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Research Article

Interval-Valued Semiprime Fuzzy Ideals of Semigroups

Sukhendu Kar,[1] Paltu Sarkar,[1] and Kostaq Hila[2]

[1] *Department of Mathematics, Jadavpur University, Kolkata, West Bengal 700032, India*
[2] *Department of Mathematics & Computer Science, University of Gjirokastra, Gjirokastra 6001, Albania*

Correspondence should be addressed to Sukhendu Kar; karsukhendu@yahoo.co.in

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We introduce the notion of (i-v) semiprime (irreducible) fuzzy ideals of semigroups and investigate its different algebraic properties. We study the interrelation among (i-v) prime fuzzy ideals, (i-v) semiprime fuzzy ideals, and (i-v) irreducible fuzzy ideals and characterize regular semigroups by using these (i-v) fuzzy ideals.

1. Introduction

Zadeh [1] first introduced the concept of fuzzy sets in 1965. After that it has become an important research tool in mathematics as well as in other fields. It has many applications in many areas like artificial intelligence, coding theory, computer science, control engineering, logic, information sciences, operations research, robotics, and others. Likewise, an idea of connecting the fuzzy sets and algebraic structures came first in Rosenfeld's mind. He first introduced the notion of fuzzy subgroup [2] in 1971 and studied many results related to groups. After that fuzzification of any algebraic structures has become a new area of research for the researchers. Some of fuzzy algebraic structures are mentioned in [3–9].

During the progress of the research on fuzzy sets, several types of extensions of fuzzy subsets were introduced. Interval-valued (in short, (i-v)) fuzzy subset is one of such extensions. In 1975, the concept of interval-valued fuzzy subset was introduced by Zadeh [10]. In this concept, the degree of membership of each element is a closed subinterval in [0,1]. Using such concept, it is possible to describe an object in a more precise way. There are many applications of (i-v) fuzzy subsets in different areas: Davvaz [11] on near rings, Hedayati [12] on semirings, Gorzałczany [13] on approximate reasoning, Turksen [14] on multivalued logic, Mendel [15] on intelligent control, Roy and Biswas [16] on medical diagnosis, and so on.

Similar to fuzzy set theory, (i-v) fuzzy set theory gradually developed on different algebraic structures. Biswas [17] defined the (i-v) fuzzy subgroups of Rosenfeld's nature and investigated some elementary properties. Narayanan and Manikantan [18] introduced the notions of (i-v) fuzzy subsemigroup and various (i-v) fuzzy ideals in semigroups. In [19], Kar et al. introduced the concept of (i-v) prime (completely prime) fuzzy ideal of semigroups and studied their properties. Khan et al. [20] introduced the concept of a quotient semigroup by an interval-valued fuzzy congruence relation on a semigroup. In [21], Thillaigovindan and Chinnadurai introduced the notion of (i-v) fuzzy interior (quasi, bi) ideals of semigroup and studied their properties. However, the concept of (i-v) semiprime (irreducible) fuzzy ideals of semigroups has not been considered so far in the best of our knowledge.

In this paper our main goal is to study the semiprime (completely semiprime) ideal of a semigroup by using (i-v) fuzzy concept and discuss their properties. Also, we prove by an example that every (i-v) semiprime fuzzy ideal may not be (i-v) prime fuzzy ideal, although the converse is true. Finally, we define (i-v) irreducible fuzzy ideal of a semigroup and discuss different relations among (i-v) prime fuzzy ideal, (i-v) semiprime fuzzy ideal, and (i-v) irreducible fuzzy ideal.

2. Preliminaries

In this section we give some basic definitions and results of fuzzy algebra which will be used in this paper.

An interval number $\tilde{a} = [a^-, a^+]$ on $[0,1]$ is defined as a closed subinterval of $[0,1]$ satisfying $0 \leq a^- \leq a^+ \leq 1$. Denote $D[0,1]$ as the set of all interval numbers on $[0,1]$ and $\tilde{0} = [0,0]$, $\tilde{1} = [1,1]$. Let $\tilde{a} = [a^-, a^+]$, $\tilde{b} = [b^-, b^+] \in D[0,1]$. Then (i) $\tilde{a} \leq \tilde{b}$ if and only if $a^- \leq b^-$ and $a^+ \leq b^+$. (ii) $\tilde{a} = \tilde{b}$ if and only if $a^- = b^-$ and $a^+ = b^+$. (iii) $\tilde{a} < \tilde{b}$ if and only if $\tilde{a} \neq \tilde{b}$ and $\tilde{a} \leq \tilde{b}$. (iv) $\mathrm{Min}^i(\tilde{a}, \tilde{b}) = [\min(a^-, b^-), \min(a^+, b^+)]$. (v) $\mathrm{Max}^i(\tilde{a}, \tilde{b}) = [\max(a^-, b^-), \max(a^+, b^+)]$. If $\tilde{b} \leq \tilde{a}$, then the difference $\tilde{a} - \tilde{b}$ is defined by $\tilde{a} - \tilde{b} = [a^- - b^-, a^+ - b^+]$, whenever, $a^- - b^- \leq a^+ - b^+$; $= [a^+ - b^+, a^- - b^-]$, whenever, $a^- - b^- > a^+ - b^+$. If $\{\tilde{a}_i : i \in \Lambda\}$ is a family of interval numbers, where $\tilde{a}_i = [a_i^-, a_i^+] \in D[0,1]$, then $\sup_{i\in\Lambda}\{\tilde{a}_i\} = [\sup_{i\in\Lambda} a_i^-, \sup_{i\in\Lambda} a_i^+]$ and $\inf_{i\in\Lambda}\{\tilde{a}_i\} = [\inf_{i\in\Lambda} a_i^-, \inf_{i\in\Lambda} a_i^+]$. In this paper, we assume that any two interval numbers in $D[0,1]$ are comparable; that is, for any two interval numbers $\tilde{a}, \tilde{b} \in D[0,1]$, we have either $\tilde{a} \leq \tilde{b}$ or $\tilde{a} > \tilde{b}$.

An (i-v) fuzzy subset of a nonempty set S is a mapping $\tilde{\mu} : S \to D[0,1]$, where $D[0,1]$ is the set of all closed subintervals of $[0,1]$. An (i-v) characteristic function $\tilde{\chi}_A$ of $A(\subseteq S)$ is an (i-v) fuzzy subset of a nonempty set S, defined by $\tilde{\chi}_A(x) = \tilde{1}$ if $x \in A$; $= \tilde{0}$ if $x \in S \setminus A$. If $\tilde{\mu}$ is an (i-v) fuzzy subset of a set $S(\neq \emptyset)$ and $[a,b] \in D[0,1]$, then a level subset of $\tilde{\mu}$, denoted by $\tilde{U}(\tilde{\mu}, [a,b])$, is defined by $\tilde{U}(\tilde{\mu}, [a,b]) = \{x \in S : \tilde{\mu}(x) \geq [a,b]\}$. It would be noted that for every (i-v) fuzzy subset $\tilde{\mu}$ of a nonempty set S, there correspond two fuzzy subsets $\mu^- : S \to [0,1]$ and $\mu^+ : S \to [0,1]$ of S such that $\tilde{\mu}(x) = [\mu^-(x), \mu^+(x)]$ for every $x \in S$ and vice versa. If $\tilde{\mu}_1$ and $\tilde{\mu}_2$ are two (i-v) fuzzy subsets of a set $S(\neq \emptyset)$, then $\tilde{\mu}_1$ is said to be subset of $\tilde{\mu}_2$, denoted by $\tilde{\mu}_1 \subseteq \tilde{\mu}_2$, if $\tilde{\mu}_1(x) \leq \tilde{\mu}_2(x)$ for all $x \in S$. For given two (i-v) fuzzy subsets $\tilde{\mu}_1$ and $\tilde{\mu}_2$ of $S(\neq \emptyset)$, $(\tilde{\mu}_1 \cup \tilde{\mu}_2)(x) = \mathrm{Max}^i(\tilde{\mu}_1(x), \tilde{\mu}_2(x))$ and $(\tilde{\mu}_1 \cap \tilde{\mu}_2)(x) = \mathrm{Min}^i(\tilde{\mu}_1(x), \tilde{\mu}_2(x))$ for all $x \in S$. If $\tilde{\mu}$ is an (i-v) fuzzy subset of a nonempty set S, then complement of $\tilde{\mu}$, denoted by $\tilde{\mu}^c$, is defined by $\tilde{\mu}^c(x) = \tilde{1} - \tilde{\mu}(x) = [1 - \mu^+(x), 1 - \mu^-(x)]$, where $\tilde{\mu}(x) = [\mu^-(x), \mu^+(x)]$ and $x \in S$. If $\tilde{\mu}_1$ and $\tilde{\mu}_2$ are two (i-v) fuzzy subsets of a semigroup S, then the product of $\tilde{\mu}_1$ and $\tilde{\mu}_2$ is an (i-v) fuzzy subset of S, defined by $(\tilde{\mu}_1 \circ \tilde{\mu}_2)(x) = \sup_{x=pq}\{\mathrm{Min}^i(\tilde{\mu}_1(p), \tilde{\mu}_2(q))\}$, whenever $x = pq$ for some $p, q \in S$; $= \tilde{0}$, otherwise. If A and B are two nonempty subsets of a semigroup S, then (i) $A \subseteq B$ if and only if $\tilde{\chi}_A \subseteq \tilde{\chi}_B$, (ii) $\tilde{\chi}_A \cap \tilde{\chi}_B = \tilde{\chi}_{A\cap B}$, and (iii) $\tilde{\chi}_A \circ \tilde{\chi}_B = \tilde{\chi}_{AB}$. A nonempty (i-v) fuzzy subset $\tilde{\mu}$ is said to be an (i-v) fuzzy left (right, two-sided, interior, bi-) ideal of a semigroup S if for any $x, y, z \in S$, $\tilde{\mu}(xy) \geq \tilde{\mu}(y)$ (resp., $\tilde{\mu}(xy) \geq \tilde{\mu}(x)$, $\tilde{\mu}(xy) \geq \tilde{\mu}(y)$ and $\tilde{\mu}(xy) \geq \tilde{\mu}(x)$, $\tilde{\mu}(xy) \geq \mathrm{Min}^i(\tilde{\mu}(x), \tilde{\mu}(y))$ and $\tilde{\mu}(xyz) \geq \tilde{\mu}(y)$, $\tilde{\mu}(xy) \geq \mathrm{Min}^i(\tilde{\mu}(x), \tilde{\mu}(y))$ and $\tilde{\mu}(xyz) \geq \mathrm{Min}^i(\tilde{\mu}(x), \tilde{\mu}(z))$. A nonempty subset A of a semigroup S is a left (right, two-sided) ideal of S if and only if $\tilde{\chi}_A$ is an (i-v) fuzzy left (resp., right, two-sided) ideal of S. A nonempty (i-v) fuzzy subset $\tilde{\mu}$ of a semigroup S is an (i-v) fuzzy left (right, two-sided) ideal of S if and only if $\tilde{\chi}_S \circ \tilde{\mu} \subseteq \tilde{\mu}$ (resp., $\tilde{\mu} \circ \tilde{\chi}_S \subseteq \tilde{\mu}$, $\tilde{\chi}_S \circ \tilde{\mu} \subseteq \tilde{\mu}$ and $\tilde{\mu} \circ \tilde{\chi}_S \subseteq \tilde{\mu}$). An (i-v) fuzzy point $x_{\tilde{a}}$ of a set $S(\neq \emptyset)$ is an (i-v) fuzzy subset of S, defined by $x_{\tilde{a}}(y) = \tilde{a}$ if $y = x$; $= \tilde{0}$ if $y \neq x$; for fixed $x \in S$ and $\tilde{a} \in D[0,1] \setminus \{\tilde{0}\}$, where $y \in S$. An (i-v) fuzzy point $x_{\tilde{a}}$ of $S(\neq \emptyset)$ is said to be contained in or to belong to a nonempty (i-v) fuzzy subset $\tilde{\mu}$, denoted by $x_{\tilde{a}} \in \tilde{\mu}$, if $\tilde{\mu}(x) \geq \tilde{a}$. We denote $\mathrm{IFP}(S)$ as the set of all (i-v) fuzzy points of a semigroup S. If $x_{\tilde{a}}, y_{\tilde{b}} \in \mathrm{IFP}(S)$, then $x_{\tilde{a}} \circ y_{\tilde{b}} = (xy)_{\mathrm{Min}^i(\tilde{a}, \tilde{b})}$. If $x_{\tilde{a}} \in \mathrm{IFP}(S)$, then (i-v) fuzzy left (right, two-sided) ideal generated by the fuzzy point $x_{\tilde{a}}$ is $\langle x_{\tilde{a}} \rangle_L = x_{\tilde{a}} \cup (\tilde{\chi}_S \circ x_{\tilde{a}})$ (resp., $\langle x_{\tilde{a}} \rangle_R = x_{\tilde{a}} \cup (x_{\tilde{a}} \circ \tilde{\chi}_S)$, $\langle x_{\tilde{a}} \rangle = x_{\tilde{a}} \cup (\tilde{\chi}_S \circ x_{\tilde{a}}) \cup (\tilde{\chi}_S \circ x_{\tilde{a}} \circ \tilde{\chi}_S) \cup (x_{\tilde{a}} \circ \tilde{\chi}_S)$).

3. (i-v) Semiprime Fuzzy Ideal of Semigroups

In this section we define (i-v) semiprime fuzzy ideals as a generalization of semiprime ideals of a semigroup and discuss its different algebraic properties.

Definition 1. A proper ideal I of a semigroup S is said to be semiprime if for any ideal A of S $AA \subseteq I$ implies $A \subseteq I$.

Proposition 2. *In a semigroup S, an ideal I of S is a semiprime ideal of S if and only if $aSa \subseteq I$ implies $a \in I$.*

Definition 3. A nonconstant (i-v) fuzzy ideal $\tilde{\mu}$ of a semigroup S is called an (i-v) semiprime fuzzy ideal of S if for any (i-v) fuzzy ideal $\tilde{\mu}_1$ of S $\tilde{\mu}_1 \circ \tilde{\mu}_1 \subseteq \tilde{\mu}$ implies $\tilde{\mu}_1 \subseteq \tilde{\mu}$.

Theorem 4. *Let I be a nonempty proper subset of a semigroup S. Then I is a semiprime ideal of S if and only if the (i-v) characteristic function $\tilde{\chi}_I$ of I is an (i-v) semiprime fuzzy ideal of S.*

Proof. Let I be a semiprime ideal of S. Then it is easy to check that $\tilde{\chi}_I$ is a nonconstant (i-v) fuzzy ideal of S. Consider $\tilde{\mu}_1$ to be an (i-v) fuzzy ideal of S such that $\tilde{\mu}_1 \circ \tilde{\mu}_1 \subseteq \tilde{\chi}_I$. Let us assume that $\tilde{\mu}_1 \nsubseteq \tilde{\chi}_I$. Then there exists $x \in S$ such that $\tilde{\mu}_1(x) \nleq \tilde{\chi}_I(x)$. Since any two interval numbers in $D[0,1]$ are comparable, $\tilde{\mu}_1(x) > \tilde{\chi}_I(x)$. This implies $\tilde{\chi}_I(x) = \tilde{0} \Rightarrow x \notin I$. Since I is a semiprime ideal of S, by Proposition 2 it follows that $xyx \notin I$ for some $y \in S$; that is, $\tilde{\chi}_I(xyx) = \tilde{0}$.

Again,

$$\begin{aligned}(\tilde{\mu}_1 \circ \tilde{\mu}_1)(xyx) &= \sup_{xyx=pq}\left\{\mathrm{Min}^i(\tilde{\mu}_1(p), \tilde{\mu}_1(q))\right\} \\ &\geq \mathrm{Min}^i(\tilde{\mu}_1(x), \tilde{\mu}_1(yx)) \\ &\geq \mathrm{Min}^i(\tilde{\mu}_1(x), \tilde{\mu}_1(x)) \\ &\quad (\text{since } \tilde{\mu}_1 \text{ is an (i-v) fuzzy ideal of } S) \\ &= \tilde{\mu}_1(x) > \tilde{\chi}_I(x) = \tilde{0} = \tilde{\chi}_I(xyx)\end{aligned} \tag{1}$$

which contradict the fact that $\tilde{\mu}_1 \circ \tilde{\mu}_1 \subseteq \tilde{\chi}_I$. Therefore, $\tilde{\mu}_1 \subseteq \tilde{\chi}_I$ and, hence, it follows that $\tilde{\chi}_I$ is an (i-v) semiprime fuzzy ideal of S.

Conversely, let $\tilde{\chi}_I$ be an (i-v) semiprime fuzzy ideal of S. Then $\tilde{\chi}_I$ is a nonconstant (i-v) fuzzy ideal of S and hence I is a proper ideal of S. Let A be an ideal of S such that $AA \subseteq I$. Then $\tilde{\chi}_A$ is an (i-v) fuzzy ideal of S and $\tilde{\chi}_A \circ \tilde{\chi}_A = \tilde{\chi}_{AA} \subseteq \tilde{\chi}_I$. Therefore, by our hypothesis, $\tilde{\chi}_A \subseteq \tilde{\chi}_I$; that is, $A \subseteq I$. Thus, I is a semiprime ideal of S. □

Proposition 5. *A nonconstant (i-v) fuzzy ideal* $\widetilde{\mu}$ *of a semigroup S is an (i-v) semiprime fuzzy ideal of S if and only if a level ideal* $\widetilde{U}(\widetilde{\mu}, \widetilde{a})$ *is a semiprime ideal of S for every* $\widetilde{a} \in \operatorname{Im}\widetilde{\mu}$.

Lemma 6. *Let I be a semiprime ideal of a semigroup S and* $\widetilde{\mu}$ *an (i-v) fuzzy subset of S defined by*

$$\widetilde{\mu}(p) = \begin{cases} \widetilde{1}, & \text{when } p \in I; \\ [\alpha, \beta], & \text{otherwise,} \end{cases} \tag{2}$$

where $[\alpha, \beta] \in D[0,1] \setminus \{\widetilde{1}\}$. *Then* $\widetilde{\mu}$ *is an (i-v) semiprime fuzzy ideal of S.*

Proof. Since I is a proper ideal of S, it is easy to verify that $\widetilde{\mu}$ is an (i-v) fuzzy ideal of S. Let $\widetilde{\mu}_1 \circ \widetilde{\mu}_1 \subseteq \widetilde{\mu}$ for any (i-v) fuzzy ideal $\widetilde{\mu}_1$ of S. If possible, let $\widetilde{\mu}_1 \nsubseteq \widetilde{\mu}$. Then $\widetilde{\mu}_1(x) \nleq \widetilde{\mu}(x)$ for some $x \in S$. Due to comparability condition of interval numbers, we can write $\widetilde{\mu}_1(x) > \widetilde{\mu}(x)$. Therefore, $\widetilde{\mu}(x) \neq \widetilde{1}$ which implies $\widetilde{\mu}(x) = [\alpha, \beta]$ implies $x \notin I$. Since I is a semiprime ideal of S, $xyx \notin I$ for some $y \in S$; that is, $\widetilde{\mu}(xyx) = [\alpha, \beta]$.

Now,

$$\begin{aligned} (\widetilde{\mu}_1 \circ \widetilde{\mu}_1)(xyx) &= \sup_{xyx = pq} \left\{\operatorname{Min}^i(\widetilde{\mu}_1(p), \widetilde{\mu}_1(q))\right\} \\ &\geq \operatorname{Min}^i(\widetilde{\mu}_1(x), \widetilde{\mu}_1(yx)) \\ &\geq \operatorname{Min}^i(\widetilde{\mu}_1(x), \widetilde{\mu}_1(x)) \\ &= \widetilde{\mu}_1(x) > \widetilde{\mu}(x) = [\alpha, \beta] = \widetilde{\mu}(xyx), \end{aligned} \tag{3}$$

a contradiction. Thus, it follows that $\widetilde{\mu}_1 \subseteq \widetilde{\mu}$ and hence $\widetilde{\mu}$ is an (i-v) semiprime fuzzy ideal of S. □

Note. Lemma 6 is an example of an (i-v) semiprime fuzzy ideal of a semigroup. Now, in the following we give an example of an (i-v) semiprime fuzzy ideal which is not an (i-v) prime fuzzy ideal, although every (i-v) prime fuzzy ideal is an (i-v) semiprime fuzzy ideal.

Example 7. Let $S = \mathbb{Z}_0^+$, set of nonnegative integers. Then, S forms a semigroup with respect to usual multiplication. Define an (i-v) fuzzy subset $\widetilde{\mu}$ of S by

$$\widetilde{\mu}(x) = \begin{cases} [\alpha, \beta], & \text{when } x = 0; \\ [\gamma, \delta], & \text{when } x \text{ is nonzero even positive integer;} \\ [\xi, \eta], & \text{otherwise,} \end{cases} \tag{4}$$

where $\widetilde{1} > [\alpha, \beta] > [\gamma, \delta] > [\xi, \eta] \geq \widetilde{0}$. Then $\widetilde{\mu}$ is an (i-v) semiprime fuzzy ideal of S. But, $\widetilde{\mu}$ is not an (i-v) prime fuzzy ideal of S, because $|\operatorname{Im}\widetilde{\mu}| \neq 2$ (see [19, Theorem 3.8]).

In the following theorem we try to extend Proposition 2 and characterize an (i-v) semiprime fuzzy ideal.

Theorem 8. *If* $\widetilde{\mu}$ *is a nonconstant (i-v) fuzzy ideal of a semigroup S, then the following conditions are equivalent.*

(i) $\widetilde{\mu}$ *is an (i-v) semiprime fuzzy ideal of S.*

(ii) *For any (i-v) fuzzy point* $x_{\widetilde{a}}$ *of S,* $x_{\widetilde{a}} \circ \widetilde{\chi}_S \circ x_{\widetilde{a}} \subseteq \widetilde{\mu}$ *implies* $x_{\widetilde{a}} \in \widetilde{\mu}$.

(iii) *For any (i-v) fuzzy point* $x_{\widetilde{a}}$ *of S,* $\langle x_{\widetilde{a}} \rangle \circ \langle x_{\widetilde{a}} \rangle \subseteq \widetilde{\mu}$ *implies* $x_{\widetilde{a}} \in \widetilde{\mu}$.

Proof. (i)⇒(ii). Let $\widetilde{\mu}$ be an (i-v) semiprime fuzzy ideal of S and $x_{\widetilde{a}} \circ \widetilde{\chi}_S \circ x_{\widetilde{a}} \subseteq \widetilde{\mu}$ for an (i-v) fuzzy point $x_{\widetilde{a}}$ of S. Then $(\widetilde{\chi}_S \circ x_{\widetilde{a}} \circ \widetilde{\chi}_S)$ is an (i-v) fuzzy ideal of S and $(\widetilde{\chi}_S \circ x_{\widetilde{a}} \circ \widetilde{\chi}_S) \circ (\widetilde{\chi}_S \circ x_{\widetilde{a}} \circ \widetilde{\chi}_S) \subseteq \widetilde{\chi}_S \circ (x_{\widetilde{a}} \circ \widetilde{\chi}_S \circ x_{\widetilde{a}}) \circ \widetilde{\chi}_S \subseteq \widetilde{\chi}_S \circ \widetilde{\mu} \circ \widetilde{\chi}_S \subseteq \widetilde{\mu}$. Therefore, by our assumption, $\widetilde{\chi}_S \circ x_{\widetilde{a}} \circ \widetilde{\chi}_S \subseteq \widetilde{\mu}$. Again, $\langle x_{\widetilde{a}} \rangle \circ \langle x_{\widetilde{a}} \rangle \circ \langle x_{\widetilde{a}} \rangle \subseteq \widetilde{\chi}_S \circ x_{\widetilde{a}} \circ \widetilde{\chi}_S \subseteq \widetilde{\mu}$, which implies $\langle x_{\widetilde{a}} \rangle \subseteq \widetilde{\mu} \Rightarrow x_{\widetilde{a}} \in \widetilde{\mu}$.

(ii)⇒(iii). Let (ii) hold and $\langle x_{\widetilde{a}} \rangle \circ \langle x_{\widetilde{a}} \rangle \subseteq \widetilde{\mu}$. Then $x_{\widetilde{a}} \circ \widetilde{\chi}_S \circ x_{\widetilde{a}} \subseteq \langle x_{\widetilde{a}} \rangle \circ \langle x_{\widetilde{a}} \rangle \subseteq \widetilde{\mu}$. Therefore, by (ii), it follows that $x_{\widetilde{a}} \in \widetilde{\mu}$.

(iii)⇒(i). Let (iii) hold and $\widetilde{\mu}_1 \circ \widetilde{\mu}_1 \subseteq \widetilde{\mu}$ for an (i-v) fuzzy ideal $\widetilde{\mu}_1$ of S. Let us assume that $\widetilde{\mu}_1 \nsubseteq \widetilde{\mu}$. Then $\widetilde{\mu}_1(x) \nleq \widetilde{\mu}(x)$ for some $x \in S$ implies $\widetilde{\mu}_1(x) > \widetilde{\mu}(x)$. Now, we can choose an interval number $\widetilde{a} \in D[0,1] \setminus \{\widetilde{0}, \widetilde{1}\}$ such that $\widetilde{\mu}_1(x) \geq \widetilde{a} > \widetilde{\mu}(x)$. This implies $x_{\widetilde{a}} \in \widetilde{\mu}_1$. Again, $\langle x_{\widetilde{a}} \rangle \circ \langle x_{\widetilde{a}} \rangle \subseteq \widetilde{\mu}_1 \circ \widetilde{\mu}_1 \subseteq \widetilde{\mu}$. By (iii), it follows that $x_{\widetilde{a}} \in \widetilde{\mu}$, that is, $\widetilde{\mu}(x) \geq \widetilde{a}$, a contradiction. Consequently, it shows that our assumption, that is, $\widetilde{\mu}_1 \nsubseteq \widetilde{\mu}$, is not true. Therefore, $\widetilde{\mu}_1 \subseteq \widetilde{\mu}$ and hence $\widetilde{\mu}$ is an (i-v) semiprime fuzzy ideal of S. □

Now, we investigate the nature of image and preimage of an (i-v) semiprime fuzzy ideal of a semigroup under homomorphism. For this reason, we first give the following definition of image and preimage of an (i-v) fuzzy set and then prove Proposition 10.

Definition 9 (see [2]). Let A and B be two nonempty sets and $f : A \to B$ a function. Let $\widetilde{\mu}$ and $\widetilde{\sigma}$ be the (i-v) fuzzy subsets of A and B, respectively. Then image $f(\widetilde{\mu})$ of $\widetilde{\mu}$ under the function f is an (i-v) fuzzy subset of B defined by

$$f(\widetilde{\mu})(y) = \begin{cases} \sup_{z \in f^{-1}(y)} \widetilde{\mu}(z), & \text{when } f^{-1}(y) \neq \emptyset; \\ \widetilde{0}, & \text{otherwise,} \end{cases} \tag{5}$$

where $y \in B$ and $f^{-1}(y) = \{x \in A : f(x) = y\}$.

Preimage $f^{-1}(\widetilde{\sigma})$ of $\widetilde{\sigma}$ under the function f is an (i-v) fuzzy subset of A defined by $f^{-1}(\widetilde{\sigma})(x) = \widetilde{\sigma}(f(x))$ for any $x \in A$.

Proposition 10. *Let* S_1 *and* S_2 *be two semigroups and* $f : S_1 \to S_2$ *an epimorphism. If* $\widetilde{\mu}$ *is an (i-v) fuzzy ideal of* S_1 *and* $\widetilde{\sigma}$ *an (i-v) fuzzy ideal of* S_2*, then*

(i) $f^{-1}(\widetilde{\sigma})$ *is an (i-v) fuzzy ideal of* S_1*;*

(ii) $f(\widetilde{\mu})$ *is an (i-v) fuzzy ideal of* S_2.

Proof. (i) Since $\widetilde{\sigma}$ is nonempty, there exists an element $y^* \in S_2$ such that $\widetilde{\sigma}(y^*) \neq \widetilde{0}$. Again, since f is surjective, $f^{-1}(y^*) \neq \emptyset$; say, $x^* \in f^{-1}(y^*) \subseteq S_1$. Therefore, $\widetilde{\sigma}(f(x^*)) = \widetilde{\sigma}(y^*) \neq \widetilde{0}$; that is, $f^{-1}(\widetilde{\sigma})(x^*) \neq \widetilde{0}$. Hence, $f^{-1}(\widetilde{\sigma})$ is nonempty. Let $x, y \in S_1$. Then $f^{-1}(\widetilde{\sigma})(xy) = \widetilde{\sigma}(f(xy)) = \widetilde{\sigma}(f(x)f(y))$ (since f is homomorphism) $\geq \widetilde{\sigma}(f(x))$ (by assumption) $= f^{-1}(\widetilde{\sigma})(x)$.

Similarly, we find that $f^{-1}(\widetilde{\sigma})(xy) \geq f^{-1}(\widetilde{\sigma})(y)$. Therefore, $f^{-1}(\widetilde{\sigma})$ is an (i-v) fuzzy ideal of S_1.

(ii) Since $\widetilde{\mu}$ is nonempty, there exists an element $x_0 \in S_1$ such that $\widetilde{\mu}(x_0) \neq \widetilde{0}$. Let $f(x_0) = y_0 \in S_2$. Then $x_0 \in f^{-1}(y_0)$ and $f(\widetilde{\mu})(y_0) = \sup_{z\in f^{-1}(y_0)}\widetilde{\mu}(z) \geq \widetilde{\mu}(x_0) > \widetilde{0}$. Hence, $f(\widetilde{\mu})$ is nonempty. Let $y_1, y_2 \in S_2$. Since f is surjective, there exist $x_1, x_2 \in S_1$ such that $f(x_1) = y_1$ and $f(x_2) = y_2$. Therefore, $y_1y_2 = f(x_1)f(x_2) = f(x_1x_2)$ and hence $x_1x_2 \in f^{-1}(y_1y_2)$. Now, $f(\widetilde{\mu})(y_1y_2) = \sup_{z\in f^{-1}(y_1y_2)}\widetilde{\mu}(z) = \sup_{f(x_1)=y_1, f(x_2)=y_2}\widetilde{\mu}(x_1x_2) \geq \sup_{x_1\in f^{-1}(y_1)}\widetilde{\mu}(x_1)$ (since $\widetilde{\mu}$ is an (i-v) fuzzy ideal of S_1) $= f(\widetilde{\mu})(y_1)$. Similarly, we get $f(\widetilde{\mu})(y_1y_2) \geq f(\widetilde{\mu})(y_2)$. Hence, it follows that $f(\widetilde{\mu})$ is an (i-v) fuzzy ideal of S_2. □

Theorem 11. *Let $f : S_1 \to S_2$ be an epimorphism from a semigroup S_1 to another semigroup S_2. If $\widetilde{\mu}$ is an (i-v) semiprime fuzzy ideal of S_2, then homomorphic preimage $f^{-1}(\widetilde{\mu})$ is an (i-v) semiprime fuzzy ideal of S_1.*

Proof. Since $\widetilde{\mu}$ is a nonconstant (i-v) fuzzy ideal of S_2, by Proposition 10, $f^{-1}(\widetilde{\mu})$ is an (i-v) fuzzy ideal of S_1 and there are two elements $y', y'' \in S_2$ such that $\widetilde{\mu}(y') \neq \widetilde{\mu}(y'')$. Since f is surjective, there exist $x', x'' \in S_1$ such that $f(x') = y'$ and $f(x'') = y''$. Therefore, $\widetilde{\mu}(f(x')) \neq \widetilde{\mu}(f(x'')) \Rightarrow f^{-1}(\widetilde{\mu})(x') \neq f^{-1}(\widetilde{\mu})(x'')$. Thus, $f^{-1}(\widetilde{\mu})$ is nonconstant. Now consider an (i-v) fuzzy point $x_{\widetilde{a}}$ of S_1 such that $x_{\widetilde{a}} \circ \widetilde{\chi}_{S_1} \circ x_{\widetilde{a}} \subseteq f^{-1}(\widetilde{\mu})$. Let us assume that $x_{\widetilde{a}} \notin f^{-1}(\widetilde{\mu})$. Then $f^{-1}(\widetilde{\mu})(x) \ngeq \widetilde{a}$; that is, $\widetilde{\mu}(f(x)) \ngeq \widetilde{a}$. This implies that $\widetilde{\mu}(f(x)) < \widetilde{a}$. Again, since $x_{\widetilde{a}} \circ \widetilde{\chi}_{S_1} \circ x_{\widetilde{a}} \subseteq f^{-1}(\widetilde{\mu})$, for any (i-v) fuzzy point $z_{\widetilde{b}}$ of S_1, $x_{\widetilde{a}} \circ z_{\widetilde{b}} \circ x_{\widetilde{a}} \in f^{-1}(\widetilde{\mu}) \Rightarrow (xzx)_{\mathrm{Min}^i(\widetilde{a},\widetilde{b})} \in f^{-1}(\widetilde{\mu}) \Rightarrow f^{-1}(\widetilde{\mu})(xzx) \geq \mathrm{Min}^i(\widetilde{a},\widetilde{b}) \Rightarrow \widetilde{\mu}(f(xzx)) \geq \mathrm{Min}^i(\widetilde{a},\widetilde{b}) \Rightarrow \widetilde{\mu}(f(x)f(z)f(x)) \geq \mathrm{Min}^i(\widetilde{a},\widetilde{b})$ (since f is homomorphism) $\Rightarrow (f(x)f(z)f(x))_{\mathrm{Min}^i(\widetilde{a},\widetilde{b})} \in \widetilde{\mu} \Rightarrow (f(x))_{\widetilde{a}} \circ (f(z))_{\widetilde{b}} \circ (f(x))_{\widetilde{a}} \in \widetilde{\mu}$. Since $z \in S_1$ is arbitrary, $f(z)$ is also arbitrary in S_2 (since f is surjective). Therefore, $(f(z))_{\widetilde{b}}$ is an arbitrary (i-v) fuzzy point in S_2. Thus, $(f(x))_{\widetilde{a}} \circ \widetilde{\chi}_{S_2} \circ (f(x))_{\widetilde{a}} \subseteq \widetilde{\mu}$. Since $\widetilde{\mu}$ is an (i-v) semiprime fuzzy ideal of S_2, by Theorem 8 it follows that $(f(x))_{\widetilde{a}} \in \widetilde{\mu}$, that is, $\widetilde{\mu}(f(x)) \geq \widetilde{a}$, which is an inconsistent result. Therefore, $x_{\widetilde{a}} \in f^{-1}(\widetilde{\mu})$ and hence $f^{-1}(\widetilde{\mu})$ is an (i-v) semiprime fuzzy ideal of S_1. □

Theorem 12. *Let $f : S_1 \to S_2$ be an epimorphism from a semigroup S_1 to another semigroup S_2. If $\widetilde{\mu}$ is an (i-v) semiprime fuzzy ideal of S_1 and also f-invariant (i.e., $f(a) = f(b) \Rightarrow \widetilde{\mu}(a) = \widetilde{\mu}(b)$ for any $a, b \in S_1$), then homomorphic image $f(\widetilde{\mu})$ of $\widetilde{\mu}$ is an (i-v) semiprime fuzzy ideal of S_2.*

Proof. Since $\widetilde{\mu}$ is nonconstant, there are two elements $x', x'' \in S_1$ such that $\widetilde{\mu}(x') \neq \widetilde{\mu}(x'')$. Again, from Proposition 10, it follows that $f(\widetilde{\mu})$ is an (i-v) fuzzy ideal of S_2. Let $f(x') = y'$ and $f(x'') = y''$ where $y', y'' \in S_2$. Therefore, $x' \in f^{-1}(y')$ and $x'' \in f^{-1}(y'')$. Again, since $\widetilde{\mu}$ is f-invariant, $f(x') = y' = f(z_1)$ for every $z_1 \in f^{-1}(y')$ implies that $\widetilde{\mu}(x') = \widetilde{\mu}(z_1)$. Also, $f(x'') = y'' = f(z_2)$ for every $z_2 \in f^{-1}(y'')$ implies $\widetilde{\mu}(x'') = \widetilde{\mu}(z_2)$. But $f(\widetilde{\mu})(y') = \sup_{z_1\in f^{-1}(y')}\widetilde{\mu}(z_1) = \widetilde{\mu}(x') \neq \widetilde{\mu}(x'') = \sup_{z_2\in f^{-1}(y'')}\widetilde{\mu}(z_2) = f(\widetilde{\mu})(y'')$. It shows that $f(\widetilde{\mu})$ is nonconstant. Now, consider an (i-v) fuzzy point $y_{\widetilde{a}}$ of S_2 such that $y_{\widetilde{a}} \circ \widetilde{\chi}_{S_2} \circ y_{\widetilde{a}} \subseteq f(\widetilde{\mu})$. Then for any (i-v) fuzzy point $w_{\widetilde{b}}$ of S_2, $y_{\widetilde{a}} \circ w_{\widetilde{b}} \circ y_{\widetilde{a}} \in f(\widetilde{\mu}) \Rightarrow (ywy)_{\mathrm{Min}^i(\widetilde{a},\widetilde{b})} \in f(\widetilde{\mu}) \Rightarrow f(\widetilde{\mu})(ywy) \geq \mathrm{Min}^i(\widetilde{a},\widetilde{b}) \Rightarrow \sup_{z\in f^{-1}(ywy)}\widetilde{\mu}(z) \geq \mathrm{Min}^i(\widetilde{a},\widetilde{b})$—(i). Since f is surjective, there exist elements $u, v \in S_1$ such that $f(u) = y$ and $f(v) = w$. Therefore, for any $z \in f^{-1}(ywy)$, $f(z) = ywy = f(u)f(v)f(u) = f(uvu)$ (f is homomorphism). Since $\widetilde{\mu}$ is f-invariant, $\widetilde{\mu}(z) = \widetilde{\mu}(uvu)$. Thus, from (i), it implies that $\sup_{f(u)=y, f(v)=w}\widetilde{\mu}(uvu) \geq \mathrm{Min}^i(\widetilde{a},\widetilde{b}) \Rightarrow \widetilde{\mu}(uvu) \geq \mathrm{Min}^i(\widetilde{a},\widetilde{b}) \Rightarrow (uvu)_{\mathrm{Min}^i(\widetilde{a},\widetilde{b})} \in \widetilde{\mu} \Rightarrow u_{\widetilde{a}} \circ v_{\widetilde{b}} \circ u_{\widetilde{a}} \in \widetilde{\mu}$ for any (i-v) fuzzy point $v_{\widetilde{b}}$ of S_1. Therefore, $u_{\widetilde{a}} \circ \widetilde{\chi}_{S_1} \circ u_{\widetilde{a}} \subseteq \widetilde{\mu}$. Since $\widetilde{\mu}$ is an (i-v) semiprime fuzzy ideal of S_1, by Theorem 8, $u_{\widetilde{a}} \in \widetilde{\mu}$, that is, $\widetilde{\mu}(u) \geq \widetilde{a}$, and $f(\widetilde{\mu})(y) = \sup_{z\in f^{-1}(y)}\widetilde{\mu}(z) = \widetilde{\mu}(u)$ (since $\widetilde{\mu}$ is f-invariant) $\geq \widetilde{a}$. This implies that $y_{\widetilde{a}} \in f(\widetilde{\mu})$ and hence, by Theorem 8, $f(\widetilde{\mu})$ is an (i-v) semiprime fuzzy ideal of S_2. □

Theorem 13. *Let S_1 and S_2 be two semigroups and $f : S_1 \to S_2$ an epimorphism. Then there is a one-to-one correspondence between the f-invariant (i-v) semiprime fuzzy ideals of S_1 and (i-v) semiprime fuzzy ideals of S_2.*

Proof. Let $\mathfrak{T}_f(S_1)$ = set of all f-invariant (i-v) semiprime fuzzy ideals of S_1 and $\mathfrak{T}(S_2)$ = set of all (i-v) semiprime fuzzy ideals of S_2.

Define a mapping $\varphi : \mathfrak{T}_f(S_1) \to \mathfrak{T}(S_2)$ by $\varphi(\widetilde{\mu}) = f(\widetilde{\mu})$, where $\widetilde{\mu} \in \mathfrak{T}_f(S_1)$. Let $\widetilde{\mu}_1, \widetilde{\mu}_2 \in \mathfrak{T}_f(S_1)$ such that $\widetilde{\mu}_1 = \widetilde{\mu}_2$. Since f is surjective, $f^{-1}(y) \neq \emptyset$ for any $y \in S_2$. Therefore, $f(\widetilde{\mu}_1)(y) = \sup_{z\in f^{-1}(y)}\widetilde{\mu}_1(z) = \sup_{z\in f^{-1}(y)}\widetilde{\mu}_2(z) = f(\widetilde{\mu}_2)(y)$. This implies that $f(\widetilde{\mu}_1) = f(\widetilde{\mu}_2) \Rightarrow \varphi(\widetilde{\mu}_1) = \varphi(\widetilde{\mu}_2)$. Hence, φ is well defined.

Let $\varphi(\widetilde{\mu}_1) = \varphi(\widetilde{\mu}_2)$ for any two $\widetilde{\mu}_1, \widetilde{\mu}_2 \in \mathfrak{T}_f(S_1)$. Then $f(\widetilde{\mu}_1) = f(\widetilde{\mu}_2) \Rightarrow f(\widetilde{\mu}_1)(y) = f(\widetilde{\mu}_2)(y)$ for any $y \in S_2 \Rightarrow \sup_{z\in f^{-1}(y)}\widetilde{\mu}_1(z) = \sup_{z\in f^{-1}(y)}\widetilde{\mu}_2(z)$—(i). Since $f^{-1}(y) \neq \emptyset$ and $\widetilde{\mu}_1$, $\widetilde{\mu}_2$ are both f-invariant, for any $z_1, z_2 \in f^{-1}(y)$, $f(z_1) = y = f(z_2)$ implies $\widetilde{\mu}_1(z_1) = \widetilde{\mu}_1(z_2)$ and $\widetilde{\mu}_2(z_1) = \widetilde{\mu}_2(z_2)$. Hence, it follows from (i) that $\widetilde{\mu}_1(x) = \widetilde{\mu}_2(x)$ for every $x \in f^{-1}(y) \subseteq S_1$. Since $y \in S_2$ is arbitrary and f is surjective, $x \in S_1$ is arbitrary. It implies that $\widetilde{\mu}_1(u) = \widetilde{\mu}_2(u)$ for all $u \in S_1$; that is, $\widetilde{\mu}_1 = \widetilde{\mu}_2$. Hence, φ is one-one.

Consider $\widetilde{\sigma} \in \mathfrak{T}(S_2)$ and $y \in S_2$. Since $f^{-1}(y) \neq \emptyset$, $f(f^{-1}(\widetilde{\sigma}))(y) = \sup_{z\in f^{-1}(y)}f^{-1}(\widetilde{\sigma})(z) = \sup_{z\in f^{-1}(y)}\widetilde{\sigma}(f(z)) = \widetilde{\sigma}(y)$. This implies that $\varphi(f^{-1}(\widetilde{\sigma})) = \widetilde{\sigma}$ where, $f^{-1}(\widetilde{\sigma}) \in \mathfrak{T}_f(S_1)$. This implies that φ is onto. Therefore, φ is bijective and the result follows. □

In the following we try to give the definition of (i-v) fuzzy p-system and characterize (i-v) semiprime fuzzy ideal using it.

Definition 14. A nonempty subset P of a semigroup S is called a p-system of S if for every $x \in P$ there exists $y \in S$ such that $xyx \in P$.

Definition 15. A nonempty (i-v) fuzzy subset $\widetilde{\mu}$ of a semigroup S is called an (i-v) fuzzy p-system of S if for any $x \in S$

and $\widetilde{a} \in D[0,1] \setminus \{\widetilde{1}\}$ $\widetilde{\mu}(x) > \widetilde{a}$ implies $\widetilde{\mu}(xyx) > \widetilde{a}$ for some $y \in S$.

Theorem 16. *A nonempty subset P of a semigroup S is a p-system of S if and only if the characteristic function $\widetilde{\chi}_P$ is an (i-v) fuzzy p-system of S.*

Proof. Let P be a p-system of S. Then $\widetilde{\chi}_P$ is nonempty. Let $x \in S$ and $\widetilde{a} \in D[0,1] \setminus \{\widetilde{1}\}$ such that $\widetilde{\chi}_P(x) > \widetilde{a}$. This implies that $\widetilde{\chi}_P(x) = \widetilde{1}$ implies $x \in P$. Therefore, by our assumption, $xyx \in P$ for some $y \in S$; that is, $\widetilde{\chi}_P(xyx) = \widetilde{1} > \widetilde{a}$. Consequently, it follows that $\widetilde{\chi}_P$ is an (i-v) fuzzy p-system of S.

Conversely, let $\widetilde{\chi}_P$ be an (i-v) fuzzy p-system of S. Then for any $x \in P$, $\widetilde{\chi}_P(x) = \widetilde{1}$. Therefore, for any $\widetilde{a} \in D[0,1] \setminus \{\widetilde{1}\}$, $\widetilde{\chi}_P(x) > \widetilde{a}$. Now, by our hypothesis, $\widetilde{\chi}_P(xyx) > \widetilde{a}$ for some $y \in S$. This implies that $\widetilde{\chi}_P(xyx) = \widetilde{1}$; that is, $xyx \in P$. Thus, it follows that P is a p-system of S. □

Theorem 17. *A nonconstant (i-v) fuzzy ideal $\widetilde{\mu}$ of a semigroup S is an (i-v) semiprime fuzzy ideal of S if and only if $\widetilde{\mu}^c$ is an (i-v) fuzzy p-system of S.*

Proof. Let $\widetilde{\mu}$ be an (i-v) semiprime fuzzy ideal of S. Since $\widetilde{\mu}$ is nonconstant, there exists $x^* \in S$ such that $\widetilde{\mu}(x^*) \neq \widetilde{1}$ and hence $\widetilde{\mu}^c(x^*) = \widetilde{1} - \widetilde{\mu}(x^*) \neq \widetilde{0}$. Therefore, $\widetilde{\mu}^c$ is nonempty. Consider an interval number $\widetilde{a} = [a^-, a^+] \in D[0,1] \setminus \{\widetilde{1}\}$ and $x \in S$ such that $\widetilde{\mu}^c(x) > \widetilde{a}$; that is, $\widetilde{1} - \widetilde{\mu}(x) > \widetilde{a}$. Then $\widetilde{\mu}(x) < \widetilde{1} - \widetilde{a}$ implies $x_{\widetilde{1}-\widetilde{a}} \notin \widetilde{\mu}$. Now, using Theorem 8 it implies that $x_{\widetilde{1}-\widetilde{a}} \circ z_{\widetilde{c}} \circ x_{\widetilde{1}-\widetilde{a}} \notin \widetilde{\mu}$ for some (i-v) fuzzy point $z_{\widetilde{c}}$, which implies $(xzx)_{\mathrm{Min}^i(\widetilde{1}-\widetilde{a},\widetilde{c})} \notin \widetilde{\mu} \Rightarrow \widetilde{\mu}(xzx) \ngeq \mathrm{Min}^i(\widetilde{1}-\widetilde{a},\widetilde{c}) \Rightarrow \widetilde{\mu}(xzx) < \mathrm{Min}^i(\widetilde{1}-\widetilde{a},\widetilde{c}) \Rightarrow \widetilde{\mu}(xzx) < \widetilde{1}-\widetilde{a} \Rightarrow \widetilde{1}-\widetilde{\mu}(xzx) > \widetilde{a}$; that is, $\widetilde{\mu}^c(xzx) > \widetilde{a}$. Consequently, it follows that $\widetilde{\mu}^c$ is an (i-v) fuzzy p-system of S.

Conversely, let $\widetilde{\mu}^c$ be an (i-v) fuzzy p-system of S and $\widetilde{\mu}_1$ an (i-v) fuzzy ideal of S such that $\widetilde{\mu}_1 \circ \widetilde{\mu}_1 \subseteq \widetilde{\mu}$. Let us choose $\widetilde{\mu}_1 \nsubseteq \widetilde{\mu}$. Then $\widetilde{\mu}_1(x) \nleq \widetilde{\mu}(x)$ for some $x \in S$; that is, $\widetilde{\mu}_1(x) > \widetilde{\mu}(x)$. Now, we can choose an interval number $\widetilde{a} \in D[0,1] \setminus \{\widetilde{0},\widetilde{1}\}$ such that $\widetilde{\mu}_1(x) > \widetilde{1}-\widetilde{a} > \widetilde{\mu}(x)$. This implies that $\widetilde{a} < \widetilde{1}-\widetilde{\mu}(x)$; that is, $\widetilde{a} < \widetilde{\mu}^c(x)$. Therefore, by our hypothesis, $\widetilde{\mu}^c(xz'x) > \widetilde{a}$ for some $z' \in S$; that is, $\widetilde{1} - \widetilde{\mu}(xz'x) > \widetilde{a} \Rightarrow \widetilde{\mu}(xz'x) < \widetilde{1} - \widetilde{a}$. Further, $\widetilde{\mu}(xz'x) \geq (\widetilde{\mu}_1 \circ \widetilde{\mu}_1)(xz'x) \geq \mathrm{Min}^i(\widetilde{\mu}_1(x), \widetilde{\mu}_1(z'x)) \geq \mathrm{Min}^i(\widetilde{\mu}_1(x), \widetilde{\mu}_1(x)) = \widetilde{\mu}_1(x) > \widetilde{1} - \widetilde{a}$, a meaningless result due to wrong assumption. Thus, it follows that $\widetilde{\mu}_1 \subseteq \widetilde{\mu}$; that is, $\widetilde{\mu}$ is an (i-v) semiprime fuzzy ideal of S. □

The concept of completely semiprime ideal (see Definition 18) is defined in [22], in which it is known as semiprime ideal. Now, we try to generalize this concept using (i-v) fuzzy points and define (i-v) completely semiprime fuzzy ideal. Also, we investigate its various properties.

Definition 18. A proper ideal I of a semigroup S is said to be completely semiprime if for any $a \in S$ $a^2 \in I$ implies $a \in I$.

Definition 19. A nonconstant (i-v) fuzzy ideal $\widetilde{\mu}$ of a semigroup S is called an (i-v) completely semiprime fuzzy ideal of S if for any (i-v) fuzzy point $x_{\widetilde{a}}$ of S $x_{\widetilde{a}} \circ x_{\widetilde{a}} \in \widetilde{\mu}$ implies $x_{\widetilde{a}} \in \widetilde{\mu}$.

Proposition 20. *A proper subset I of a semigroup S is a completely semiprime ideal of S if and only if $\widetilde{\chi}_I$ is an (i-v) completely semiprime fuzzy ideal of S.*

Proof. Let I be a completely semiprime ideal of S. Then I is a proper ideal of S and hence $\widetilde{\chi}_I$ is a nonconstant (i-v) fuzzy ideal of S. Consider an (i-v) fuzzy point $x_{\widetilde{a}}$ of S such that $x_{\widetilde{a}} \circ x_{\widetilde{a}} \in \widetilde{\chi}_I$. This implies that $(x^2)_{\widetilde{a}} \in \widetilde{\chi}_I \Rightarrow \widetilde{\chi}_I(x^2) \geq \widetilde{a} \Rightarrow \widetilde{\chi}_I(x^2) = \widetilde{1} \Rightarrow x^2 \in I$. Therefore, by our assumption, $x \in I$ implies $\widetilde{\chi}_I(x) = \widetilde{1} \geq \widetilde{a}$ implies $x_{\widetilde{a}} \in \widetilde{\chi}_I$. Thus, it follows that $\widetilde{\chi}_I$ is an (i-v) completely semiprime fuzzy ideal of S.

Conversely, let $\widetilde{\chi}_I$ be an (i-v) completely semiprime fuzzy ideal of S. Then $\widetilde{\chi}_I$ is a nonconstant (i-v) fuzzy ideal of S and hence I is a proper ideal of S. Let $x^2 \in I$ for any $x \in S$. Then $\widetilde{\chi}_I(x^2) = \widetilde{1} \geq \widetilde{a}$ for any interval number $\widetilde{a} \in D[0,1] \setminus \{\widetilde{0}\}$. Therefore, $(x^2)_{\widetilde{a}} \in \widetilde{\chi}_I \Rightarrow x_{\widetilde{a}} \circ x_{\widetilde{a}} \in \widetilde{\chi}_I$. Hence, by our assumption, $x_{\widetilde{a}} \in \widetilde{\chi}_I$. This implies that $\widetilde{\chi}_I(x) \geq \widetilde{a} \Rightarrow \widetilde{\chi}_I(x) = \widetilde{1} \Rightarrow x \in I$. This shows that I is a completely semiprime ideal of S. □

Proposition 21. *A nonconstant (i-v) fuzzy ideal $\widetilde{\mu}$ of a semigroup S is an (i-v) completely semiprime fuzzy ideal of S if and only if a level ideal $\widetilde{U}(\widetilde{\mu},\widetilde{a})$ is completely semiprime ideal of S for every $\widetilde{a} \in \mathrm{Im}\,\widetilde{\mu}$.*

Proposition 22. *A nonconstant (i-v) fuzzy ideal $\widetilde{\mu}$ of a semigroup S is an (i-v) completely semiprime fuzzy ideal of S if and only if $\widetilde{\mu}(x) = \widetilde{\mu}(x^2)$ for every $x \in S$.*

Proof. Let $\widetilde{\mu}$ be an (i-v) completely semiprime fuzzy ideal of S and $x \in S$. If $\widetilde{\mu}(x^2) = \widetilde{a}$ for some $\widetilde{a} \in D[0,1]$, then $(x^2)_{\widetilde{a}} \in \widetilde{\mu}$ implies $x_{\widetilde{a}} \circ x_{\widetilde{a}} \in \widetilde{\mu}$. Hence, by our assumption, $x_{\widetilde{a}} \in \widetilde{\mu}$. This implies that $\widetilde{\mu}(x) \geq \widetilde{a} = \widetilde{\mu}(x^2) \geq \widetilde{\mu}(x)$ (since $\widetilde{\mu}$ is an (i-v) fuzzy ideal of S). Thus, $\widetilde{\mu}(x) = \widetilde{\mu}(x^2)$.

Conversely, let $\widetilde{\mu}(x) = \widetilde{\mu}(x^2)$ for all $x \in S$ and consider an (i-v) fuzzy point $x_{\widetilde{a}}$ of S such that $x_{\widetilde{a}} \circ x_{\widetilde{a}} \in \widetilde{\mu}$. Therefore, $(x^2)_{\widetilde{a}} \in \widetilde{\mu} \Rightarrow \widetilde{\mu}(x^2) \geq \widetilde{a} \Rightarrow \widetilde{\mu}(x) \geq \widetilde{a} \Rightarrow x_{\widetilde{a}} \in \widetilde{\mu}$. This shows that $\widetilde{\mu}$ is an (i-v) completely semiprime fuzzy ideal of S. □

Proposition 23. *Let $f : S_1 \rightarrow S_2$ be a semigroup epimorphism. Then the following statements are true.*

(i) *If $\widetilde{\mu}$ is an (i-v) completely semiprime fuzzy ideal of S_2, then homomorphic preimage $f^{-1}(\widetilde{\mu})$ is an (i-v) completely semiprime fuzzy ideal of S_1.*

(ii) *If $\widetilde{\theta}$ is an f-invariant (i-v) completely semiprime fuzzy ideal of S_1, then homomorphic image $f(\widetilde{\theta})$ is an (i-v) completely semiprime fuzzy ideal of S_2.*

Proof. (i) It is clear that $f^{-1}(\widetilde{\mu})$ is an (i-v) fuzzy ideal of S_1. Now, for any $x \in S_1$, $f^{-1}(\widetilde{\mu})(x^2) = \widetilde{\mu}(f(x^2)) = \widetilde{\mu}((f(x))^2)$ (f is a homomorphism) $= \widetilde{\mu}(f(x))$ (by our assumption) $= f^{-1}(\widetilde{\mu})(x)$. Thus, Proposition 22 implies that $f^{-1}(\widetilde{\mu})$ is an (i-v) completely semiprime fuzzy ideal of S_1.

(ii) From Proposition 10, it follows that $f(\widetilde{\theta})$ is an (i-v) fuzzy ideal of S_2. Let $y \in S_2$. Since f is onto, $f^{-1}(y) \neq \emptyset$. Let $x \in f^{-1}(y)$. Therefore, $y^2 = (f(x))^2 = f(x^2)$ (since

f is homomorphism), which implies $x^2 \in f^{-1}(y^2)$. Now, $f(\widetilde{\theta})(y^2) = \sup_{z\in f^{-1}(y^2)}\widetilde{\theta}(z) = \widetilde{\theta}(x^2)$ (since $\widetilde{\theta}$ is f-invariant) $= \widetilde{\theta}(x)$ (using Proposition 22) $= \sup_{z\in f^{-1}(y)}\widetilde{\theta}(z) = f(\widetilde{\theta})(y)$. Hence, it follows that $f(\widetilde{\theta})$ is an (i-v) completely semiprime fuzzy ideal of S_2. □

Proposition 24. *Every (i-v) completely semiprime fuzzy ideal of a semigroup S is an (i-v) semiprime fuzzy ideal of S.*

Proof. Let $\widetilde{\mu}$ be an (i-v) completely semiprime fuzzy ideal of S and consider $\widetilde{\mu}_1 \circ \widetilde{\mu}_1 \subseteq \widetilde{\mu}$ for an (i-v) fuzzy ideal $\widetilde{\mu}_1$ of S. Let us choose $\widetilde{\mu}_1 \nsubseteq \widetilde{\mu}$. Then $\widetilde{\mu}_1(x) \nleq \widetilde{\mu}(x)$ for some $x \in S$; that is, $\widetilde{\mu}_1(x) > \widetilde{\mu}(x)$. Now, we take an interval number $\widetilde{a} \in D[0,1] \setminus \{\widetilde{0},\widetilde{1}\}$ such that $\widetilde{\mu}_1(x) \geq \widetilde{a} > \widetilde{\mu}(x)$. This implies that $x_{\widetilde{a}} \in \widetilde{\mu}_1$ and hence $x_{\widetilde{a}} \circ x_{\widetilde{a}} \in \widetilde{\mu}_1 \circ \widetilde{\mu}_1 \subseteq \widetilde{\mu}$; that is, $x_{\widetilde{a}} \circ x_{\widetilde{a}} \in \widetilde{\mu}$. Hence, by our assumption, $x_{\widetilde{a}} \in \widetilde{\mu}$ which implies $\widetilde{\mu}(x) \geq \widetilde{a}$, an absurd result. Consequently, it follows that $\widetilde{\mu}_1 \subseteq \widetilde{\mu}$ and so $\widetilde{\mu}$ is an (i-v) semiprime fuzzy ideal of S. □

Note. But the converse of Proposition 24 is not always true; that is, every (i-v) semiprime fuzzy ideal of a semigroup S may not be an (i-v) completely semiprime fuzzy ideal of S.

Example 25. Consider $S = \{\left(\begin{smallmatrix} x & y \\ 0 & z\end{smallmatrix}\right) : x, y, z \in \mathbb{Z}\}$. Then S is a semigroup with respect to matrix multiplication. Define an (i-v) fuzzy subset $\widetilde{\mu}$ of S by

$$\widetilde{\mu}(I) = \begin{cases} \widetilde{a}, & \text{when } I \text{ is a null matrix;} \\ \widetilde{b}, & \text{otherwise,} \end{cases} \tag{6}$$

where $\widetilde{a}, \widetilde{b} \in D[0,1]$ such that $\widetilde{0} \leq \widetilde{b} < \widetilde{a} \leq \widetilde{1}$. Then it is easily shown that $\widetilde{\mu}$ is an (i-v) semiprime fuzzy ideal of S. But, if we take a matrix $I_1 = \left(\begin{smallmatrix} 0 & a \\ 0 & 0\end{smallmatrix}\right) \in S$ such that $a \neq 0$, then we see that $I_1^2 = \left(\begin{smallmatrix} 0 & a \\ 0 & 0\end{smallmatrix}\right)\left(\begin{smallmatrix} 0 & a \\ 0 & 0\end{smallmatrix}\right) = \left(\begin{smallmatrix} 0 & 0 \\ 0 & 0\end{smallmatrix}\right)$, a null matrix in S. Therefore, $\widetilde{\mu}(I_1^2) = \widetilde{a} \neq \widetilde{b} = \widetilde{\mu}(I_1)$. Thus, from Proposition 22, it follows that $\widetilde{\mu}$ is not an (i-v) completely semiprime fuzzy ideal of S.

Proposition 26. *In a commutative semigroup S, every (i-v) semiprime fuzzy ideal is an (i-v) completely semiprime fuzzy ideal of S.*

Proof. Let $\widetilde{\mu}$ be an (i-v) semiprime fuzzy ideal of S and $x_{\widetilde{a}}$ an (i-v) fuzzy point of S such that $x_{\widetilde{a}} \circ x_{\widetilde{a}} \in \widetilde{\mu}$. Then $(x^2)_{\widetilde{a}} \in \widetilde{\mu}$ implies $\widetilde{\mu}(x^2) \geq \widetilde{a}$. Now, for any $p \in S$, we have

$$(\langle x_{\widetilde{a}}\rangle \circ \langle x_{\widetilde{a}}\rangle)(p) = \begin{cases} \widetilde{a}, & \text{when } p \in \langle x\rangle\langle x\rangle; \\ \widetilde{0}, & \text{otherwise.} \end{cases} \tag{7}$$

If $p \notin \langle x\rangle\langle x\rangle$, then $(\langle x_{\widetilde{a}}\rangle \circ \langle x_{\widetilde{a}}\rangle)(p) = \widetilde{0} \leq \widetilde{\mu}(p)$. Again, if $p \in \langle x\rangle\langle x\rangle$, then $p \in \{x^2\} \cup x^2S$ (since S is commutative). Thus, for both cases $\widetilde{\mu}(p) \geq \widetilde{\mu}(x^2) \geq \widetilde{a} = (\langle x_{\widetilde{a}}\rangle \circ \langle x_{\widetilde{a}}\rangle)(p)$. Thus, it follows from the above that $\langle x_{\widetilde{a}}\rangle \circ \langle x_{\widetilde{a}}\rangle \subseteq \widetilde{\mu}$, which implies $\langle x_{\widetilde{a}}\rangle \subseteq \widetilde{\mu}$ implies $x_{\widetilde{a}} \in \widetilde{\mu}$. This shows that $\widetilde{\mu}$ is an (i-v) completely semiprime fuzzy ideal of S. □

Corollary 27. *In a commutative semigroup S, a nonconstant (i-v) fuzzy ideal $\widetilde{\mu}$ of S is an (i-v) semiprime fuzzy ideal of S if and only if $\widetilde{\mu}(x) = \widetilde{\mu}(x^2)$ for every $x \in S$.*

Theorem 28. *A commutative semigroup S is regular if and only if every nonconstant (i-v) fuzzy ideal of S is an (i-v) semiprime fuzzy ideal of S.*

Proof. Let $\widetilde{\mu}$ be a nonconstant (i-v) fuzzy ideal of a regular semigroup S and $\widetilde{\mu}_1 \circ \widetilde{\mu}_1 \subseteq \widetilde{\mu}$ for any (i-v) fuzzy ideal $\widetilde{\mu}_1$ of S. Then $\widetilde{\mu}_1 = \widetilde{\mu}_1 \cap \widetilde{\mu}_1 = \widetilde{\mu}_1 \circ \widetilde{\mu}_1$ (since S is regular) $\subseteq \widetilde{\mu}$. This implies that $\widetilde{\mu}$ is an (i-v) semiprime fuzzy ideal of S.

Conversely, let every nonconstant (i-v) fuzzy ideal of S be an (i-v) semiprime fuzzy ideal of S. Let $x \in S$. Since $\langle x^2\rangle$ is a principal ideal of S generated by x^2, $\widetilde{\chi}_{\langle x^2\rangle}$ is an (i-v) fuzzy ideal of S and hence, by our assumption, $\widetilde{\chi}_{\langle x^2\rangle}$ is an (i-v) semiprime fuzzy ideal of S. Therefore, by Corollary 27, we can write $\widetilde{\chi}_{\langle x^2\rangle}(x) = \widetilde{\chi}_{\langle x^2\rangle}(x^2) = \widetilde{1}$. This implies $x \in \langle x^2\rangle = \{x^2\} \cup xSx$ (since S is commutative). Thus, for both cases there always exists $y \in S$ such that $x = xyx$ and hence S is regular. □

Proposition 29. *If $\widetilde{\mu}$ is an (i-v) completely semiprime fuzzy ideal of a semigroup S, then $\widetilde{\mu}(ab) = \widetilde{\mu}(ba)$ for all $a, b \in S$.*

Proof. Since $\widetilde{\mu}$ is an (i-v) completely semiprime fuzzy ideal of S, for any $a, b \in S$, $\widetilde{\mu}(ab) = \widetilde{\mu}((ab)^2) = \widetilde{\mu}((ab)(ab)) = \widetilde{\mu}(a(ba)b) \geq \widetilde{\mu}(ba)$ (since $\widetilde{\mu}$ is an (i-v) fuzzy ideal of S) $= \widetilde{\mu}((ba)^2) = \widetilde{\mu}((ba)(ba)) = \widetilde{\mu}(b(ab)a) \geq \widetilde{\mu}(ab)$. This implies that $\widetilde{\mu}(ab) = \widetilde{\mu}(ba)$ and hence the result follows. □

Definition 30 (see [19]). A nonconstant (i-v) fuzzy ideal $\widetilde{\mu}$ of a semigroup S is called an (i-v) completely prime fuzzy ideal of S if for any two (i-v) fuzzy points $x_{\widetilde{a}}, y_{\widetilde{b}}$ of S $x_{\widetilde{a}} \circ y_{\widetilde{b}} \in \widetilde{\mu}$ implies either $x_{\widetilde{a}} \in \widetilde{\mu}$ or $y_{\widetilde{b}} \in \widetilde{\mu}$.

Theorem 31 (see [19]). *Let $\widetilde{\mu}$ be an (i-v) prime fuzzy ideal of a semigroup S. Then $\widetilde{\mu}$ is an (i-v) completely prime fuzzy ideal of S if and only if for any two (i-v) fuzzy points $x_{\widetilde{a}}$ and $y_{\widetilde{b}}$ of S $x_{\widetilde{a}} \circ y_{\widetilde{b}} \in \widetilde{\mu}$ implies $y_{\widetilde{b}} \circ x_{\widetilde{a}} \in \widetilde{\mu}$.*

Theorem 32. *Let $\widetilde{\mu}$ be an (i-v) prime fuzzy ideal of a semigroup S. Then $\widetilde{\mu}$ is an (i-v) completely prime fuzzy ideal of S if and only if $\widetilde{\mu}$ is an (i-v) completely semiprime fuzzy ideal of S.*

Proof. Let $\widetilde{\mu}$ be an (i-v) completely prime fuzzy ideal of S. Then it is clear that $\widetilde{\mu}$ is an (i-v) completely semiprime fuzzy ideal of S.

Conversely, let $\widetilde{\mu}$ be an (i-v) completely semiprime fuzzy ideal of S and consider two (i-v) fuzzy points $x_{\widetilde{a}}$ and $y_{\widetilde{b}}$ of S such that $x_{\widetilde{a}} \circ y_{\widetilde{b}} \in \widetilde{\mu}$. Then $(xy)_{\mathrm{Min}^i(\widetilde{a},\widetilde{b})} \in \widetilde{\mu}$. This implies that $\widetilde{\mu}(xy) \geq \mathrm{Min}^i(\widetilde{a},\widetilde{b}) \Rightarrow \widetilde{\mu}(yx) \geq \mathrm{Min}^i(\widetilde{b},\widetilde{a})$ (by Proposition 29) $\Rightarrow y_{\widetilde{b}} \circ x_{\widetilde{a}} \in \widetilde{\mu}$. Hence, by Theorem 31, it follows that $\widetilde{\mu}$ is an (i-v) completely prime fuzzy ideal of S. □

Definition 33 (see [22]). A semigroup S is called intra-regular if for each element $x \in S$ there exist elements $y, z \in S$ such that $x = yx^2z$.

Theorem 34. *In a semigroup S, the following statements are equivalent.*

(i) *S is intra-regular.*

(ii) *S is a semilattice of simple semigroups.*

(iii) *Every ideal of S is completely semiprime.*

(iv) *Every (i-v) fuzzy ideal of S is an (i-v) completely semiprime fuzzy ideal of S.*

Proof. The equivalence conditions (i), (ii), and (iii) follow from [22, Theorem 4.4].

(i)⇒(iv). Let S be intra-regular. Then, for any $x \in S$, there exist $y, z \in S$ such that $x = yx^2z$. If $\widetilde{\mu}$ is an (i-v) fuzzy ideal of S, $\widetilde{\mu}(x) = \widetilde{\mu}(yx^2z) \geq \widetilde{\mu}(x^2) \geq \widetilde{\mu}(x)$. This implies that $\widetilde{\mu}(x) = \widetilde{\mu}(x^2)$ for all $x \in S$. Hence, Proposition 22 implies that $\widetilde{\mu}$ is an (i-v) completely semiprime fuzzy ideal of S.

(iv)⇒(i). Let (iv) hold and $x \in S$. Since $\langle x^2 \rangle$ is a principal ideal of S generated by x^2, $\widetilde{\chi}_{\langle x^2 \rangle}$ is an (i-v) fuzzy ideal of S and hence, by our assumption, $\widetilde{\chi}_{\langle x^2 \rangle}$ is an (i-v) completely semiprime fuzzy ideal of S. Now, using Proposition 22, we can write $\widetilde{\chi}_{\langle x^2 \rangle}(x) = \widetilde{\chi}_{\langle x^2 \rangle}(x^2) = \widetilde{1}$. This implies that $x \in \langle x^2 \rangle = \{x^2\} \cup x^2S \cup Sx^2 \cup Sx^2S$. Therefore, for any possible form of $x \in S$, it implies that $x \in Sx^2S$. Hence, S is intra-regular. □

Theorem 35. *The following conditions are equivalent in a semigroup S.*

(i) *S is intra-regular.*

(ii) *Every (i-v) fuzzy interior ideal of S is an (i-v) completely semiprime fuzzy ideal of S.*

(iii) *For every (i-v) fuzzy interior ideal $\widetilde{\mu}$ of S and $x \in S$, $\widetilde{\mu}(x) = \widetilde{\mu}(x^2)$.*

Proof. Since proof is simple, we omit the proof. □

4. (i-v) Irreducible Fuzzy Ideal of Semigroups

In this section we have defined (i-v) irreducible fuzzy ideal of a semigroup S and we study its several properties.

Definition 36. A proper ideal A of a semigroup S is called an irreducible ideal of S if for any two ideals B and C of S $B \cap C = A$ implies either $B = A$ or $C = A$.

Definition 37. A nonconstant (i-v) fuzzy ideal $\widetilde{\mu}$ of a semigroup S is called an (i-v) irreducible fuzzy ideal of S if for any two (i-v) fuzzy ideals $\widetilde{\mu}_1$ and $\widetilde{\mu}_2$ of S $\widetilde{\mu}_1 \cap \widetilde{\mu}_2 = \widetilde{\mu}$ implies either $\widetilde{\mu}_1 = \widetilde{\mu}$ or $\widetilde{\mu}_2 = \widetilde{\mu}$.

Definition 38. A proper ideal A of a semigroup S is called a strongly irreducible ideal of S if for any two ideals B and C of S $B \cap C \subset A$ implies either $B \subset A$ or $C \subset A$.

Definition 39. A nonconstant (i-v) fuzzy ideal $\widetilde{\mu}$ of a semigroup S is called an (i-v) strongly irreducible fuzzy ideal of S if for any two (i-v) fuzzy ideals $\widetilde{\mu}_1, \widetilde{\mu}_2$ of S $\widetilde{\mu}_1 \cap \widetilde{\mu}_2 \subset \widetilde{\mu}$ implies either $\widetilde{\mu}_1 \subset \widetilde{\mu}$ or $\widetilde{\mu}_2 \subset \widetilde{\mu}$.

Proposition 40. *Every (i-v) strongly irreducible fuzzy ideal of a semigroup S is an (i-v) irreducible fuzzy ideal of S. But the converse is not true in general.*

Proof. For the converse part, we give a counter example.

Consider a semigroup $(S, *)$ where $S = \{a, b, c, d\}$ and a binary operation "$*$" on S is defined by

$*$	a	b	c	d
a	a	a	a	a
b	a	b	a	a
c	a	b	c	a
d	a	a	a	a

(8)

Then $I_1 = \{a\}$, $I_2 = \{a, b\}$, $I_3 = \{a, b, c\}$, $I_4 = \{a, b, d\}$, and S are the ideals of S in which, I_3, I_4 are the irreducible ideals of S, but not the strongly irreducible ideals of S. Define an (i-v) fuzzy subset $\widetilde{\mu}$ of S such that

$$\widetilde{\mu}(x) = \begin{cases} \widetilde{1}, & \text{when } x \in I, \text{ an irreducible ideal of } S; \\ \widetilde{a}, & \text{otherwise,} \end{cases} \tag{9}$$

where $\widetilde{a} \in D[0,1] \setminus \{\widetilde{1}\}$. Then $\widetilde{\mu}$ is an irreducible fuzzy ideal of S, but not an (i-v) strongly irreducible fuzzy ideal of S. □

Proof of Propositions 41–45 is straightforward and so we omit the proof.

Proposition 41. *Let $\widetilde{\mu}$ be an (i-v) irreducible fuzzy ideal of a semigroup S and $x \in S$. If $\widetilde{\mu}(x) = [\mu^-(x), \mu^+(x)]$ for some fuzzy ideals μ^- and μ^+ of S, then μ^- and μ^+ are both irreducible fuzzy ideals of S.*

Proposition 42. *A proper ideal A of a semigroup S is an irreducible ideal of S if and only if the characteristic function $\widetilde{\chi}_A$ is an (i-v) irreducible fuzzy ideal of S.*

Proposition 43. *An (i-v) fuzzy ideal $\widetilde{\mu}$ of a semigroup S is an (i-v) irreducible fuzzy ideal of S if and only if a level ideal $\widetilde{U}(\widetilde{\mu}, \widetilde{a})$ is an irreducible ideal of S for every $\widetilde{a} \in \operatorname{Im} \widetilde{\mu}$.*

Proposition 44. *If $\widetilde{\mu}_1$ and $\widetilde{\mu}_2$ are two (i-v) irreducible fuzzy ideals of a semigroup S, then $\widetilde{\mu}_1 \cap \widetilde{\mu}_2$ is an (i-v) irreducible fuzzy ideal of S, provided $\widetilde{\mu}_1 \cap \widetilde{\mu}_2$ is nonempty.*

Proposition 45. *Let S and T be two nonempty sets and $\phi : S \to T$ a function. If $\widetilde{\mu}_1$, $\widetilde{\mu}_2$ and $\widetilde{\theta}_1$, $\widetilde{\theta}_2$ are the (i-v) fuzzy ideals of S and T, respectively, then the following statements are true.*

(i) $\phi(\phi^{-1}(\widetilde{\theta}_1)) = \widetilde{\theta}_1$.

(ii) $\phi^{-1}(\phi(\widetilde{\mu}_1)) = \widetilde{\mu}_1$, *provided $\widetilde{\mu}_1$ is ϕ-invariant.*

(iii) $\phi(\widetilde{\mu}_1 \cap \widetilde{\mu}_2) = \phi(\widetilde{\mu}_1) \cap \phi(\widetilde{\mu}_2)$, *provided $\widetilde{\mu}_1$ and $\widetilde{\mu}_1$ are ϕ-invariant.*

(iv) $\phi^{-1}(\widetilde{\theta}_1 \cap \widetilde{\theta}_2) = \phi^{-1}(\widetilde{\mu}_1) \cap \phi^{-1}(\widetilde{\mu}_2)$.

In the following theorem we try to find the homomorphic image and preimage of an (i-v) irreducible fuzzy ideal of a semigroup.

Theorem 46. *Let $\phi : S \to T$ be a semigroup epimorphism and $\widetilde{\mu}$, $\widetilde{\theta}$ (i-v) irreducible fuzzy ideals of S and T, respectively.*

Then

(i) *homomorphic image $\phi(\tilde{\mu})$ is an (i-v) irreducible fuzzy ideal of T, provided $\tilde{\mu}$ is ϕ-invariant;*

(ii) *homomorphic preimage $\phi^{-1}(\tilde{\theta})$ is an (i-v) irreducible fuzzy ideal of S, provided every (i-v) fuzzy ideal of S is ϕ-invariant.*

Proof. (i) Let $\tilde{\mu}$ be ϕ-invariant (i-v) irreducible fuzzy ideal of S. Since $\tilde{\mu}$ is a nonconstant (i-v) fuzzy ideal of S, by Proposition 10, $\phi(\tilde{\mu})$ is a nonconstant (i-v) fuzzy ideal of T. Let $\tilde{\mu}_1$ and $\tilde{\mu}_2$ be two (i-v) fuzzy ideals of T such that $\tilde{\mu}_1 \cap \tilde{\mu}_2 = \phi(\tilde{\mu})$. Since $\tilde{\mu}$ is ϕ-invariant, by Proposition 45, $\tilde{\mu} = \phi^{-1}(\phi(\tilde{\mu})) = \phi^{-1}(\tilde{\mu}_1 \cap \tilde{\mu}_2) = \phi^{-1}(\tilde{\mu}_1) \cap \phi^{-1}(\tilde{\mu}_2)$. Since $\phi^{-1}(\tilde{\mu}_1)$ and $\phi^{-1}(\tilde{\mu}_2)$ are (i-v) fuzzy ideals of S (by Proposition 10) and $\tilde{\mu}$ is an (i-v) irreducible fuzzy ideal of S, either $\phi^{-1}(\tilde{\mu}_1) = \tilde{\mu}$ or $\phi^{-1}(\tilde{\mu}_2) = \tilde{\mu}$. This follows that either $\tilde{\mu}_1 = \phi(\phi^{-1}(\tilde{\mu}_1)) = \phi(\tilde{\mu})$ or $\tilde{\mu}_2 = \phi(\phi^{-1}(\tilde{\mu}_2)) = \phi(\tilde{\mu})$. Hence, $\phi(\tilde{\mu})$ is an (i-v) irreducible fuzzy ideal of T.

(ii) Let every (i-v) fuzzy ideal of S be ϕ-invariant. Since $\tilde{\theta}$ is a nonconstant (i-v) fuzzy ideal of T, by Proposition 10 $\phi^{-1}(\tilde{\theta})$ is a nonconstant (i-v) fuzzy ideal of S. Let $\tilde{\theta}_1$ and $\tilde{\theta}_2$ be two (i-v) fuzzy ideals of S such that $\tilde{\theta}_1 \cap \tilde{\theta}_2 = \phi^{-1}(\tilde{\theta})$. Then, by Proposition 45, $\tilde{\theta} = \phi(\phi^{-1}(\tilde{\theta})) = \phi(\tilde{\theta}_1 \cap \tilde{\theta}_2) = \phi(\tilde{\theta}_1) \cap \phi(\tilde{\theta}_2)$. Since $\tilde{\theta}$ is an (i-v) irreducible fuzzy ideal of T, either $\phi(\tilde{\theta}_1) = \tilde{\theta}$ or $\phi(\tilde{\theta}_2) = \tilde{\theta}$. This follows that either $\tilde{\theta}_1 = \phi^{-1}(\phi(\tilde{\theta}_1)) = \phi^{-1}(\tilde{\theta})$ or $\tilde{\theta}_2 = \phi^{-1}(\phi(\tilde{\theta}_2)) = \phi^{-1}(\tilde{\theta})$. Hence, $\phi^{-1}(\tilde{\theta})$ is an (i-v) irreducible fuzzy ideal of S. □

Theorem 47. *If $\phi : S \to T$ is a semigroup epimorphism, then there is a one-to-one correspondence between the set of ϕ-invariant (i-v) irreducible fuzzy ideals of S and the set of all (i-v) irreducible fuzzy ideals of T.*

Proof. Let $\mathfrak{IRF}_\phi$ be the set of all ϕ-invariant (i-v) irreducible fuzzy ideals of S and $\mathfrak{IRF}$ the set of all (i-v) irreducible fuzzy ideals of T. Now, we define a mapping $\psi : \mathfrak{IRF}_\phi \to \mathfrak{IRF}$ by $\psi(\tilde{\mu}) = \phi(\tilde{\mu})$. Then, it is a well-defined map. Let $\psi(\tilde{\mu}_1) = \psi(\tilde{\mu}_2)$, where $\tilde{\mu}_1, \tilde{\mu}_2 \in \mathfrak{IRF}_\phi$. This implies that $\phi(\tilde{\mu}_1) = \phi(\tilde{\mu}_2) \Rightarrow \sup_{z\in\phi^{-1}(y)}\tilde{\mu}_1(z) = \sup_{z\in\phi^{-1}(y)}\tilde{\mu}_2(z)$ for all $y \in T \Rightarrow \tilde{\mu}_1(z^*) = \tilde{\mu}_2(z^*)$ for all $z^* \in \phi^{-1}(y)$ (since $\tilde{\mu}_1$ and $\tilde{\mu}_2$ are ϕ-invariant). Since ϕ is onto and y is arbitrary, z^* is also arbitrary and hence $\tilde{\mu}_1 = \tilde{\mu}_2$. Therefore, it follows that ψ is one-to-one. □

Theorem 48. *Every (i-v) prime fuzzy ideal of a semigroup S is an (i-v) irreducible fuzzy ideal of S.*

Proof. Let $\tilde{\mu}$ be an (i-v) prime fuzzy ideal of S. Consider $\tilde{\mu}_1$ and $\tilde{\mu}_2$ to be two (i-v) fuzzy ideals of S such that $\tilde{\mu}_1 \cap \tilde{\mu}_2 = \tilde{\mu}$. Then $\tilde{\mu} \subseteq \tilde{\mu}_1$ and $\tilde{\mu} \subseteq \tilde{\mu}_2$. Again, since $\tilde{\mu}_1$ and $\tilde{\mu}_2$ are (i-v) fuzzy ideals of S, $\tilde{\mu}_1 \circ \tilde{\mu}_2 \subseteq \tilde{\mu}_1 \cap \tilde{\mu}_2 = \tilde{\mu}$. Therefore, by our assumption, either $\tilde{\mu}_1 \subseteq \tilde{\mu}$ or $\tilde{\mu}_2 \subseteq \tilde{\mu}$. This implies that either $\tilde{\mu}_1 = \tilde{\mu}$ or $\tilde{\mu}_2 = \tilde{\mu}$. Hence, $\tilde{\mu}$ is an (i-v) irreducible fuzzy ideal of S. □

For converse part, we set an example as follows.

Consider a semigroup $\mathbb{N}$ with usual multiplication and an ideal $4\mathbb{N}$ of $\mathbb{N}$. Then $4\mathbb{N}$ is an irreducible ideal of $\mathbb{N}$, but not a prime ideal of $\mathbb{N}$. Now, if we define an (i-v) fuzzy subset $\tilde{\mu}$ of $\mathbb{N}$ by

$$\tilde{\mu}(x) = \begin{cases} \tilde{1}, & \text{when } x \in 4\mathbb{N}; \\ \tilde{b}, & \text{otherwise}, \end{cases} \tag{10}$$

where $\tilde{b} \in D[0,1] \setminus \{\tilde{1}\}$, then it is easily shown that $\tilde{\mu}$ is an (i-v) irreducible fuzzy ideal of $\mathbb{N}$, but not an (i-v) prime fuzzy ideal of $\mathbb{N}$.

Remark 49. But the converse of Theorem 48 is true in a regular semigroup.

Theorem 50. *Every (i-v) irreducible fuzzy ideal of a regular semigroup S is an (i-v) prime fuzzy ideal of S.*

Proof. Let $\tilde{\mu}$ be an (i-v) irreducible fuzzy ideal of S and consider $\tilde{\mu}_1$ and $\tilde{\mu}_2$ to be two (i-v) fuzzy ideals of S such that $\tilde{\mu}_1 \circ \tilde{\mu}_2 \subseteq \tilde{\mu}$. Since S is regular, for every $x \in S$, there exists $y \in S$ such that $x = xyx$. Therefore, $\tilde{\mu}(x) \geq (\tilde{\mu}_1 \circ \tilde{\mu}_2)(x) \geq \mathrm{Min}^i(\tilde{\mu}_1(x), \tilde{\mu}_2(x)) = (\tilde{\mu}_1 \cap \tilde{\mu}_2)(x)$ which implies $\tilde{\mu}_1 \cap \tilde{\mu}_2 \subseteq \tilde{\mu}$. But $\tilde{\mu} = \tilde{\mu} \cup (\tilde{\mu}_1 \cap \tilde{\mu}_2) = (\tilde{\mu} \cup \tilde{\mu}_1) \cap (\tilde{\mu} \cup \tilde{\mu}_2)$. Thus, by our assumption, either $\tilde{\mu} \cup \tilde{\mu}_1 = \tilde{\mu}$ or $\tilde{\mu} \cup \tilde{\mu}_2 = \tilde{\mu}$. This implies either $\tilde{\mu}_1 \subseteq \tilde{\mu}$ or $\tilde{\mu}_2 \subseteq \tilde{\mu}$. Hence, $\tilde{\mu}$ is an (i-v) prime fuzzy ideal of S. □

Theorem 51. *A nonconstant (i-v) fuzzy ideal $\tilde{\mu}$ of a semigroup S is an (i-v) prime fuzzy ideal of S if and only if $\tilde{\mu}$ is an (i-v) irreducible fuzzy ideal as well as an (i-v) semiprime fuzzy ideal of S.*

Proof. Let $\tilde{\mu}$ be an (i-v) prime fuzzy ideal of S. Then $\tilde{\mu}$ is an (i-v) semiprime fuzzy ideal of S and, by Theorem 48, $\tilde{\mu}$ is also an (i-v) irreducible fuzzy ideal of S.

Conversely, let $\tilde{\mu}$ be an (i-v) irreducible fuzzy ideal as well as an (i-v) semiprime fuzzy ideal of S. Consider $\tilde{\mu}_1$ and $\tilde{\mu}_2$ to be two (i-v) fuzzy ideals of S such that $\tilde{\mu}_1 \circ \tilde{\mu}_2 \subseteq \tilde{\mu}$. Then, $(\tilde{\mu}_1 \cap \tilde{\mu}_2) \circ (\tilde{\mu}_1 \cap \tilde{\mu}_2) \subseteq \tilde{\mu}_1 \circ \tilde{\mu}_2 \subseteq \tilde{\mu}$. But $\tilde{\mu}$ is an (i-v) semiprime fuzzy ideal of S. Therefore, $\tilde{\mu}_1 \cap \tilde{\mu}_2 \subseteq \tilde{\mu}$. Now, $\tilde{\mu} = \tilde{\mu} \cup (\tilde{\mu}_1 \cap \tilde{\mu}_2) = (\tilde{\mu} \cup \tilde{\mu}_1) \cap (\tilde{\mu} \cup \tilde{\mu}_2)$ which implies either $\tilde{\mu} \cup \tilde{\mu}_1 = \tilde{\mu}$ or $\tilde{\mu} \cup \tilde{\mu}_2 = \tilde{\mu}$ implies either $\tilde{\mu}_1 \subseteq \tilde{\mu}$ or $\tilde{\mu}_2 \subseteq \tilde{\mu}$. Hence, $\tilde{\mu}$ is an (i-v) prime fuzzy ideal of S. □

Definition 52. A set $\mathfrak{C}$ of (i-v) fuzzy ideals of a semigroup S forms an i-chain if $\tilde{\mu}_1 \subseteq \tilde{\mu}_2$ or $\tilde{\mu}_2 \subseteq \tilde{\mu}_1$ for any two (i-v) fuzzy ideals $\tilde{\mu}_1, \tilde{\mu}_2 \in \mathfrak{C}$. If such a set exists in S, then we say S has an i-chain.

Definition 53. An (i-v) fuzzy ideal $\tilde{\mu}$ of a semigroup S is called maximal if, for any (i-v) fuzzy ideal $\tilde{\theta}$ of S, $\tilde{\mu} \subseteq \tilde{\theta}$ implies $\tilde{\theta} = \tilde{\mu}$.

Theorem 54 shows the existence of an irreducible ideal corresponds to an ideal of a semigroup. In Theorem 55, we try to generalize this result in terms of interval-valued fuzzy concept.

Theorem 54. *Let A be an ideal of a semigroup S and $a \in S$ such that $a \notin A$. Then, there exists an irreducible ideal I of S such that $A \subseteq I$ and $a \notin I$.*

Theorem 55. *Let $\tilde{\mu}$ be an (i-v) fuzzy ideal of a semigroup S and $x \in S$ such that $\tilde{\mu}(x) > \tilde{0}$. Then, there exists an (i-v) irreducible fuzzy ideal $\tilde{\theta}$ of S such that $\tilde{\mu} \subseteq \tilde{\theta}$ and $\tilde{\theta}(x) = \tilde{\mu}(x)$.*

Proof. Let A be a set of all (i-v) fuzzy ideals $\tilde{\delta}$ of S such that $\tilde{\mu} \subseteq \tilde{\delta}$ and $\tilde{\delta}(x) = \tilde{\mu}(x) > \tilde{0}$. Then $A \neq \phi$ and, under inclusion, A is a poset. Now, if we consider an i-chain $\mathfrak{C} = \{\tilde{\gamma} : \tilde{\gamma} \in A\}$ of A, then it is easy to show that the (i-v) fuzzy ideal $\bigcup \tilde{\gamma}$ of S is an upper bound of $\mathfrak{C}$. Therefore, from Zorn's Lemma, we can say that A has a maximal element, say, $\tilde{\theta}$, which is an (i-v) fuzzy ideal of S containing $\tilde{\mu}$ such that $\tilde{\theta}(x) = \tilde{\mu}(x)$. Let $\tilde{\theta}_1$ and $\tilde{\theta}_2$ be two (i-v) fuzzy ideals of S such that $\tilde{\theta} = \tilde{\theta}_1 \cap \tilde{\theta}_2$. Then either $\tilde{\theta} \subseteq \tilde{\theta}_1$ or $\tilde{\theta} \subseteq \tilde{\theta}_2$. Therefore, by maximality condition, it implies that either $\tilde{\theta}_1 = \tilde{\theta}$ or $\tilde{\theta}_2 = \tilde{\theta}$. Hence, $\tilde{\theta}$ is an (i-v) irreducible fuzzy ideal of S. □

Theorem 56. *The following conditions are equivalent in a semigroup S.*

(i) *S is regular.*

(ii) *Every (i-v) fuzzy ideal $\tilde{\mu}$ of S is idempotent.*

(iii) *Every (i-v) fuzzy ideal of S is an (i-v) semiprime fuzzy ideal of S.*

(iv) *Every (i-v) fuzzy ideal of S is an intersection of (i-v) prime fuzzy ideals of S containing it.*

Proof. (i)⇒(ii). Let S be regular and $\tilde{\mu}$ an (i-v) fuzzy ideal of S. Then $\tilde{\mu} = \tilde{\mu} \cap \tilde{\mu} = \tilde{\mu} \circ \tilde{\mu}$. This shows that $\tilde{\mu}$ is an (i-v) idempotent fuzzy ideal of S.

(ii)⇒(i). Let (ii) hold and consider $\tilde{\mu}_1$ and $\tilde{\mu}_2$ to be two (i-v) fuzzy ideals of S. Then $\tilde{\mu}_1 \circ \tilde{\mu}_2 \subseteq \tilde{\mu}_1 \cap \tilde{\mu}_2$ and $\tilde{\mu}_1 \cap \tilde{\mu}_2$ is an (i-v) fuzzy ideal of S. Therefore, by (ii), $\tilde{\mu}_1 \cap \tilde{\mu}_2 = (\tilde{\mu}_1 \cap \tilde{\mu}_2) \circ (\tilde{\mu}_1 \cap \tilde{\mu}_2) \subseteq \tilde{\mu}_1 \circ \tilde{\mu}_2$. This implies that $\tilde{\mu}_1 \cap \tilde{\mu}_2 = \tilde{\mu}_1 \circ \tilde{\mu}_2$ and hence S is regular.

(ii)⇒(iii). Let every (i-v) fuzzy ideal of S be idempotent. Let $\tilde{\mu}$ be an (i-v) fuzzy ideal of S and $\tilde{\mu}_1 \circ \tilde{\mu}_1 \subseteq \tilde{\mu}$ for any (i-v) fuzzy ideal $\tilde{\mu}_1$ of S. Then, by our assumption, $\tilde{\mu}_1 = \tilde{\mu}_1 \circ \tilde{\mu}_1 \subseteq \tilde{\mu}$; that is, $\tilde{\mu}_1 \subseteq \tilde{\mu}$. Hence, $\tilde{\mu}$ is an (i-v) semiprime fuzzy ideal of S.

(iii)⇒(iv). Let (iii) hold and consider $\tilde{\mu}$ to be an (i-v) fuzzy ideal of S. Then, for any $x \in S$, it follows from Theorem 55 that there exists an (i-v) irreducible fuzzy ideal $\tilde{\theta}$ of S such that $\tilde{\mu} \subseteq \tilde{\theta}$ and $\tilde{\mu}(x) = \tilde{\theta}(x)$. Now, if we consider the set of all those (i-v) irreducible fuzzy ideals $\tilde{\theta}_i$ $(i \in \Lambda)$ of S containing $\tilde{\mu}$, then $\bigcap_{i\in\Lambda} \tilde{\theta}_i$ is the smallest (i-v) irreducible fuzzy ideal of S containing $\tilde{\mu}$. Also, $(\bigcap_{i\in\Lambda} \tilde{\theta}_i)(x) = \inf_{i\in\Lambda}\{\tilde{\theta}_i(x)\} \leq \tilde{\theta}(x) = \tilde{\mu}(x) \leq (\bigcap_{i\in\Lambda} \tilde{\theta}_i)(x)$. This implies that $(\bigcap_{i\in\Lambda} \tilde{\theta}_i)(x) = \tilde{\mu}(x)$ for arbitrary $x \in S$. Hence $\tilde{\mu} = \bigcap_{i\in\Lambda} \tilde{\theta}_i$. But (iii) implies every $\tilde{\theta}_i$ $(i \in \Lambda)$ is an (i-v) semiprime fuzzy ideal of S. Hence, by Theorem 51, every $\tilde{\theta}_i$ $(i \in \Lambda)$ is an (i-v) prime fuzzy ideal of S. Thus, it follows that $\tilde{\mu}$ is an intersection of (i-v) prime fuzzy ideals of S containing $\tilde{\mu}$.

(iv)⇒(ii). Let (iv) hold and consider $\tilde{\mu}$ to be an (i-v) fuzzy ideal of S. Then $\tilde{\mu} \circ \tilde{\mu} \subseteq \tilde{\mu}$. Again, since $\tilde{\mu} \circ \tilde{\mu}$ is an (i-v) fuzzy ideal of S, by (iv), $\tilde{\mu} \circ \tilde{\mu} = \bigcap\{\tilde{\vartheta} : \tilde{\vartheta}$ is an (i-v) prime fuzzy ideal of S containing $\tilde{\mu} \circ \tilde{\mu}\}$. This implies that $\tilde{\mu} \circ \tilde{\mu} \subseteq \tilde{\vartheta}$ for every $\tilde{\vartheta}$. But each $\tilde{\vartheta}$ is an (i-v) prime fuzzy ideal of S. Hence, $\tilde{\mu} \subseteq \tilde{\vartheta}$. Therefore, $\tilde{\mu} \subseteq \bigcap\{\tilde{\vartheta}\} = \tilde{\mu} \circ \tilde{\mu}$; that is, $\tilde{\mu} \subseteq \tilde{\mu} \circ \tilde{\mu}$. Thus, it follows that $\tilde{\mu} = \tilde{\mu} \circ \tilde{\mu}$; that is, $\tilde{\mu}$ is an (i-v) idempotent fuzzy ideal of S. □

Theorem 57. *Every (i-v) fuzzy ideal of a semigroup S is an (i-v) prime fuzzy ideal of S if and only if S is regular and all (i-v) fuzzy ideals of S form an i-chain.*

Proof. Let every (i-v) fuzzy ideal of S be an (i-v) prime fuzzy ideal of S. Then each of these ideals is also an (i-v) semiprime fuzzy ideal of S and hence, by Theorem 56, S is regular. Again, for any two (i-v) fuzzy ideals $\tilde{\mu}_1$ and $\tilde{\mu}_2$ of S, $\tilde{\mu}_1 \circ \tilde{\mu}_2 \subseteq \tilde{\mu}_1 \cap \tilde{\mu}_2$ and $\tilde{\mu}_1 \cap \tilde{\mu}_2$ is an (i-v) fuzzy ideal of S. Therefore, by our assumption, $\tilde{\mu}_1 \cap \tilde{\mu}_2$ is an (i-v) prime fuzzy ideal of S. Thus, it follows that either $\tilde{\mu}_1 \subseteq \tilde{\mu}_1 \cap \tilde{\mu}_2$ or $\tilde{\mu}_2 \subseteq \tilde{\mu}_1 \cap \tilde{\mu}_2$; that is, either $\tilde{\mu}_1 \subseteq \tilde{\mu}_2$ or $\tilde{\mu}_2 \subseteq \tilde{\mu}_1$. Thus, the set of all (i-v) fuzzy ideals of S forms an i-chain.

Conversely, let S be regular and all (i-v) fuzzy ideals form an i-chain. Let $\tilde{\mu}$ be an (i-v) fuzzy ideal of S and $\tilde{\mu}_1 \circ \tilde{\mu}_2 \subseteq \tilde{\mu}$ for any two (i-v) fuzzy ideals $\tilde{\mu}_1, \tilde{\mu}_2$ of S. Then, from chain property, either $\tilde{\mu}_1 \subseteq \tilde{\mu}_2$ or $\tilde{\mu}_2 \subseteq \tilde{\mu}_1$. Again, since S is regular, by Theorem 56, $\tilde{\mu}_1 = \tilde{\mu}_1 \circ \tilde{\mu}_1$ and $\tilde{\mu}_2 = \tilde{\mu}_2 \circ \tilde{\mu}_2$. Now, if $\tilde{\mu}_1 \subseteq \tilde{\mu}_2$, then $\tilde{\mu}_1 = \tilde{\mu}_1 \circ \tilde{\mu}_1 \subseteq \tilde{\mu}_1 \circ \tilde{\mu}_2 \subseteq \tilde{\mu}$ and if $\tilde{\mu}_2 \subseteq \tilde{\mu}_1$, then $\tilde{\mu}_2 = \tilde{\mu}_2 \circ \tilde{\mu}_2 \subseteq \tilde{\mu}_1 \circ \tilde{\mu}_2 \subseteq \tilde{\mu}$. Thus it follows that either $\tilde{\mu}_1 \subseteq \tilde{\mu}$ or $\tilde{\mu}_2 \subseteq \tilde{\mu}$. Hence $\tilde{\mu}$ is an (i-v) prime fuzzy ideal of S. □

5. Conclusion

Interval-valued fuzzy ideals are new tools to study fuzzy algebra. Interval-valued semiprime fuzzy ideals may be used to further study of fuzzy semigroups and fuzzy semirings and certainly give some important aspects of fuzzy algebra as a whole.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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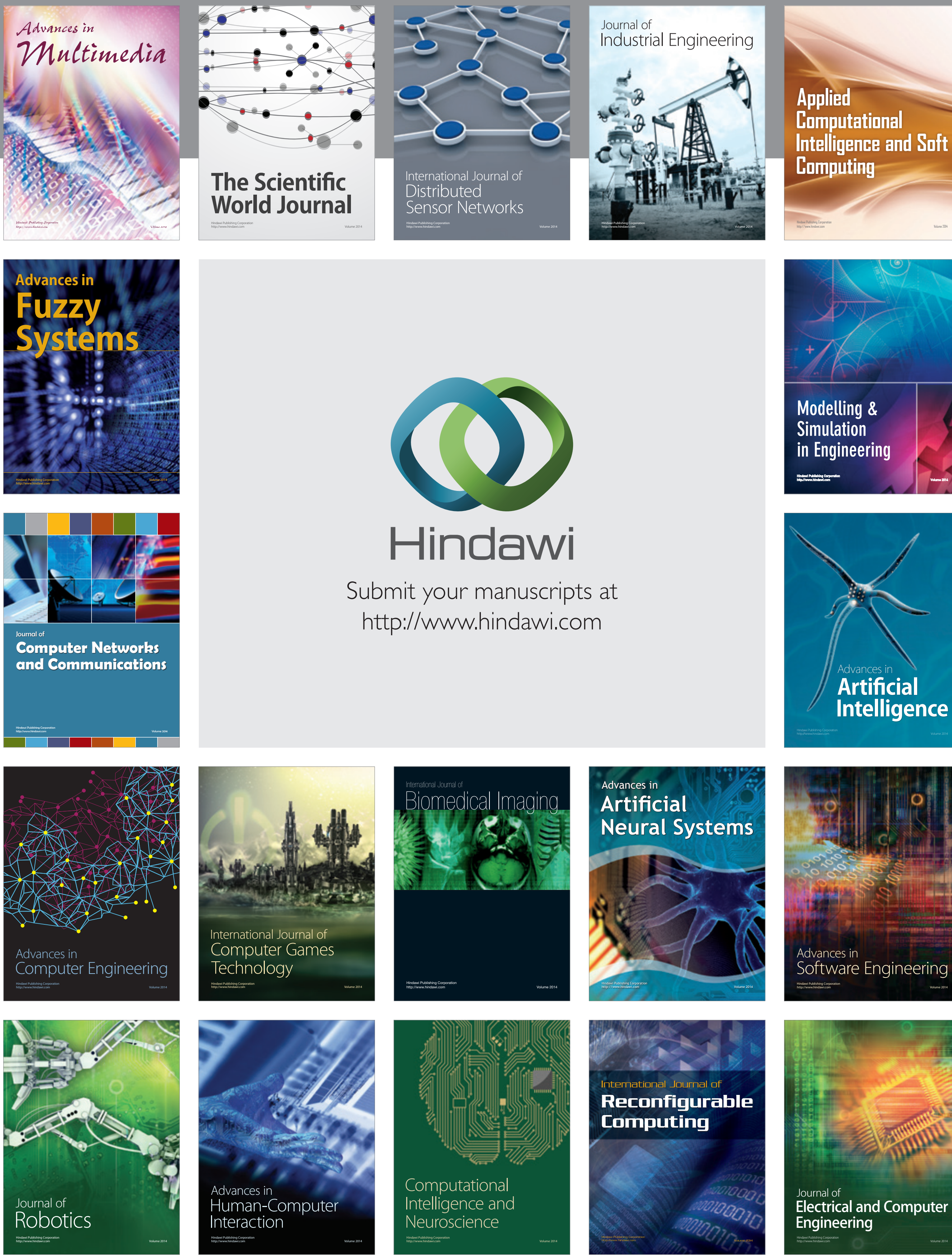
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