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Research Article

Group Decision Making Process for Supplier Selection with TOPSIS Method under Interval-Valued Intuitionistic Fuzzy Numbers

Mohammad Izadikhah

Department of Mathematics, Islamic Azad University, Arak-Branch, Arak, Iran

Correspondence should be addressed to Mohammad Izadikhah, m_izadikhah@yahoo.com

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Supplier selection is a fundamental issue of supply chain area that heavily contributes to the overall supply chain performance, and, also, it is a hard problem since supplier selection is typically a multicriteria group decision problem. In many practical situations, there usually exists incomplete and uncertain, and the decision makers cannot easily express their judgments on the candidates with exact and crisp values. Therefore, in this paper an extended technique for order preference by similarity to ideal solution (TOPSIS) method for group decision making with Atanassov's interval-valued intuitionistic fuzzy numbers is proposed to solve the supplier selection problem under incomplete and uncertain information environment. In other researches in this area, the weights of each decision maker and in many of them the weights of criteria are predetermined, but these weights have been calculated in this paper by using the decision matrix of each decision maker. Also, the normalized Hamming distance is proposed to calculate the distance between Atanassov's interval-valued intuitionistic fuzzy numbers. Finally, a numerical example for supplier selection is given to clarify the main results developed in this paper.

1. Introduction

Supplier selection, the process of finding the right suppliers who are able to provide the buyer with the right quality products and/or services at the right price, at the right time, and in the right quantities, is one of the most critical activities for establishing an effective supply chain.

Selecting the wrong supplier could be enough to deteriorate the whole supply chains financial and operational position. In todays highly competitive, global operating environment, it is impossible to produce low-cost, high-quality products successfully without satisfactory suppliers [1, 2]. The success of a supply chain is highly dependent on selection of good suppliers.

Supplier selection is a fundamental issue of supply chain area that heavily contributes to the overall supply chain performance. Particularly for companies that spend a high percentage of their sales revenue on parts and material supplies and whose material costs represent a larger portion of total costs, savings from supplies are of particular importance. These strongly urge for a more systematic and transparent approach to purchasing decision making, especially regarding the area of supplier selection. Selecting the suppliers significantly reduces the purchasing cost and improves corporate competitiveness, and that is why many experts believe that the supplier selection is the most important activity of a purchasing department. Supplier selection is the process by which suppliers are reviewed evaluated, and chosen to become part of the companys supply chain. The overall objective of supplier selection process is to reduce purchase risk, maximize overall value to the purchaser, and build the closeness and long-term relationships between buyers and suppliers [3].

In other words, the major aims of supply chain management are to reduce supply chain risk, reduce production costs, maximize revenue, improve customer service, and

Table 1: Excerpt of the questionnaire for evaluating criteria.

No.	Supplier selection criteria	The importance of criteria for evaluating construction companies 1 Very unimportant	2	3 Unimportant	4	5 Normal	6	7 Important	8	9 Very important
1	Company's suggested net price									
2	Company's qualitative capabilities									
3	After construction services									
4	Company's delivery on-time capabilities									
5	Company's geographical situation									
6	Company's organization and management									
7	Company's financial status									
8	Company's capacity and production facilities									
9	Company's partnership antecedents									
10	Company's technical capacity									
11	Future potential purchases from Company									
12	Company's operational control (including reporting, quality control, and inventory control system)									
13	Company's status in related industry									
14	Company's individuals antecedents									
15	Company's organizational behavior									
16	Company's eagerness to cooperate									
17	Company's policy of guarantee and legal claims									
18	Company's adaptation with the purchaser's procedures and instruments									
19	Company's performance antecedents									

optimize inventory levels, business processes, and cycle times, resulting in increased competitiveness, customer satisfaction, and profitability [4–9].

Indeed supplier selection is a multiple criteria decision-making (MCDM) problem affected by several conflicting factors such as price, quality, and delivery.

Several factors affect a supplier performance. Dickson [10], Ellram [11], Roa and Kiser [12], Stamm and Golhar [13] identified, respectively, 60, 18, 13, and 23 criteria for supplier selection. One of the well-known studies on supplier selection belongs to Dickson who identified 23 important evaluation criteria for supplier selection. Weber et al. [14] reviewed and classified 74 articles that addressed the supplier selection problem.

Over the years, several techniques have been developed to solve the problem efficiently. Supply chain management has received recently considerable attention in both academia and industry.

De Boer et al. [15] identified four stages for supplier selection including definition of the problem, formulation of criteria, qualification, and final selection, respectively. They reviewed and classified MCDM approaches for supplier selection. Several methodologies have been proposed for the supplier selection problem. The systematic analysis for supplier selection includes categorical method, weighted point method [16, 17], matrix approach [18], vendor performance matrix approach [19], vendor profile analysis [20], analytic hierarchy process (AHP) [21–23], analytic network process (ANP) [24], mathematical programming [25–28], and multiple objective programming (MOP) [29–33].

Essential, the supplier selection problem in supply chain system is a group decision making combination of several and different criteria with different forms of uncertainty [34]. Hence, this problem is a kind of multiple criteria decision making problem (MCDM) that requires MCDM methods for an effective problem solving. The supplier selection process is often influenced by uncertainty in practice [35, 36].

Several influence factors are often not taken into account in the decision making process, such as incomplete information, additional qualitative criteria, and imprecision preferences [3, 37]. Therefore, the fuzzy set theory has been applied to supplier selection recently. Li et al. [38] and Holt [39] discussed the application of the fuzzy set theory in supplier selection. Chen et al. [3] extended the concept of TOPSIS method to develop a methodology for solving supplier selection problems in fuzzy environment. Haq and Kannan [40] presented a structured model for evaluating the supplier selection for the rubber industry using AHP, and the model is verified with the fuzzy AHP. Bayrak et al. [41] presented a fuzzy multicriteria group decision making

Table 2: Final result for evaluating criteria and italicized criteria.

No.	Company (supplier) selection criteria	Average	Ranking
	Company's performance		
1	*Company's suggested net price*	7.91	2
2	*Company's qualitative capabilities*	7.85	3
3	After construction services	7	7
4	*Company's delivery on-time capabilities*	8	1
5	Company's geographical situation	6.41	8
6	Company's organization and management	5.77	13
	Company's status		
7	Company's financial status	6.21	11
8	Company's capacity and production facilities	5.36	15
9	*Company's partnership antecedents*	7.56	5
10	Company's technical capacity	5.42	14
11	Future potential purchases from Company	5.32	16
12	Company's operational control (including reporting, quality control, and inventory control system)	4.13	19
13	*Company's status in related industry*	7.32	6
14	Company's individuals antecedents	4.7	18
15	Company's organizational behavior	5	17
16	Company's eagerness to cooperate	6.24	10
17	Company's policy of guarantee and legal claims	6.37	9
18	Company's adaptation with the purchaser's procedures and instruments	5.86	12
19	*Company's performance antecedents*	7.73	4

Table 3: Importance weight as linguistic variables.

Linguistic terms	IVIF numbers
Extremely important (EI)	$\langle[1.00, 1.00], [0.00, 0.00]\rangle$
Very important (VI)	$\langle[0.80, 0.90], [0.05, 0.10]\rangle$
Important (I)	$\langle[0.65, 0.75], [0.10, 0.20]\rangle$
Medium (M)	$\langle[0.45, 0.55], [0.35, 0.45]\rangle$
Unimportant (U)	$\langle[0.25, 0.35], [0.55, 0.65]\rangle$
Very unimportant (VU)	$\langle[0.00, 0.10], [0.80, 0.90]\rangle$

approach to supplier selection based on fuzzy arithmetic operation. Chou and Chang [4] presented strategy-aligned fuzzy simple multiattribute rating technique (SMART) approach for solving the supplier selection problem from the perspective of strategic management of the supply chain. Chan et al. [42] presented fuzzy AHP to efficiently tackle both quantitative and qualitative decision factors involved in the selection of global supplier. Önüt et al. [43] developed a supplier evaluation approach based on ANP and TOPSIS methods for the supplier selection.

This paper proposes an Atanassov's interval-valued intuitionistic fuzzy multicriteria group decision making with TOPSIS method for supplier selection problem.

The technique for order preference by Similarity to an ideal solution (TOPSIS) method is presented in Chen and Hwang [44], with reference to [45]. The basic principle is that the chosen alternative should have the shortest distance from the ideal solution and the farthest distance form the negative-ideal solution. In the process of TOPSIS, the performance ratings and the weights of the criteria are given as exact values. In real-world situation, because of incomplete or nonobtainable information, the data (attributes) are often not so deterministic, and therefore they usually are fuzzy/imprecise. Therefore, some researches try to use TOPSIS method for fuzzy/imprecise data. For example, Tsaur et al. [46] first convert a fuzzy MCDM problem into a crisp problem via centroid defuzzification and then solve the nonfuzzy MCDM problem using the TOPSIS method. Chen and Tzeng [47] transform a fuzzy MCDM problem into a nonfuzzy MCDM using fuzzy integral. In [48] Chu proposed a fuzzy TOPSIS approach for selecting plant location, where the ratings of various alternative locations under various criteria and the weights of various criteria are assessed in linguistic terms represented by fuzzy numbers. In the proposed method, the ratings and weights assigned by decision makers are averaged and normalised into a comparable scale. The membership function of each normalised weighted rating can be developed by interval arithmetic of fuzzy numbers. Byun and Lee [49] provide a decision support system for the selection of a rapid prototyping process using the modified TOPSIS method. Recently, in some researches, TOPSIS method is considered for extension. For example, Chen [50] extends the concept of TOPSIS to develop a methodology for solving multiperson multicriteria decision making problems in fuzzy environment. Abo-sinna et al. [51] extend the TOPSIS method to solve multiobjective

Table 4: Linguistic terms for rating the alternatives.

Linguistic terms	IVIF numbers
Extremely good (EG)/extremely high (EH)	$\langle[1.00, 1.00], [0.00, 0.00]\rangle$
Very very good (VVG)/very very high (VVH)	$\langle[0.80, 0.90], [0.05, 0.10]\rangle$
Very good (VG)/very high (VH)	$\langle[0.70, 0.80], [0.05, 0.10]\rangle$
Good (G)/high (H)	$\langle[0.60, 0.70], [0.15, 0.20]\rangle$
Medium good (MG)/medium high (MH)	$\langle[0.50, 0.60], [0.25, 0.30]\rangle$
Fair (F)/medium (M)	$\langle[0.40, 0.50], [0.35, 0.40]\rangle$
Medium bad (MB)/medium low (ML)	$\langle[0.30, 0.40], [0.45, 0.50]\rangle$
Bad (B)/low (L)	$\langle[0.15, 0.25], [0.55, 0.60]\rangle$
Very bad (VB)/very low (VL)	$\langle[0.00, 0.10], [0.70, 0.75]\rangle$
Very very bad (VVB)/very very low (VVL)	$\langle[0.00, 0.10], [0.85, 0.90]\rangle$

Table 5: Rating of the alternatives from decision maker 1 (DM_1).

	Criterion 1 C_1	Criterion 2 C_2	Criterion 3 C_3	Criterion 4 C_4	Criterion 5 C_5	Criterion 6 C_6
Supplier S_1	ML	MB	G	M	VH	VG
Supplier S_2	VH	VG	VVG	G	M	ML
Supplier S_3	M	VVG	VG	G	MB	L
Supplier S_4	VVH	V	VG	VG	VVG	G
Weight of each criteria	I	EI	VI	M	I	I

Table 6: Rating of the alternatives from decision maker 2 (DM_2).

	Criterion 1 C_1	Criterion 2 C_2	Criterion 3 C_3	Criterion 4 C_4	Criterion 5 C_5	Criterion 6 C_6
Supplier S_1	L	B	VB	M	H	VG
Supplier S_2	H	VVG	VG	M	G	M
Supplier S_3	ML	VG	G	G	B	VB
Supplier S_4	VH	VVG	VB	B	VG	VG
Weight of each criteria	VI	VI	EI	VI	I	U

Table 7: Rating of the alternatives from decision maker 3 (DM_3).

	Criterion 1 C_1	Criterion 2 C_2	Criterion 3 C_3	Criterion 4 C_4	Criterion 5 C_5	Criterion 6 C_6
Supplier S_1	MH	MB	VG	VVG	H	G
Supplier S_2	VVH	VG	VG	G	G	M
Supplier S_3	L	VG	VG	G	MB	VVG
Supplier S_4	VVH	VG	VVG	MG	VG	G
Weight of each criteria	I	VI	VI	U	I	I

Table 8: Rating of the alternatives from decision maker 4 (DM_4).

	Criterion 1 C_1	Criterion 2 C_2	Criterion 3 C_3	Criterion 4 C_4	Criterion 5 C_5	Criterion 6 C_6
Supplier S_1	L	VB	MB	VB	L	MB
Supplier S_2	VVH	G	VB	VG	G	VB
Supplier S_3	H	MB	VB	MB	VB	MB
Supplier S_4	VH	VB	G	VB	B	VB
Weight of each criteria	M	I	I	U	U	I

Table 9: Ideal matrix.

	Company 1	Company 2	Company 3	Company 4
	S_1	S_2	S_3	S_4
C_1	$\langle[0.291, 0.394], [0.429, 0.482]\rangle$	$\langle[0.737, 0.843], [0.066, 0.119]\rangle$	$\langle[0.385, 0.490], [0.338, 0.394]\rangle$	$\langle[0.755, 0.859], [0.050, 0.100]\rangle$
C_2	$\langle[0.197, 0.298], [0.528, 0.579]\rangle$	$\langle[0.709, 0.814], [0.066, 0.119]\rangle$	$\langle[0.665, 0.779], [0.087, 0.150]\rangle$	$\langle[0.505, 0.643], [0.187, 0.274]\rangle$
C_3	$\langle[0.462, 0.576], [0.220, 0.294]\rangle$	$\langle[0.634, 0.755], [0.097, 0.165]\rangle$	$\langle[0.564, 0.678], [0.127, 0.198]\rangle$	$\langle[0.606, 0.729], [0.127, 0.198]\rangle$
C_4	$\langle[0.482, 0.613], [0.256, 0.331]\rangle$	$\langle[0.588, 0.692], [0.141, 0.200]\rangle$	$\langle[0.540, 0.643], [0.197, 0.251]\rangle$	$\langle[0.402, 0.517], [0.263, 0.341]\rangle$
C_5	$\langle[0.551, 0.659], [0.158, 0.221]\rangle$	$\langle[0.557, 0.659], [0.185, 0.238]\rangle$	$\langle[0.197, 0.298], [0.528, 0.579]\rangle$	$\langle[0.648, 0.766], [0.091, 0.157]\rangle$
C_6	$\langle[0.602, 0.709], [0.114, 0.179]\rangle$	$\langle[0.291, 0.394], [0.443, 0.495]\rangle$	$\langle[0.413, 0.551], [0.305, 0.387]\rangle$	$\langle[0.532, 0.643], [0.168, 0.234]\rangle$

Table 10: Weights of each decision maker.

Decision maker	DM_1	DM_2	DM_3	DM_4
Weight	0.279	0.257	0.284	0.180

nonlinear programming problems. Also, Jahanshahloo et al. [52, 53] and Izadikhah [54] extended the TOPSIS method for decision making problems with interval and fuzzy data.

In the fuzzy sets theory, the membership of an element to a fuzzy set is only a single value between zero and one [55]. However, in reality, the degree of nonmembership of an element in a fuzzy set is not certainly equal to 1 minus the degree of membership. That is to say, there may be some hesitation degree. Therefore, Atanassov [56] extended Zadehs fuzzy sets to intuitionistic fuzzy sets, which is a generalization of the concept of fuzzy sets. The theory of intuitionistic fuzzy sets is characterized by a membership degree, a non-membership degree, and a hesitation degree. Atanassov and Gargov [57] also proposed the concept of Atanassov's interval-valued intuitionistic fuzzy sets as a further generalization of fuzzy set theory.

In Boran et al. [58], the TOPSIS method combined with the intuitionistic fuzzy set is proposed to select appropriate supplier in group decision making environment. Also in Ye [59], an extended TOPSIS method for group decision making with Atanassov's interval-valued intuitionistic fuzzy numbers is proposed to solve the partner selection problem under incomplete and uncertain information environment.

Group decision making involves weighted aggregation of all individual decisions to obtain a single collective decision. The weights of DMs play an important role in the processes of weighted aggregation.

In this paper an extended technique for order preference by similarity to ideal solution (TOPSIS) method for group decision making with Atanassov's interval-valued intuitionistic fuzzy numbers is proposed to solve the supplier selection problem under incomplete and uncertain information environment. In other researches in this area, the weights of each decision maker and in many of them the weights of criteria are predetermined (see [50, 59], e.g.,), but these weights have been calculated in this paper in new method by using the decision matrix of each decision maker. In the presented method first, we select the most important criteria by Delphi method, then we define a new similarity measure to obtain the weights of each decision maker and by use of these weights, we can determine the weights of criteria. Also, the normalized Hamming distance is proposed to calculate the distance between Atanassov's interval-valued intuitionistic fuzzy numbers.

The rest of the paper is organized as follows: In this Section 2 a brief description about the following concepts is given.

First, the concept of Atanassov's interval-valued intuitionistic fuzzy sets and then some basic operations on IVIF sets are explained. After that TOPSIS method and the Delphi method are reviewed.

In Section 3 we will focus on the proposed method. In Section 4 a numerical example is demonstrated. In this section we consider an example where the managerial board of a university has to outsource construction of their new building, and then we will introduce some useful insights. Some conclusions are drawn for the study in Section 5.

2. Basic Concept

Since the fuzzy set was pioneered by Zadeh [60], a variety of sets involving imprecision have been developed. Due to knowledge limitation and time pressure, hesitancy occurs when an individual faces a problem with uncertainty. The expression of uncertainty and vagueness has been widely discussed. The interval-valued fuzzy set (IVFS) defined by Zadeh [61] is shown by the membership function within a closed subinterval of $[0, 1]$. Atanassov [56] introduced the intuitionistic fuzzy set, which is characterized by the membership function, nonmembership function, and hesitancy function. IVFSs and Atanassov's intuitionistic fuzzy sets are regarded as flexible and practical tools for dealing with fuzziness and uncertainty. Atanassov and Gargov [57] further introduced the interval-valued intuitionistic fuzzy set (IVIFS), a generalization of IVFSs and IFSs that provides the membership function and non-membership function with intervals rather than exact numbers.

2.1. The Concept of Atanassov's Interval-Valued Intuitionistic Fuzzy Sets. In 1986 Atanassov extended Zadeh's fuzzy sets to intuitionistic fuzzy sets, which is a generalization of the concept of fuzzy sets. The following definitions will be needed throughout the paper.

Table 11: Aggregated IVIF decision matrix.

	Company 1	Company 2	Company 3	Company 4
	S_1	S_2	S_3	S_4
C_1	$\langle[0.308, 0.411], [0.416, 0.468]\rangle$	$\langle[0.733, 0.839], [0.066, 0.119]\rangle$	$\langle[0.360, 0.464], [0.364, 0.419]\rangle$	$\langle[0.761, 0.865], [0.050, 0.100]\rangle$
C_2	$\langle[0.215, 0.316], [0.513, 0.564]\rangle$	$\langle[0.715, 0.820], [0.061, 0.113]\rangle$	$\langle[0.688, 0.799], [0.074, 0.134]\rangle$	$\langle[0.530, 0.666], [0.168, 0.252]\rangle$
C_3	$\langle[0.484, 0.599], [0.199, 0.272]\rangle$	$\langle[0.667, 0.784], [0.081, 0.144]\rangle$	$\langle[0.599, 0.709], [0.107, 0.172]\rangle$	$\langle[0.617, 0.740], [0.120, 0.190]\rangle$
C_4	$\langle[0.519, 0.648], [0.228, 0.302]\rangle$	$\langle[0.579, 0.682], [0.153, 0.211]\rangle$	$\langle[0.557, 0.660], [0.183, 0.236]\rangle$	$\langle[0.437, 0.551], [0.235, 0.311]\rangle$
C_5	$\langle[0.577, 0.684], [0.140, 0.201]\rangle$	$\langle[0.552, 0.654], [0.190, 0.243]\rangle$	$\langle[0.215, 0.316], [0.513, 0.564]\rangle$	$\langle[0.677, 0.791], [0.077, 0.138]\rangle$
C_6	$\langle[0.621, 0.726], [0.102, 0.163]\rangle$	$\langle[0.313, 0.415], [0.426, 0.477]\rangle$	$\langle[0.433, 0.574], [0.285, 0.369]\rangle$	$\langle[0.561, 0.670], [0.150, 0.213]\rangle$

Table 12: Weights of criteria.

	Weight
C_1	$\langle[0.671, 0.780], [0.105, 0.194]\rangle$
C_2	$\langle[0.779, 0.882], [0.057, 0.113]\rangle$
C_3	$\langle[0.779, 0.882], [0.057, 0.113]\rangle$
C_4	$\langle[0.510, 0.638], [0.262, 0.363]\rangle$
C_5	$\langle[0.598, 0.703], [0.136, 0.247]\rangle$
C_6	$\langle[0.575, 0.681], [0.053, 0.270]\rangle$

Definition 1. Let $X = \{x_1, \ldots, x_n\}$ be a finite universal set. An intuitionistic fuzzy set A in X is an object having the following form:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle ; x \in X\}, \quad (1)$$

where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ are degrees of membership and nonmembership of an element $x \in X$, respectively, with the condition $0 \le \mu_A(x) + \nu_A(x) \le 1$.

$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called the intuitionistic fuzzy index of $x \in A$. It represents the degree of indeterminacy or hesitation of $x \in A$. For each $x \in X$, $0 \le \pi_A(x) \le 1$.

In some cases, because of the complexity and uncertainties of the objective things, it is suitable and convenient to express their data in Atanassov's interval-valued intuitionistic fuzzy sets.

In 1989 Atanassov and Gargov proposed the following definition of Atanassov's interval-valued intuitionistic fuzzy sets.

Definition 2. Let $X = \{x_1, \ldots, x_n\}$ be a finite universal set. An interval-valued intuitionistic fuzzy set A in X is an object having the following form:

$$A = \{\langle x, [\mu_{Al}(x), \mu_{Au}(x)], [\nu_{Al}(x), \nu_{Au}(x)]\rangle ; x \in X\}, \quad (2)$$

where $0 \le \mu_{Au}(x) + \nu_{Au}(x) \le 1$ and $\mu_{Al}(x) \ge 0$, $\nu_{Al}(x) \ge 0$.

For the simplicity of notation, we write IVIF sets instead of Atanassov's interval-valued intuitionistic fuzzy sets and IVIF numbers instead of Atanassov's interval-valued intuitionistic fuzzy numbers.

2.2. Basic Operations on IVIF Sets. For convenience, Xu [62] denoted by $A = \langle [a,b],[c,d]\rangle$ an IVIF number, where $[a,b] \subset [0,1]$, $[c,d] \subset [0,1]$, and $b + d \le 1$.

To aggregate IVIF numbers, we use the following operations (see Xu [62] for more details).

Definition 3. Let $A_1 = \langle [a_1,b_1],[c_1,d_1]\rangle$ and $A_2 = \langle [a_2,b_2],[c_2,d_2]\rangle$ be two IVIF numbers, then multiplication between A_1 and A_2 is defined as follows:

$$A_1 \otimes A_2 = \langle [a_1 a_2, b_1 b_2], [c_1 + c_2 - c_1 c_2, d_1 + d_2 - d_1 d_2]\rangle. \quad (3)$$

Definition 4. Let $A_1 = \langle [a_1,b_1],[c_1,d_1]\rangle$ and $A_2 = \langle [a_2,b_2],[c_2,d_2]\rangle$ be two IVIF numbers; then summation between A_1 and A_2 is defined as follows:

$$A_1 + A_2 = \langle [a_1 + a_2 - a_1 a_2, b_1 + b_2 - b_1 b_2], [c_1 c_2, d_1 d_2]\rangle. \quad (4)$$

Definition 5. Let $A = \langle [a,b],[c,d]\rangle$ be an IVIF number and $\lambda \in \mathbb{R}$ an arbitrary positive real number; then

$$\lambda A = \left\langle \left[1 - (1-a)^{\lambda}, 1 - (1-b)^{\lambda}\right], \left[c^{\lambda}, d^{\lambda}\right]\right\rangle. \quad (5)$$

By the above definitions, we have the following.

Corollary 6 (Xu [62]). *Let $\{A_1, \ldots, A_n\}$ be the set of n IVIF numbers, where $A_j = \langle [a_j,b_j],[c_j,d_j]\rangle$, $j = 1, \ldots, n$. The weighted arithmetic average of them is defined by*

$$\sum_{j=1}^{n} \lambda_j A_j = \left\langle \left[1 - \prod_{j=1}^{n}\left(1-a_j\right)^{\lambda_j}, 1 - \prod_{j=1}^{n}\left(1-b_j\right)^{\lambda_j}\right], \left[\prod_{j=1}^{n}\left(c_j\right)^{\lambda_j}, \prod_{j=1}^{n}\left(d_j\right)^{\lambda_j}\right]\right\rangle, \quad (6)$$

where λ_j is the weight of A_j, $j = 1, \ldots, n$, $\lambda_j \in [0,1]$, and $\sum_{j=1}^{n} \lambda_j = 1$.

Definition 7. Let $A = \langle [a,b],[c,d]\rangle$ be an IVIF number. Xu defined a score function s to measure the IVIF number A as follows:

$$s(A) = \frac{1}{2}(a - c + b - d). \quad (7)$$

Table 13: Weighted decision matrix.

	Company 1 S_1	Company 2 S_2	Company 3 S_3	Company 4 S_4
C_1	$\langle[0.206, 0.320], [0.477, 0.571]\rangle$	$\langle[0.491, 0.655], [0.164, 0.290]\rangle$	$\langle[0.241, 0.362], [0.431, 0.532]\rangle$	$\langle[0.511, 0.675], [0.150, 0.275]\rangle$
C_2	$\langle[0.168, 0.279], [0.541, 0.613]\rangle$	$\langle[0.557, 0.723], [0.114, 0.214]\rangle$	$\langle[0.535, 0.705], [0.127, 0.232]\rangle$	$\langle[0.413, 0.587], [0.215, 0.337]\rangle$
C_3	$\langle[0.377, 0.528], [0.244, 0.354]\rangle$	$\langle[0.519, 0.691], [0.133, 0.241]\rangle$	$\langle[0.466, 0.625], [0.157, 0.266]\rangle$	$\langle[0.480, 0.653], [0.170, 0.282]\rangle$
C_4	$\langle[0.264, 0.412], [0.431, 0.556]\rangle$	$\langle[0.295, 0.434], [0.375, 0.497]\rangle$	$\langle[0.284, 0.420], [0.397, 0.514]\rangle$	$\langle[0.223, 0.351], [0.436, 0.562]\rangle$
C_5	$\langle[0.345, 0.481], [0.257, 0.399]\rangle$	$\langle[0.330, 0.460], [0.300, 0.430]\rangle$	$\langle[0.129, 0.222], [0.579, 0.672]\rangle$	$\langle[0.405, 0.557], [0.203, 0.351]\rangle$
C_6	$\langle[0.357, 0.494], [0.149, 0.389]\rangle$	$\langle[0.180, 0.283], [0.456, 0.618]\rangle$	$\langle[0.249, 0.391], [0.323, 0.540]\rangle$	$\langle[0.323, 0.456], [0.195, 0.426]\rangle$

Obviously, $-1 \leq s(A) \leq 1$, and the larger the value of $s(A)$, the higher the A.

Definition 8. Let $A = \langle[a, b], [c, d]\rangle$ be an IVIF number. Xu defined an accuracy function to measure the IVIF number A as follows:

$$h(A) = \frac{1}{2}(a + c + b + d), \tag{8}$$

where $h(A) \in [0, 1]$. The larger the value of $h(A)$ is, the higher the degree of accuracy of the IVIFN of A is.

The separation between alternatives can be measured by the Hamming distance. In this paper, we use the normalized Hamming distance proposed by Park et al. [63].

Definition 9. Let $A^* = (A_1^*, \ldots, A_n^*)$ be a vector of n IVIF numbers, such that $A_j^* = \langle[a_j^*, b_j^*], [c_j^*, d_j^*]\rangle, j = 1, \ldots, n$ and $B_i = (B_{i1}, \ldots, B_{in}), i = 1, \ldots, m$, m vectors of n IVIF numbers, such that $B_{ij} = \langle[a_{ij}, b_{ij}], [c_{ij}, d_{ij}]\rangle, i = 1, \ldots, m, j = 1, \ldots, n$; then the separation measure between B_i's and A^* based on the normalized Hamming distance is defined as follows:

$$d_i = \frac{1}{4m} \sum_{j=1}^{n} \left\{ \left| a_{ij} - a_j^* \right| + \left| b_{ij} - b_j^* \right| + \left| c_{ij} - c_j^* \right| + \left| d_{ij} - d_j^* \right| \right\}, \quad i = 1, \ldots, m. \tag{9}$$

By a similar manner, we can define a distance measure between two matrices.

Definition 10. Let $X^1 = (x_{ij}^1)$ and $X^2 = (x_{ij}^2), i = 1, \ldots, m, j = 1, \ldots, n$ be two matrices where $x_{ij}^1 = \langle[a_{ij}^1, b_{ij}^1], [c_{ij}^1, d_{ij}^1]\rangle$ and $x_{ij}^2 = \langle[a_{ij}^2, b_{ij}^2], [c_{ij}^2, d_{ij}^2]\rangle$ are IVIF numbers. One can define a distance measure between X^1 and X^2 based on the Hamming distance as follows:

$$d(X^1, X^2) = \frac{1}{4mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \left\{ \left| a_{ij}^1 - a_{ij}^2 \right| + \left| b_{ij}^1 - b_{ij}^2 \right| + \left| c_{ij}^1 - c_{ij}^2 \right| + \left| d_{ij}^1 - d_{ij}^2 \right| \right\}. \tag{10}$$

2.3. TOPSIS Method. The TOPSIS (technique for order preference by similarity to an ideal solution) method is presented in Chen and Hwang [44], with reference to Hwang and Yoon [45]. TOPSIS is a multiple criteria method to identify solutions from a finite set of alternatives. The basic principle is that the chosen alternative should have the shortest distance from the positive ideal solution and the farthest distance from the negative ideal solution. In the process of TOPSIS, the performance ratings and the weights of the criteria are given as exact values. In real-world situation, because of incomplete or nonobtainable information, the data (attributes) are often not so deterministic; therefore they usually are fuzzy/imprecise. Therefore, some researches try to use the TOPSIS method for fuzzy/imprecise data.

2.4. Delphi Method to Select Most Important Criteria. We can select the most important criteria by the Delphi method. Based on Saaty [64], if there are more than seven factors at the same level, there are too many selections on the questionnaires and it is tough for participants to make a choice. This problem can be overcome by the way of elimination or combination. The Delphi method is to reduce the number of criteria while keeping real important attributes. The process is summarized as follows [65].

Step 1. Formation of a team to study the subject, and the panelists are experts in the area to be investigated.

Step 2. Development of the first round Delphi questionnaire.

Step 3. Transmission of the results of the first questionnaire to the panelists and analysis of the first round responses.

Step 4. Preparation of the second round questionnaire.

Step 5. Transmission of the results of the second round questionnaire to the panelists and analysis of the second round responses (Steps 4 and 5 are reiterated as long as desired or necessary to achieve stability in the results).

Step 6. Preparation of a report to present the conclusions.

3. The Proposed Method

In many practical decision making problems, such as the supplier selection or selection of a partner for an enterprise in the field of supply chain management, military system efficiency evaluation, and so on, decision makers usually need to provide their preferences over alternatives. Consider that the socioeconomic environment becomes more complex, the preference information provided by decision makers is usually imprecise, that is, there may be hesitation or uncertainty about preferences because a decision should be made under time pressure and lack of knowledge or data or the decision makers have limited attention and information processing capacities. In such cases, it is suitable and convenient to express the decision makers' preferences in interval-valued intuitionistic fuzzy sets (IVIFSs). The fundamental characteristic of the IVIFS is that the values of its membership function and nonmembership function are intervals rather than exact numbers. Therefore, it is necessary and interesting to pay attention to the group decision making problems with interval-valued intuitionistic preference information.

Our main goal is to select the most appropriate supplier among m alternatives $A = \{A_1, \ldots, A_m\}$. Assume that a committee (group) of K decision makers $D = \{D_1, \ldots, D_k\}$ has been formed to select the most suitable supplier. So, the procedure for Atanassov's interval-valued intuitionistic fuzzy TOPSIS method has been given as the following steps.

Step 1. Determine the most important criteria.

Usually, there are many criteria (attributes) in supplier selection where some of them are not necessary important or may be very unimportant. Therefore, we select the most important criteria by the Delphi method. Let $C = \{C_1, \ldots, C_n\}$ be the set of criteria selected by Delphi method.

The importance of each criteria and the rating of each alternative $A_i, i = 1, \ldots, m$, with respect to the criterion $C_j, j = 1, \ldots, n$, must be determined by decision group. Because of incomplete and uncertain information about the alternatives, decision maker $D_t, t = 1, \ldots, k$, assigns an interval-valued intuitionistic fuzzy number x_{ij}^t to estimate his/her judgment on alternative A_i with respect to criterion C_j. For a better decision, we need to specify the importance of the decision of each decision maker. Therefore, we determine the weight of DMs.

Step 2. Determine the weight of decision makers

As mentioned above, decision maker D_t cannot easily estimate an exact value to alternative A_i with respect to criterion C_j. Let $X^t = (x_{ij}^t)$ be decision matrix of the tth decision maker, where $x_{ij}^t = \langle [\mu_{ijl}^t, \mu_{iju}^t], [\nu_{ijl}^t, \nu_{iju}^t] \rangle$ is an Atanassov's interval-valued intuitionistic fuzzy number.

Now, we obtain the matrix X^*, namely ideal matrix. Consider that $X^* = (x_{ij}^*) = \langle [\mu_{ijl}^*, \mu_{iju}^*], [\nu_{ijl}^*, \nu_{iju}^*] \rangle$, where $x_{ij}^* = (1/k) \sum_{t=1}^k x_{ij}^t$, and therefore, by formula (6) we have:

$$x_{ij}^* = \left\langle \left[1 - \prod_{t=1}^{k} \left(1 - \mu_{ijl}^t\right)^{1/k}, 1 - \prod_{t=1}^{k} \left(1 - \mu_{iju}^t\right)^{1/k} \right], \left[\prod_{t=1}^{k} \left(\nu_{ijl}^t\right)^{1/k}, \prod_{t=1}^{k} \left(\nu_{iju}^t\right)^{1/k} \right] \right\rangle. \tag{11}$$

Clearly, $x_{ij}^* = \langle [\mu_{ijl}^*, \mu_{iju}^*], [\nu_{ijl}^*, \nu_{iju}^*] \rangle$ is also an Atanassov's interval-valued intuitionistic fuzzy number.

In order to determine the weights of decision makers, we use the following concept.

The less the distance of decision of D_t from the ideal decision is the higher importance for D_t is expected.

Therefore, by using formula (10) we can define the following similarity measure between decision matrix of D_t and the ideal decision matrix:

$$S(X^t, X^*) = \frac{d(X^t, X^{c*})}{d(X^t, X^*) + d(X^t, X^{c*})}. \tag{12}$$

Then, we can determine the weights of decision makers according to the above similarity measure. Assume that the weight of D_t is denoted by λ_t. Hence, we can determine each $\lambda_t, t = 1, \ldots, k$, as follows:

$$\lambda_t = \frac{S(X^t, X^*)}{\sum_{i=1}^k S(X^i, X^*)}. \tag{13}$$

Clearly, $\lambda_t \geq 0$ and $\sum_{t=1}^k \lambda_t = 1$.

Step 3. Construct the aggregated Atanassov's interval-valued intuitionistic fuzzy decision matrix.

In order to aggregate all the individual decisions and construct one group decision we need to construct aggregated interval-valued intuitionistic fuzzy decision matrix. We denote the aggregated Atanassov's interval-valued intuitionistic fuzzy decision matrix by X, where $X = \sum_{t=1}^k \lambda_t X^t$, and, therefore, $X = (x_{ij}) = \langle [\mu_{ijl}, \mu_{iju}], [\nu_{ijl}, \nu_{iju}] \rangle$ where

$$x_{ij} = \left\langle \left[1 - \prod_{t=1}^{k} \left(1 - \mu_{ijl}^t\right)^{\lambda_t}, 1 - \prod_{t=1}^{k} \left(1 - \mu_{iju}^t\right)^{\lambda_t} \right], \left[\prod_{t=1}^{k} \left(\nu_{ijl}^t\right)^{\lambda_t}, \prod_{t=1}^{k} \left(\nu_{iju}^t\right)^{\lambda_t} \right] \right\rangle. \tag{14}$$

Step 4. Determine the weights of criteria.

Let us denote by $W = (w_1, \ldots, w_n)$ the vector of weights of criteria, where w_j indicates the relative importance of criterion c_j. These weights are expressed as interval-valued intuitionistic fuzzy numbers. In order to obtain W, we must gather the decision makers opinions to get the aggregated Atanassov's interval-valued intuitionistic fuzzy weight of criteria. Let the fuzzy importance weight of jth criterion with respect to tth decision maker be $w_j^t = \langle [\alpha_{jl}^t, \alpha_{ju}^t], [\beta_{jl}^t, \beta_{ju}^t] \rangle$. Then, the weights of criteria are calculated by formula (6) as follows:

$$\begin{aligned} w_j &= \lambda_1 w_j^1 + \cdots + \lambda_k w_j^k \\ &= \left\langle \left[1 - \prod_{t=1}^{k} \left(1 - \alpha_{jl}^t\right)^{\lambda_t}, 1 - \prod_{t=1}^{k} \left(1 - \alpha_{ju}^t\right)^{\lambda_t} \right], \left[\prod_{t=1}^{k} \left(\beta_{jl}^t\right)^{\lambda_t}, \prod_{t=1}^{k} \left(\beta_{ju}^t\right)^{\lambda_t} \right] \right\rangle, \quad j = 1, \ldots, n. \end{aligned} \tag{15}$$

By formula (15) we see that each w_j is an Atanassov's interval-valued intuitionistic fuzzy number. For simplicity of notation, let us assume that $w_j = \langle[\alpha_{jl}, \alpha_{ju}], [\beta_{jl}, \beta_{ju}]\rangle$.

Step 5. Determine the weighted decision matrix.

Our next claim is to determine the weighted decision matrix. We can construct the weighted decision matrix by (3). Let R be the weighted decision matrix such that $R = (r_{ij})$; then $r_{ij} = w_j \otimes x_{ij} = \langle[a_{ij}, b_{ij}, c_{ij}, d_{ij}]\rangle$.

Step 6. Determine the Atanassov's interval-valued intuitionistic fuzzy PIS and NIS.

The underlying logic of TOPSIS method is to define the positive-ideal solution (PIS) and the negative-ideal solution (NIS). The PIS is the solution that maximizes the benefit criteria and minimizes the cost criteria, whereas the NIS is the solution that minimizes the benefit criteria and maximizes the cost criteria. The optimal alternative is the one with the shortest distance from the positive solution and the farthest distance from the negative solution.

We denote the set of benefit criteria as B and the set of cost criteria as C. Then, we can determine the IVIF PIS as $A^+ = (r_1^+, \ldots, r_n^+)$, where

$$r_j^+ = \begin{cases} \left\langle\left[\max_i a_{ij}, \max_i b_{ij}, \min_i c_{ij}, \min_i d_{ij}\right]\right\rangle, & j \in B, \\ \left\langle\left[\min_i a_{ij}, \min_i b_{ij}, \max_i c_{ij}, \max_i d_{ij}\right]\right\rangle, & j \in C. \end{cases} \quad (16)$$

And we can determine the IVIF NIS as $A^- = (r_1^-, \ldots, r_n^-)$, where

$$r_j^- = \begin{cases} \left\langle\left[\min_i a_{ij}, \min_i b_{ij}\right], \left[\max_i c_{ij}, \max_i d_{ij}\right]\right\rangle, & j \in B, \\ \left\langle\left[\max_i a_{ij}, \max_i b_{ij}\right], \left[\min_i c_{ij}, \min_i d_{ij}\right]\right\rangle, & j \in C. \end{cases} \quad (17)$$

Without loss of generality we can assume that for $j = 1, \ldots, n$ we have $r_j^+ = \langle[a_j^+, b_j^+], [c_j^+, d_j^+]\rangle$ and $r_j^- = \langle[a_j^-, b_j^-], [c_j^-, d_j^-]\rangle$.

Step 7. Construct the separation measures.

The separation measure degree between candidate A_i and the IVIF positive-ideal solution is defined using the normalized Hamming distance as follows:

$$d_i^+ = \frac{1}{4m}\sum_{j=1}^{n}\left\{\left|a_{ij} - a_j^+\right| + \left|b_{ij} - b_j^+\right| + \left|c_{ij} - c_j^+\right| + \left|d_{ij} - d_j^+\right|\right\}, \quad i = 1, \ldots, m. \quad (18)$$

And, also, the separation measure degree between candidate A_i and the IVIF negative-ideal solution is defined as follows:

$$d_i^- = \frac{1}{4m}\sum_{j=1}^{n}\left\{\left|a_{ij} - a_j^-\right| + \left|b_{ij} - b_j^-\right| + \left|c_{ij} - c_j^-\right| + \left|d_{ij} - d_j^-\right|\right\}, \quad i = 1, \ldots, m. \quad (19)$$

Step 8. Calculate the closeness coefficient.

The closeness coefficient (relative closeness) of the alternative A_i with respect to the IVIF PIS A^* is defined as follows:

$$C_i = \frac{d_i^-}{d_i^- + d_i^+}, \quad \text{where } 0 \le C_i \le 1. \quad (20)$$

Step 9. Rank the alternatives.

According to the relative closeness, we can determine the ranking order of all alternative in descending order of C_i's.

4. Application

In the process of MADM with intuitionistic fuzzy information, sometimes, the attribute values take the form of intuitionistic fuzzy numbers and the information about attribute weights is incompletely known or completely unknown. In such cases, it is suitable and convenient to express the decision makers' preferences in an interval-valued intuitionistic fuzzy number (IVIFN). Therefore, it is necessary and interesting to pay attention to the group decision-making problems with interval-valued intuitionistic preference information. Therefore, it is necessary to pay attention to this issue.

In order to illustrate the proposed method, we consider an example where the managerial board of a university has to outsource construction of their new building. There are some construction companies as possible alternatives, and also, there are many factors for evaluating these companies. To hedge risks, a committee of four experts (decision makers), D_1, D_2, D_3, and D_4, has been formed to select the most suitable construction companies.

Step 1. Determine the Most Important Criteria.

In the first part of the study, the most important factors for evaluating suppliers (construction companies) are examined. The concept of the Delphi method is used to generate a consensus of opinions among experts and to extract the most important criteria. A nine-point scale is used in the questionnaires to collect experts opinions, with preferences of very unimportant, unimportant, normal, important, and very important (scores of 1, 3, 5, 7, and 9, resp.), and the value of 2, 4, 6 and 8 is the mid-opinion between 1, 3, 5, 7 and 9. An excerpt of the questionnaire is as shown in Table 1.

Eleven experts are asked to fill out the questionnaire. We arbitrarily set threshold at 64%, and 6 main criteria are selected from each part.

The result is shown in Table 2, and the selected criteria for evaluating companies are italicized. Six main attributes (criteria) are taken into consideration by decision makers during the supplier selection process. These criteria are as follows:

C_1: (Price) Company's suggested net price.

C_2: (Quality) Company's qualitative capabilities.

C_3: (Delivery time) Company's delivery on-time capabilities.

C_4: (Performance history) Company's performance antecedents.

C_5: (Economic status) Company's financial (economic) status.

C_6: (Relation with industry) Company's status in related industry (including credit and leadership).

Therefore, one cost criterion, C_1, and five benefit criteria, $C_2, \ldots, C_6$, are considered. After preliminary screening, four companies (S_1, S_2, S_3, S_4) remain for further evaluation. By using the results from the Delphi method, a hierarchy is developed for incorporating the criteria into the company evaluation process. This hierarchy is as shown in Figure 1.

Since experts may not identify the importance of factors clearly, the results of questionnaires may be biased. To consider the vagueness of experts opinions, the data are considered as IVIF numbers. With the proposed model, the university can have a better understanding of the capabilities that a company must possess and can evaluate and select the most suitable construction company for cooperation. Four decision makers use the linguistic weighting variables shown in Table 3 to assess the importance of the criteria. Also the decision makers use the linguistic rating variables shown in Table 4 to evaluate the ratings of candidates with respect to each criterion.

The ratings of the five companies by the decision makers under the various criteria and the importance weights of the criteria determined by these four decision makers are shown in Tables 5, 6, 7, and 8. The linguistic evaluations shown in Tables 5, 6, 7, and 8 are converted into IVIF numbers.

Step 2. Determine the weight of decision makers.

Now, in order to determine the weight of decision makers we must construct the ideal matrix. It is done by formula (11). The ideal matrix is shown in Table 9. We Use formulas (12) and (13) in order to determine the weights of each decision makers. The result is shown in Table 10.

Table 10 shows that the decision of DM_3 is more important than those of other decision makers.

Step 3. Construct the Aggregated Atanassov's Interval-Valued Intuitionistic Fuzzy Decision Matrix.

In order to aggregate all individual decisions and construct one group decision we use formula (14) and construct the aggregated IVIF decision matrix. Table 11 shows the result.

Table 14: Atanassov's interval-valued intuitionistic fuzzy PIS and NIS.

	IVIF PIS	IVIF NIS
C_1	$\langle[0.206, 0.320], [0.477, 0.571]\rangle$	$\langle[0.511, 0.675], [0.150, 0.275]\rangle$
C_2	$\langle[0.557, 0.723], [0.114, 0.214]\rangle$	$\langle[0.168, 0.279], [0.541, 0.613]\rangle$
C_3	$\langle[0.519, 0.691], [0.133, 0.241]\rangle$	$\langle[0.377, 0.528], [0.244, 0.354]\rangle$
C_4	$\langle[0.295, 0.434], [0.375, 0.497]\rangle$	$\langle[0.223, 0.351], [0.436, 0.562]\rangle$
C_5	$\langle[0.405, 0.556], [0.203, 0.351]\rangle$	$\langle[0.129, 0.222], [0.579, 0.672]\rangle$
C_6	$\langle[0.357, 0.494], [0.149, 0.389]\rangle$	$\langle[0.180, 0.283], [0.456, 0.618]\rangle$

Table 15: Distance of each alternative from IVIF PIS and IVIF NIS.

	Distance from IVIF PIS	Distance from IVIF NIS	Closeness coefficient	Ranking
S_1	0.162	0.212	0.567	4
S_2	0.155	0.219	0.585	3
S_3	0.144	0.230	0.615	1
S_4	0.149	0.225	0.603	2

Step 4. Determine the weights of criteria.

By applying formula (15) and Tables 5, 6, 7, 8, and 10, the importance of the criteria can be calculated. See Table 12.

Step 5. Determine the weighted decision matrix.

We can construct the weighted decision matrix by formula (3). Table 13 shows the weighted decision matrix.

Step 6. Determine the Atanassov's interval-valued intuitionistic PIS and NIS.

By formulas (16) and (17) we can determine the IVIF PIS and IVIF NIS as in Table 14.

Step 7. Construct the Separation Measures.

In this step, we calculate the separation of each alternative from Atanassov's interval-valued intuitionistic fuzzy positive ideal solution and from Atanassov's interval-valued intuitionistic fuzzy negative ideal solution, using formulas (18) and (19), respectively. Table 15 shows the distance of each alternative from IVIF PIS and IVIF NIS.

Step 8. Calculate the Closeness Coefficient.

Closeness coefficient of each alternative can be calculated by formula (20). The fourth column of Table 15 shows the closeness coefficient of all alternatives.

Step 9. Rank the Alternatives.

According to the closeness coefficient, the ranking order of all alternatives is shown in the final column of Table 15.

As can be seen, S_3, company 3, is selected as appropriate construction company among the alternatives.

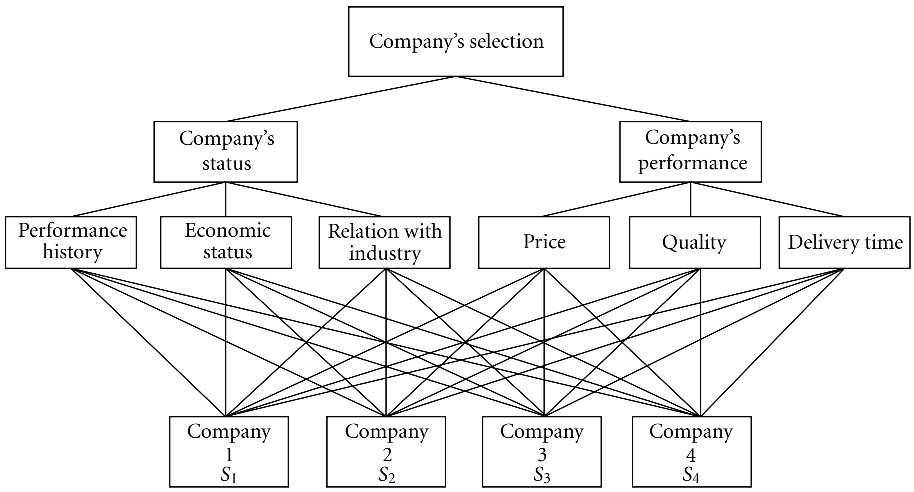

Figure 1: The Hierarchical Structure.

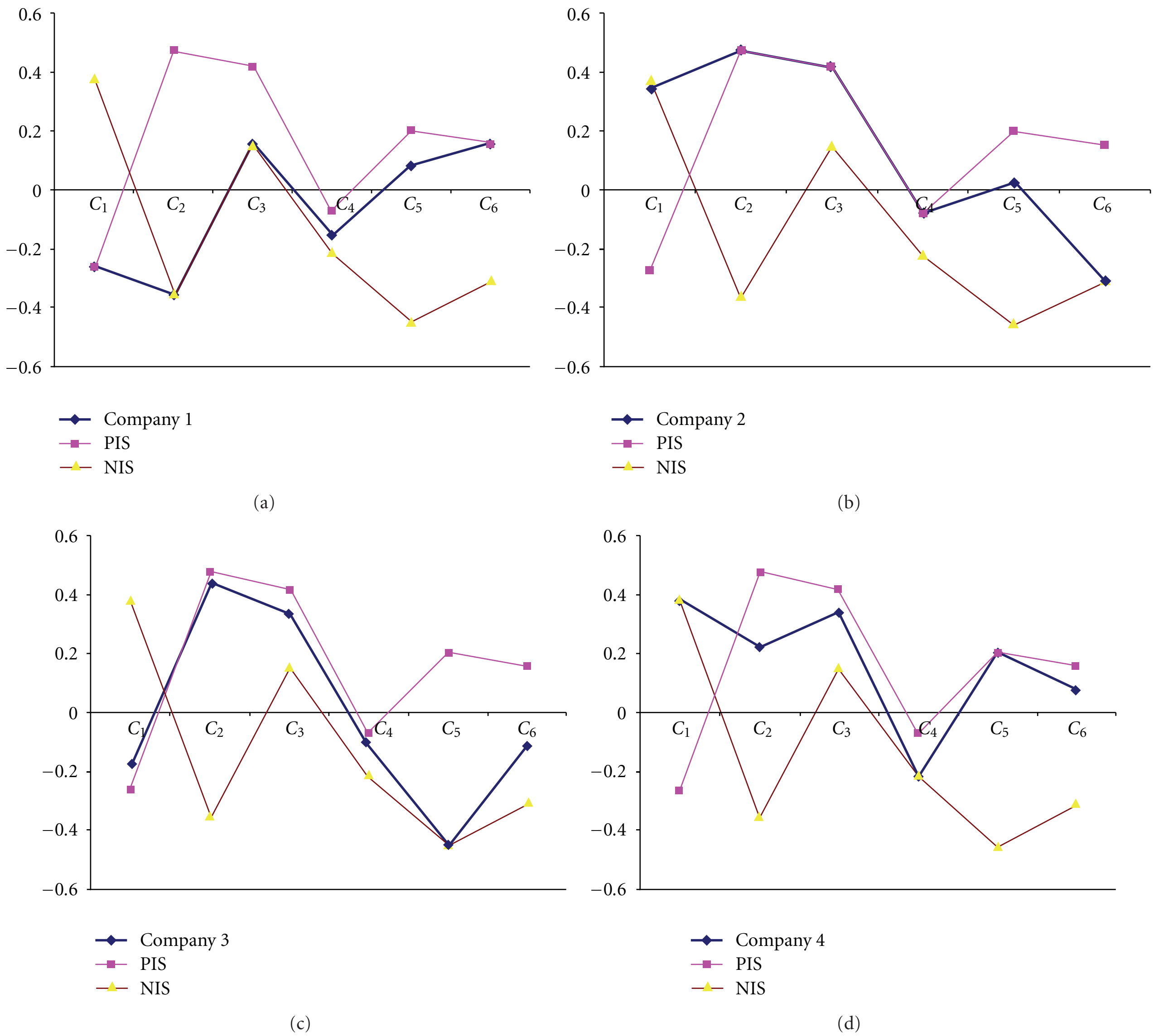

Figure 2: Comparison between each company and IVIF PIS and IVIF NIS.

4.1. More Insight. In this section, for the purpose of comparison we calculate the score function of each Atanassov's interval-valued intuitionistic fuzzy number of weighted decision matrix and IVIF PIS and IVIF NIS. After that, performances of companies with respect to each criterion are calculated and are shown in Figure 2. As can be seen from Figure 2 company 3 performs relatively better than the other three companies under most of the criteria, and is closer to IVIF PIS than other companies. On the other hand, company 1 performs relatively worse than the other three companies under most of the criteria and is closer to NIS than the other companies. To summarize, company 3 should be selected for cooperation.

5. Conclusion

During the recent years, how to determine suitable suppliers in the supply chain has become a key strategic consideration. In supplier selection process the supplier's information and performances are usually incomplete and uncertain. Therefore, the decision makers are unable (or unwilling) to express their judgment on the suppliers with exact and crisp values and the evaluations are very often expressed in linguistic terms. In such situation fuzzy set theory is an appropriate tool to deal with this kind of problems. In this paper the linguistic is characterized by Atanassov's interval-valued intuitionistic fuzzy numbers and an extended TOPSIS method for group decision making with IVIF numbers is developed to solve the supplier selection problem.

The proposed approach first uses the Delphi method to select the most important criteria. Next, by means of judgment matrix of each decision maker calculates the weight of each decision maker and the weight of each criteria. Then the weighted decision matrix is constructed, and, then, the IVIF PIS and the IVIF NIS are determined.

Based on the normalized Hamming distance, we calculate the relative closeness of each alternatives to the IVIF PIS and rank the alternative according to the relative closeness and select the most desirable one(s).

The proposed method is very flexible. Using this method not only enables us to determine outranking order of suppliers but also assess and rate the supplier and assess and rate the decision makers.

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