

Research Article

First-Principles Study of the Quasi-Particle and Excitonic Effect in o-BC₂N: The GW + BSE Study

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Ternary boron-carbon-nitride compounds are the hardest, chemically stable, and most applicable semiconductors in optoelectronic devices. We investigate the quasi-particle and excitonic properties of type II o-BC₂N using many-body perturbation theory (MBPT). The state-of-the-art GW and BSE methods were used to determine the accurate band gap and excited-state characteristics of this material. We simulate the convergence test and structural optimization in DFT, which is the starting point for the GW calculation. We also compute the convergence test of the parameters in GW and BSE. As a result, the bandgap of our system is found to be 2.31 eV and 1.95 eV using the GW approximation and DFT-PBE, respectively. Since the valence and conduction band edges are located at different Brillouin zones, we decide that o-BC₂N is an indirect bandgap semiconductor. In addition, by applying the scissor operator, we corrected the quasi-particle bandgap, which shows almost the same result as the GW approximation. Furthermore, using the BSE algorithm, we calculate the optical bandgap of type II o-BC₂N to be 4.0 eV with the excitonic effect and 4.4 eV without the excitonic effect. The highest peaks of the imaginary dielectric function with the excitonic effect shift to a lower energy level at 11 eV than without the excitonic effect at 13.5 eV. The electron charge distribution is computed by fixing the hole position. Finally, we suggest that type II o-BC₂N is promising for the application of optoelectronic semiconductors.

1. Introduction

Triplet, boron-carbon-nitrogen (B-C-N) substances have recently garnered significant interest since they represent a new type of higher strength material with exceptionally high hardness comparable to diamond [1–3] and outstanding thermochemical stability as cubic boron nitride (c-BN) [4]. Furthermore, B-C-N compounds are expected to operate for electrical and optoelectronic devices because their bandgaps are considered to be midway between semimetallic graphite and insulator hexagonal BN (hBN) and may be controlled by the atomic content and atomic structure [5, 6]. In addition to the normal characteristics, B-C-N materials may display a

variety of other notable properties, including vibrational and thermal properties [7].

Diamond-like BC₂N (c-BC₂N) has received a lot of interest among triple B-C-N systems. Despite the fact that various purported cubic BC₂N compounds have been effectively produced [8, 9], the crystalline phase of BC₂N has yet to be confirmed definitively. As a result, the majority of theoretical and empirical investigations focused on BC₂N's structural forms and mechanical properties.

Numerous structures have already been proposed so far, such as zinc-blende [10], chalcopyrite [11], tetragonal [12], wurtzite [13], body-centered BC₂N [14], low-density [15], and type II or orthorhombic structures [16] of BC₂N.

Among these structures, the zinc-blende phase was assumed to be accurate due to its simulated X-ray diffraction (XRD) spectrum, which agreed well with the experimental data [12], and the tetragonal phase was predicted to be another candidate because the simulated XRD and Raman patterns agreed well with the experimental results.

Density functional theory (DFT) [17, 18] is a single-particle technique that is extremely successful in determining the ground-state electrical characteristics of many electron materials. The Kohn–Sham (KS) DFT formalism is widely used to understand and predict the structural, electronic, magnetic, and other properties of a wide range of molecules and condensed matter systems. When using the local density approximation (LDA) or generalized gradient approximation (GGA) functionals, depending on the KS energy levels (or bands) results in consistent 30–100% underestimation of the electronic bandgap (E_{gap}) of semiconducting compounds and insulators [19].

This effect occurs to be independent of the nature of the system [20], and the bandgap’s underestimation has been attributed to their inherent lack of derivative discontinuity [21, 22] and delocalization errors [23]. As a result, it is clear that precise prediction of bandgaps is one of the crucial problems in DFT, with potentially broad applications in many scientific fields such as photocatalytic processes and optoelectronic devices.

Semiempirical approaches, such as DFT + U [24–26] and hybrid density functionals (HSE) [27], which were originally developed to improve the description of the ground-state energies of small molecules, have been shown to improve the description of bandgaps in semiconductor systems [28]. Furthermore, the prediction of quasi-particles necessitates the use of many-body techniques, such as the many-body perturbation theory (MBPT) based on the GW method, to describe the energies of solid-state electronic excitation spectra. Despite its high computational cost, GW accurately predicts the quasiparticle energies of the materials [29].

The whole excitation spectrum of systems may be obtained by using time-dependent density functional theory (TDDFT) [30]. However, when common (semi) local functionals [31] are applied, they fail in most cases to reproduce charge-transfer excitations. These problems opened the way for the achievement of range-separated hybrid functionals [27, 32] that seem to be accurate for both local and charge-transfer excitations [33], despite the fact that transferability of variables influencing short and long-range exchange contributions [34, 35] is still a difficult problem.

Recently, another method derived from many-body perturbation theory (MBPT) within Green’s function formalism, known as the GW approximation [36, 37] and the Bethe–Salpeter equation (BSE) [38] algorithm, which were originally developed for bulk semiconductors, has been successfully applied to the problem of charge-transfer excitations.

Moreover, the GW approach has lately become a widely used method for calculating bandgaps and charged excitation energies for extended [39, 40] and finite systems [29, 41]. Many-body perturbation theory (MBPT) [42] presents a natural framework for an ab initio, parameter-free explanation

of photo-ionization processes and charged excitations [43] in the GW technique for electron self-energy [36, 44, 45]. Due to the electrostatic Coulomb interaction, an electron stimulated to a conduction band state in a semiconductor or an insulator may form a bonded state with the hole it leaves behind in the valence band state. The electrically neutral quasi-particle (QP), also known as an exciton [46], has a hydrogenic wave function with a much smaller binding energy and a much larger particle size than a hydrogen atom due to the relatively small effective masses of the excited electron and hole, as well as the screening of the Coulomb force by other electrons in the system. The attractive interaction between electrons and holes, which is commonly represented by the exciton binding energy, contributes to the creation of excitons [46], trions (charged excitons) [47, 48], and biexcitons [49].

In semiconductor-based optoelectronic devices, such as light-emitting diodes and laser diodes, the excitonic effect is essential [50–52]. Radiative recombination of excitons is thought to be far more efficient than that of free electrons and holes. Materials having packed and stable excitons are thus promising for low-threshold lasing.

In this study, we apply many-body perturbation theory (MBPT) methods, such as the GW approximation (GWA) in conjunction with the Bethe–Salpeter equation (BSE) formalism to calculate the quasi-particle bandgap and the characteristics of two body electron-hole bound states associated with neutral excitations of bulk type II o-BC₂N semiconductors. To the best of our knowledge, there is no scientific paper that shows the quasi-particle bandgap and the excitonic effect of type II o-BC₂N using GW and BSE. The convergence test for both GWA and BSE is computed carefully.

2. Computational Details

2.1. Ground-State Calculations. The DFT calculation was performed with the Quantum ESPRESSO software [53, 54]. The generalized gradient approximation (GGA) with the Perdew–Burke–Ernzerhof (PBE) [55, 56] functional describes the exchange correlation. We employ the norm-conserving Vanderbilt pseudopotential [57], as used in electron-ion interactions. We expanded the one-electron wave functions on a plane-wave basis, with the plane-wave energy cutoff set at 544 eV to assure total energy convergence to 0.001 eV per primitive cell. The Broyden–Fletcher–Goldfarb–Shanno (BFGS) [58] method was used for structural optimization. The ground-state convergence threshold is set at 10^8 eV between two subsequent simulation steps, and all atoms may be completely relaxed throughout geometric optimization till the Hellmann–Feynman force acting on each atom is less than 0.01 eV. The many-body effect is realized within the GWA by employing ground-state Kohn–Sham (KS) wave functions and corresponding eigenvalues (G0W0) [59].

The convergence criteria were set at 3.6×10^9 Ry/cell in energy and 0.4 for mixing. Following relaxation, the Brillouin zone (BZ) was integrated using a $7 \times 7 \times 7k$ -point grid. Our model began with a DFT calculation for the plane-wave self-consistent field and then continued to crystal structure

optimization. We utilized this result as input for the Yambo code by applying the command `p2y`, and then, the excitation-state calculation was performed.

2.2. Excited-State Calculation: Bandgap Correction and Optical Properties. Electronic excitations are not sufficiently described by fundamental KS theory since DFT calculates the ground state, and quasi-particle (QP) effects must be incorporated. A well-known disadvantage of DFT is the underestimation of the bandgap. To circumvent this problem, we employed the cutting-edge GWA, which works very well for calculating the bandgap in most semiconductors.

The Yambo code [60, 61] was used to compute QP adjustments at G0W0 and self-consistent GW (evGW), which were then utilized for BSE computations. The plasmon-pole approximation (PPA) [62, 63] was used to calculate the inverse dielectric matrix for the GWA. After we computed the convergence test of the parameters, we set the following parameters as the starting point for our calculation. A 40 Ry energy cutoff for the exchange component of self-energy (the number of G-vectors in the exchange), a 4 Ry cutoff for the correlation part (energy cutoff in the screening) or response block size, 30 bands in the independent response function, 30 GW bands to expand Green's function, and Newton linearization are used to calculate the GWA. In addition, the fourth round of GW self-consistency on eigenvalue alone (evGW) was utilized.

The BSE computation was performed after the GW calculation, and evGW was utilized throughout the BSE calculation. The convergence test of the parameters in the BSE calculation is also essential. Once we test their convergence, we decided to use a 4 Ry cutoff energy (screened interaction (W) or electron-hole interaction), 30 Ry Hartree potential (V) components, 4–12 Bethe–salpeter (BS) bands (5 occupied and 4 unoccupied states), a diagonalization solver (calculates all exciton energies and composition in terms of electron-hole pair) for BSE, and a $12 \times 12 \times 4$ k -point mesh for both GW and BSE calculations.

3. Mathematical Formulation

The mathematical derivation of the GW and BSE calculation is as follows. We start from the generalized gradient approximation of the Kohn–Sham (KS) equation, which is given as

$$\left[-\frac{1}{2}\nabla^2 + V_{eff}(\mathbf{r}) \right] \varphi_{nk}^{KS}(\mathbf{r}) = E_{nk}^{KS} \varphi_{nk}^{KS}(\mathbf{r}), \quad (1)$$

where $V_{eff} = V_{ext}(\mathbf{r}) + V_H(\mathbf{r}) + V_{xc}(\mathbf{r})$, ∇^2 is the kinetic energy, V_{ext} is the external potential, V_H is the Hartree potential, and V_{xc} is the exchange-correlation potential.

The perturbatively corrected GW QP eigenvalues are obtained by replacing the exchange-correlation contribution to the KS eigenvalues (Equation (1)) with $\sum_{xc}(\mathbf{r}, \mathbf{r}', E)$ self-energy (calculated by the GWA) that accounts for many-body exchange-correlation effects. As a result, Equation (1) can be rewritten as

$$\left[-\frac{1}{2}\nabla^2 + V_{ext}(\mathbf{r}) + V_H(\mathbf{r}) + \int \sum_{xc}(\mathbf{r}, \mathbf{r}'; E_{nk}^{qp}) |\varphi_i^{qp}(\mathbf{r}') d\mathbf{r}' = E_{nk}^{qp} \varphi_{nk}^{qp}(\mathbf{r}), \quad (2)$$

where $\sum_{xc}(\mathbf{r}, \mathbf{r}'; E_{nk}^{qp})$ is the energy-dependent self-energy operator.

The quasi-particle (QP) energy with the GW approximation is given by

$$E_{nk}^{qp} = E_{nk}^{KS} + M_{nk} \left[\text{Re} \sum_{nk} (E_{nk}^{KS}) - V_{nk}^{xc} \right], \quad (3)$$

where E_{nk}^{KS} is the KS eigenvalue, Σ_{nk} and V_{nk}^{xc} are the diagonal matrix elements of self-energy and the exchange-correlation (xc) potential that is employed in the single-particle KS Hamiltonian. The QP renormalization factor M_{nk} accounts for the energy dependence of self-energy and is given by

$$M_{nk} = \left[1 - \frac{d \sum_{nk}(\omega)}{d\omega} \Big|_{\omega=E_{nk}^{KS}} \right]^{-1}. \quad (4)$$

For $M_{nk} \ll 1$, we may deduce that φ_{nk}^{KS} is not a (correct) QP state. The two possible explanations for this are as follows: The first explanation is that electrons are tightly correlated, and hence, the QP image does not apply; the second one is that φ_{nk}^{KS} is a poor approximation to the genuine QP wave function φ_{nk}^{qp} . While the former argument is based on the physics of the underlying electron system, the latter argument simply states that the Kohn–Sham orbital does not adequately reflect actual many-body excitations. For instance, where the QP image is totally correct, that is, all QP states have norms extremely near to 1 or 0, yet simple noninteracting orbitals do not offer a decent approximation to them [64].

The self-energy operator in Equation (2) includes all interactions beyond the Hartree contribution given by

$$\sum_{xc}(\mathbf{r}, \mathbf{r}'; E_{nk}^{qp}) = iGW, \quad \sum_{xc}(\mathbf{r}, \mathbf{r}'; E_{nk}^{qp}) = \frac{i}{2\pi} \int G(\mathbf{r}, \mathbf{r}'; E_{nk}^{qp} + \omega) W(\mathbf{r}, \mathbf{r}'; \omega) \exp^{i\omega\theta} d\omega, \quad (5)$$

where G is Green's function and W is dynamically screened Coulomb potential and are given as

$$G(\mathbf{r}, \mathbf{r}'; E_{nk}^{qp} + \omega) = \sum_n \sum_{\mathbf{k}} \frac{\varphi_n(\mathbf{r})\varphi_n^*(\mathbf{r}')}{E_{nk}^{qp} - \varepsilon_n + \vartheta \text{sgn}(\varepsilon_n - E_F)}, W(\mathbf{r}, \mathbf{r}'; \omega) = \int \varepsilon^{-1}(\mathbf{r}, \mathbf{r}''; \omega) V(\mathbf{r}', \mathbf{r}'') d\mathbf{r}''. \quad (6)$$

φ_n and ε_n are the input single-particle eigenstates/eigenenergies derived from DFT Kohn–Sham computations, respectively. The Fermi level is denoted by E_F , while the bare Coulomb potential is denoted by V in the KS calculation. The inverse dynamical dielectric matrix, ε^{-1} , is derived below inside the random-phase approximation, and χ_o is the independent-electron susceptibility. To ensure convergence, the infinitesimally tiny positive value (ϑ) is added while performing a Fourier transformation from time to frequency space.

$$\chi_{G,G'}(\mathbf{q}, \omega) = \chi_{G,G'}^o(\mathbf{q}, \omega) + \sum_{G_1, G_2} \chi_{G,G_1}(\mathbf{q}, \omega) [V_{G_1}(\mathbf{q})\delta_{G_1, G_2} + f_{G_1, G_2}^{xc}] \chi_{G_2, G'}(\mathbf{q}, \omega), \quad (9)$$

where f_{G_1, G_2}^{xc} is the xc kernel that accounts for the exchange and correlation effects.

In PPA, the frequency dependence of the dielectric function $\varepsilon_{G,G'}^{-1}(\mathbf{q}, \omega)$ is modeled as a single-pole approximation:

$$\varepsilon_{G,G'}^{-1}(\mathbf{q}, \omega) = R_{GG}(\mathbf{q}) \left(\frac{1}{\omega - \omega_{G,G'}(\mathbf{q}) + i\vartheta} - \frac{1}{\omega + \omega_{G,G}(\mathbf{q}) - i\vartheta} \right), \quad (10)$$

where $\omega_{G,G'}$ and $R_{G,G'}(\mathbf{q})$ are the plasmon frequency and the real spectral function, respectively.

For optical calculations, we employed a variety of approximations, including the independent-particle approximation with local-field effects and the Bethe–Salpeter equation.

Before delving into the data, we will go through the theoretical approximations that have been utilized to compute optical absorption. The optical spectra are derived at the level of the Bethe–Salpeter equation (BSE) starting from the Kohn–Sham wave functions and quasi-particles [65–68]:

$$(E_{c\mathbf{k}}^{qp} - E_{v\mathbf{k}}^{qp}) B_{v\mathbf{k}}^S + \sum_{\mathbf{k}'c'} \langle v\mathbf{k} | \kappa_{eh} | v'c' \mathbf{k}' \rangle B_{v'\mathbf{k}'}^S = \Omega^S B_{v\mathbf{k}}^S, \quad (11)$$

where $B_{v\mathbf{k}}^S$ and Ω^S are the excited eigenstates and eigenvalues (energy of excited states) for the S^{th} exciton. Electronic excitations are represented in terms of electron-hole pairs (i.e., vertical excitations at a particular k -point from a valence band state (v) with quasi-particle energy $E_{v\mathbf{k}}^{qp}$ to a conduction band state (c) with energy $E_{c\mathbf{k}}^{qp}$). The interaction kernel κ_{eh} defines the screened Coulomb interaction between electrons and holes as well as the exchange interaction, which includes local-field effects.

In this framework, the electron-hole amplitude in the real space (or the wave function of excitons) is given by the Tamm–Dancoff approximation as follows [66, 69–71]:

The microscopic inverse dielectric function in the reciprocal vector is given by [36, 37]

$$\varepsilon_{G,G'}^{-1}(\mathbf{q}, \omega) = \delta_{G,G'} + V_G(\mathbf{q}) \chi_{G,G'}(\mathbf{q}, \omega), \quad (7)$$

where the bare Coulomb interaction in a reciprocal vector is

$$V_G(\mathbf{q}) = \frac{4\pi}{|\mathbf{q} + \mathbf{G}| |\mathbf{q} + \mathbf{G}'|}. \quad (8)$$

The polarization susceptibility $\chi_{G,G'}(\mathbf{q}, \omega)$ is given by

$$\Phi_S(\mathbf{r}_e, \mathbf{r}_h) = \sum_{\mathbf{k}} \sum_{v'} \sum_c B_{v\mathbf{k}}^S \phi_{c\mathbf{k}}(\mathbf{r}_e) \phi_{v\mathbf{k}}^*(\mathbf{r}_h), \quad (12)$$

where $\phi_{c\mathbf{k}}(\mathbf{r}_e)$ and $\phi_{v\mathbf{k}}^*(\mathbf{r}_h)$ are the wave function of the electron in the conduction band and hole in the valence band at the wave vector \mathbf{k} in the real space.

The exciton state is represented by the expansion of the following equation [72]:

$$|S\rangle = \sum_{\mathbf{k}} \sum_{v'} \sum_c B_{v\mathbf{k}}^S |v\mathbf{k}\rangle. \quad (13)$$

If the interaction kernel κ_{eh} is missing, Equation (11) merely returns $\Omega^S = E_{c\mathbf{k}}^{qp} - E_{v\mathbf{k}}^{qp}$, indicating that the system's excitations belong to separate electron-hole pairs.

The imaginary portion of the dielectric function (ε_2) gives the optical absorption spectrum, which may be computed as [68]

$$\varepsilon_2(\hbar\omega) \propto \sum_S \left| \sum_{c\mathbf{k}} B_{v\mathbf{k}}^S \frac{\langle c\mathbf{k} | \eta | v\mathbf{k} \rangle}{E_{c\mathbf{k}} - E_{v\mathbf{k}}} \right| \delta(\Omega^S - \hbar\omega - \gamma), \quad (14)$$

where the dipole matrix elements for electronic transitions from valence to conduction states are $\langle c\mathbf{k} | \eta | v\mathbf{k} \rangle$ and γ is widening of energy. We only examine light absorption with polarization, and hence, the orientation of the dipole operator η is obtained.

4. Results and Discussion

4.1. Parameter Optimization and Structural Stability. We model primitive unit cells of type II orthorhombic BC_2N . Then, we compute the convergence test with respect to the plane-wave kinetic energy cutoff and the k -point grid density using Quantum ESPRESSO. Because bad convergence tests usually result in an inaccurate converged total energy, these parameters are the primary starting points for any plane-wave self-consistent field (PWSCF) computations. The

convergence test calculation demonstrates that changes in the energy cutoff and k -points become stable at 544 eV and $7 \times 7 \times 7$, respectively. Hence, raising the energy cutoff above 544 eV and k -points above $7 \times 7 \times 7$ has no significant influence on total energy.

The optimization of the type II o-BC₂N crystal structure was calculated by considering the above convergence test. As a result, we found that the bond lengths of C-C, C-N, and C-B are 1.52, 1.56, and 1.58 Å, respectively. We also obtain the lattice constants $a = 2.52$, $b = 2.549$, and $c = 3.62$ Å, which are in good agreement with the earlier theoretical values of 2.527 Å, 2.533 Å, and 3.605 Å [11, 16]. Figures 1(a)–1(c) show the optimized crystal structure of type II o-BC₂N in $2 \times 2 \times 3$ supercells (see Figure 1(a)) and primitive unit cells (Figures 1(b)) and 1(c)) with space group Pmm2. The green, brown, and light blue colors correspond to boron, carbon, and nitrogen atoms, as depicted in the figure, respectively.

To confirm the structural stability of orthorhombic BC₂N (o-BC₂N), we computed phonon dispersion and found that this system is stable.

4.2. Electronic Properties. There are different computational methods to calculate the electrical properties of materials. Among these, DFT(GGA) is a computational method to evaluate the bandgap of the system, even though it has inaccuracy with respect to the experimental results. For a more accurate calculation of the quasi-particle bandgap, we apply the GW approximation. Before the calculation of the band structure of type II o-BC₂N, we compute the self-consistency test of GW on eigenvalues. This is because one-shot GW (G0W0) is insufficient to determine the accurate bandgap of the system. The importance of self-consistency by upgrading the eigenenergies of both G and W has been proved to be critical for many semiconductors in minimizing the effect of the starting point dependency on G0W0 [73]. To do this, we utilize the Yambo code, and our results are discussed as follows.

4.2.1. Self-Consistency of GW Eigenvalue Only and the Quasi-Particle Band Structure of Type II o-BC₂N. The G0W0 technique frequently produces unsatisfactory results for molecular systems and many solids. The main reason for this failure is that the DFT-starting point with local or semilocal exchange and correlation functionals results in an insufficiently small gap when compared to the experimental one, and a single-shot GW cannot compensate for this inaccuracy. This method has been used successfully to improve the G0W0 bandwidths and bandgaps of a wide range of solid materials [73]. This self-consistency of GW is calculated up to four iterations until the difference between consistent GW is 0.01 eV. We used up to four iterations because the bandgap is converged at this point, which means that taking above this level does not change the result of the bandgap; instead, it increases our computational cost, as shown in Figure 2(a)). There are several levels of the GW approach to enhance consistency with experiments. According to [74], partial self-consistency on Green's function G alone (GW0) and

both G and the screened Coulomb interaction W (GW) can also widen the bandgap.

We determine the self-consistency of GW on eigenvalues only (evGW) as follows: First, from DFT, the KS eigenvalues and eigenstates are used in the first GW iterations to compute the expectation value of self-energy. Then, multiple GW iterations are performed, replacing the KS eigenvalues with the most recent GW QP values while leaving the KS eigenstates fixed. This technique is repeated until the GW gap is converged. Based on this result, QP energies and KS eigenvectors are then utilized to construct the BSE electron-hole Hamiltonian for the simulation of excitonic effects. In our case, Figure 2(a)) depicts the convergence test for four iterations of an evGW computation. Taking the fourth iteration into consideration, the quasi-particle HOMO-LUMO gap opened from the DFT-PBE calculation to the GW calculation.

We investigate the quasi-particle band structure of type II orthorhombic BC₂N (o-BC₂N) taking evGW into consideration. The band structure was computed along the path Γ (0.0, 0.0, 0.0) – X (0.5, 0.0, 0.0, 0.0) – S (0.5, 0.5, 0.0) – Y (0.0, 0.5, 0.0) – Γ (0.0, 0.0, 0.0) – Z (0.0, 0.0, 0.5) – U (0.5, 0.0, 0.5) – R (0.5, 0.5, 0.5) – T (0.0, 0.5, 0.5) – Z (0.0, 0.0, 0.5) at the Brillouin zone, which are high-symmetric points of the orthorhombic unit cell, as shown in Figure 2(b)), and the Fermi level is adjusted to zero. The results show that o-BC₂N possesses semiconductor qualities with a bandgap of 1.95 eV, 1.97 eV, and 2.31 eV applying DFT-PBE, G0W0, and evGW (G4W4), respectively. The top of the valence band and bottom of the conduction band are positioned in different locations, suggesting that type II o-BC₂N belongs to the indirect bandgap family. As we can observe from Figure 2(b)), the evGW (G4W4) correction shifts the valence band down and the conduction band up. First-principles calculations are performed by other academics accord well with our findings. Mattesini and Matar, for example, observed an indirect bandgap of 1.69 eV for type II o-BC₂N by employing LDA [16], and Bao et al. calculated 1.734 eV using GGA-PW91 [75], demonstrating the trustworthiness of our computational technique, since LDA is very poor in determining the bandgap of the system.

In addition, to correct the quasi-particle bandgap, we apply the scissor operator and renormalize the conduction and/or valence band, as shown in Figure 3. The impact of GW self-energy is an opening of the gap and a linear stretching of conduction/valence bands, which may be approximated by performing a linear fit of positive and negative energies (the zero is set at the top of the valence band). Hence, we found approximately the same result as that of GW QP calculation.

4.3. Optical Properties: Excitonic Effect. When a valence electron crosses a bandgap to the conduction band, it does not always escape the hole it creates. In some materials, Coulomb forces are strong enough to keep negatively charged electrons and positively charged holes together in neutral electron-hole pairs known as excitons. The existence and behavior of this quasi-particle influence the electrical

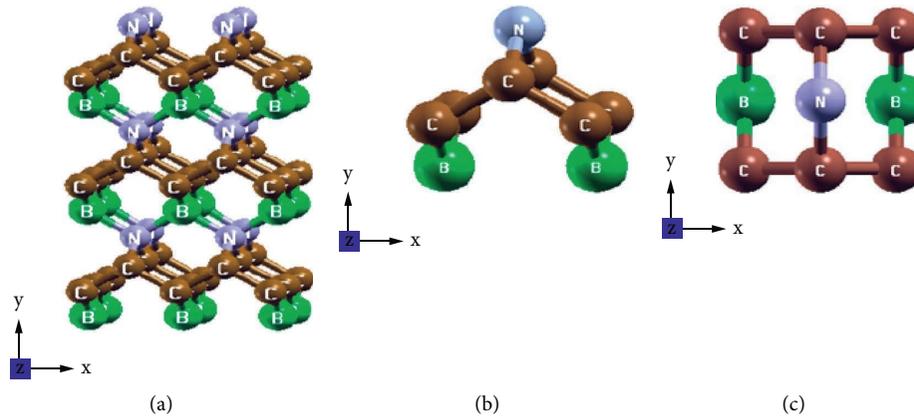


FIGURE 1: Optimized crystal structure of type II orthorhombic BC_2N : (a) supercells of $o\text{-}BC_2N$ of $22 \times 2 \times 3$, (b) a side view of unit cells of $o\text{-}BC_2N$, and (c) a top view of unit cell crystal structures of $o\text{-}BC_2N$, with the green color representing boron (B), the brown color representing carbon (C), and the light blue color representing nitrogen (N).

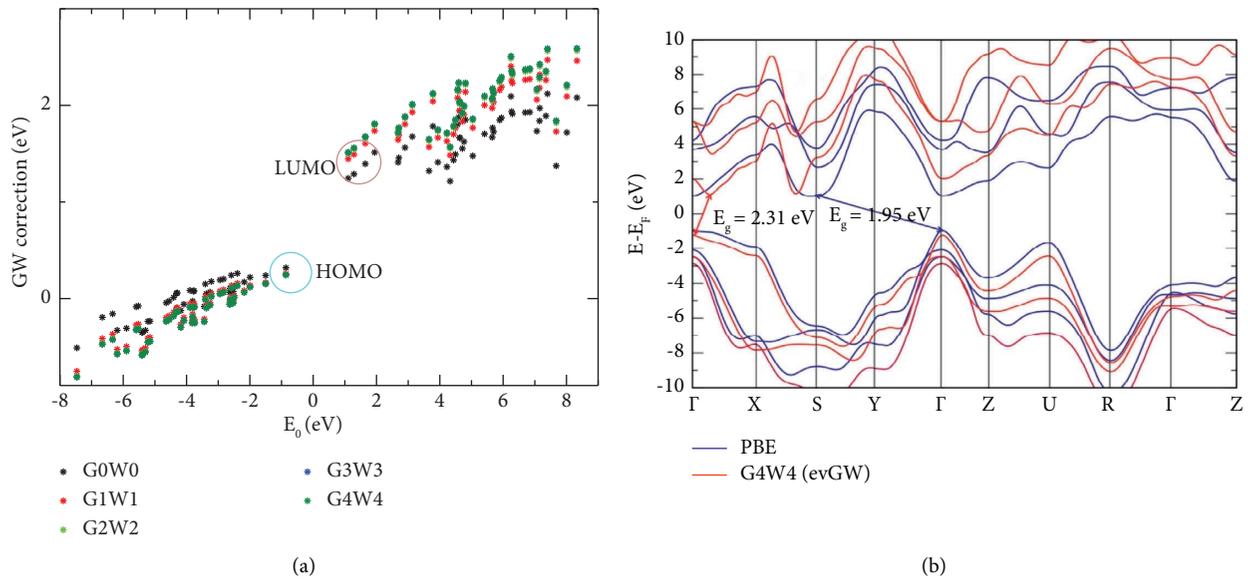


FIGURE 2: Quasi-particle bandgap calculation: (a) the convergence test of self-consistent GW on eigenvalues only for four iterations and (b) QP band structure calculation using evGW and DFT (PBE) methods.

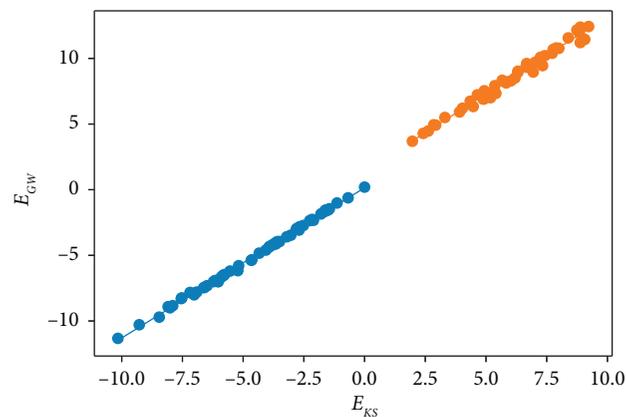


FIGURE 3: Scissor operator correction in QP and renormalization of conduction and valence bands. E_{GW} and E_{KS} are in electron volt (eV).

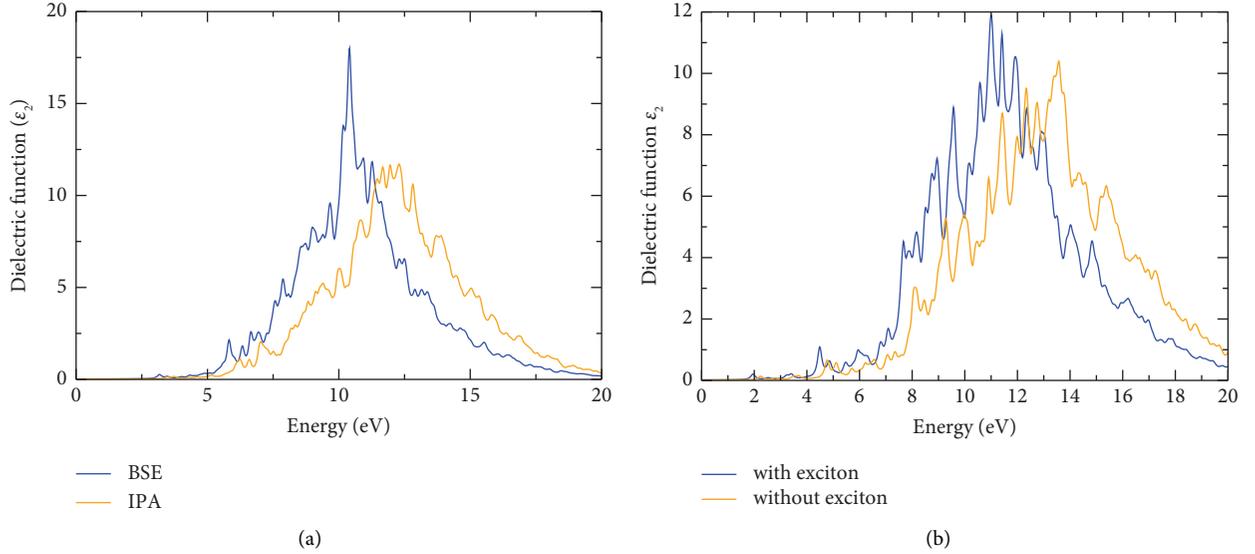


FIGURE 4: Optical spectra of imaginary dielectric function with arbitrary units: (a) the BSE and IPA result with respect to photon energy and (b) the excitonic effect in o-BC₂N using the BSE algorithm in the polarization direction xyz (/or (111)).

and optoelectronic properties of materials in which they reside.

The optical spectra of type II o-BC₂N are computed using the BSE algorithm based on our previous evGW (G4W4) result. Figure 4(a) shows the calculated spectra of o-BC₂N with comparison of the BSE and independent particle approximation (IPA). The threshold frequency, commonly known as the optical gap, shifts to the lower energy for BSE than for IPA. The highest peaks were found at 10.2 eV and 11 eV for BSE and IPA, respectively. In addition to the threshold frequency, there are multiple nonvanishing and/or noticeable peaks. Cutting-edge analysis of concise physical and chemical representations in the atom-dominated bandgap, atom-orbital singularities, and strong optical responses are used to realize the optical excitation process. Figure 4(b) clearly shows the excitonic effect in type II o-BC₂N. We can exhibit the band edge optical spectrum of 4.0 eV and 4.4 eV with exciton (electron-hole interaction) and without excitonic effects, respectively. Furthermore, the predicted complex dielectric function ($\epsilon_2(\omega)$) has two main peaks with the exciton effect and without the excitonic effect, which are positioned at 11 eV and 13.5 eV, respectively. Due to its broad bandgap, the light response range of o-BC₂N with and without excitonic effects is restricted to the ultraviolet area, limiting solar energy usage.

Figure 5 depicts the absorption spectra of type II o-BC₂N by computing BSE and the renormalized (valence and conduction band) scissor operator. This makes a difference in peak positions, distribution, and intensity. Besides a simple shift, we renormalized the bandwidth of the valence and conduction bands. As a result, we found that opening the QP bandgap using a scissor operator shows a red shift with a high peak at 10 eV.

Figure 6 shows the amplitude and the strength of excitons of o-BC₂N. The strength of bright excitons is at 1, as

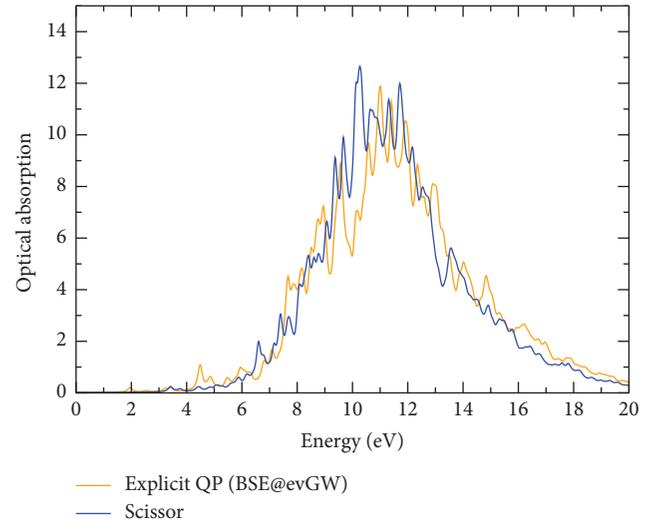


FIGURE 5: The absorption (arbitrary unit) of type II o-BC₂N using the BSE and scissor operator after renormalization of the valence band and conduction band in the polarization direction xyz (/or (111)).

shown in Figure 6(b), which is related to the highest peak shown in Figure 4(b), with (blue) excitonic effects.

The contribution of valence and conduction band states for the exciton is shown in Figure 7. The measured optical gap is composed of the transitions at the Γ point, as shown in Figure 7, which is corresponding to the strongest peak of ϵ_2 in Figure 4(b), as a result of the excitonic effect.

In the absence of exciton-phonon coupling, photon emission or absorption entails excited states $\Phi^S q$ (Equation (12)) of the total wave vector q corresponding to photon momentum, which is henceforth represented as $\mathbf{q}=0$. To address the exciton localization for o-BC₂N, we compute the exciton probability $|\Phi^S(r_h, r_e)|^2$ for a given

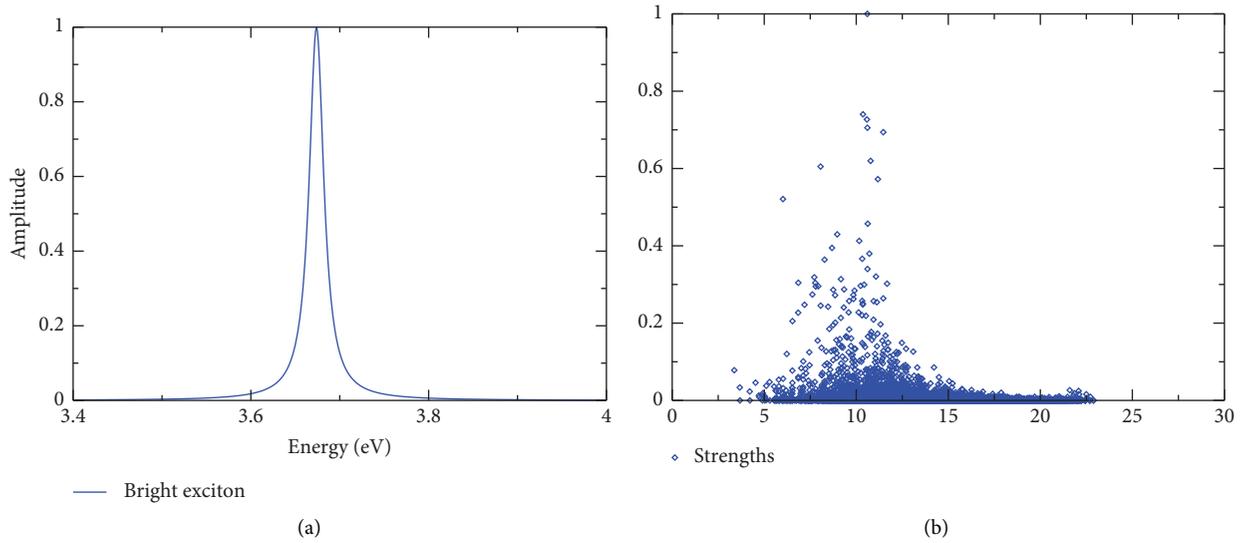


FIGURE 6: Exciton amplitude (a) and its strength (b) of the bright exciton.

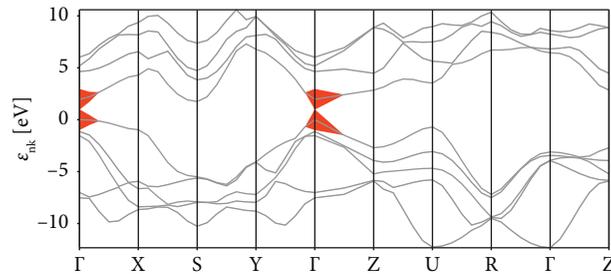


FIGURE 7: The contribution of the exciton at the valence band and conduction band edge.

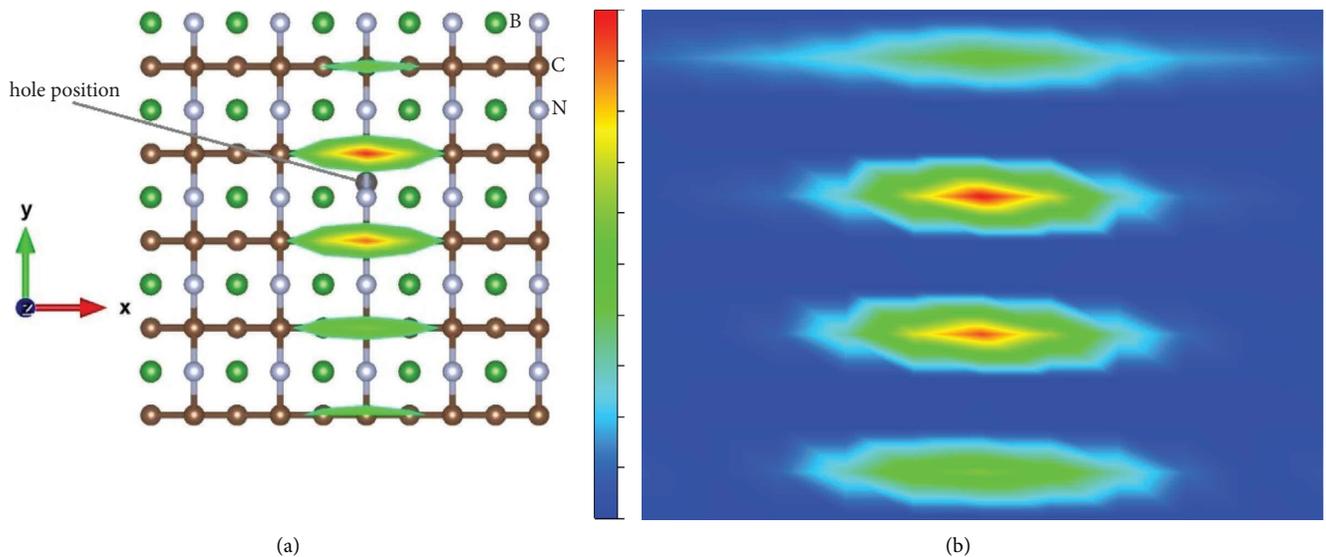


FIGURE 8: Exciton spatial distribution as the hole position fixed: (a) a top view of $5 \times 5 \times 1$ supercells of o-BC₂N with the green color representing boron (B), the brown color representing carbon (C), and light blue representing nitrogen (N) and (b) a two-dimensional contour plot of the probability density of finding electrons.

hole position \mathbf{r}_h and illustrate the electron distribution relative to the hole for the excited state at 11 eV. Such a distribution is shown in Figure 8 for a hole located just

above a nitrogen atom. The electron is delocalized on nitrogen atoms, as shown in Figure 8(a), and the likelihood of locating the electron on a nitrogen atom is

greatest for the boron atom's nearest neighbors where the hole is localized.

Similarly, in Figure 8(b), we calculate the exciton wave function, representing the electronic charge distribution $|\Phi^S(\mathbf{r}_h, \mathbf{r}_e)|^2$ for a fixed position of the hole \mathbf{r}_h , for the lowest energy exciton in the o-BC₂N system, which is visible for light polarized along the (111) axis. In accordance with earlier BSE computations [76, 77], we discover a charge-transfer exciton in o-BC₂N: the electronic charge is mostly concentrated on the nearest neighbor atoms with regard to the location of the hole. The charge-transfer nature of the exciton in o-BC₂N is caused by the interaction of the exchange electron-hole interaction with band dispersion, which causes the hopping term to be big enough to induce substantial mixing between Frankel and charge-transfer states.

When a single photon is absorbed, it produces a bright exciton. In the case of dark excitons, electrons and holes are connected by an optically forbidden transition, implying that the electron did not reach the conduction band only by photon absorption, which also required phonon scattering. Dark excitons are challenging to investigate using conventional optical methods because they can generate or recombine via a variety of paths.

5. Conclusion

In conclusion, we predicted the quasi-particle bandgap and the excitonic effect in type II orthorhombic BC₂N (o-BC₂N). To investigate the electronic and optical properties of this system, we performed the convergence test and structure optimization using state-of-the-art DFT and GW-BSE. We also calculate the self-consistent GW eigenvalue alone (evGW) for a better prediction of the quasi-particle bandgap, since the one-shot GW (G₀W₀) approximation is insufficient to approximate an accurate bandgap. Then, afterward, using DFT-PBE and GWA (evGW), we computed the quasi-particle bandgap and found that o-BC₂N has an indirect bandgap of 1.95 eV and 2.31 eV, respectively. This shows that the GWA has a better approximate bandgap than PBE. This disparity is due to the DFT-well-known PBE's flaw of underestimating the bandgap. In addition, we tried to correct the quasi-particle bandgap using the scissor operator, which shows almost the same result as the GW approximation. The valence band maximum (or HOMO) and conduction band minimum (or LUMO) are found at different Brillouin zones, and as a result, o-BC₂N is an indirect bandgap semiconductor. Furthermore, we calculated the optical properties of type II o-BC₂N with and without the electron-hole interaction (exciton). As a result, we observed the impact of the exciton, which changes the material properties from its bare (without exciton) o-BC₂N. According to the computed complex part of the dielectric function, the highest peaks occur at 11 eV and 13.5 eV with and/or without excitonic effects, respectively. This demonstrates that the dielectric function with excitons exhibits the optical spectrum shifts to lower photon energy than without the exciton effect. The highest peak in the presence of excitons also describes the strength of excitons. In the absence

of exciton-phonon coupling, we calculated the electronic charge distribution by fixing the hole position. By establishing the position of the hole on the top of the N atom, we calculate the spatial distribution of the wave function of the electron in the real space. These findings recommend that o-BC₂N is a promising material for advanced applications in optoelectronic devices.

Data Availability

All data that support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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