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Research Article

The Study of Transport Properties of (III−Mn) V Diluted Magnetic Semiconductors

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We investigated the transport properties of diluted magnetic semiconductors (DMSs) theoretically by using the Heisenberg and Ruderman–Kittel–Kasuya–Yosida (RKKY) exchange interaction models by considering both spin and charge disorder. The formalism is applied to the specific case of $Ga_{1-x}Mn_xAs$. Using the Heisenberg model and the Green function formalism the total thermal excitation of the magnon is calculated. The magnetization and Curie temperature of Mn-doped GaAs is calculated. The theoretical calculation of T_C of $Ga_{1-x}Mn_xAs$ at $x = 0.08$ has a good agreement with the experimental calculation at $x = 0.08$ (i.e., 162 k). The exchange interaction constant and spin-dependent relaxation time is calculated by using RKKY interaction. The electrical conductivity and hole mobility are calculated by using the Boltzmann transport equation and the spin-dependent relaxation time. The electrical conductivity of Mn-doped III–V DMS is exponentially increased with temperature and magnetic impurity concentration. Hole mobility of Mn-doped III–V diluted magnetic semiconductor is increased with the magnetic impurity concentration.

1. Introduction

The modern electronic industry heavily relies on diluted magnetic semiconductors (DMSs). In DMSs, semiconducting and magnetic properties coexist. The semiconductor property processes information using the charge of electrons and holes, while the magnetic property stores information using the spin of magnetic ions/atoms. Since magnetic materials and semiconductor applications have developed independently, it makes sense to integrate their capabilities to create spintronics, which have greater functionality [1, 2]. The fundamental goal is to regulate the electrons' degree of spin freedom in specific semiconducting materials. The most practical way to add spin degrees of freedom to semiconductors is to incorporate magnetic ions into them. These semiconductors can be defined as semimagnetic or DMSs [3, pp. 20–22].

According to the current studies on DMSs, the presence of magnetic ions significantly alters the electron transport phenomenon [3, pp. 20–22]. Beginning in the 1970s, it was realized that one material (DMS) might have both magnetic and semiconducting properties. The antiferromagnetic exchange between the magnetic ions (Mn) spins regulates the magnetic interaction in II–VI-based DMSs, leading to paramagnetic, spin glass, and finally long-range antiferromagnetic behavior. Nowadays, ferromagnetic properties are found in p-type II–VI DMS heterostructures, but they only exist at Curie temperatures (T_C), which are typically below 2.0 K. As a result, installing ferromagnetic-based III–V semiconductors with magnetic ion, Mn, which stay ferromagnetic to significantly higher T_C, has made very substantial progress [1]. The recent discovery that ferromagnetic ordering in III–V DMSs with phase transition temperatures of annealed DMS $Ga_{1-x}Mn_xAs$ could exceed 110 K; roughly approaching 170 K has greatly increased interest in those materials [4, p. 356; 5]. Although encouraging, the greatest temperature (173 K) in Mn-doped GaAs for $0.08 \leq x \geq 0.01$ obtained utilizing low-temperature annealing procedures is still too low for the practical applications [6].

A lot of theoretical and experimental work has been done on raising T_C. Currently, the highest reported value is just about 190 K. There are numerous ways to calculate this temperature. According to Vašek et al. [7], measuring magnetization as a function of temperature is the most suitable

approach. It is much easier to measure conductivity (resistivity) than magnetization, and T_C can also be found by looking at how resistance changes with temperature. Studying these transport characteristics theoretically is fairly difficult. Since the exchange interaction between the carriers and the localized moments, which results in the ferromagnetic transition, affects these features, it is necessary to take this into nonperturbative consideration [6, 8]. The magnetic impurity concentration and temperature have a significant impact on the experimentally determined dc resistivity in the DMS materials, which exhibits interesting characteristics. According to Hwang and Sarma [8], there is a metal–insulator transition in the material (Ga, Mn)As between insulating samples with low-manganese concentrations and metallic samples with higher concentrations. In this study, the theoretical computation of the DMS transport properties was carried out using the Ruderman–Kittel–Kasuya–Yosida (RKKY) interaction, the Heisenberg model, and the Green Function formalism.

2. Theoretical Models

2.1. RKKY Model Interaction Approaches for Spin Dependent Relaxation Time. In this case, the RKKY model, characterized by the energy J_{ij} of exchange coupling, is typically used to describe spin–spin interactions [9]. This interaction at the localized moment is determined by the spin density of the electrons [10, pp. 213–218]. Formulation of the spin density of electrons in terms of the given field operators, the coupling between the spin S_j at r_j and the conduction electron is

$$H_{p-d} = -2J\, S_j .\, S_i(r_j) = -J \sum_{\alpha\beta} S_j .\, \sigma_{\alpha\beta} \left(\widehat{\psi}^{+}{}_{\alpha}(r_j)\widehat{\psi}_{\beta}(r_j)\right), \tag{1}$$

where $\widehat{\psi}$ is the field operator states expanded in terms of the block states. By using the Fourier transform of field operators, the number density operator is expressed by

$$n(q) = \frac{1}{V} \sum_{kk'\sigma} \int e^{-i(k+q-k').r} u_k^{\times}(r) u_{k'}(r) c_{k\sigma}^{+} c_{k'\sigma} dr, \tag{2}$$

where $c_{k\sigma}$ and $c_{k\sigma}^{+}$ are the annihilation and creation operators of electrons, respectively. If we translate Equation (2) into terms of the spin density, the overall spin is conserved in the interaction, although conduction electrons and localized spins may exchange positions. The electron of wave vector k is scattered in state k', and this state determines the coupling strength. Assume that the k dependance may be ignored. The interaction with conduction electrons is determined by

$$H_{p-d} = -\frac{J}{V} \sum_{l=1}^{N_i} \sum_{k,k'} \sum_{k,k'} e^{i(k-k').r} \left\{ S_l^{+} c_{k'\downarrow}^{+} c_{k\uparrow} + S_l^{-} c_{k'\uparrow}^{+} c_{k\downarrow} + S_l^{z} \left(c_{k'\uparrow}^{+} c_{k\uparrow} + c_{k'\downarrow}^{+} c_{k\downarrow} \right) \right\}. \tag{3}$$

The interaction of localized spin Mn ions with electrons rises to the creation or annihilation of an electron–hole pair. By considering elastic transitions and the common energy $E_i = E_f$, denoted by E_0, the Hamiltonian is written as follows:

$$\langle f|H_{\text{eff}}|i\rangle = -\sum_j \frac{\langle f|\lambda H_{\text{eff}}|j\rangle \langle j|\lambda H_{\text{eff}}|i\rangle}{E_j - E_0}, \tag{4}$$

where $|i\rangle$ is the ground state, $|f\rangle$ the state of the scattering of the electron, λ is the coupling constant, and H_{eff} is the effective Hamiltonian which is equal to H_{p-d}. Since the flip of the impurity spin is accompanied by the flip of an electron spin, the collision term of the distribution function of spin-up and spin-down electrons is

$$\left(\frac{\partial f(k)}{\partial t}\right)_{\text{coll}} = \sum_{k'} W_{k\uparrow,k'\uparrow} \left\{ f_{\uparrow}(k') \left[1 - f_{\uparrow}(k)\right] - f_{\uparrow}(k) \left[1 - f_{\uparrow}(k')\right] \right\} + \sum_{k'} W_{k\uparrow,k'\downarrow} \left\{ f_{\downarrow}(k') \left[1 - f_{\uparrow}(k)\right] - f_{\uparrow}(k) \left[1 - f_{\downarrow}(k')\right] \right\}, \tag{5}$$

where $W_{k,k'}$ is the transition probability, $f(k)$ and $f(k')$ are the distribution function of the initial states and the scattered states, respectively. Then by substituting and using the distribution function for the spin of the electron–hole pair, the matrix element of the operator H is $H = -\sum_{ij} J(r_i - r_j) S_i . S_j$. With the help of some integral notation, the quadratic dispersion relation that holds true for free electrons, and the notations, we can calculate the exchange coupling strength between magnetic impurities and delocalized charge carriers. This exchange interaction constant, also known as the effective exchange interaction, is given by

$$J(r) = \frac{m_e J^2 k_{\text{F}}^4}{\hbar^2 \pi^3} F(2k_{\text{F}} r), \tag{6}$$

where k_{F} is the Fermi wave vector, and $F(x) = \text{xcosx} - \text{sinx}/x$, $x = 2k_{\text{F}} r$. The well-known RKKY interaction is provided by Equation (6). The interaction in the system can be either ferromagnetic or antiferromagnetic depending on the value of the oscillation function $F(2k_{\text{F}} r)$. J_{ij} is used to calculate different temperature properties such as Curie temperature, spin-dependent relaxation time, spin stiffness constants, and magnon energies. Then, by adding up all possible spin orientations, the transition probability in J is given by:

$$W(k\uparrow, k'\uparrow) = W(k\uparrow, k'\downarrow) = \frac{N_i 2\pi J^2 S(S+1)}{3\hbar} [1 + 4Jg(E_k)] \delta(E_k - E_{k''}), \tag{7}$$

where N_i is the concentration of magnetic impurities and $g(E_k)$ is the singular function in the third-order correction. By using Fermi–Dirac distribution and for electron close to the Fermi surface at low temperature (i.e., $E_k > k_B T$) and substituting this into collision integral, the spin-dependent relaxation time τ is the inverse of the transition probability

which is written as follows:

$$\tau_s(E_F) = \tau_0 e^{\hbar\beta\Delta E_k}, \tag{8}$$

where $\tau_0 = 3E_F/2\pi N_i J_{pd}^2 s(s+1)\beta$ is constant, ΔE is the energy gap between the impurity band and valence band. It is also known as activation energy. The thermal excitation of holes from the impurity band to the significantly more conductive valence band is used to compute this energy. The maximum and minimum energy positions of the Fermi level are observed to commonly differ by no more than 1 EV in III–V semiconductors. In the case of GaAs specifically, the conduction band is found at $E_{FS} + 0.9$eV and the valence band at $E_{FS} - 0.5$eV [1].

2.2. Heisenberg Model Approaches for Thermal Excitation of Magnon. The energy difference ΔE is the central parameter for estimating magnetic transition temperature in the Heisenberg model [2]. If the Heisenberg exchange interaction term $S_i.S_j$ describes the interaction between the atomic spins at lattice locations r_i and r_j and the exchange coupling, which depends on spin separation, may be used to describe the strength of the interaction, then the Heisenberg Hamiltonian is $H = -\sum_{i,j} j_{ij} S_i.S_j$. Atomic spins tend to align themselves in a preferred direction and take on an ordered structure at low temperatures, where the exchange interaction typically does not leave them independent [11]. The simplest of them is ferromagnetic order, where J is a positive sign in the exchange interaction. Heisenberg Hamiltonian H has the following notation if the external field is applied in the z-direction:

$$H = -\sum_{ij} J_{ij} S_i.S_j - g\mu_B \boldsymbol{B} \sum_i S_i^z - g\mu_B \boldsymbol{B} \sum_j S_j^z, \tag{9}$$

where g is the lande g-factor of the localized moment, μ_B is the Bohr magneton, and $\boldsymbol{B}$ is the applied magnetic field. If we denote the ground state by $|0\rangle$, an excited state is obtained by the series of raising S^+ and lowering S^- operators to the ground states. Then the eigenstate of the Hamiltonian possessing translational symmetry may be characterized by a wave vector k, we shall seek the proper eigenstate in the form $|n_k\rangle = 1/\left(\sqrt{2S}\right)\left(\sqrt{N}\right)\sum_l e^{ik.r_l} S_l^+ |0\rangle$.Where N is the number of spins and S is the spins of localized magnetic ions. To transform spin operators to the deviation of creation and annihilation operators $a_i^+$and a_i, respectively, the wave functions of the spin–wave are created and annihilated by

$$\begin{aligned} a_k^+ &= \frac{1}{\sqrt{2SN}} \sum_l e^{ik.r_l} S_l^+ \\ a_k &= \frac{1}{\sqrt{2SN}} \sum_l e^{ik.r_l} S_l^-, \end{aligned} \tag{10}$$

where S_l^+ and S_l^- are spin-raising and lowering operators, respectively. If the operators a_i^+ and a_i were boson operators, the operators in the lattice representation would also behave as boson operators. At low temperatures, the z-components of the spins can be well approximated by $-S$. By using the lattice representation of bosons inverse Fourier transforms, the spin operators are written as follows:

$$\begin{aligned} S_{ia}^+ &= \sqrt{2S} a_i^+, S_{ia}^- = \sqrt{2S} a_i, S_{ia}^z = -S + a_i^+ a_i \\ S_{jb}^+ &= \sqrt{2S} b_j^+, S_{jb}^z = -S + b_j^+ bj, S_{jb}^- = \sqrt{2S} b_j, \end{aligned} \tag{11}$$

where a_i^+, a_i and b_j^+, b_j are creation and annihilation operators of i^{th} and j^{th}atom on sublattice i and j, respectively. It is convenient to make a transition from the atomic, a_i^+, a_i and b_j^+, b_j to the magnon variables c_k^+, c_k and d_k^+, d_k for two sublattices, respectively. These magnon variables, for two sublattices are defined as follows:

$$\begin{aligned} c_{k\sigma}^+ = c_k^+ &= \frac{1}{\sqrt{N}} \sum_i a_{i\sigma}^+ e^{ik.R_i}, c_k = \frac{1}{\sqrt{N}} \sum_i a_{i\sigma} e^{-ik'.R_i} \\ d_{k'}^+ &= \frac{1}{\sqrt{N}} \sum_j b_{j\sigma}^+ e^{ik'.R_j}, d_{k'} = \frac{1}{\sqrt{N}} \sum_j b_{j\sigma} e^{-ik'.R_j}, \end{aligned} \tag{12}$$

where σ is the spin of the atoms, R_i and R_j are the position vectors of atoms i and j. Then using an inverse Fourier transform of these new magnon variables $\vec{s}_i.\vec{s}_j$ is calculated as follows:

$$\vec{s}_i.\vec{s}_j = \vec{s}_{ai}.\vec{s}_{bj} = \frac{1}{2}\left(s_{ai}^+ s_{bj}^- + s_{ai}^- s_{bj}^+\right) + s_{iz}^a s_{jz}^b. \tag{13}$$

Then by substituting Equation (13) into Equation (9), and using the Dirac delta function, the Hamiltonian of excited state H_{mag} becomes

$$H_{mag} = \sum_{ij} SzJ_{ij} \left[\sum_k \left[c_k^+ c_k + d_k^+ d_k - c_k^+ d_k \exp(ik.l) - d_k^+ c_k \sum_l \exp(-ik.l)\right)\right] - g\mu_B B/(N)\left[\sum_k c_k^+ c_k + d_k^+ d_k\right]. \tag{14}$$

By considering exchange interactions of strength J between nearest neighbors only and taking the average of this equation over J and substituting into Equation (14) we obtain:

$$\begin{aligned} H_{mag} = {} & 2SzxJ\left[\sum_k \left[c_k^+ c_k + d_k^+ d_k - \gamma_k\left(c_k^+ d_k + d_k^+ c_k\right)\right]\right] \\ & -g\mu_B B/(N)\left[\sum_k c_k^+ c_k + d_k^+ d_k\right], \end{aligned} \tag{15}$$

where x is the concentrations of the impurity added to the semiconductors, z is the number of nearest neighbors, and $\gamma_k = 1/z\sum_l \exp(ik.l)$. According to Solyam [12], the leading correction of γ_k in the long wavelength limit is of the order k^2, which puts it extremely close to the unity. When using the simple cubic lattice with the lattice constant α and small values of the wave numbers k, along with two new creation and annihilation operators that combine the operators of the two sublattices, along with the inverse transform of these two operators in Equation (15), the excitation energy $\hbar\omega_k$ for ferromagnetic materials is

$$\hbar\omega_k = 2SxJk^2a^2 - g\mu_B\boldsymbol{B}. \tag{16}$$

The ground state of a simple ferromagnetic has all spins parallel. At long wavelengths, $ka << 1$ so that, the frequency of magnon ω_k is proportional to k^2, in the same limit the frequency of a phonon is proportional to k. The second term in Equation (16) is small, for $\boldsymbol{B} \to 0$, the excitation energy for ferromagnetic material is expressed as follows:

$$\hbar\omega_k = 2SxJk^2a^2. \tag{17}$$

2.3. The Green's Function Formalism for the Heisenberg Model. To answer the question of how a quantum mechanical system reacts to an external perturbation, such as an electrical or magnetic field, Green's functions naturally appear as response functions; the corresponding response functions would then describe the electrical conductivity of a system. For the quasiparticle spectrum of the system described by the Hamiltonian in Equation (15), we consider the following retarded Green's function, at $\tau' = 0, \tau = t$

$$G_k(t) = \langle\langle a_k(t); a_k(0)\rangle\rangle = -i\theta(t)\langle\left[a_k(t), a_{k'}^+\right]\rangle. \tag{18}$$

By using the momentum space creation and annihilation operators and differentiating Equation (18) with respect to t and by taking its Fourier transform we get

$$\begin{aligned} &-i\omega\int dt\left\langle\left\langle a_q(t); a_{q'}^+\right\rangle\right\rangle_{\omega^+} \\ &= -i\left\langle\left[a_q(t), a_q^+\right]_{\mp}\right\rangle - i\left\langle\left\langle\left[\left[a_q(t), H\right]; a_q^+\right]_{\mp}\right\rangle\right\rangle_{\omega^+.} \end{aligned} \tag{19}$$

Therefore, the equation of motion for fermions and bosons for finite temperature ($T > 0$) is

$$i\omega\left\langle\left\langle a_k(t); a_{k'}^+\right\rangle\right\rangle = \left\langle\left[a_k(t), a_{k'}^+\right]_{\mp}\right\rangle + \left\langle\left\langle[a_k(t), H']; a_k^+\right)_{\mp}\right\rangle\right\rangle, \tag{20}$$

where $H = \langle H_{\text{magnon}}\rangle = \omega_k\langle a_k(t)a_{k'}^+(t)\rangle$ and ω is the frequency of excitation energy. The correlation function $\langle a_k(t), a_{k'}^+(t)\rangle$ is related to the analytic property of Green's function by

$$\left\langle\left\langle a_k^+(t), a_{k'}(t')\right\rangle\right\rangle = \lim_{\delta\to 0} i\frac{\int_{-\infty}^{\infty}\left[\left\langle\left\langle a_k^+(t), a_{k'}(t')\right\rangle\right\rangle\omega + i\delta - \left\langle\left\langle a_k^+(t), a_{k'}(t')\right\rangle\right\rangle\omega - i\delta\right]e^{i(\omega t - t')}}{e^{\beta\omega} - 1}. \tag{21}$$

By using Schrodinger picture, taking the Fourier transformation of Equation (24), and for $k = k'$ and $\delta_{kk'} = 1$, Equation (21) becomes $G_{kk'(\omega)} = \frac{1}{2\pi(\omega - \omega_k)}$. Again by considering this equation and taking $t = t'$ equal time correlation gives the number operator which is

$$\left\langle\left\langle a_k a_{k'}^+\right\rangle\right\rangle = \frac{\lim_{\delta\to 0} i\int\delta(\omega - \omega_k)d\omega}{e^{\beta\omega} - 1}. \tag{22}$$

By using Dirac delta function in Equation (22), the total number of magnons in all modes excited at temperature T can be calculated as follows:

$$\sum_k \langle n_k\rangle = \frac{1}{V}\sum_k \frac{1}{e^{\beta\omega} - 1}. \tag{23}$$

Then by using the integral function, the relation between the volume of the primitive cell and its wave function k which is $V = 4\pi/3k^3$, the long magnon dispersion in ferromagnetic materials which is $\omega = 2Jsxa^2k^2$, and by assuming $\hbar = 1$, the total excitation number of magnon is

$$\sum_k \langle n_k\rangle = \frac{1}{\pi^2}\int_0^{\infty}\frac{k^2}{e^{\beta 2Jsxa^2k^2} - 1}dk. \tag{24}$$

To solve Equation (24) let's take, $y = 2Jxsa^2k^2\beta$, $k^2 = (y/jxsa^2\beta)$, and $dk = dy/2y^{1/2}(2jsxa^2\beta)^{1/2}$ gives

$$\sum_k \langle n_{\text{k}}\rangle = \frac{0.050661}{(2jsxa^2\beta)^{\frac{3}{2}}}\int_0^{\infty}\frac{y^{1/2}}{e^y - 1}dy = \frac{0.0468}{(Jsxa^2)^{\frac{3}{2}}}(k_BT)^{\frac{3}{2}}. \tag{25}$$

Equation (25) is the total excitation number of magnon which is obtained by introducing zeta and gamma function $\zeta(z) = 1/\Gamma(z)\int_0^{\infty} y^{z-1}dy/e^y - 1$ where $\Gamma(z) = (z-1)!$, $\int_0^{\infty}\sqrt{y}dy/e^y - 1 = \zeta(3/2) = 2.612$. Then the total excitation energy of magnons $U = \Delta E$ becomes

$$H_{\text{mag}} = U = \sum_k \omega_k \langle n_k \rangle = \frac{V}{2\pi^2} \int_0^\infty \frac{2jsxa^2k^4}{e^{2\beta jsxa^2k^2} - 1} dk. \tag{26}$$

To solve Equation (26) let us introduce variable $y = \beta A k^2$, $dy = 2\beta A k dk = \left(2(\beta A y)^{1/2}\right) dk, k = (y/\beta A)^{1/2}$. Then by substituting these values in Equation (26), the total excitation energy is

$$U = \frac{V}{2\pi^2 \beta^{\frac{5}{2}} A^{\frac{3}{2}}} \int_0^\infty \frac{y^{\frac{3}{2}}}{e^y - 1} dy. \tag{27}$$

By introducing zeta and gamma function $\int_0^\infty y^{3/2} dy/e^y - 1\zeta(5/2) = 1.341$, then the total excitation energy (activation energy) of magnon becomes

$$U = 1.341 \frac{V}{2\pi^2 A^{\frac{3}{2}}} k_B^{\frac{5}{2}} T^{\frac{5}{2}} = 0.06793 a^3 \left(\frac{k_B^5}{A^3}\right)^{\frac{1}{2}} T^{\frac{5}{2}}, \tag{28}$$

where $A = 2jsxa^2$ and a is the lattice constant which is given by Vegard's law, $a = (0.566(1 - x) + 0.598x)$nm [13]. Equation (28) indicates the relation between temperature, concentration, and energy gap of the DMSs. As the concentration of magnetic impurity, x, increases the total excitation energy decreases. That means the energy gap is decreased and the charge carriers are moved simply. The electrical properties of the semiconductors are changed. This total excitation energy, Equation (28), is used to calculate the transport properties of DMSs.

2.3.1. Magnetization. Several methods have been developed to precisely determine T_C, the temperature at which a material changes from a ferromagnetic state to a paramagnetic one. Measuring of magnetization as a function of temperature is the most effective technique. To calculate the magnetization of ferromagnetic material and its alignment in the z-direction at a temperature T, we use the relation between magnetic moment and magnetization as follows:

$$M(T) = \frac{N}{V\langle \mu_z \rangle} = \frac{N}{V} g\mu_B \langle S^z \rangle, \tag{29}$$

where N, V, g, and μ_B are the total number of particles, the volume, the Lande g-Factor and the Bohr magneton, respectively. Then by using $S_i^z = S - a_i^+ a_i$ and Equation (25), Equation (29) becomes

$$\frac{M(T)}{M(0)} = 1 - fT^{\frac{3}{2}}, \tag{30}$$

where $f = (0.0468)(V/NS)(k_B/JSxa^2)^{3/2}$ is constant, $M(0)$ is the magnetization at absolute zero where all spins are parallel and V is the volume of the unit cell. The ratio of magnetization at temperature T and magnetization at absolute zero is called reduced magnetization. Equation (30) indicates that the total magnetization decreases by one unit of spin with the excitation (or creation) of a magnon and the temperature dependance of decrease in magnetization. Mn-doped GaAs has an FCC lattice structure with $n = 4/a^3$ with a lattice constant of about $a = 5.65A^0$ and $s = 5/2$ spin of Mn^{+2}.

2.3.2. Critical Temperature. By the limit of $M(T)/M(0)$ approaches zero, $T \to T_c$, then the critical temperature for ferromagnetic material is

$$T_c = \left(\frac{Ns}{0.0468V}\right)^{\frac{2}{3}} \left(\frac{jxsa^2}{k_B}\right). \tag{31}$$

The critical temperature of ferromagnetic GaMnAs increases as the concentration of magnetic ions impurity increases. Equation (31) indicates the relation between the transition temperatures and the concentration of magnetic ions in ferromagnetic materials.

2.3.3. Electrical Conductivity. For Mn-doped GaAs, the scattering mechanism is the ionized magnetic impurity scattering. The Boltzmann transport theory is used to determine the electrical conductivity of the carriers when the predominant scattering mechanism is the scattering by ionized impurities because it is effective in treating along-range disorder [14]. According to Shinjo [3], the density of spin-up and spin-down in ferromagnetic materials is almost similar (i.e., $n_+ \sim n_-$). Supposing the electric field lies along the z-direction$E = (0, 0, E_z)$ and the magnetic field is zero, then using Equation (8), the rate of change of the probability $f_k^{\pm}$, Equation (5), with which the state $K_\pm$ is occupied due to the collision with the localized spins becomes $f(k) = f_0 - (e/m^\times)\tau \partial f_0/\partial E E_x$. Then using the hole density of particles $\langle n_k \rangle$ with spin σ, the Fermi Dirac distribution function and parabolic energy, the current density is given by

$$j_\pm = \left(\frac{2e^2 n E_x}{3k_B T m^*}\right) \left(\frac{\int_0^\infty E^{\frac{3}{2}} \tau f_0(1 - f_0) dE}{\int_0^\infty E^{\frac{1}{2}} f_0(E) dE}\right). \tag{32}$$

If Fermi energy is located in the valence band, the term $\partial f_0/\partial E = -1/k_B T f_0(1 - f_0)$ has a large value near the Fermi energy only; it may be approximated by Dirac delta function $\delta(E - E_F)$. Then the total current density becomes $j_\pm = e^2 n \tau(E_F) E/m^*$. Where $\tau(E_F)$ is the relaxation time at $E = E_F$. Then the electrical conductivity of spin-up and spin-down electrons is expressed as follows:

$$\sigma_\pm = e^2 n \frac{\tau(E_F)}{m^*}. \tag{33}$$

Since Mn-doped GaAs has a zinc blende crystal structure similar to GaAs, the optical thresh hold at $\omega = E_g/\hbar$ directly gives the band gap [11]. Where ω is the thermal energy excitation of a single magnon. Then the total thermal energy excitation is related to the energy gap by the relation $U = E_g/\hbar$. Where in our case by assuming $\hbar = 1$, $\Delta E \approx E_g = U$, which is the total excitation energy of magnon. As the concentration of magnetic impurity increases the energy

gap is reduced. By substituting the spin-dependent relaxation time Equations (8) and (28) in Equation (33), the temperature dependance electrical conductivity becomes

$$\sigma(T) = \sigma_0 e^{\left(0.0679a^3\left(\frac{k_B^3}{A^3}\right)^{\frac{1}{2}}T^{\frac{3}{2}}\right)}. \quad (34)$$

Where $\sigma_0 = e^2 p\tau_0/m_h^{\times}$, $A = 2JxSa^2$. $Ga_{1-x}Mn_xAs$ is a p-type semiconductor, thus we utilize the hole concentration $p = N/V$ rather than n. According to Dietl et al. [1], the optimal formula for the hole concentration, p, is $p = xN_0$, where N_0 is the concentration of Ga sites in GaAs, which is equal to $2.2x10^{22}cm^{-3}$, and x is the concentration of the impurity. According to Blakemore [15], for (Ga,Mn)As, the hole effective mass is $m_h^{\times} = 0.5m_0$, where $m_0 = m_e$ is the electron mass.

2.3.4. Hole Mobility. Since Mn-doped GaAs is a p-type semiconductor, the mobility we calculated is hole mobility. By using the relation between electrical conductivity and mobility, the mobility of impurity is:

$$\mu(T) = \frac{e}{m_h^{\times}}\tau_0 e^{0.0679a^3\left(\frac{k_B^3}{4\pi^4A^3}\right)^{\frac{1}{2}}T^{\frac{3}{2}}}, \quad (35)$$

where $A = 2JxSa^2$. From the relation of spin-dependent relaxation time, we have $\mu(T) \propto 1/N_{\mathrm{Mn}}$. Where N_{Mn} is the impurity density, J is the p–d exchange interaction constant and s is the spin of magnetic impurity. The hole mobility is exponentially proportional to $T^{3/2}$.

3. Results and Discussion

Different transport properties of DMSs are computed in this work. The transport properties of p-type Mn-doped GaAs have been investigated theoretically. The theoretical calculation is done using different methods and models. Throughout this study, for better numerical estimations, we use the exchange integral of local magnetic moments, at different sites separated approximately by a distance of GaAs lattice constant ($a = 5.65A^0$), $J_{p-d} = 31.195x10^{-23}\,J$, the spin of the Mn atom S = 5/2, the Boltzmann constant $k_B = 1.38x\,10^{-23}J/K$, reduced Plank's constant $\hbar = 1$, hole effective mass $m_h = 0.5m_0$, electron charge $e = 1.6022x10^{19}c$, and mass of an electron $m_0 = 9.1094x10^{-31}$kg. The Curie temperature for this article, as determined theoretically, is around 162 K. Using this value, the transport properties of ferromagnetic GaMnAs are discussed as follows:

At $x = 0.08$, the magnetization of the the $Ga_{1-x}Mn_xAs$ decreased with the temperature of $T^{\frac{3}{2}}$, as seen in Equation (30) and Figure 1. This demonstrates that the magnetization of this material reduces to zero as the temperature rises. When there is no applied external field, the spontaneous magnetization of ferromagnetic GaMnAs occurs in the ground state. This indicates that the spins are positioned parallel to one another. The orientations of these spins are altered in the presence of an external field. At high temperatures, this disruption in spin orientation results in a reduction in spontaneous magnetization. As a result, the spontaneous magnetization is zero above the Curie temperature. There is no spontaneous magnetization at high temperatures.

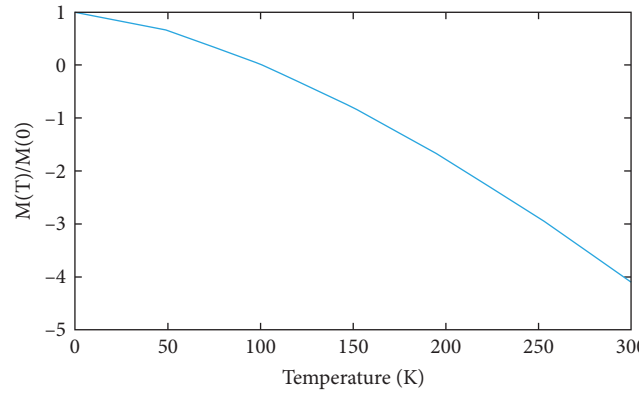

FIGURE 1: Reduced magnetization versus temperature of $Ga_{0.92}Mn_{0.08}As$.

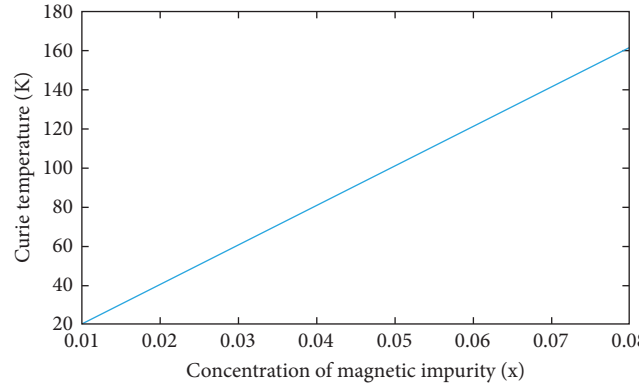

FIGURE 2: Curie temperature versus concentration of magnetic impurity of $Ga_{0.92}Mn_{0.08}As$.

In Equation (31), we determine the concentration dependance of the Curie temperature for Mn-doped GaAs DMS. As shown in Figure 2, the transition temperature rises as magnetic ion concentration does. We determine the GaMnAsT_c value for $0.08 \leq x \geq 0.01$. The estimated curie temperature in accord well when compared to experimental and theoretical expectations. While our calculation places the Curie temperature of $Ga_{1-x}Mn_xAs$ at $x = 0.08$ around 162 k, the experimental result for this value is 173 k [6]. Within a reasonable concentration range (about 10%), high T_c is very challenging to achieve for homogeneous DMS systems. Therefore, as the Mn content x increases, the concentration of Mn occupying the Ga sites (MnGa) increases, leading to higher T_C for $Ga_{1-x}Mn_xAs$. By increasing the Mn content and/or the free hole concentration in the alloy, the varied values of T_c in GaMnAs might be enhanced. It has been demonstrated experimentally that T_C in $Ga_{1-x}Mn_xAs$ rises with increasing hole concentration. Due to the fact that T_C goes to zero when holes are compensated and

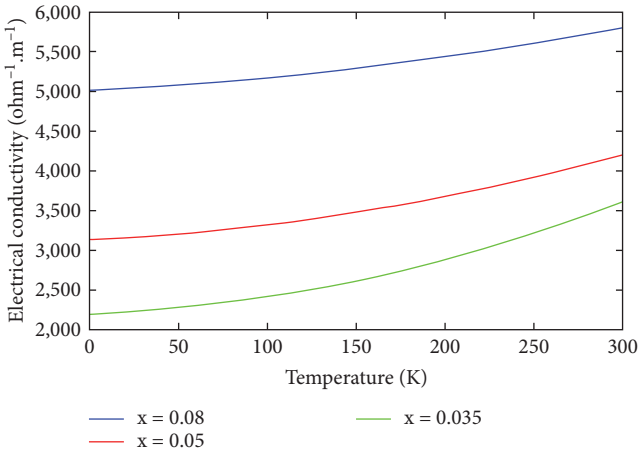

FIGURE 3: Electrical conductivity of $Ga_{0.92}Mn_{0.08}As$ versus temperature.

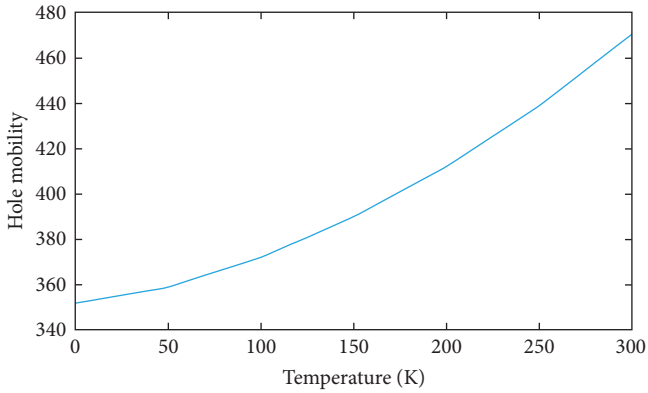

FIGURE 4: Hole mobility of $Ga_{0.95}Mn_{0.05}As$ versus temperature.

that T_C of $(Ga, Mn)As$ increases with the increasing hole concentrations.

Using Equation (34), Figure 3 illustrates the electrical conductivity of $Ga_{1-x}Mn_xAs$ for $x = 0.035$, $x = 0.05$, and $x = 0.08$. A doped or extrinsic semiconductor's electrical conductivity increases with increased temperatures and magnetic impurity concentration.

According to this idea, the electrical conductivity of DMSs, which are semiconductors with magnetic impurities added into them, increases exponentially with both temperature and magnetic ion concentration [11]. A small addition of a magnetic impurity has a significant impact on the electrical conductivity, as shown in Figure 3. We can see from the graph that electrical conductivity increases as magnetic impurity concentration does. Due to the fact that modern electronics are made by adding magnetic impurities (introducing spin) to semiconductors, or "Spintronics," the electrical conductivity of semiconductors is altered by the presence of magnetic ions. The conductivity is high at $x = 0.08$, which corresponds to a high concentration, and low at $x = 0.035$. Electrical conductivity increases along with the magnon's thermal excitation.

Since GaMnAs is a p-type semiconductor we discuss the hole mobility. From the relation of mobility and relaxation time in Equation (35) we can find the hole mobility of $Ga_{1-x}Mn_xAs$ for $0.08 \leq x \geq 0.01$. The hole mobility is exponentially increased with temperature as shown in the Figure 4. This can be understood qualitatively by considering that with increasing temperature, electrons can travel faster and this makes it easier to escape the ionized impurities. Figure 4 shows the hole mobility at $x = 0.05$. Holes are more mobile when they experience less scattering, i.e., the time between collision (relaxation time) is larger and the effective mass is smaller. As the temperature increase the hole mobility is also increase.

4. Conclusion

In this study, we studied the theoretical transport properties of DMS with a focus on $Ga_{1-x}Mn_xAs$. The $III_{1-x}Mn_xV$ ferromagnetic semiconductor known as $Ga_{1-x}Mn_xAs$ has undergone the most extensive study to date. The concentration of magnetic dopants and temperature have a significant impact on the interesting transport characteristics of DMSs. Since the transport properties are depend on the density of states, the impurity added to the semiconductor increases this density of state. Magnons are thermally excited when an electromagnetic field is applied. We investigated the Curie temperature, magnetization, electrical conductivity, and hole mobility of $Ga_{1-x}Mn_xAs$ using this thermal excitation of a magnon.

By using these theoretical work T_C of $Ga_{1-x}Mn_xAs$ at $x = 0.08$ is 162 K. From experimental work, T_C of $Ga_{1-x}Mn_xAs$ at $x = 0.08$ is 173 K. Therefore, the theoretical calculation is more approximated to the experimental work. The Curie temperature of $Ga_{1-x}Mn_xAs$ is increasing with increasing concentration. Since it is a ferromagnetic material, it has spontaneous magnetization at low temperatures. The magnetization of this material is reduced to zero as $T \to T_C$. The electrical conductivity of $Ga_{1-x}Mn_xAs$ is exponentially increased with an increase in temperature from 0 to 300 K and concentration of magnetic ions in the range of $0.08 \leq x \geq 0.01$. Since hole mobility is directly proportional to the relaxation time (i.e., $\mu_h \propto \tau$) it exponentially increases with increasing temperature.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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