Hindawi
Advances in Condensed Matter Physics
Volume 2017, Article ID 2435857, 8 pages
https://doi.org/10.1155/2017/2435857

Research Article

The Tunnelling Current through Oscillating Resonance and the Sisyphus Effect

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Received 19 February 2017; Accepted 29 March 2017; Published 11 April 2017

Academic Editor: Sergei Sergeenkov

The tunnelling current through an oscillating resonance level is thoroughly investigated exactly numerically and with several approximations—analytically. It is shown that while the oscillations can increase the tunnelling current (and in several cases the increase is exponentially large), their main effect is to reduce it dramatically at certain energies. In fact, the current in the presence of the oscillations cannot increase the maximum current of the adiabatic solution. That is why, while the elevator effect does occur in this system, the Sisyphus effect is the more dominant and prominent one.

1. Introduction

Resonant tunnelling (RT) is a surprising quantum effect despite being common in quantum heterostructures [1–7]. When a quantum particle propagates through an opaque barrier, it tunnels, that is, penetrates the barrier, with low probability [8]. However, if the barrier has a local quasi-bound state, as a result of an attractive point impurity, that is, a local potential well, and the particle's energy is equal to the resonance energy, the particle can resonantly pass through the barrier via the local well with a very high probability, which in principle can be as high as unity [9–11]. This effect occurs in any number of dimensions (for 2D see [12, 13]). It was natural that this peculiar conduct would be utilized for the generation of quantum transistor, where small variations in the resonance energy level (the gate) can dramatically change the current through the barrier [7, 10, 14]. It was then realized that when the well varies during tunnelling, the particle can absorb vibrating quanta from the well. As a consequence, the particle can be activated to much higher energies (close to the barrier's height) and penetrates through the well with much higher probability. This dynamic resonant tunnelling was investigated in a variety of fields: from the foundations of quantum mechanics [15–30] via nano- and microelectronics [14, 31–33] to biochemistry and biology [34–40].

But it was then realized that dynamic resonant tunnelling is even more interesting. For example, when the perturbation's (the changing well) time-scale is shorter than the quasi-bound-state's life-time, the particle can be trapped inside the well, at the resonance level. As a result, when the potential well changes, the particle can be lifted energetically. This process was termed eigenstate assisted activation (EAA) and the elevator effect (EE) by Azbel [15, 17]. However, recently, it has been shown that the eigenstate's presence can *suppress* the activation and reduce the transmission [20, 30, 41]. Despite the fact that an additional energy is introduced to the system via the potential oscillations, the net effect is *current reduction*; therefore, the *Sisyphus effect* (after the mythological hero) is a more adequate name for the effect. In fact, when there is a destructive interference between the two time events, in which the well's eigenenergy crosses the incoming particles energy, the particle cannot dwell in the well and activation is suppressed. This effect occurs for specific energies, and in very narrow spectral bands, and therefore it was suggested that it can be used for a frequency effect transistor, where the current is controlled by the *frequency* of the gate [20, 41].

It is the object of this paper to investigate the current (and not only the activation energy) dependence on the resonance energy and oscillation's frequency and to show that current suppression is the main effect of the varying eigenstate. The

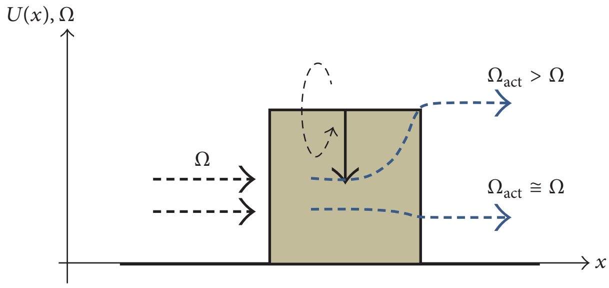

Figure 1: Schematic illustration of the system's dynamics. For most energies activation occurs; that is, $\Omega_{\text{act}} > \Omega$; however, for specific energies, that is, $\Omega = \Omega_m$, activation is suppressed and then $\Omega_{\text{act}} \cong \Omega$.

largest current increase occurs when the incoming particle's energy is lower than the minimum resonance energy level. But even then the activated current is always *lower* than the maximum adiabatic current. Beyond the maximum point, the main effect is current reduction at certain energies due to the Sisyphus effect.

2. The Model

The system is presented in Figure 1. It consists of an opaque potential barrier with an oscillating delta function well $-(f_0 + \Delta f \cos(\omega t))\delta(x - x_0)$ in its center.

The system's Schrödinger equation is then

$$-\frac{\partial^2}{\partial x^2}\psi(x,t) + [U(x) - (f_0 + \Delta f \cos(\omega t))\,\delta(x - x_0)]\,\psi(x,t) = i\frac{\partial \psi(x,t)}{\partial t}. \tag{1}$$

Hereinafter for simplicity we adopt the units, where the electron mass is half and the reduced Planck constant is unity; that is, $m = 1/2$ and $\hbar = 1$. $U(x)$ is the barrier potential, which can in principle have an arbitrary shape provided it is bounded in space, but for simplicity in the simulations we adopt the well-known rectangular barrier

$$U(x) = \begin{cases} U & |x| < L \\ 0 & |x| \geq L. \end{cases} \tag{2}$$

Let $\varphi_\omega^\pm(x)$ be the solutions of the stationary homogenous (without the delta function well) Schrödinger equation; that is,

$$-\frac{\partial^2}{\partial x^2}\varphi_\omega^\pm(x) + [U(x) - \omega]\,\varphi_\omega^\pm(x) = 0, \tag{3}$$

which can be written as "right going" $\varphi_\omega^+(x)$ and "left going" $\varphi_\omega^-(x)$ solutions:

$$\varphi_\omega^+(x) = \begin{cases} \exp(i\sqrt{\omega}x) + r_\omega \exp(-i\sqrt{\omega}x) & x \longrightarrow -\infty \\ t_\omega \exp(i\sqrt{\omega}x) & x \longrightarrow \infty, \end{cases} \tag{4}$$

$$\varphi_\omega^-(x) = \begin{cases} t_\omega \exp(-i\sqrt{\omega}x) & x \longrightarrow -\infty \\ \exp(-i\sqrt{\omega}x) + r_\omega \exp(i\sqrt{\omega}x) & x \longrightarrow \infty, \end{cases} \tag{5}$$

where t_ω and r_ω are the transmission and reflection coefficients for the energy ω, respectively.

3. The General Solution

Given the initial energy of the incoming particle is Ω, we seek a solution for (1) of the form

$$\psi(x,t) = \begin{cases} \varphi_{\Omega+n\omega}^+(x)\exp(-i\Omega t) + \displaystyle\sum_{n=-\infty}^{\infty} R_n \frac{\varphi_{\Omega+n\omega}^-(x)}{\varphi_{\Omega+n\omega}^-(x_0)} \exp(-i(\Omega + n\omega)t) & x < x_0 \\ \displaystyle\sum_{n=-\infty}^{\infty} S_n \frac{\varphi_{\Omega+n\omega}^+(x)}{\varphi_{\Omega+n\omega}^+(x_0)} \exp(-i(\Omega + n\omega)t) & x > x_0. \end{cases} \tag{6}$$

After matching the wave function and its derivative at $x = x_0$, that is, for $\varepsilon \longrightarrow 0$

$$\begin{aligned} \psi(x_0 + \varepsilon, t) &= \psi(x_0 - \varepsilon, t), \\ \psi'(x_0 + \varepsilon, t) - \psi'(x_0 - \varepsilon, t) &= -(f_0 + \Delta f \cos(\omega t))\,\psi(x_0, t) \end{aligned} \tag{7}$$

(the tags stand for spatial derivatives), then it is straightforward to see that the coefficients S_n satisfy the difference equation

$$S_n \frac{2}{q_n} - [S_{n+1} + S_{n-1}] = 2\varphi_\Omega^+(x_0)\left[\frac{f_0}{\Delta f} + \frac{1}{q_0}\right]\delta(n), \tag{8}$$

where

$$q_n \equiv \frac{\Delta f}{1/G_{\Omega+n\omega}^+(x_0, x_0) - f_0} \tag{9}$$

and $G_\omega^+(x, x')$ is the outgoing Green function; that is, in terms of the eigenstates (4) it can be written as

$$\frac{1}{G_\omega^+(x_0, x_0)} = \frac{\varphi_\omega^{-\prime}(x_0)}{\varphi_\omega^-(x_0)} - \frac{\varphi_\omega^{+\prime}(x_0)}{\varphi_\omega^+(x_0)}. \tag{10}$$

With this terminology the mean activation energy $\langle\omega_{\text{act}}\rangle$ and the mean current

$$\langle j\rangle = \left\langle 2\mathfrak{I}\left(\frac{\psi^*\partial\psi}{\partial x}\right)\right\rangle \tag{11}$$

can be derived:

$$\begin{aligned}\langle\omega_{\text{act}}\rangle &= \frac{\sum_{n=-\infty}^{\infty}(\Omega+n\omega)\left|S_n\right|^2\left|\varphi^+_{\Omega+n\omega}(x)/\varphi^+_{\Omega+n\omega}(x_0)\right|^2}{\sum_{n=-\infty}^{\infty}\left|S_n\right|^2\left|\varphi^+_{\Omega+n\omega}(x)/\varphi^+_{\Omega+n\omega}(x_0)\right|^2},\end{aligned} \tag{12}$$

$$\begin{aligned}\langle j\rangle &= 2\sum_{n=-\infty}^{\infty}\left|S_n\right|^2\frac{\mathfrak{I}\left(\varphi^{+*}_{\Omega+n\omega}(x)\,\varphi^{+\prime}_{\Omega+n\omega}(x)\right)}{\left|\varphi^+_{\Omega+n\omega}(x_0)\right|^2}\\ &= \sum_{n=-\infty}^{\infty}\left|S_n\right|^2\frac{j_n}{\left|\varphi^+_{\Omega+n\omega}(x_0)\right|^2},\end{aligned} \tag{13}$$

respectively, where

$$j_n \equiv 2\mathfrak{I}\left(\varphi^{+*}_{\Omega+n\omega}(x)\,\varphi^{+\prime}_{\Omega+n\omega}(x)\right) = 2k_n\left|t_n\right|^2 \tag{14}$$

is the current of the nth dynamic mode.

4. The Adiabatic Solution

The stationary state solution for (1) when the point defect's potential varies adiabatically [when $f(t) \equiv f_0 + \Delta f\cos(\omega t)$] is

$$\begin{aligned}\psi(x) &= \frac{\varphi^+_\omega(x>x_0)}{1-f(t)\,G^+_\omega(x_0,x_0)}\\ &= \frac{\varphi^+_\omega(x>x_0)}{1-f(t)/\left(\varphi^{-\prime}_\omega(x_0)/\varphi^-_\omega(x_0)-\varphi^{+\prime}_\omega(x_0)/\varphi^+_\omega(x_0)\right)}.\end{aligned} \tag{15}$$

Therefore, the wave function beyond the barrier in the adiabatic approximation is

$$\psi(x>L,t) = \frac{t_\Omega\exp\left(i\sqrt{\omega}x\right)}{1-\left(f_0+\Delta f\cos(\omega t)\right)G^+_\Omega(x_0,x_0)}. \tag{16}$$

That is, the quasi-bound-state energy varies in time

$$\Omega^*(t) \cong U - \frac{\left(f_0+\Delta f\cos(\omega t)\right)^2}{4}, \tag{17}$$

and therefore the instantaneous current is equal to

$$j(x,t) = \frac{j_s\mathfrak{I}\left\{(1/q_0)/\left(1-q_0^*\cos(\omega t)\right)\right\}}{\mathfrak{I}\left\{1/q_0\right\}}, \tag{18}$$

where

$$j_s \equiv \frac{2\sqrt{\Omega}\left|t_\Omega\right|^2}{\left|1-f_0G^+_\Omega(x_0,x_0)\right|^2} \tag{19}$$

is the stationary current and (following (9))

$$q_0 = \frac{\Delta f}{1/G^+_\Omega(x_0,x_0)-f_0}. \tag{20}$$

From (18) the average current, that is, $\langle j\rangle = T^{-1}\int_T j(x,t)dt$, where the average is taken over the period $T = 2\pi/\omega$, is finally

$$\langle j\rangle = \frac{j_s\mathfrak{I}\left\{(1/q_0)/\sqrt{1-q_0^{*2}}\right\}}{\mathfrak{I}\left\{1/q_0\right\}}. \tag{21}$$

Therefore, the ratio between the average current and the stationary one depends on a single complex parameter q_0 (20).

The maximum average current ($\langle j\rangle_{\max}$) is reached, within this adiabatic approximation, when the lower value of the resonance eigenbound state is equal to the incoming particle's energy, that is, for $\Omega = U - (f_0+\Delta f)^2/4$, or $\Delta f = 2K - f_0$, for which case

$$\begin{aligned}\frac{\langle j\rangle_{\max}}{j_s} &\cong \frac{\exp(3KL)}{8\sin(\operatorname{atan}(k/K))}\\ &= \frac{\exp(3KL)}{8}\frac{\sqrt{1+(k/K)^2}}{k/K}\\ &= \frac{\exp(3KL)}{8}\frac{\sqrt{U}}{k}.\end{aligned} \tag{22}$$

This current is exponentially larger than the stationary tunnelling current (18).

Below and above the resonance, the current can be approximated by the following.

Below the resonance value, that is, for $\Delta f < 2K - f_0$, the current can be approximated by

$$\langle j\rangle \cong j_s\frac{1}{\left[1-\left(\Delta f/(2K-f_0)\right)^2\right]^{3/2}}. \tag{23}$$

Beyond the resonance, that is, for $\Delta f > 2K - f_0$, the current can be approximated by

$$\begin{aligned}\langle j\rangle &\cong j_s\frac{(1-f_0/2K)\exp(2KL)}{2\sin(2\operatorname{atan}(k/K))\left[\left(\Delta f/(2K-f_0)\right)^2-1\right]^{1/2}}.\end{aligned} \tag{24}$$

In Figure 2 the adiabatic solution is plotted with these approximations.

These approximations are very useful, especially for the lower values of the amplitude, since it is independent of the exact shape of the barrier and even independent of its width.

5. Resonances and Antiresonances

In Figure 3 the exact numerical solution of (1) and (13) is plotted (solid curve) along with the adiabatic solution (dashed curve). Three regimes appear.

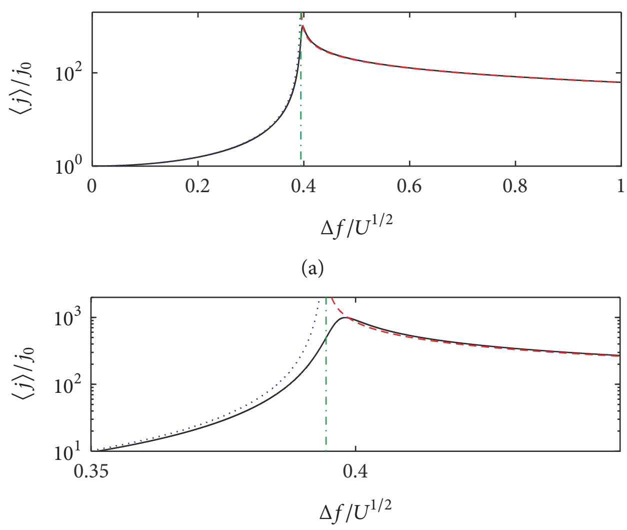

Figure 2: The adiabatic approximation (solid curve) and its approximations (the dotted and dashed curves). The lower panel is a zoom-in of the transition zone. The simulations parameters were $L\sqrt{U} = 7$, $f_0/\sqrt{U} = 0.5$, $\Omega/U = 0.8$, $\omega/U = 0.01$, and $\Delta f_r = 2K - f_0 \cong 0.3944$, presented by the vertical line.

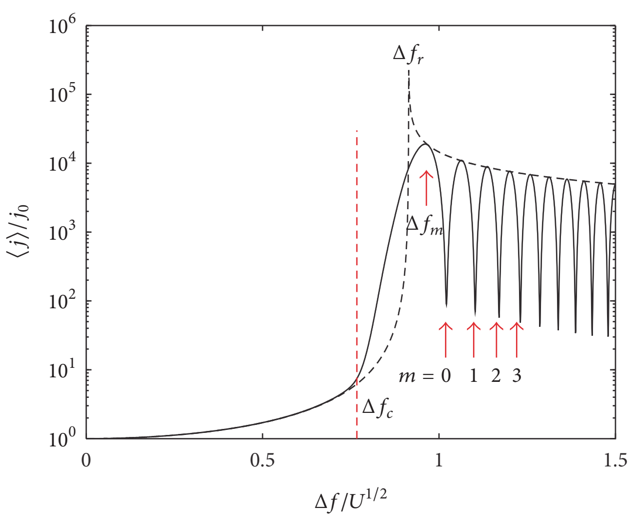

Figure 3: The exact numerical solution of the current (solid curve) and the adiabatic approximation (dotted curve). The simulations parameters were $L\sqrt{U} = 7$, $f_0/\sqrt{U} = 0.5$, $\Omega/U = 0.5$, $\omega/U = 0.01$, and $\Delta f_r = 2K - f_0 \cong 0.9142$.

In the weak modulation regime, that is, $0 < \Delta f < \Delta f_c$, activation is negligible where the particle tunnels out with its initial energy; that is, $\langle \Omega_{\mathrm{act}} \rangle \cong \Omega$. In this regime the adiabatic approximation is an excellent evaluation of the current.

Beyond the transition threshold $\Delta f > \Delta f_c$, the current exceeds the adiabatic value, but it cannot reach the resonance of the adiabatic approximation at Δf_r. In fact, beyond $\Delta f_r < \Delta f$ the exact current is lower than the adiabatic value for most modulation values Δf. When $\Omega > U/2$ the adiabatic approximation can be used as an upper bound value for the real current.

This fact suggests a peculiar behaviour that an EAA occurs mainly when the incoming particle's energy Ω is lower than the minimum eigenenergy of the quasi-bound state; that is, $\Omega < U - (f_0 + \Delta f)^2/4$. That means that the particle did not experience an ordinary resonant tunnelling to the well and only then elevated to higher energies, but rather gained energy quanta from the oscillations to mitigate the tunnelling.

When the oscillations amplitude increases beyond the resonance level, that is, when the minimum value of the eigenstate energy is lower than the incoming energy, then the quasi-eigenstate mostly *decreases* the current. When there is a destructive interference inside the well, then the particle cannot dwell there and the current is substantially reduced.

It was shown [30] that the destructive interference occurs for

$$\int_{t_1}^{t_2} dt' \left[\Omega^*\left(t'\right) - \Omega\right] = \pi\left(2m + \frac{3}{2}\right) \qquad \text{for } m = 0, 1, 2, \ldots \tag{25}$$

when $\Omega^*(t) \cong U - (f_0 + \Delta f \cos(\omega t))^2/4$ is the instantaneous resonance state, and t_1 and t_2 are the time events, in which the incoming particle's energy crosses the resonance state; that is, $\Omega = \Omega^*(t_1) = \Omega^*(t_2)$. Therefore, after substituting (17) in (25) and calculating for t_1 and t_2, the amplitude values $\Delta f_{\min}$ for which activation suppression occurs, and a corresponding substantial decrease in current, are

$$\Delta f_{\min}^{(m)} = 2K - f_0 + \left[\frac{3\omega}{4K}\sqrt{2\left(2K - f_0\right)}\left(m + \frac{3}{4}\right)\pi\right]^{2/3}. \tag{26}$$

The four first minima, that is, $m = 0, 1, 2$, and 3, are marked with arrows in Figure 3.

Using the same logic, the maxima of the current occur for

$$\Delta f_{\max}^{(m)} \cong 2K - f_0 + \left[\frac{3\omega}{4K}\sqrt{2\left(2K - f_0\right)}\left(m + \frac{1}{4}\right)\pi\right]^{2/3}. \tag{27}$$

And the largest maximum $\Delta f_{\max}^{(m=0)}$ is marked as Δf_m in Figure 3.

Since, beyond the resonance value Δf_r, the adiabatic approximation is a good approximation for the current maxima, we can substitute $\Delta f_m \equiv \Delta f_{\max}^{(m=0)}$ in (24) to achieve a compact expression for the maximum current

$$\langle j_{\max} \rangle = j_s \cdot \frac{\left(1 - f_0/2K\right)\exp\left(2KL\right)}{2\sin\left(2\,\mathrm{atan}\left(k/K\right)\right)\sqrt{2}\left[3\pi\omega\sqrt{2}/16K\left(2K - f_0\right)\right]^{1/3}}. \tag{28}$$

Clearly, this expression is exponentially larger than the stationary current j_s but is exponentially *smaller* than the adiabatic maximum (22).

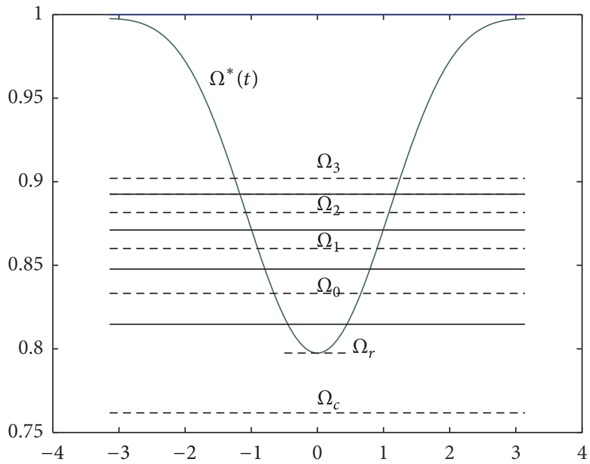

FIGURE 4: The energies for which constructive (solid horizontal lines) and destructive interference (dashed horizontal lines) occur. The curve stands for $\Omega^*(t)$, its minimum value is Ω_r, and $\Omega_0 - \Omega_r = \Omega_r - \Omega_c$.

It is also of interest to mention that the smaller the oscillating frequency ω, the larger the maximum current. Clearly, this agrees with the fact that the adiabatic maximum is larger. However, this approximation fails when ω is exponentially small and the adiabatic approximation is restored.

The deviation from the adiabatic approximation occurs when the oscillation amplitude reaches the quasi-resonance Ω_c, where $\Omega_0 - \Omega_r = \Omega_r - \Omega_c$ (see Figure 4). In terms of the amplitude,

$$\Delta f_c = 2K - f_0 - \left[\frac{3\omega}{4K}\sqrt{2(2K - f_0)}\frac{5}{4}\pi\right]^{2/3}. \quad (29)$$

At this amplitude value the current can be evaluated by substituting (29) in (23):

$$j_c = \langle j\rangle(\Delta f_c) \cong j_s\frac{4K(2K - f_0)}{15\omega\pi}. \quad (30)$$

This increase is relatively mild and depends on the energy distance from the resonance energy level.

From this reasoning it is possible to formulate an approximation for the frequency dependence of the mean current. The main contributions to the wave function (and therefore, for the current) come from the points, in which the incoming energy Ω crosses the resonance energy $\Omega^*(t)$. Since in this regime the particle is trapped to the well, the phase difference between the two contributions is

$$\Delta\Phi \equiv \frac{C}{\omega} + \frac{\pi}{2}, \quad (31)$$

where $C \equiv \int_{-\xi_0}^{\xi_0} d\xi'[\Omega^*(\xi') - \Omega]$ is a constant, $\Omega^*(\xi) \cong U - (f_0 + \Delta f\cos(\xi))^2/4$ is the resonance energy, and $\xi_0 = \arccos[(2K - f_0)/\Delta f]$ is the crossing normalized time.

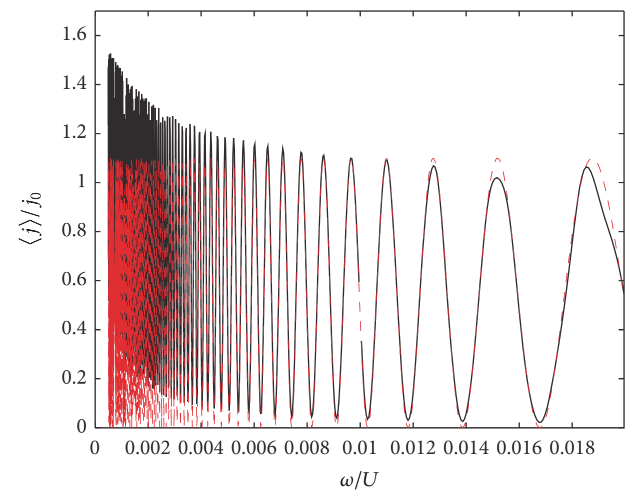

FIGURE 5: The dependence of the mean current on the oscillating frequency. The solid line represents the exact numerical solution, while the dashed (red) curve stands for the approximation. Equation (32) is multiplied by 1.1. The simulations parameters were $L\sqrt{U} = 6$, $f_0/\sqrt{U} = 0.8$, $\Omega/U = 0.5$, and $\Delta f/\sqrt{U} = 1.1$.

Therefore,

$$\langle j_{\max}\rangle \cong \frac{j_s(\exp(i\Delta\Phi/2) + \exp(-i\Delta\Phi/2))^2}{4} = j_s\cos^2\left(\frac{C}{2\omega} + \frac{\pi}{4}\right). \quad (32)$$

Thus, the phase is inversely proportional to the oscillating frequency. This simple property is clearly seen in Figure 5, where there is a high agreement between the exact solution and the approximation (32).

6. The Convergence to the Adiabatic Regime

The process, in which the solution converges to the adiabatic one, is illustrated in Figure 6. When ω decreases the deviation point from the adiabatic solution (Δf_c) and the largest maximum (Δf_m) converges both toward Δf_r.

Similarly, the oscillations frequencies, for which activation is suppressed, are

$$\omega_m = \frac{4(\Omega - U + (f_0 + \Delta f)^2/4)^{3/2}}{3\pi(m + 3/4)\sqrt{\Delta f(f_0 + \Delta f)}} \quad \text{for } m = 0, 1, 2, \ldots, \quad (33)$$

where full activation occurs for

$$\omega_m^{(\mathrm{act})} = \frac{4(\Omega - U + (f_0 + \Delta f)^2/4)^{3/2}}{3\pi(m + 1/4)\sqrt{\Delta f(f_0 + \Delta f)}} \quad \text{for } m = 0, 1, 2, \ldots. \quad (34)$$

These minima and maxima are clearly shown in Figure 7. Moreover, it is clearly shown that when the frequency is

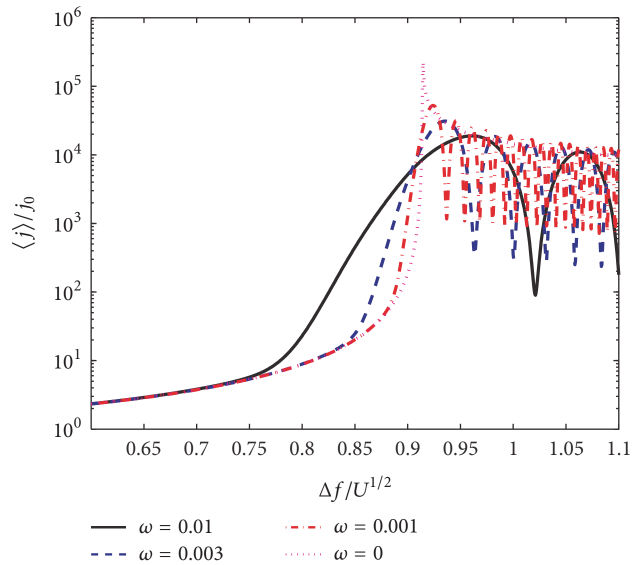

FIGURE 6: The average current as a function of the oscillation's amplitude for different oscillation's frequencies (same numerical parameters as in Figure 3).

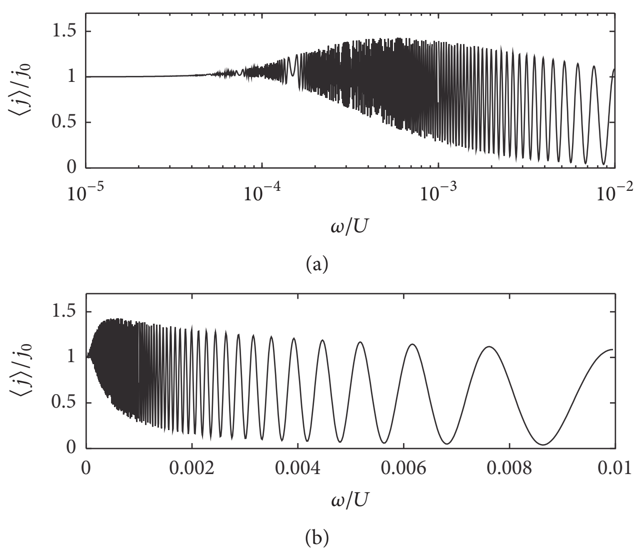

FIGURE 7: The ratio between the mean current and the stationary current as a function of the perturbation frequency in linear (b) and logarithmic (a) scales. The parameters are $L\sqrt{U} = 6$, $f_0/\sqrt{U} = 0.5$, $\Omega/U = 0.5$, and $\Delta f/\sqrt{U} = 1.2$. The adiabatic transition is approximately at $\omega/U \sim 10^{-4}$.

lower than the adiabatic threshold, that is, for $\omega/U \ll \exp(-KL)$, which is approximately $\sim 10^{-4}$ in the case presented in Figure 7, the average current is very close to the adiabatic approximation; that is, $\langle j \rangle \cong j_0$. When the frequency increases beyond this value the average begins to oscillate with the frequency. Within this activation zone, the current can exceed the adiabatic value considerably $\langle j \rangle > j_0$; however, as the frequency increases even further the maxima converges again to $\langle j \rangle \to j_0$.

7. Physical Realization

Due to the sensitivity of the current on the oscillating frequency, it is natural to identify such processes in microscopic tunnelling structure, such as odour receptors (see, e.g., [40, 41]).

However, current nanoscopic electronics allow fabricating such devices, where the current is controlled by the bias frequency. A possible realization of this device is a semiconductors heterostructure, where AlGaAs and GaAs are used alternately for the wells and the conductors/well. When the aluminium mole fraction is about 0.4, then the barrier height (between the two materials) is approximately 0.4 eV (see, e.g., [7]). Therefore, since the effective electron mass in AlGaAs is approximately $m \cong 0.07 m_0$ (where m_0 is the free electron mass), then if a 3 nm width well is connected to an ac voltage source, an approximately $\Delta V \cong 0.3$ V amplitude is sufficient to reach the resonance level (note that $\Delta V = \Delta f / w$, where w is the width of the well, which we took to be $w \cong 3$ nm). In this case, if the well is modulated at frequency $\omega / 2\pi \cong 10$ GHz, then (26) and (27) can be approximated (measured in volts):

$$\begin{aligned} \Delta V_{\min}^{(m)} &\cong 0.3 \left[1 + 0.003 \left(m + \frac{3}{4} \right)^{2/3} \right] [\mathrm{V}], \\ \Delta V_{\max}^{(m)} &\cong 0.3 \left[1 + 0.003 \left(m + \frac{1}{4} \right)^{2/3} \right] [\mathrm{V}], \end{aligned} \quad (35)$$

respectively.

Thus, an approximately 1 mV variation in the amplitude of the oscillating voltage will increase/decrease the current by at least a factor of $\sim \exp(2KL) \cong 8000$ (for a barrier's width of $2L \cong 10$ nm).

These performances suggest that such a device can be used as a frequency effect transistor. These devices can be much more accurate than ordinary transistors since frequency is a parameter, which can be controlled with great precision (much greater than voltage, e.g.).

8. Summary

The current through an opaque barrier with an oscillating well was calculated both exactly numerically and approximately analytically for different regimes. In particular the exact solution was compared to the adiabatic solution. The main conclusions are as follows.

Despite the fact that the adiabatic analysis neglects activation processes, the adiabatic approximation is a good evaluation of the *upper* limit for the current. In fact, it is shown that the presence of the quasi-bound eigenstate usually suppresses the activation and therefore *decreases* the current. A substantial current increase beyond the adiabatic level occurs mainly when the incoming energy is *lower* than the minimum resonance energy. In this regime the activation is not due to the elevator effect [15, 17] since the particle cannot be trapped in the well in this energy regime. Moreover, the current is *always smaller* than the maximum value of the adiabatic approximation.

When the incoming energy crosses the resonance energy, the incoming particle can be trapped in the well and then activated to higher energies (EE); however, when it comes to the current, the increase is relatively small. The main effect of the eigenbound state is current reduction when there is a destructive interference inside the well (the Sisyphus effect).

In general, the dependence of the mean current on the oscillation amplitude Δf follows the adiabatic solution, except for few regimes, which can be fully characterized by the following values: the point where the two solutions depart, that is, Δf_c; the resonance value (i.e., the maximum value) of the adiabatic approximation Δf_r, and the activation minima $\Delta f_{\min}^{(m)}$ and maxima $\Delta f_{\max}^{(m)}$.

These results suggest that activation is not an optimal method to increase the mean current; however, they do show that the current can easily be controlled by changes in the frequency and therefore may be used in frequency effect devices [20, 41].

Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

The author would like to thank Chene Tradonsky for helping with the derivation of the mean adiabatic value (21).

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