that the alternative cA_2 is closer to the ideal

Table 3: Fermatean fuzzy linguistic decision matrix.

Decision experts	Alternatives	FF linguistic term for the criteria				
		cC1	cC2	cC3	cC4	cC5
DE_1	cA_1	G	M	B	VVB	MB
	cA_2	MG	EB	VVG	VG	VB
	cA_3	VVG	G	VVB	EB	MG
	cA_4	VVB	B	VG	VB	M
	cA_5	EG	MB	EG	VVG	G
	cA_6	M	VB	MG	VG	B
DE_2	cA_1	M	VVG	MB	G	B
	cA_2	EB	VG	VB	MG	VVG
	cA_3	G	EB	MG	VVG	VVB
	cA_4	B	VB	M	VVB	VG
	cA_5	MB	VVG	G	EG	EG
	cA_6	VB	VG	B	M	MG
DE_3	cA_1	MB	B	M	G	VVB
	cA_2	VB	VVG	EB	MG	VG
	cA_3	MG	VVB	G	VVG	EB
	cA_4	M	VG	B	VVB	VB
	cA_5	G	EG	MB	EG	VVG
	cA_6	B	MG	VB	M	VG

Table 4: FFDM.

DE	Alternatives	cC1	cC2	cC3	cC4	cC5
DE_1	cA_1	(0.76, 0.37)	(0.56, 0.57)	(0.38, 0.77)	(0.19, 0.99)	(0.43, 0.69)
	cA_2	(0.64, 0.49)	(0, 1)	(0.95, 0.37)	(0.85, 0.39)	(0.28, 0.89)
	cA_3	(0.95, 0.37)	(0.76, 0.37)	(0.19, 0.99)	(0, 1)	(0.64, 0.49)
	cA_4	(0.19, 0.99)	(0.38, 0.77)	(0.85, 0.39)	(0.28, 0.89)	(0.56, 0.57)
	cA_5	(1, 0)	(0.43, 0.69)	(1, 0)	(0.95, 0.37)	(0.76, 0.37)
	cA_6	(0.56, 0.57)	(0.28, 0.89)	(0.64, 0.49)	(0.85, 0.39)	(0.38, 0.77)
DE_2	cA_1	(0.56, 0.57)	(0.19, 0.99)	(0.43, 0.69)	(0.76, 0.37)	(0.38, 0.77)
	cA_2	(0, 1)	(0.85, 0.39)	(0.28, 0.89)	(0.64, 0.49)	(0.95, 0.37)
	cA_3	(0.76, 0.37)	(0, 1)	(0.64, 0.49)	(0.95, 0.37)	(0.19, 0.99)
	cA_4	(0.38, 0.77)	(0.28, 0.89)	(0.56, 0.57)	(0.19, 0.99)	(0.85, 0.39)
	cA_5	(0.43, 0.69)	(0.95, 0.37)	(0.76, 0.37)	(1, 0)	(1, 0)
	cA_6	(0.28, 0.89)	(0.85, 0.39)	(0.38, 0.77)	(0.56, 0.57)	(0.64, 0.49)
DE_3	cA_1	(0.43, 0.69)	(0.38, 0.77)	(0.56, 0.57)	(0.76, 0.37)	(0.19, 0.99)
	cA_2	(0.28, 0.89)	(0.95, 0.37)	(0, 1)	(0.64, 0.49)	(0.85, 0.39)
	cA_3	(0.64, 0.49)	(0.19, 0.99)	(0.76, 0.37)	(0.95, 0.37)	(0, 1)
	cA_4	(0.56, 0.57)	(0.85, 0.39)	(0.38, 0.77)	(0.19, 0.99)	(0.28, 0.89)
	cA_5	(0.76, 0.37)	(1, 0)	(0.43, 0.69)	(1, 0)	(0.95, 0.37)
	cA_6	(0.38, 0.77)	(0.64, 0.49)	(0.28, 0.89)	(0.56, 0.57)	(0.85, 0.39)

Table 5: NFFDM.

DE	Alternatives	cC1	cC2	cC3	cC4	cC5
DE_1	cA_1	(0.37, 0.76)	(0.56, 0.57)	(0.38, 0.77)	(0.99, 0.19)	(0.69, 0.43)
	cA_2	(0.49, 0.64)	(0, 1)	(0.95, 0.37)	(0.39, 0.85)	(0.89, 0.28)
	cA_3	(0.37, 0.95)	(0.76, 0.37)	(0.19, 0.99)	(1, 0)	(0.49, 0.64)
	cA_4	(0.99, 0.19)	(0.38, 0.77)	(0.85, 0.39)	(0.89, 0.28)	(0.57, 0.56)
	cA_5	(0, 1)	(0.43, 0.69)	(1, 0)	(0.37, 0.95)	(0.37, 0.76)
	cA_6	(0.57, 0.56)	(0.28, 0.89)	(0.64, 0.49)	(0.39, 0.85)	(0.77, 0.38)

Table 5: Continued.

DE	Alternatives	cC1	cC2	cC3	cC4	cC5
DE_2	cA_1	(0.57, 0.56)	(0.19, 0.99)	(0.43, 0.69)	(0.37, 0.76)	(0.77, 0.38)
	cA_2	(1, 0)	(0.85, 0.39)	(0.28, 0.89)	(0.49, 0.64)	(0.37, 0.95)
	cA_3	(0.37, 0.76)	(0, 1)	(0.64, 0.49)	(0.37, 0.95)	(0.99, 0.19)
	cA_4	(0.77, 0.38)	(0.28, 0.89)	(0.56, 0.57)	(0.99, 0.19)	(0.39, 0.85)
	cA_5	(0.69, 0.43)	(0.95, 0.37)	(0.76, 0.37)	(0, 1)	(0, 1)
	cA_6	(0.89, 0.28)	(0.85, 0.39)	(0.38, 0.77)	(0.57, 0.56)	(0.49, 0.64)
DE_3	cA_1	(0.69, 0.43)	(0.38, 0.77)	(0.56, 0.57)	(0.37, 0.76)	(0.99, 0.19)
	cA_2	(0.89, 0.28)	(0.95, 0.37)	(0, 1)	(0.49, 0.64)	(0.39, 0.85)
	cA_3	(0.49, 0.64)	(0.19, 0.99)	(0.76, 0.37)	(0.37, 0.95)	(1, 0)
	cA_4	(0.57, 0.56)	(0.85, 0.39)	(0.38, 0.77)	(0.99, 0.19)	(0.89, 0.28)
	cA_5	(0.37, 0.76)	(1, 0)	(0.43, 0.69)	(0, 1)	(0.37, 0.95)
	cA_6	(0.77, 0.38)	(0.64, 0.49)	(0.28, 0.89)	(0.57, 0.56)	(0.39, 0.85)

Table 6: NCFFDM.

Alternatives	cC1	cC2	cC3	cC4	cC5
cA_1	(0.57, 0.61; 0.25)	(0.43, 0.81; 0.30)	(0.47, 0.69; 0.15)	(0.71, 0.67; 0.55)	(0.36, 0.84; 0.23)
cA_2	(0.85, 0.46; 0.48)	(0.79, 0.72; 0.84)	(0.66, 0.84; 0.68)	(0.46, 0.72; 0.14)	(0.79, 0.65; 0.57)
cA_3	(0.42, 0.80; 0.18)	(0.53, 0.88; 0.56)	(0.62, 0.72; 0.50)	(0.72, 0.83; 0.88)	(0.45, 0.89; 0.46)
cA_4	(0.81, 0.43; 0.30)	(0.61, 0.74; 0.42)	(0.66, 0.62; 0.32)	(0.96, 0.23; 0.09)	(0.65, 0.68; 0.42)
cA_5	(0.50, 0.80; 0.54)	(0.86, 0.50; 0.52)	(0.80, 0.50; 0.54)	(0.26, 0.98; 0.26)	(0.91, 0.32; 0.33)
cA_6	(0.77, 0.44; 0.23)	(0.67, 0.66; 0.45)	(0.48, 0.75; 0.31)	(0.52, 0.69; 0.21)	(0.68, 0.60; 0.34)

Table 7: ACFFDM.

Alternatives	$CFFWA_Q$	$CFFWA_P$	$CFFWG_Q$	$CFFWG_P$
cA_1	(0.51, 0.74; 0.27)	(0.51, 0.74; 0.33)	(0.47, 0.77; 0.27)	(0.47, 0.77; 0.33)
cA_2	(0.78, 0.65; 0.57)	(0.78, 0.65; 0.72)	(0.75, 0.69; 0.57)	(0.75, 0.69; 0.72)
cA_3	(0.54, 0.84; 0.44)	(0.54, 0.84; 0.59)	(0.51, 0.85; 0.44)	(0.51, 0.85; 0.59)
cA_4	(0.76, 0.57; 0.33)	(0.76, 0.57; 0.38)	(0.69, 0.65; 0.33)	(0.69, 0.65; 0.38)
cA_5	(0.82, 0.54; 0.45)	(0.82, 0.54; 0.49)	(0.69, 0.74; 0.45)	(0.69, 0.74; 0.49)
cA_6	(0.67, 0.61; 0.33)	(0.67, 0.61; 0.37)	(0.65, 0.64; 0.33)	(0.65, 0.64; 0.37)

Table 8: CFFCDM and CFFEDM.

Distance measures	Aggregation operators	cA_1	cA_2	cA_3	cA_4	cA_5	cA_6
CFFCDM	$CFFWA_Q$	0.64424	0.29431	0.58872	0.38019	0.29947	0.44474
	$CFFWA_P$	0.62492	0.24777	0.53840	0.36178	0.28197	0.43097
	$CFFWG_Q$	0.67386	0.33570	0.60643	0.45052	0.44240	0.47217
	$CFFWG_P$	0.65455	0.28416	0.55610	0.43211	0.42940	0.45840
CFFEDM	$CFFWA_Q$	0.59835	0.36282	0.56236	0.44875	0.36243	0.49423
	$CFFWA_P$	0.57904	0.31128	0.51203	0.43034	0.34944	0.48047
	$CFFWG_Q$	0.61433	0.38979	0.57348	0.49529	0.47268	0.50780
	$CFFWG_P$	0.59502	0.33825	0.52315	0.47688	0.45969	0.49403

solution. As a result, cA_2 will be the best option of electronic autorickshaw pertaining to proposed AOs and DMs in the CFF set environment.

5.2. Visualisation of Results and Comparison Analysis. In order to check for accuracy, we compare and display the aggregated values as charts in this subsection. Figure 3 showcases the connections between all of the aggregated values.

Various CFFCDM and CFFEDM are demonstrated in Figure 4.

Figure 5 exhibits the association between $CFFWA_P$ with $CFFWG_P$ and $CFFWA_Q$ with $CFFWG_Q$.

The ranks obtained by CFFCDM and CFFEDM are displayed in Figure 6 as follows.

From the results, we obtain the ranking $cA_2 > cA_5 > cA_4 > cA_6 > cA_3 > cA_1$, and also, we can see that the results are consistent with the suggested methods.

Table 9: Ranking of alternatives by proposed methods and existing methods.

Distance measures	Aggregation operators	Ranks
CFFCDM	$CFFWA_Q$	$cA_2 > cA_5 > cA_4 > cA_6 > cA_3 > cA_1$
	$CFFWA_P$	$cA_2 > cA_5 > cA_4 > cA_6 > cA_3 > cA_1$
	$CFFWG_Q$	$cA_2 > cA_5 > cA_4 > cA_6 > cA_3 > cA_1$
	$CFFWG_P$	$cA_2 > cA_5 > cA_4 > cA_6 > cA_3 > cA_1$
CFFEDM	$CFFWA_Q$	$cA_5 > cA_2 > cA_4 > cA_6 > cA_3 > cA_1$
	$CFFWA_P$	$cA_2 > cA_5 > cA_4 > cA_6 > cA_3 > cA_1$
	$CFFWG_Q$	$cA_2 > cA_5 > cA_4 > cA_6 > cA_3 > cA_1$
	$CFFWG_P$	$cA_2 > cA_5 > cA_4 > cA_6 > cA_3 > cA_1$
Ranking by existing methods		
FFCDM	FFWA	$cA_5 > cA_2 > cA_4 > cA_6 > cA_3 > cA_1$
	FFWG	$cA_2 > cA_5 > cA_4 > cA_6 > cA_3 > cA_1$
	FFWPA	$cA_2 > cA_5 > cA_4 > cA_6 > cA_3 > cA_1$
	FFWPG	$cA_2 > cA_5 > cA_4 > cA_6 > cA_3 > cA_1$
FFEDM	FFWA	$cA_5 > cA_2 > cA_4 > cA_6 > cA_3 > cA_1$
	FFWG	$cA_2 > cA_5 > cA_4 > cA_6 > cA_3 > cA_1$
	FFWPA	$cA_2 > cA_5 > cA_4 > cA_6 > cA_3 > cA_1$
	FFWPG	$cA_2 > cA_5 > cA_4 > cA_6 > cA_3 > cA_1$

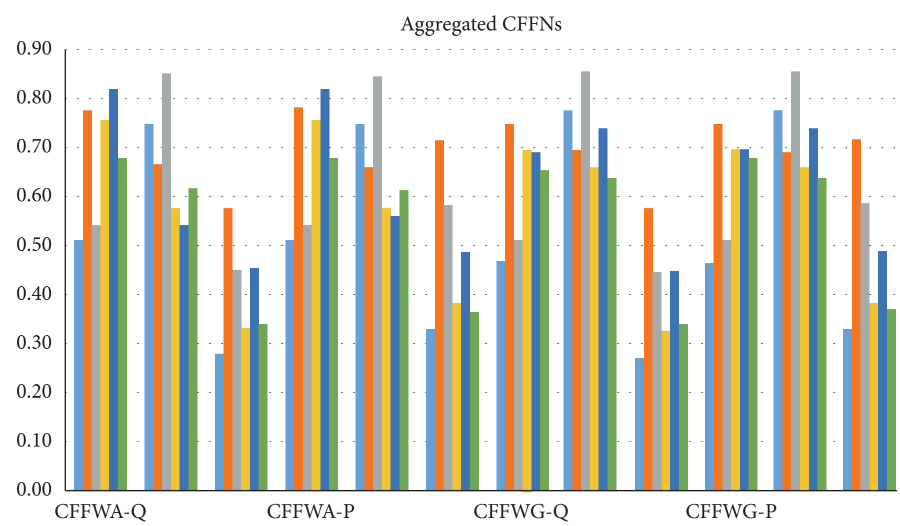

Figure 3: Graphical representation of aggregated CFFNs.

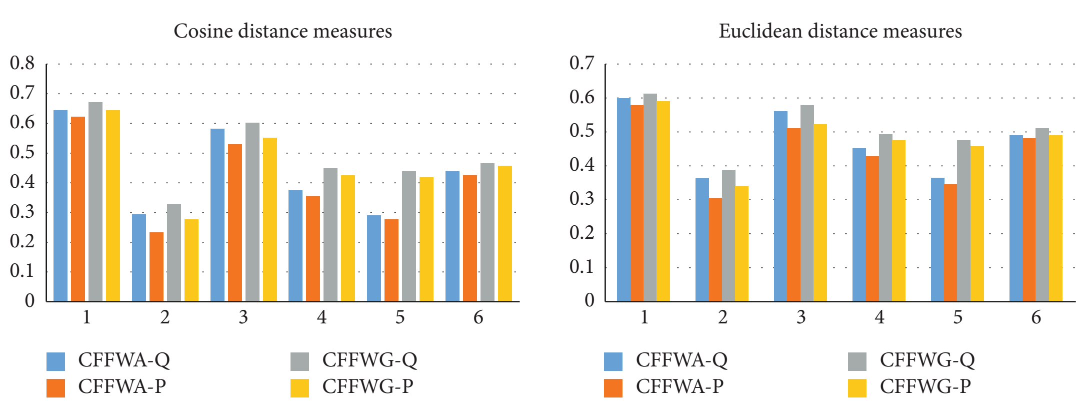

Figure 4: Comparison of CFF cosine and CFF Euclidean measures.

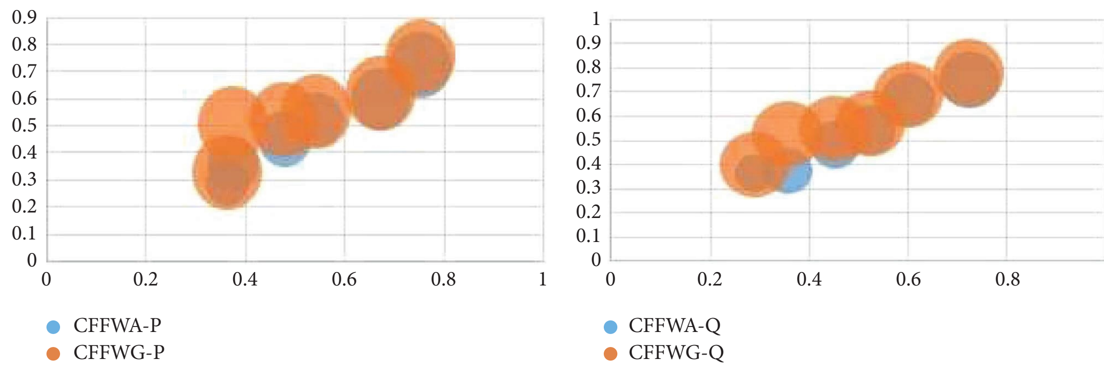

Figure 5: Representation of values of CFFWA over CFFWG.

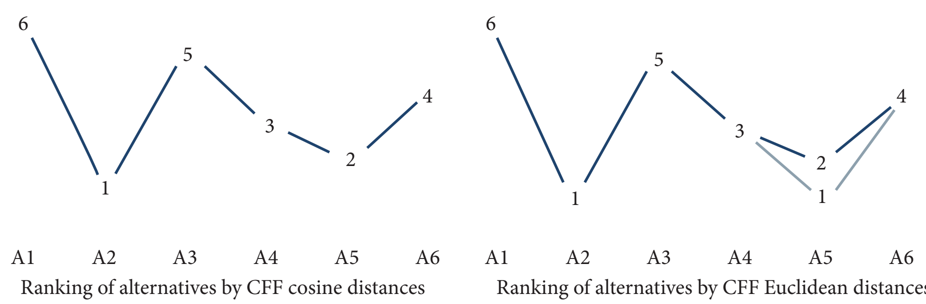

Figure 6: Ranking by distance measures.

While this approach offers a number of benefits, it also has limitations. The circle's radius will grow significantly, and its centre will stray from its true value if there is a significant difference in one of the decision makers. It can be overcome by eliminating such diverse values.

6. Conclusion and Future Work

The circular Fermatean fuzzy set has been put forwarded in this paper. The main objective of a CFF set is its circular framework. The CFF set is a circle with centre AV and NAV with radius not greater than $\sqrt{2}$. So, the CFF set in two dimensions can deal high-order imprecise data by wrapping in a circle. The algebraic operations and their properties among CFF sets are explored. By implementing the FF t-norm and t-conorm, CFF weighted and geometric operations are provided in the claimed CFF set environment. CFFCDM and CFFEDM are further discussed for the purpose of computing the distance between the aggregated value and the ideal solution. Proposed AOs and DMs serve as tools to choose electric autorickshaw based on multiple considerations. Visualisation and comparison ensure the reliability of the proposed AOs and DMs.

In the FFS and IVFFS settings, the input data are a point and an interval that are in one dimension, whereas in the CFF set environment, the input data are a circle in two dimensions. Due to its extensive range compared to FFS and IVFFS, CFF sets are extremely advantageous in managing uncertainty. As an instance, it can be applied far more effectively in pattern classification, deep learning, machine learning, and other areas such as medical diagnostics.

Future research will be focused on the study of various possible t-norm and t-conorm operators for aggregation of CFF sets. Also, different distance and similarity metrics can be defined and will be applied between the clusters consisting of FFS by converting them to the CFF set in computational image analysis and medical treatment. Also, circular representation can be applied in the generalized fuzzy set like quasirung orthopair fuzzy set and for its specific values $q = 1$ and $q = 4$. The aggregation operators of developed sets will be used on group decision-making.

Data Availability

No data were used to support the findings of this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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