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Research Article

Effect of Noise on the Decoherence of a Central Electron Spin Coupled to an Antiferromagnetic Spin Bath

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We analyze the influence of a two-state autocorrelated noise on the decoherence and on the tunneling Landau-Zener (LZ) transitions during a two-level crossing of a central electron spin (CES) coupled to a one dimensional anisotropic-antiferomagnetic spin, driven by a time-dependent global external magnetic field. The energy splitting of the coupled spin system is found through an approach that computes the noise-averaged frequency. At low magnetic field intensity, the decoherence (or entangled state) of a coupled spin system is dominated by the noise intensity. The effects of the magnetic field pulse and the spin gap antiferromagnetic material used suggest to us that they may be used as tools for the direct observation of the tunneling splitting through the LZ transitions in the sudden limit. We found that the dynamical frequencies display basin-like behavior decay with time, with the birth of entanglement, while the LZ transition probability shows Gaussian shape.

1. Introduction

The physics of decoherence including LZ transition in a superconducting quantum spin devices is a subject of fundamental importance and of great current interest, for example, in the context of prospective "coupling-CES devices" [1, 2]. Quantum decoherence remains the most important obstacle to overcome in order to build a quantum computer with these devices. Decoherence due to noise or entanglement with uncontrollable degrees of freedom of the environment is responsible for the decay of the coherent superposition of qubit states. The CES driven by the external noise field has stimulated a lot of interest in the LZ tunneling in periodic structures, although this phenomenon was originally discussed with respect to Bloch oscillations of a crystal of electrons in a strong electric field [3, 4]. Nowadays the most successful experimental systems are semiconductor superlattices [5] that allow access to many different aspects of LZ tunneling [6, 7] including decoherence when associated to noise or interaction with the environmental degrees of freedom [8–10]. Thus, it is possible to manipulate the values of the environment or system parameters at any time such that the qubits undergo one-qubit or two-qubit gate operations [11–13].

It is found in the review of many previous works that the decoherence of electron spins coupled to a baths of nuclear spins in quantum dots [14–16] or solid-state impurity centers [17, 18] is a key issue in spin-based quantum information processing because of the long coherence time of the quantum spin systems [19–23]. In modern quantum technologies, when the relevant environment is of nanometer size, its quantum nature becomes important [24]. Environments can be modeled as either baths of harmonic oscillators [25] or spins, argued to represent distinct types of environmental modes [26]. The simplest system-environment models consist of a CES coupled to the environment, for example, the spin-boson model [10] that has applications to decoherence of qubits for quantum information processing.

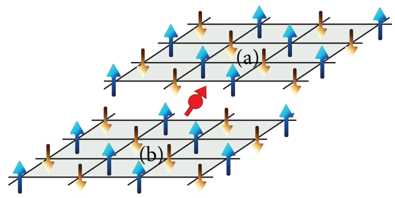

Figure 1: Central electron spin coupled to a one-dimensional anisotropic-antiferromagnetic spin systems. The spin configuration is compatible with the spin state in a sublattice (i.e., the spins on each sublattice are antiparallel). (a) and (b) represent physical configuration of a two-interpenetrating quantum spin systems.

Antiferromagnetic superconducting materials have been reported recently to have applications in the area of quantum information processing [27–32].

Therefore in such a system, decoherence of a cluster-spin degrees of freedom is expected to arise mainly from the hyperfine coupling with the nuclear spins [20, 21, 33].

This paper considers a CES coupled to two-interpenetrating spins systems (Figure 1).

Within the framework of dissipative Landau-Zener transitions of qubits [29], we propose a scheme to study decoherence and LZ transition of CES coupled to two-interpenetrating spins systems in state of an antiferromagnetic material with long-range order.

To tackle this problem and tailor the dynamic of the noisy CES coupled spin system, the frequency and the transition probability become figures of merit [21]. The choice of the anisotropic-antiferromagnetic spin Bath makes use of the fact that, considering a two separate sublattices with a certain gap of energy, each sublattice can be expanded in the basis states that are products of single system when coupling with the CES. To solve this problem, we make use of the well-known spin wave approximation (SWA) formalism [1, 2].

In [1], the authors investigated the influence of a constant external magnetic field on the parameters characterizing the decoherence of a CES coupled to an antiferromagnetic environment, where the decoherence factor, which displays a Gaussian shape decay with time, depends on the strength of the external magnetic field and on the crystal anisotropy field [1]. This work was extended in our recent paper [2] in which we showed that the two different magnon modes, resulting from the frequency splitting via the application of the variable B-field (VBF), exhibit each a resonance peak of similar amplitude at different time ranges. Taking into account the transition induced by noise in a solid states system [34], we now consider the effect of a noise field which was not considered in works [1, 2]. This effect is comparable to the two-state autocorrelated (TSAC) noises [35].

The plan of the paper is as follows. In Section 2, we present the model Hamiltonian. Though this problem was studied in our previous article [2], it is important to give a simple analysis of the influence of the TSAC noise on the dynamics of a CES system by evaluating the average frequencies of oscillations. In Section 3 in the light of LZ scenario, the LZ tunneling transition probability is found. We then close the work with the conclusion.

2. Model Hamiltonian

Decoherence rates for different superposition vibration states of a spin-1/2 particle at low temperatures in a time VBF may conveniently be modeled by the "CES" model, which couples a central spin S to a spin-bath B of N-antiferromagnetic spin particles in the presence of a TSAC noise [35]. In view of the forgoing argument, we propose the following Hamiltonian:

$$H = H_S + H_B + H_{SB}, \tag{1}$$

with

$$H_S = -g\mu_B B S_0^z + \xi(t) S_0^z, \tag{2}$$

$$H_{SB} = -\frac{J_0 S_0^z}{\sqrt{N}} \sum_i \left(S_{a,i}^z + S_{b,i}^z\right), \tag{3}$$

$$\begin{aligned} H_B = {} & -g\mu_B (B + B_A) \sum_i S_{a,i}^z - g\mu_B (B - B_A) \sum_j S_{b,j}^z \\ & + J \sum_{i,\delta} S_{a,i} S_{b,i+\delta} + J \sum_{j,\delta} S_{b,j} S_{a,j+\delta} \\ & + \xi(t) \left(\sum_i S_{a,i}^z + \sum_j S_{b,j}^z \right), \end{aligned} \tag{4}$$

$$B = B_0 \sin \omega t, \tag{5}$$

where H_S and H_B are, respectively, the Hamiltonian of the CES and of the environment. H_{SB} is the Hamiltonian of the interaction [1, 2, 36]. g is the gyromagnetic factor and μ_B the Bohr magneton. For simplicity, significant interaction (3) between the CES and the environment is assumed to be of the Ising type (for a more detailed consideration see [37]).

In the presence of the external magnetic field B applied in the Z-direction, the spin orientation changes as the direction of the anisotropy field changes. The presence of noise induces fluctuation of the system as well as the spin wave excitations [23]. In [38] the crossover from the case of a single link of the spin system to the bath was analyzed, where the CES is frustrated according to the noise and uniformly coupled to all the spins of the bath.

In line with the transformations used in [1, 2] and considering that the magnetic field frequency is weak (i.e., $\vec{B} = B_0\omega t\vec{z}$), the Hamiltonians H_S, H_{SB}, and H_B can be rewritten as [36, 39]

$$H(t) = H_S + H'_B + H'_{SB}, \tag{6}$$

$$H_S = -\frac{1}{2}(\varepsilon(t) - \xi(t))\sigma^z; \qquad \varepsilon(t) = g\mu_B B_0 \omega t, \tag{7}$$

$$H'_{SB} = -\frac{J_0}{2\sqrt{N}}\sum_k (\beta_k^+\beta_k - \alpha_k^+\alpha_k)\sigma^z, \tag{8}$$

$$\begin{aligned} H'_B = E'_0 &+ \sum_k \omega_k^+\left(\alpha_k^+\alpha_k + \frac{1}{2}\right) \\ &+ \sum_k \omega_k^-\left(\beta_k^+\beta_k + \frac{1}{2}\right), \end{aligned} \tag{9}$$

where $\alpha_k^+(\alpha_k)$ and $\beta_k^+(\beta_k)$ are the creation (annihilation) operators of the two different magnons with wave vector k and frequency $\omega_k^+(\omega_k^-)$, respectively.

Considering the Hamiltonian in (6) and taking the small k approximation, the dynamical frequencies of the coupled spin system are presented as

$$\begin{aligned} \omega_k^\pm = 2MsJ\sqrt{\left(\frac{1+g\mu_B B_A}{2MSJ}\right)^2 - \frac{1+2k^2l^2}{M}} \\ \pm g\mu_B B \mp 2\langle\xi(t)\rangle, \end{aligned} \tag{10}$$

where l is the side length of cubic primitive cell of each sublattice and M the number of nearest neighboring atoms. The slow fluctuations of the coupled spin system correspond to low temperatures $T \ll T_N$, where T_N is the Néel temperature [40]. The frequencies for the two magnon modes plotted in Figures 1 and 2 display a stochastic behavior due to the presence of noise (see (10)).

In analogy to the dressed atom picture [41], the eigenfrequencies $\omega_k^\pm$ can be associated with dressed states, that is, the oscillator frequencies of state $|\uparrow k\rangle$ and $|\downarrow k\rangle$ in the presence of mutual coupling. There is a characteristic anticrossing with the energy gap due to the frequency splitting of $\Lambda = 4(\varepsilon_0 - \langle\xi(t)\rangle)$, where ε_0 is the contribution of the Zeeman effect. The splitting increases with the increase of the time VBF and decreases with an increase of the noise intensity.

The entangled state (see Figure 2) is the consequence of decoherence induced in a system. This figure shows also that the noise favors transitions by reducing the bandwidth gap energy level between the two networks. The basin like behavior (see Figure 3) shows how the dispersion relation proceeds with two extreme values corresponding to the high cut-off frequency and the low cut-off frequency. In this work we find that, when the external magnetic field exceeds a critical value B_c, we have $\omega_k^- < 0$ which indicates that the corresponding branch of magnons is no longer stable due to effects of noise. As a result, we noted that when $\omega_k^- < 0$, the antiferromagnetic polarization flips perpendicular to the field; that is, the magnetic field induces spin flop transition while the presence of noise in the system enhances oscillations.

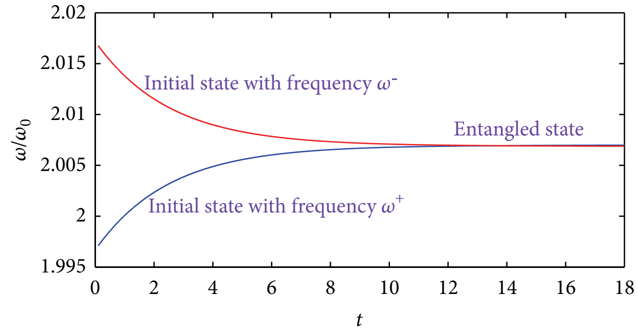

Figure 2: Plot of the dispersion relation versus time. Red curves for the spin branches with frequency ω^-; blue curve for the spin branches with frequency ω^+. Both curves entangled at a certain time scales in the presence of noise.

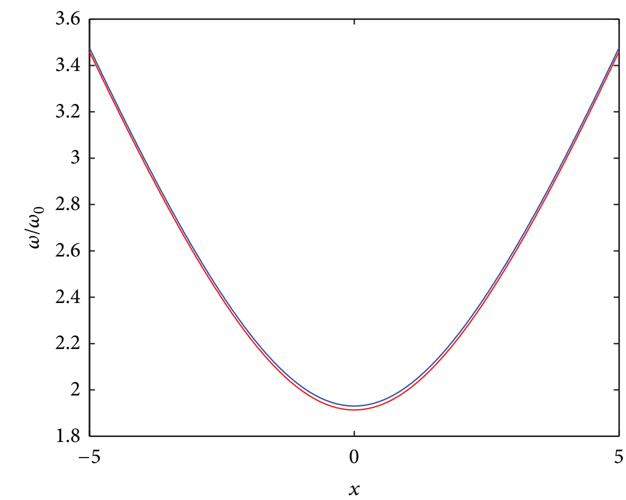

Figure 3: Plot of the dispersion relation versus the wave vector ($x = kl$). Red curves for the spin branches with frequency ω^-; blue curve for the spin branches with frequency ω^+. Both curves show basin-like behavior.

3. Landau-Zener Tunneling Transition Probability

As in the so-called "tunneling model" of two level system (TLS) defects [42], in the atomic-scale materials defect resides in a potential energy landscape that displays two local minima separated by a small energy gap. For a sufficiently large tunneling matrix element, the eigenstates are appropriate superposition of wave functions localized in the two potential minima.

The characteristic anticrossing occurs in the system with the antiferromagnetic band gap energy due to the frequency splitting or bandwidth frequency ($\hbar = 1$) of $\Lambda = 4(\varepsilon_0 - \langle\xi(t)\rangle)$, the eigenmodes of the oscillators with frequencies ω^+, ω^- translate the system into a two-level system coupled by a constant magnetic field. In analogy to the case in [43] where the coupling is via a spring constant, whereby in the light of LZ scenario, the frequency difference of the oscillator changes

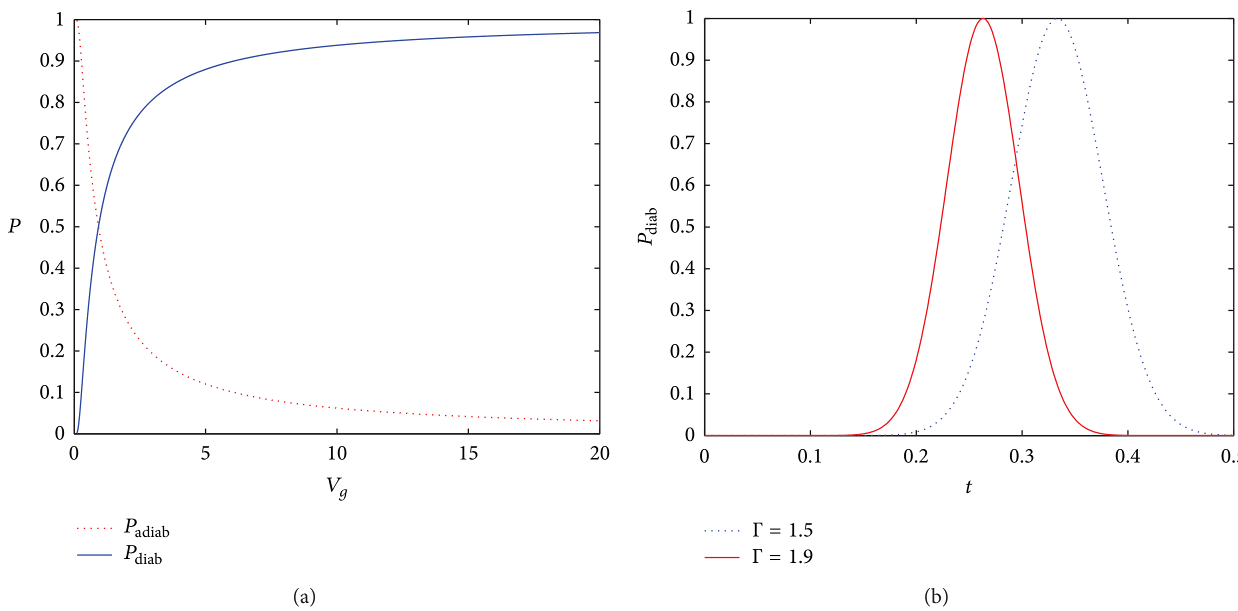

FIGURE 4: (a) LZ transition probability in the sudden (12) versus the group velocity, for diabatic (dot curves) and for adiabatic cases (solid curve); (b) LZ transition probability versus time for different values of the noise factor Γ.

linearly in time, the probability for level crossing (diabatic transition) at infinite large time is

$$P_{\text{diab}} = \exp\left[-\pi\frac{\Lambda^2}{2V_g}\right]. \tag{11}$$

Thus attributing this thought to this present issue (6) may be pictured as describing the TLS in which defects exist in both configurational states and corresponds to local minima in the antiferromagnetic potential energy landscape. In the low temperature regime, the dynamics of the system is dominated by the TSAC noise frequency, Γ. In this case, the incoherent tunneling rate from $|0\rangle$ to $|1\rangle$ is given by the bandwidth frequency Λ which changes linearly in time. Thus, with the help of (11), in the consideration of this work, the transition probability for level crossing (diabatic transition) in the sudden limit is

$$P_{\text{diab}} = \exp\left[-\pi\frac{\left(4\left(\varepsilon_0 - \langle\xi(t)\rangle\right)\right)^2}{2V_g}\right]. \tag{12}$$

V_g is the transition speed given by the dispersion relation $V_g = \partial\omega/\partial k$ and k the wave vector. If $a = 0$ or $\Gamma \to +\infty$, then $V_g \approx \varepsilon_0$, yielding (12) which is exactly the LZ transition probability in a completely coherent sudden limit [44].

In Figures 4(a) to 5(b), considering the stochastic effect of the frequencies due to the TSAC noise, we plot the range of values of time for which the survival probability amplitude is maximum, as it exhibits a resonant peak. There is shrinkage in the width of this resonant peak for large value of the Zeeman energy. This implies that the initially prepared state of the environmental frequency mode could be tailored with precision if certain values of the Zeeman energy ε_0, noise amplitude a, and noise phase Γ are used.

Consequently, Figure 6(a) shows the diabatic survival probability versus the distribution velocity, for different values of the noise phase. In Figure 6(b), the time-dependent survival probability is also plotted. We find that the diabatic probability increases with the increase of the distribution velocity while the noise phase decreases.

As is expected in Figure 5(a), the transition probability plotted as a function of noise phase factor Γ shows phase decoherence in the system. The birth of entanglement in the system (see Figure 2) occurs after some time (maximum value) that corresponds to state degeneracy. This time given in (13) is identified as an autocorrelation time (or decoherence time):

$$\tau = \tau_r \ln\left(\frac{a}{\varepsilon_0}\right), \tag{13}$$

where τ_c is a relaxation time. From (13) we find that the long coherence regime of the system corresponds to the high magnetic field intensity and for small value of the noise amplitude a.

4. Conclusion

In this paper, we have investigated theoretically decoherence and the LZ transition of a superconducting-qubit coupled spin system, where the system exhibit bistability mediated by the CES trapped by the vibronic coupling. As presented in our results, the two magnon modes in the system displayed a stochastic behavior according to (10). It is observed that the environment frequency modes can be tailored with precision by controlling the Zeeman energy, noise amplitude, and noise phase. In the light of LZ scenario, the existence of the energy gap as a tunneling matrix element couples with the Bath

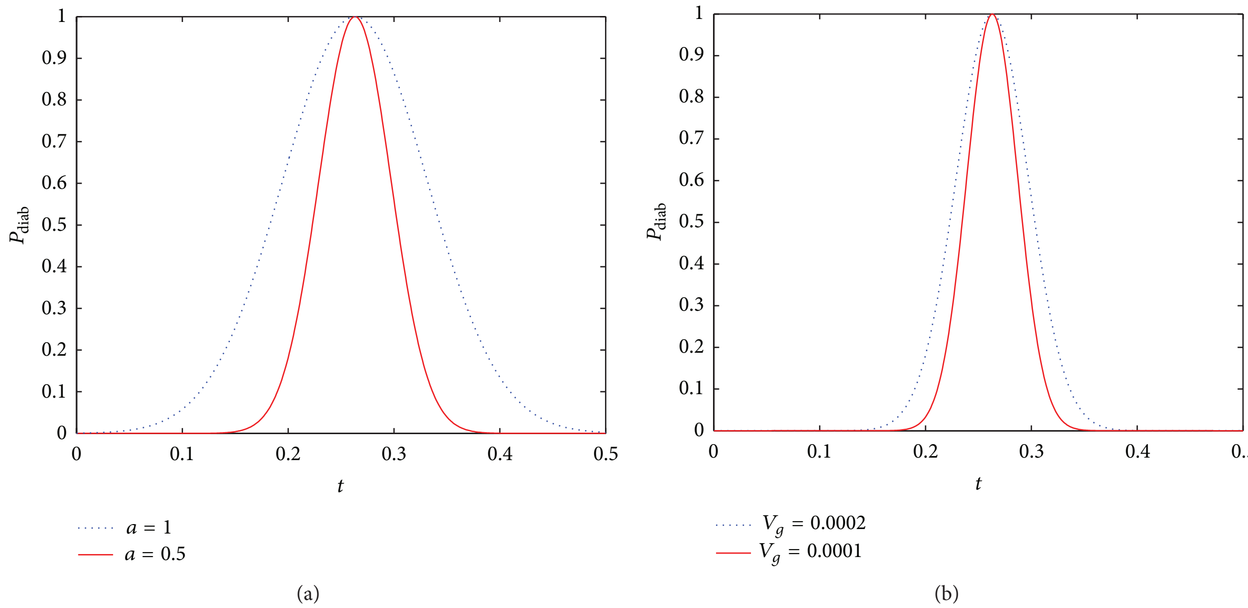

FIGURE 5: Plots of the LZ transition probability versus time, in the sudden limit expressed in (12). The curves inserted in Figure 5(a) show the behavior for different values of the noise amplitude a and in Figure 5(b) for different values the group velocity.

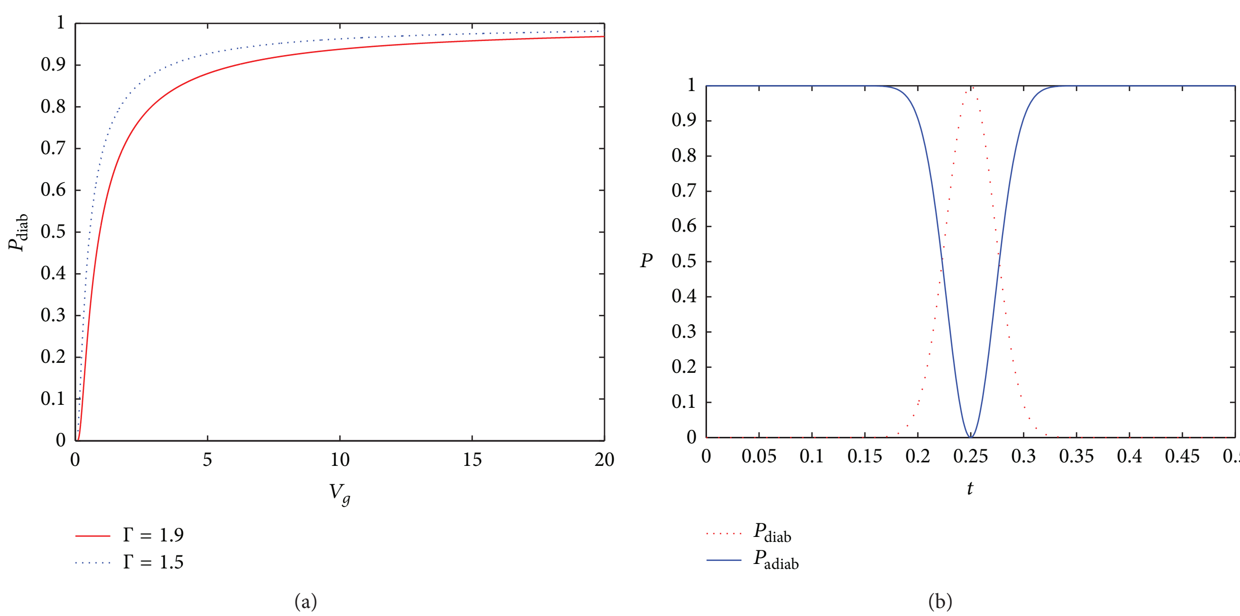

FIGURE 6: Plots of the LZ transition probability in the sudden limit expressed in (12). Figure 6(a) shows the LZ transition probability in function of the group velocity for different values of the noise factor Γ while Figure 6(b) shows the LZ transition probability in function of time in a diabatic (dot curve) and in adiabatic cases (solid curve).

networks; a probability for level crossing occurs. It is shown in this paper that the presence of noise as well as the driving field induces fluctuations in energy levels. These fluctuations modify the transition rate in two ways: (1) due to the TSAC noise, the modifications tends in the direction in which the nonadiabaticity is effectively increased and the probability becomes limited by the upper and the lower bounds. These effects amplify decoherence of the CES. (2) The driving field modifies the transitions in the direction in which the nonadiabaticity decreases and thus this leads to the reduction of decoherence. So, in a high magnetic field regime, the CES is strongly coupled to the spin bath and this contributes to delay decoherence effects in the coupled system. Both dynamical frequencies and LZ transition probability depend on the strength of the time VBF, the noise amplitude, and the crystal anisotropy field.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publishing of this paper.

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