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Review Article

Topological Excitations in Quantum Spin Systems

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The origin and significance of topological excitations in quantum spin models in low dimensions are presented in detail. Besides a general review, our own work in this area is described in great depth. Apart from theoretical analysis of the existence and properties of spin vortices and antivortices, the possible experimental consequences and signatures are also highlighted. In particular, the distinguishing features between the even and odd charged topological excitations are brought out through a detailed analysis of the topological term in the quantum action. Moreover, an interesting symmetry property is predicted between the excitations from a ferromagnetic model and an antiferromagnetic model. Through a novel approach of ours, a bridge is established between field theoretical formalism and the well-known statistical mechanical treatment of Berezinskii-Kosterlitz-Thouless (BKT) transition involving these topological excitations. Furthermore, a detailed phenomenological analysis of the experimentally observed static and dynamic magnetic properties of the layered magnetic materials, possessing XY anisotropy in the in-plane spin-spin couplings, is undertaken to test the theoretical predictions regarding the behaviour of these excitations. The importance and the crucial role of quantum spin fluctuations in these studies are also brought out very clearly by our analysis.

1. Introduction

The question of existence of nonconventional excitations in spin systems has been probed for the last four decades. The seminal papers of Kosterlitz and Thouless (1973) and by Berezinskii (1971, 1972) laid a foundation for this investigation [1–3]. The models considered there involved classical spins situated on two dimensional (2D) spatial lattice. Since then, developments in various branches of condensed matter physics have prompted investigation with quantum spin models in low-dimensional systems as well.

Recent progress in the field of low-dimensional strongly correlated electronic systems and in particular their magnetic and transport properties both from theoretical and experimental standpoints has stimulated and motivated the urgent need for the search of non-conventional spin excitations in purely quantum systems [4–8].

Many of these quantum systems exhibit topological character in their excitations, single particle and/or collective. It is therefore very important that a proper understanding of their origin and properties is taken up.

The collective excitations in quantum spin systems can broadly be divided into two categories: (i) conventional ones, namely, spin waves (quantised as magnons) and (ii) topological ones, namely, vortices and merons. The spin wave modes exist in the magnetically long range ordered phases of a Heisenberg magnet and are very stable and well defined in the low temperature regime. They are low energy excitations and represent a coherent precessional motion of all the spins around the direction of spontaneous magnetization on a lattice. In the short range ordered phase, these spin wave like modes become highly damped and fragile. In contrast, vortices and merons are high energy excitations and generally occur at higher temperatures. They exhibit large deformations and twists of the planar components and slow tilting of the out of plane component of the spins over a finite region of the lattice. Besides, the spin configurations contained in these topological excitations have the property that they cannot be shrunk to a point.

As a first step towards the understanding of these topological excitations, it would be appropriate to investigate the two-dimensional quantum XY or XY-anisotropic Heisenberg

models. With the results of the analysis for the corresponding classical models being known, it is very instructive as well as natural to probe the quantum effects arising particularly in the low spin cases [9–11]. A pertinent step in this direction would be a semiclassical approach based on phenomenology. Parallel to this, a full fledged quantum treatment involving combination of field theoretic approach and conventional many body techniques can also be pursued.

2. Methodology

We had originally adopted a phenomenological approach to $S = 1/2$ quantum Heisenberg ferromagnet and antiferromagnet on square lattice in two dimensions [12]. For K_2CuF_4, a spin-1/2 ferromagnet with small XY anisotropy, the static results from experiments and classical numerical calculations agree quite well with the Berezinski-Kosterlitz-Thouless (BKT) proposed phenomenology [1–3]. For La_2CuO_4, however, a spin-1/2 antiferromagnet, the static results from the experiments and classical renormalization group calculations exhibit a substantial disagreement with BKT-like phenomenology. The spin dynamics display even more interesting features in the above two systems. The inelastic neutron scattering experiments performed on the above two materials show a "central peak", that is, a peak at $\omega = 0$ in the constant q-scan for the dynamical structure function for both of them [13–16]. In the first system it occurs in the low q-regime, whereas for the second material, it takes place somewhat in the midway towards the zone boundary.

The application of the BKT inspired phenomenological treatment reproduces the central peak as well for the first system; however, the dynamical structure factor becomes negative in a large regime of the q-space and hence exhibits unphysical behaviour [17]. This signals the failure and limitation of the semiclassical treatment involving BKT (see Figure 1) and calls for a fully fledged quantum treatment [17]. In the second system, the results from BKT-like phenomenology for the experimental observations from inelastic neutron scattering experiment involving staggered spin field and the RG calculational results available for the very weak XY-anisotropy limit disagree quantitatively by a large factor (about 4) [12]. This calls for a thorough investigation. The experimental results for static and dynamic correlations are presented in Figures 2 and 3, respectively [12]. The origin of this discrepancy might as well be in the presence of enormous quantum fluctuations in spin 1/2 quantum antiferromagnet on 2D lattice and the failure of the classical/semiclassical treatment. This is more vigorous than that expected for a ferromagnet on a 2D lattice. However, at zero temperature, the quantum simulation performed by several theoretical groups brings out the existence of topological excitations (bound vortex-antivortex pair) even for an XY-like ferromagnet on 2D lattice [18–20]. On the other hand, even classical Monte-Carlo simulation allowing tunnelling between different topological sectors seems to wash out the BKT transition [21].

In view of the above developments and confusion, a long programme was taken up by the present authors to systematically analyse the interacting quantum spin models, namely,

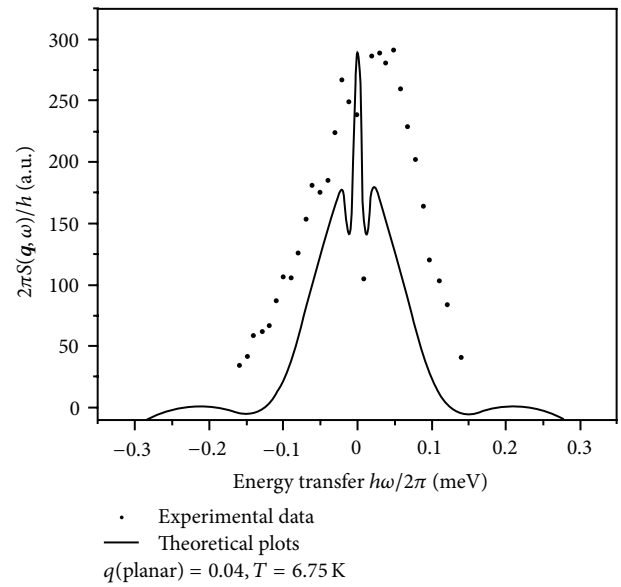

Figure 1: Total dynamical structure function versus energy for K_2CuF_4.

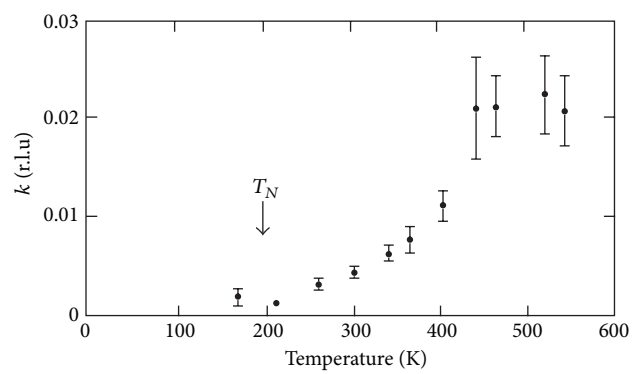

Figure 2: Inverse of correlation length versus temperature in 2D for La_2CuO_4.

XY-anisotropic quantum Heisenberg spin systems, on low-dimensional lattices. Our method is based on the application of the path integral technique to derive an effective action corresponding to the spin Hamiltonian. The major aim was to explore whether a nonvanishing "topological term" arises in this effective action and if so, to what extent it indicates a possibility for the existence of topological excitations in the system. The presence of such a term in fact indicates a very strong possibility for the existence of topological excitations in the system and furthermore the numerical magnitude of this term (always an integer) represents the "winding number" for such configurations.

The approach we adopted is the following [12, 17, 22–26].

(1) Calculate the partition function of the quantum spin model by making use of Trotter's formula and working in the spin coherent state basis.

(2) Generate the corresponding "action" by taking a quasicontinuum approximation.

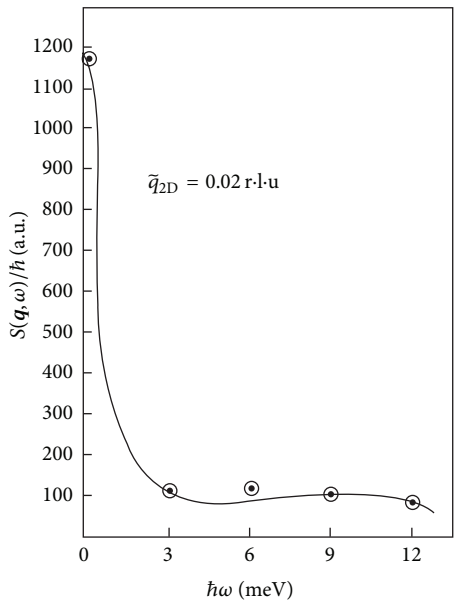

FIGURE 3: Convoluted dynamical structure function $S(\mathbf{q}, \omega)$ versus ω from neutron scattering experiment (for La_2CuO_4) in the constant q-scan in 2D.

(3) Rearrange and combine the various terms and contributions to extract out an effective nonlinear sigma model type part and a so-called topological part.

(4) The sigma model part essentially represents spin wave modes, whereas the topological part implies the breaking up of the configurations into distinct topological sectors. For antiferromagnetic models, the above decomposition into two parts can be done easily. For ferromagnetic models, however, it is found that there can be another additional 3rd piece which is nontopological but has an interesting geometrical interpretation.

(5) Evaluate the topological part both in the long wavelength and medium wavelength limits. In the case of nonvanishing contribution, the topological term has been explicitly calculated for vortices (antivortices) in an XY-anisotropic quantum Heisenberg ferromagnet in the low-q regime and antiferromagnet in the medium wavelength limit, on a square lattice.

(6) When the net topological term is nonzero, we expect to see its signature in the partition function of the corresponding spin model. This also leads and motivates us to study the static and the dynamic spin-spin correlation functions in this model theoretically and compare them with the experimental results from materials which are very good realizations of these models.

(7) Besides, various important physical and mathematical properties of the topological term and the topological excitations were studied. In fact, it was found that they carry enormous physical significance like magnitude of the vortex excitation energy, the thermodynamic feasibility of existence of these excitations, and last but not the least the characterisation of the even and odd winding number sectors. This evaluation of the properties backed by the suitable calculations also threw light on many more dramatic and fundamental issues, as will be discussed later.

(8) For the time being for pragmatic reasons, the calculations for correlation functions have so far been based on combinations of both analytical and numerical techniques. Moreover, the phenomenological as well as the microscopic approaches have been made use of in this investigation.

(9) The temperature dependence of the experimentally obtained static correlation functions has been studied thoroughly. The materials which are candidates for exhibiting BKT transitions are in reality very often layered or quasi-two dimensional. Moreover, the spin-spin couplings in the real materials often have additional contributions with the theoretical model being investigated. It is therefore very important to look into the right temperature regime to extract the pure spatial two-dimensional effects and those appropriate to the theoretical spin model.

(10) The above procedure is followed for dynamical correlation function as well. In this case, the detailed energy dependence of dynamical structure factor $S(\mathbf{q}, \omega)$ in constant q-scans from experiments is analysed very thoroughly. The instrumental resolution function is also taken into account while making comparison with the experimental data from inelastic neutron scattering experiments.

3. Analytical Description of the Quantum Action

It is algebraically quite complicated to write down the partition function of a quantum spin system in low dimensions containing huge amount of quantum fluctuations, using conventional angular momentum basis. However, making use of coherent spin state basis, one can get around these difficulties. Furthermore, one can formulate a path integral representation of the quantum partition function by introducing coherent spin states at each lattice point [22]. It is only this formulation where the topological term, that is, the Wess-Zumino (WZ) type term, becomes manifested explicitly in the action corresponding to the spin system. This formulation was first established by Fradkin and Stone in the long wave length limit of an isotropic quantum Heisenberg antiferromagnetic chain [27]. In this limit, the system takes the form of a nonlinear sigma model with a Hopf term, that is, the WZ term in 1 + 1 dimensions, and distinguishes the striking difference between the spectra corresponding to integer and half-integer spins [28, 29].

Making use of Trotter formula

$$Z = \mathrm{Tr}\, e^{-\beta H} = \lim_{N_t \to \infty, \delta t \to 0} \left(e^{-\delta t H}\right)^{N_t} \tag{1}$$

and inserting resolution of identity $I = \int d\mu(\mathbf{n})|\mathbf{n}\rangle\langle\mathbf{n}|$, one can write down the partition function as (chapter 5 in [22] and references therein).

$Z = \int \mathcal{D}\mathbf{n} e^{-\mathcal{S}_E[\mathbf{n}]}$, where the quantum Euclidean action $\mathcal{S}_E$ for the coherent spin fields $\mathbf{n}(\mathbf{r}, t)$ is given by

$$\mathcal{S}_E = -is \sum_{\mathbf{r}} \mathcal{S}_{\mathrm{WZ}}[\mathbf{m}(\mathbf{r}, t)] + \int_0^\beta dt \mathcal{H}(\mathbf{n}), \tag{2}$$

where "s" is the magnitude of the spin ($s = 1/2$ in the present case), $\mathbf{r}$ is the position vector of a lattice site, and $\mathcal{H}(n)$ is the expectation value of the spin Hamiltonian ($\mathcal{H}(S)$) ((3) and (6) below) in a coherent state on the lattice [23, 24].

The variable t denotes the pseudo-time (Euclidean time) appropriate to the above fields and has the dimension of inverse temperature. The parameter β stands for $1/kT$ as usual; T is the real thermodynamic temperature of the spin system. The quantity $\mathcal{S}_{\mathrm{WZ}}[\mathbf{m}(\mathbf{r}, t)]$ is the Wess-Zumino term (WZ term) (see (4) below), corresponding to a single spatial lattice point $\mathbf{r}$ at a fixed time t [25]. At this stage, let us mention that the WZ term in the action is analytically obtained in this way and thus it predicts the possibility of topological excitations in the quantum spin system. The spin operator $S(\mathbf{r}, t)$ is connected with the coherent spin variable $\mathbf{n}(\mathbf{r}, t)$ in the following fashion [22]:

$$\begin{aligned} \langle \mathbf{n}|\mathbf{S}|\mathbf{n}\rangle &= s\mathbf{n}, \\ \mathcal{H}(\mathbf{n}) &= \langle \mathbf{n}|\mathcal{H}(\mathbf{S})|\mathbf{n}\rangle, \\ |\mathbf{n}\rangle &= \prod_{\mathbf{r}} |\mathbf{n}(\mathbf{r}, t)\rangle, \end{aligned} \tag{3}$$

where $\prod$ denotes the direct product of all coherent spin states over the spatial lattice and $|\mathbf{n}\rangle$ stands for a coherent state on the lattice. The variable t denotes the pseudo-time having the dimension of inverse temperature with $\beta = 1/kT$ as usual; T is the real thermodynamic temperature of the spin system. The WZ term S_{WZ} corresponding to a single spatial lattice point $\mathbf{r}$ at a fixed time "t" is given as follows:

$$\begin{aligned} \mathcal{S}_{\mathrm{WZ}}[\mathbf{m}(\mathbf{r}, t)] = \int_0^\beta dt \int_0^1 d\tau \mathbf{m}(\mathbf{r}, t, \tau) \\ \cdot \partial_t \mathbf{m}(\mathbf{r}, t, \tau) \wedge \partial_\tau \mathbf{m}(\mathbf{r}, t, \tau) \end{aligned} \tag{4}$$

with $\mathbf{m}(\mathbf{r}, t, 0) \equiv \mathbf{n}(\mathbf{r}, t)$, $\mathbf{m}(\mathbf{r}, t, 1) \equiv \mathbf{n}_0(\mathbf{r})$, and $\mathbf{m}(\mathbf{r}, 0, \tau) \equiv \mathbf{m}(\mathbf{r}, \beta, \tau)$, $t \in [0, \beta]$, $\tau \in [0, 1]$, and β is the usual inverse temperature.

The expression in (4) is the area of the cap bounded by the trajectory Γ parametrized by $\mathbf{n}(\mathbf{r}, t)[\equiv (n_1(\mathbf{r}, t), n_2(\mathbf{r}, t), n_3(\mathbf{r}, t))]$ on the sphere:

$$\mathbf{n}(\mathbf{r}, t) \cdot \mathbf{n}(\mathbf{r}, t) = 1. \tag{5}$$

Furthermore, the fields $\mathbf{m}(\mathbf{r}, t, \tau)$ are the fields in the higher dimensional (t, τ)-space and the boundary values $\mathbf{n}(\mathbf{r}, t)$ are the coherent spin fields. The field $\mathbf{n}_0(\mathbf{r})$ is the fixed point $(0, 0, 1)$ on the above sphere and the state vector $|\mathbf{n}(\mathbf{r}, t)\rangle$ appearing on the right-hand side of (3) is the spin coherent state at a single lattice point $\mathbf{r}$. For the two-dimensional lattice, we express $\mathbf{n}(\mathbf{r}, t) \equiv \mathbf{n}(ia, ja, t)$ as $\mathbf{n}(ia, ja)$ for brevity, with a being the lattice spacing. The spin Hamiltonian $\mathcal{H}(S)$ is given by

$$\mathcal{H}(\mathbf{S}) = -g \sum_{\langle \mathbf{r}, \mathbf{r}' \rangle} \tilde{\mathbf{S}}(\mathbf{r}) \cdot \tilde{\mathbf{S}}(\mathbf{r}') - g\lambda_z \sum_{\langle \mathbf{r}, \mathbf{r}' \rangle} S_z(\mathbf{r}) S_z(\mathbf{r}'), \tag{6}$$

where $g > 0$ corresponding to anisotropic Heisenberg spin system of XXZ type with ferromagnetic coupling and $g < 0$ corresponding to that with antiferromagnetic coupling. We represent here the explicit calculational procedure for the quantity $\mathcal{S}_{\mathrm{WZ}}[\mathbf{m}(\mathbf{r}, t)]$ given by (4).

It is possible to evaluate only the "difference" of $\mathcal{S}_{\mathrm{WZ}}[\mathbf{m}(ia, ja)]$ terms from two neighbouring lattice sites in terms of the coherent spin fields $\mathbf{n}(ia, ja)$. Thus, to extract the topological-like contribution from $\mathcal{S}_{\mathrm{WZ}}^{\mathrm{stagg}}$, we use the following expressions for the above "difference" [22]:

$$\begin{aligned} \delta_x \mathcal{S}_{\mathrm{WZ}}[\mathbf{m}(\mathbf{r})] &\equiv \mathcal{S}_{\mathrm{WZ}}[\mathbf{m}(ia, ja)] - \mathcal{S}_{\mathrm{WZ}}[\mathbf{m}((i-1)a, ja)] \\ &= \int_0^\beta dt \left[\delta_x \mathbf{n} \cdot (\mathbf{n} \wedge \partial_t \mathbf{n})\right](\mathbf{r}), \\ \delta_y \mathcal{S}_{\mathrm{WZ}}[\mathbf{m}(\mathbf{r})] &\equiv \mathcal{S}_{\mathrm{WZ}}[\mathbf{m}(ia, ja)] - \mathcal{S}_{\mathrm{WZ}}[\mathbf{m}(ia, (j-1)a)] \\ &= \int_0^\beta dt \left[\delta_y \mathbf{n} \cdot (\mathbf{n} \wedge \partial_t \mathbf{n})\right](\mathbf{r}), \end{aligned} \tag{7}$$

where

$$\begin{aligned} \delta_x \mathbf{n}(ia, ja) &\equiv \mathbf{n}(ia, ja) - \mathbf{n}((i-1)a, ja), \\ \delta_y \mathbf{n}(ia, ja) &\equiv \mathbf{n}(ia, ja) - \mathbf{n}(ia, (j-1)a) \end{aligned} \tag{8}$$

with $\mathbf{r} \equiv (ia, ja)$, and $i, j = 1, 2, \ldots, 2N$.

Now, the total Wess-Zumino term on the lattice is as given in the right-hand side of (2) and has to be evaluated in the case of ferro- or antiferromagnet.

It is in the case of antiferromagnet that we can calculate the term $\mathcal{S}_{\mathrm{WZ}}^{\mathrm{stagg}}$ (see (9)) exactly since it appears with difference of two terms in the neighbouring lattice points owing to staggering. This is not so in the case of ferromagnet since we will not get alternating positive and negative signs in the the lattice sum of the WZ term as there is no staggering. However, we can extract the topological information content from the Wess-Zumino term as demonstrated by (10) and (11) (for more details see [23, 24])

$$\begin{aligned} 2\mathcal{S}_{\mathrm{WZ}}^{\mathrm{stagg}} &= 2\sum_{i,j} (-1)^{i+j} \mathcal{S}_{\mathrm{WZ}}[\mathbf{m}(ia, ja)] \\ &= \sum_{i,j} (-1)^{i+j} \int_0^\beta dt \left([\mathbf{n}(i, j) - \mathbf{n}(i, j-1)]\right. \\ &\qquad \cdot (\mathbf{n} \wedge \partial_t \mathbf{n})(i, j) \\ &\qquad + [\mathbf{n}(i, j) - \mathbf{n}(i-1, j)] \end{aligned}$$

$$\cdot (\mathbf{n} \wedge \partial_t \mathbf{n})(i, j) - [\mathbf{n}(i, j-1) - \mathbf{n}(i-1, j-1)] \cdot (\mathbf{n} \wedge \partial_t \mathbf{n})(i, j-1) - [\mathbf{n}(i-1, j) - \mathbf{n}(i-1, j-1)] \cdot (\mathbf{n} \wedge \partial_t \mathbf{n})(i-1, j)). \tag{9}$$

In the case of ferromagnet, we have the following expression for the WZ term:

$$\begin{aligned} \mathcal{S}_{\mathrm{WZ}} &= \sum_{i,j=1}^{2N} \mathcal{S}_{\mathrm{WZ}}[\mathbf{m}(ia, ja)] \\ &= \sum_{i,j=1}^{2N} \Big[\frac{1}{2} \mathcal{S}_{\mathrm{WZ}}[\mathbf{m}((i-1)a, ja)] \\ &\quad + \frac{1}{2} \mathcal{S}_{\mathrm{WZ}}[\mathbf{m}(ia, (j-1)a)] \\ &\quad + \frac{1}{2} \delta_x \mathcal{S}_{\mathrm{WZ}}[\mathbf{m}(ia, ja)] \\ &\quad + \frac{1}{2} \delta_y \mathcal{S}_{\mathrm{WZ}}[\mathbf{m}(ia, ja)] \Big]. \end{aligned} \tag{10}$$

Now, one can extract the quantity which can be written in terms of the coherent spin fields obtained from difference of WZ terms at two neighbouring lattice sites. In the case of one-dimensional antiferromagnetic system, this turns out to be exactly the winding number in the long wavelength limit [22, 27–29]. For a unified description of Heisenberg ferromagnetic and antiferromagnetic spin systems in one and two spatial dimensions, we refer to [25]. In more precise terms the difference between WZ terms from two neighbouring lattice sites is analogous to the surface density of the above winding number in terms of coherent spin field $\mathbf{n}$ expressed in a discretized form.

$$\begin{aligned} 2\mathcal{S}_{\mathrm{WZ}} &= \sum_{Q} N_Q \sum_{i=1}^{(Q+1)^2} \mathcal{A}_i \\ &\quad - \sum_{i,j=1}^{2N} \Big\{ \int_0^\beta dt \mathbf{n}((i-1)a, ja) \cdot (\mathbf{n} \wedge \partial_t \mathbf{n})(ia, ja) \\ &\quad + \int_0^\beta dt \mathbf{n}(ia, (j-1)a) \\ &\quad \cdot (\mathbf{n} \wedge \partial_t \mathbf{n})(ia, ja) \Big\} \\ &= \mathcal{S}_{\mathrm{WZ}}^{n\mathrm{top}} + \mathcal{S}_{\mathrm{WZ}}^{\mathrm{top}}, \end{aligned} \tag{11}$$

where

$$\mathcal{S}_{\mathrm{WZ}}^{n\mathrm{top}} = \sum_{Q} N_Q \sum_{i=1}^{(Q+1)^2} \mathcal{A}_i$$

$$\mathcal{S}_{\mathrm{WZ}}^{\mathrm{top}} = -\sum_{i,j=1}^{2N} \Big\{ \int_0^\beta dt \mathbf{n}((i-1)a, ja) \cdot (\mathbf{n} \wedge \partial_t \mathbf{n})(ia, ja) + \int_0^\beta dt \mathbf{n}(ia, (j-1)a) \cdot (\mathbf{n} \wedge \partial_t \mathbf{n})(ia, ja) \Big\}. \tag{12}$$

The term $\mathcal{A}_i$ in the right-hand side of the first equation in (12) above stands for the "area of the cap" at the ith vertex within a vortex of topological charge Q and N_Q is the number of such vortices. By topological charge Q, we mean that if we go around the vortex (antivortex) once, the spin at any vertex undergoes a rotation of $2\pi Q$ $(-2\pi Q)$.

It should be kept in mind, however, that the above arguments involving the low temperature approximation to the nontopological terms do not interfere with our general formalism for studying the topological excitations, which is valid at all temperatures. The term $\mathcal{S}_{\mathrm{WZ}}^{\mathrm{top}}$ represents the topological content in the WZ term in the case of ferromagnet, whereas in the case of antiferromagnet (9) represents the topological content in the entire WZ term. The existence of a nonvanishing topological term on the lattice implies that the entire spin configuration is fragmented into distinct topological sectors characterised by winding numbers or topological charges.

4. The Quantum Action

The quantum action corresponding to antiferromagnet is given by [23]

$$\begin{aligned} \mathcal{S}_E^{\mathrm{stagg}} &= -is \sum_{i,j} (-1)^{i+j} \mathcal{S}_{\mathrm{WZ}}[\mathbf{m}(i, j)] \\ &\quad + \int_0^\beta dt \Big\{ -|g| s^2 \sum_{i,j} [\tilde{\mathbf{n}}(i, j) \cdot \tilde{\mathbf{n}}(i+1, j) \\ &\quad + \tilde{\mathbf{n}}(i, j) \cdot \tilde{\mathbf{n}}(i, j+1)] \\ &\quad - |g| \lambda_z s^2 \sum_{(i,j)} [n_3(i, j) n_3(i+1, j) \\ &\quad + n_3(i, j) n_3(i, j+1)] \Big\} \\ &\quad - \int_0^\beta dt \sum_{(i,j)} a^2 \lambda_{i,j} [\mathbf{n}^2(i, j) - 1], \end{aligned} \tag{13}$$

where we have assumed the spin configurations to have correlations exhibiting a bipartite symmetry when the temperature is not too high [26].

The quantum action for ferromagnet is given by [24]

$$\begin{aligned}\mathcal{S}_E = -is\sum_{i,j}\mathcal{S}_{\mathrm{WZ}}\left[\mathbf{m}\left(i,j\right)\right] &+ \int_0^\beta dt\left[-\left|g\right|s^2\sum_{\langle(i,j),(i',j')\rangle}\tilde{\mathbf{n}}\left(i,j\right)\cdot\tilde{\mathbf{n}}\left(i',j'\right)\right.\\ &\left.-\left|g\right|\lambda_z s^2\sum_{\langle(i,j),(i',j')\rangle}n_3\left(i,j\right)n_3\left(i',j'\right)\right]\\ &+\frac{1}{2}\int_0^\beta dt\sum_{(i,j)}a^2\lambda_{i,j}\left[\mathbf{n}^2\left(i,j\right)-1\right].\end{aligned} \tag{14}$$

The above actions can be derived by making use of the equations (2) to (6) (and chapter 5 in [22]). The integral containing the expression $\sum_{(i,j)} a^2\lambda_{i,j}[\mathbf{n}^2(i,j)-1]$ on the right-hand sides of both the actions corresponds to the constraint given by (5). The term $\sum_{i,j}\mathcal{S}_{\mathrm{WZ}}[\mathbf{m}(i,j)]$ in (14) acts as a topological charge measuring quantity for vortices (antivortices) which are constructed using usual coherent states for ferromagnets [24]. The object $\sum_{i,j}(-1)^{i+j}\mathcal{S}_{\mathrm{WZ}}[\mathbf{m}(i,j)]$ in (13) acts as a topological charge measuring quantity for vortices (antivortices) which are constructed using staggered coherent states for antiferromagnets [23]. One important result which came out from this investigation is that *the vortices and antivortices in the case of Heisenberg ferromagnet behave as antivortices and vortices, respectively, in the case of Heisenberg antiferromagnet*.

It can be shown that the above actions lead to existence of topological objects, even when $\lambda_z \to 0$ with $\lambda_z \neq 0$, and the WZ term represents the topological charge [23–25]. The topological objects are solely vortices and antivortices in the $\lambda_z \to 0$ limit when the 3rd component of the coherent spin field n_3 is vanishingly small. The WZ term is evaluated on a vortex on the lattice by equations of motion and local periodic boundary conditions (see appendix in [23, 24]). For illustration, we present features of 2-vortices and 3-vortices to bring out essential properties of vortices and antivortices with *even* or *odd valued* topological charges. These are essentially frozen (not mobile) vortices assuming that the system is below T_{BKT}. For detailed descriptions, we refer to the above.

5. Analysis of 2-Vortex

For a typical 2-vortex, we refer to Figure 4. Here **a**, **b**, **c,** and **d**, each carry topological charge +1 (for a 2-antivortex, we have the spins at the outer boundary oriented in the opposite direction and consequently each of the corresponding **a**, **b**, **c**, and **d** will carry a charge −1). The horizontal and vertical spins are precisely the expectation values of the operators S_x, S_y in the coherent states at the lattice point (coherent state at a lattice point $\mathbf{r}$: $|\mathbf{n}(\mathbf{r},t)\rangle = \cos(\theta(\mathbf{r},t)/2)|1/2\rangle + (e^{-i\phi(\mathbf{r},t)})\sin(\theta(\mathbf{r},t)/2)|-1/2\rangle$ for $s = 1/2$ in the present case), with the S_z expectation being small.

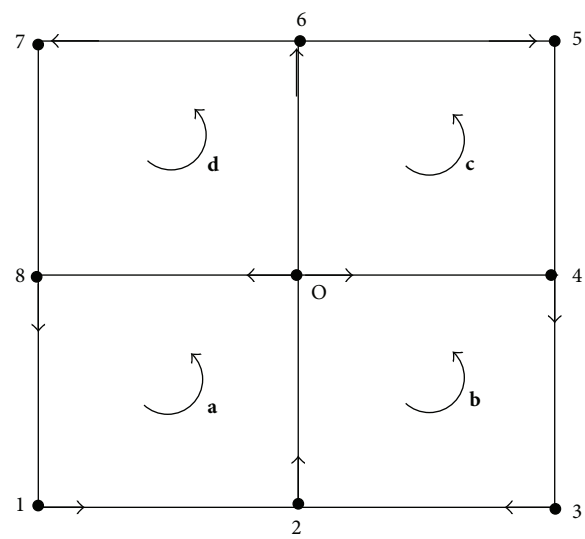

Figure 4: 2-vortex.

However, the spin at the central lattice point of the vortex (the point O in Figure 4) becomes nonunique, as is clear from the construction of the subvortices. Thus, the central point situated on a lattice site turns out to be a defect or a singular point, and cancellation of the contribution of $\mathcal{S}_{\mathrm{WZ}}^{\mathrm{stagg}}$ along the common bonds does not take place and therefore one does not have a consistent spin field configuration. The same statements hold for $\mathcal{S}_{\mathrm{WZ}}^{\mathrm{stagg}}$ in the case of antiferromagnet, where the spin fields are now staggered spin fields and vortices (antivortices) of ferromagnet become antivortices (vortices). In other words, the contributions of the WZ term on the boundary of the 2-vortex are not well defined and the construction of the 2-vortex in this scheme becomes problematic. The scenario persists in all the vortices (antivortices) possessing *even-valued* topological charges.

6. Analysis of 3-Vortex

For 3-vortex, we have a consistent spin field configuration (Figure 5). We have now an elementary anti-vortex plaquette at the central region with well-defined usual or staggered spin field configurations, and the contributions of the WZ term (i.e., of $\mathcal{S}_{\mathrm{WZ}}$ or $\mathcal{S}_{\mathrm{WZ}}^{\mathrm{stagg}}$) along the common bonds exactly cancel each other, giving rise to consistent spin field configurations, while along the boundary, it has a well defined finite value. It may be pointed out that in contrast to the case with even-valued charge, we now have a subvortex with opposite charge occupying the central region.

7. Phenomenology Based Analysis

So far, we have carried out the analysis, more of a mathematical nature, assuming the physical existence of topological excitations in these models. In order to substantiate the feasibility of this physical scenario, we now look at (i) the energetics and (ii) dynamics of these excitations.

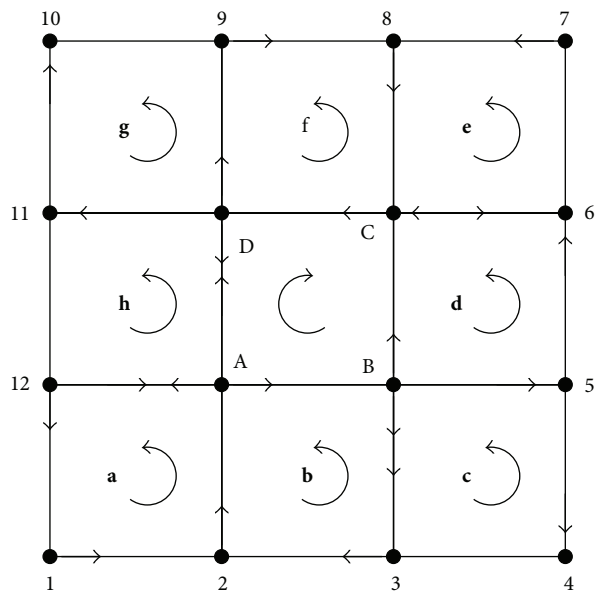

Figure 5: 3-vortex.

Simple calculations based on the Hamiltonian show that the excitation energy of a 1-vortex, as measured from the ground state, is approximately $+12|g|s^2$ both for ferromagnet and antiferromagnet [23, 24]. Please note that $s = 1/2$ in our case. It is worthwhile to mention that the ground state spin configuration is obtained by taking the coherent spin vectors $(\mathbf{n}(ia, ja))$ aligned in the "same direction" (as appropriate to ferromagnet or antiferromagnet). Typically, these excitations are of much higher energy than the average thermal energy. Therefore, these topological excitations can be produced at very low temperature predominantly by means of quantum fluctuations, thereby maintaining a steady vortex antivortex pair density even at temperature close to zero.

The next important step in this direction would be to identify signature of vortex dynamics (a) in the theoretical calculation for the dynamical structure factor $S(\mathbf{q}, \omega)$ corresponding to our quantum spin models and (b) in the observed data from inelastic neutron scattering experiment corresponding to the appropriate materials. We have made a theoretical study analyzing available results from inelastic neutron scattering experiment performed on a quasi-two-dimensional spin-1/2 ferromagnetic material K_2CuF_4 and spin-1/2 antiferromagnetic material La_2CuO_4 [12, 17].

8. Concluding Remarks

(i) Our field theoretical approach will have to be extended to the case of finite λ_z, keeping in mind the relevant materials. Making use of the theoretical results one, can in principle evaluate the static and dynamic spin correlations for spin 1/2 anisotropic quantum Heisenberg magnets at any temperature in two dimensions. These would contain contributions from both merons (antimerons) and magnons [30]. The merons (antimerons) will emerge from the equations of motion satisfied by the quantum mechanical action. The spin-spin (in our case this spin is the coherent spin field, we have introduced) correlations would contain contributions from both merons (antimerons) and magnons. Some of the characteristics, distinguishing even and odd topological charge sectors, are likely to be reflected in our results for spin correlation functions in the case of nonvanishing λ_z.

(ii) The aim of our analysis has been to formulate a consistent theory which can explain the occurrence of central peak in the neutron scattering experiments. The evaluation of the static and dynamic spin correlations will play an important role in the theoretical determination of the dynamical structure factor $S(\mathbf{q}, \omega)$ for low-dimensional quantum ferromagnets. The occurrences of "central peak" in the experimentally observed $S(\mathbf{q}, \omega)$ strongly indicate possible existence and dynamics of topological excitations of the kind described above. The "central peak" refers to the peak occurring at $\omega = 0$ in the plot of $S(\mathbf{q}, \omega)$ versus ω in the constant q-scan. Generally, this is an important signature for the spin dynamics driven also by the translational motion of the topological excitations and defects. These will play a crucial role in the quantitative verification of quantum BKT scenerio by making detailed comparison with the available experimental results.

Our approach finds similarity with treatments in many other areas of condensed matter physics like quantum Hall effect and strongly correlated electron systems [31–34].

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References

[1] J. M. Kosterlitz and D. J. Thouless, "Ordering, metastability and phase transitions in two-dimensional systems," *Journal of Physics C*, vol. 6, no. 7, pp. 1181–1203, 1973.

[2] V. L. Berezinskii, "Destruction of long-range order in one-dimensional and two-dimensional systems having a continuous symmetry group—I. Classical Systems," *Journal of Experimental and Theoretical Physics*, vol. 32, no. 3, pp. 493–500, 1971.

[3] V. L. Berezinskii, "Destruction of long-range order in one-dimensional and two-dimensional systems with a continuous symmetry group—II. Quantum systems," *Journal of Experimental and Theoretical Physics*, vol. 34, no. 3, pp. 610–616, 1972.

[4] T. K. Ng, "Topological spin excitations of heisenberg antiferromagnets in two dimensions," *Physical Review Letters*, vol. 82, pp. 3504–3507, 1999.

[5] A. A. Katanin and V. Y. Irkhin, "Magnetic order and spin fluctuations in low-dimensional insulating systems," *Physics-Uspekhi*, vol. 50, no. 6, pp. 613–635, 2007.

[6] Y. il Shin, "Creation of 2D skyrmions in spin-1 polar Bose-Einstein condensates," in *Proceedings of the ULT*, KAIST, Daejeon, Republic of Korea, 2011.

[7] C. Timm and K. H. Benneman, "Doping dependence of the Néel temperature in Mott-Hubbard antiferromagnets: effect of vortices," *Physical Review Letters*, vol. 84, pp. 4994–4997, 2000.

[8] J. Des Cloizeaux and J. J. Pearson, "Spin-wave spectrum of the antiferromagnetic linear chain," *Physical Review*, vol. 128, no. 5, pp. 2131–2135, 1962.

[9] H. J. Mikeska, "Solitons in a one-dimensional magnet with an easy plane," *Journal of Physics C*, vol. 11, article L29, 1978.

[10] G. M. Wysin and A. R. Bishop, "Quantum Monte Carlo and transfer-matrix calculations for one-dimensional easy-plane ferromagnets," *Physical Review B*, vol. 34, no. 5, pp. 3377–3387, 1986.

[11] C. Kawabata and A. R. Bishop, "Monte Carlo simulation of the two-dimensional classical Heisenberg model with easy-plane anisotropy," *Solid State Communications*, vol. 42, no. 8, pp. 595–600, 1982.

[12] R. Chaudhury, "High temperature superconductivity—current status, our theoretical and experimental work," *Indian Journal of Physics A: Special Issue on High Temperature Superconductivity*, vol. 66, pp. 159–180, 1992.

[13] K. Hirakawa, H. Yoshizawa, and K. Ubukoshi, "Neutron scattering study of the phase transition in two-dimensional planar ferromagnet K_2CuF_4," *Journal of the Physical Society of Japan*, vol. 51, no. 7, pp. 2151–2158, 1982.

[14] K. Hirakawa, "Neutron scattering study of spin dynamics at the magnetic phase transition in two-dimensional planar ferromagnet K_2CuF_4," *Journal of the Physical Society of Japan*, vol. 52, pp. 4220–4230, 1983.

[15] Y. Endoh, "Static and dynamic spin correlations in pure and doped La_2CuO_4," *Physical Review B*, vol. 37, pp. 7443–7453, 1988.

[16] K. Yamada, "Spin dynamics in the two-dimensional quantum antiferromagnet La_2CuO_4," *Physical Review B*, vol. 40, pp. 4557–4565, 1989.

[17] S. Sarkar, S. K. Paul, and R. Chaudhury, "Theoretical analysis of neutron scattering results for quasi-two dimensional ferromagnets," *European Physical Journal*, vol. 85, no. 11, p. 380, 2012.

[18] E. Loh Jr., D. J. Scalapino, and P. M. Grant, "Monte Carlo studies of the quantum XY model in two dimensions," *Physical Review B*, vol. 31, no. 7, pp. 4712–4714, 1985.

[19] R. H. Swendsen, "First- and second-order phase transitions in the $d = 2$ XY model," *Physical Review Letters*, vol. 49, article 1302, 1982.

[20] D. D. Betts, F. C. Salevsky, and J. Rogiers, "Vortices in the two-dimensional s = 1/2 XY model," *Journal of Physics A*, vol. 14, no. 2, pp. 531–538, 1981.

[21] R. Gupta, "Is Kosterlitz Thouless transition possible in 2D XY model?" Los Alamos National Laboratory preprint, 1985.

[22] E. Fradkin, *Field Theories of Condensed Matter Systems*, Addision-Wesley, San Francisco, Calif, USA, 1991.

[23] R. Chaudhury and S. K. Paul, "Physical realization and possible identification of topological excitations in quantum Heisenberg anti-ferromagnet on a two dimensional lattice," *European Physical Journal B*, vol. 69, no. 4, p. 491, 2009.

[24] R. Chaudhury and S. K. Paul, "Physical realization of topological excitations in quantum Heisenberg ferromagnet on lattice," *European Physical Journal B*, vol. 76, no. 4, p. 391, 2010.

[25] R. Chaudhury and S. K. Paul, "Possible existence of topological excitations in quantum spin models in low dimensions," *Physical Review B*, vol. 60, pp. 6234–6237, 1999.

[26] R. Chaudhury and S. K. Paul, "Effective theory for a quantum spin system in low dimensions -beyond the long wavelength limit," *Modern Physics Letters B*, vol. 16, p. 251, 2002.

[27] E. Fradkin and M. Stone, "Topological terms in one- and two-dimensional quantum Heisenberg antiferromagnets," *Physical Review B*, vol. 38, pp. 7215–7218, 1988.

[28] F. D. M. Haldane, "O(3) Nonlinear σ model and the topological distinction between integer- and half-integer-spin antiferromagnets in two simensions," *Physical Review Letters*, vol. 61, pp. 1029–1032, 1988.

[29] T. Senthil and M. P. A. Fisher, "Competing orders, nonlinear sigma models, and topological terms in quantum magnets," *Physical Review B*, vol. 74, Article ID 064405, 11 pages, 2006.

[30] P. W. Anderson, S. John, G. Baskaran, B. Doucot, and S. D. Liang, Princeton University Preprint, 1988.

[31] A. M. M. Pruisken, "On localization in the theory of the quantized hall effect: a two-dimensional realization of the θ-vacuum," *Nuclear Physics B*, vol. 235, pp. 277–298, 1984.

[32] R. B. Laughlin, "The relationship between high-temperature superconductivity and the fractional quantum hall effect," *Science*, vol. 242, no. 4878, pp. 525–533, 1988.

[33] A. M. Finkelstein and A. I. Larkin, "Two coupled chains with Tomonaga-Luttinger interactions," *Physical Review B*, vol. 47, pp. 10461–10473, 1993.

[34] R. Chaudhury and D. Gangopadhyay, "Interacting fermions, scaling and possible departure from fermi liquid behavior," *Modern Physics Letters B*, vol. 9, no. 25, p. 1657, 1995.