Hindawi Publishing Corporation
Advances in Fuzzy Systems
Volume 2016, Article ID 6475403, 12 pages
http://dx.doi.org/10.1155/2016/6475403

Research Article

An Efficient Ranking Technique for Intuitionistic Fuzzy Numbers with Its Application in Chance Constrained Bilevel Programming

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Received 22 November 2015; Revised 24 March 2016; Accepted 3 April 2016

Academic Editor: Kemal Kilic

The aim of this paper is to develop a new ranking technique for intuitionistic fuzzy numbers using the method of defuzzification based on probability density function of the corresponding membership function, as well as the complement of nonmembership function. Using the proposed ranking technique a methodology for solving linear bilevel fuzzy stochastic programming problem involving normal intuitionistic fuzzy numbers is developed. In the solution process each objective is solved independently to set the individual goal value of the objectives of the decision makers and thereby constructing fuzzy membership goal of the objectives of each decision maker. Finally, a fuzzy goal programming approach is considered to achieve the highest membership degree to the extent possible of each of the membership goals of the decision makers in the decision making context. Illustrative numerical examples are provided to demonstrate the applicability of the proposed methodology and the achieved results are compared with existing techniques.

1. Introduction

The concept of bilevel programming problem (BLPP) was first introduced by Candler and Townsley [1]. The BLPP is considered as a class of optimization problems where two decision makers (DMs) locating at two different hierarchical levels independently control a set of decision variables paying serious attention to the benefit of the others in a highly conflicting decision making situation. The upper level DM is termed as leader and the lower level DM as follower. There are several applications of BLPP in many real life problems such as agriculture, biofuel production, economic systems, finance, engineering, banking, management sciences, and transportation problem. Several methods were proposed to solve BLPPs by different researchers [2, 3] in the past. But these traditional approaches are unable to provide a satisfactory solution if the parameter values involved with a BLPP inevitably contain some uncertain data or linguistic information. Stochastic programming (SP) and fuzzy programming (FP) are two powerful techniques to handle such type of problems. Using probability theory, Dantzig [4] introduced SP. The SP was developed in various directions like chance constrained programming (CCP), recourse programming, multiobjective SP, and so forth. Charnes and Cooper [5] developed the concept of CCP.

Again, from the viewpoint of uncertainty or fuzziness involved in human's judgments, Zimmermann [6] first applied fuzzy set theory [7] in decision making problems with several conflicting objectives. The concept of membership functions in BLPPs was introduced by Lai and Hwang [8]. Lai's solution concept was then extended by Shih et al. [9] and a supervised search procedure with the use of max-min operator of Bellman and Zadeh [10] was proposed. The basic concept of this procedure is that the follower optimizes his/her objective function, taking into consideration leader's goal. Recently, Lodwick and Kacprizyk [11] developed a methodology for solving decision making problems under fuzziness. The main difficulty of FP approach is that the

objectives of the DMs are conflicting. So there is possibility of rejecting the solution again and again by the DMs and the solution process is continued by redefining the membership functions repeatedly until a satisfactory solution is obtained. This makes the solution process a very lengthy and tedious one. To remove these difficulties fuzzy goal programming (FGP) [12–14] is used as an efficient tool for making decision in an imprecisely defined multiobjective decision making (MODM) arena. Baky [15] developed a FGP technique for solving multiobjective multilevel programming problem.

Also it is observed that the fuzzy sets (FSs) are not always capable of dealing with lack of knowledge with respect to degrees of membership. Realizing the fact Atanassov [16–18] introduced the concept of intuitionistic FSs (IFSs) by implementing a nonmembership degree which can handle the drawback of FSs and express more abundant and flexible information than the FSs. In recent years, there is a growing interest in the study of decision making problems with intuitionistic fuzzy numbers (IFNs) [19–21] and with interval-valued intuitionistic fuzzy information [22–25].

The ranking of IFNs [26, 27] plays an important role in dealing with IFNs as ranking of fuzzy numbers (FNs). Grzegoraewski [28] suggested some methods for measuring distances between IFNs and interval-valued FNs, based on Hausdorff metric. The methodology for solving intuitionistic fuzzy linear programming problems with triangular IFNs (TIFNs) was developed by Dubey and Mehra [29] by converting the model into crisp linear programming problem. Li [30] developed a ratio ranking method for the TIFNs. Nehi [31] put forward a new ordering method of IFNs in which two characteristic values for IFNs are defined by the integral of the inverse fuzzy membership and nonmembership functions multiplied by the grade with powered parameters. Recently, numerous ranking methods for IFNs have been proposed in literature to rank IFNs [32–34]. Although many defuzzification methods have already been proposed so far, no method gives a right effective defuzzification output. Most of the existing defuzzification methods tried to make the estimation of IFNs in an objective way. This paper proposes a method using the concept probability density function of IFNs and Mellin's transform [35, 36] to find the ranking of normal IFNs. This ranking method removes the ambiguous outcomes and distinguishes the alternatives clearly.

In fuzzy BLPP [37] it is sometimes realized that the concept of membership function does not provide satisfactory solutions in a highly conflicting decision making situation. In this context IFNs can be used to capture both the membership and nonmembership degrees of uncertainties of both the DMs. Also there are some real world situations, where randomness and fuzziness occur simultaneously. The decision making problem having such type of ambiguous information is known as fuzzy stochastic programming problem. Many researchers [38–41] derived different methods to solve such type of decision making problems. But FGP approach for solving fuzzy stochastic linear BLPP with IFNs is yet to appear in the literature.

In the present study FGP process is adopted for solving bilevel intuitionistic FP problems where the parameters are expressed in terms of normal IFNs. A ranking technique for normal IFNs is first proposed and then using the proposed technique the model is converted into a deterministic problem. The individual optimal value of each objective is found in isolation to construct the fuzzy membership goals of each of the objectives. Finally, FGP model is developed for the achievement of highest degree of each of the defined membership goals to the extent possible by minimizing group regrets in the decision making context. To explore the potentiality of the proposed approach, two illustrative examples are considered and solved and the achieved solutions are compared with the predefined technique developed by Dubey and Mehra [29].

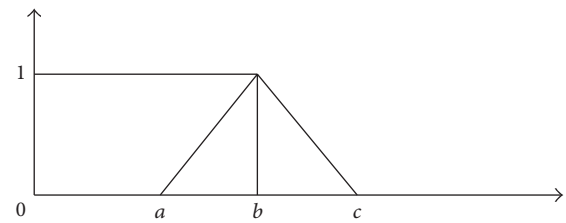

Figure 1: TFN.

2. Preliminaries

In this section some basic concepts on FNs, triangular FNs (TFNs), IFNs, and triangular IFNs (TIFNs) are discussed.

2.1. FN [42]. A fuzzy set $\widetilde{A}$ defined on the set of real numbers, $\mathbb{R}$, is said to be an FN if its membership function $\mu_{\widetilde{A}}(x)$ satisfies the following characteristics:

(i) $\mu_{\widetilde{A}} : \mathbb{R} \to [0,1]$ is continuous.

(ii) $\mu_{\widetilde{A}}(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$.

(iii) $\mu_{\widetilde{A}}(x)$ is strictly increasing on $[a,b]$ and strictly decreasing on $[c,d]$.

(iv) $\mu_{\widetilde{A}}(x) = 1$ for all $x \in [b,c]$, where $a \le b \le c \le d$.

2.2. TFN [43]. An FN $\widetilde{A} = (a,b,c)$ is said to be TFN if its membership function $\mu_{\widetilde{A}}(x)$ is given by

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0 & \text{if } x < a \text{ or } x > c \\ \dfrac{x-a}{b-a} & \text{if } a \le x \le b \\ \dfrac{c-x}{c-b} & \text{if } b \le x \le c. \end{cases} \tag{1}$$

The TFN can be expressed in the form of Figure 1.

2.3. IFN [44]. An IFN $\widetilde{A}$ is

(i) an IFS defined on $\mathbb{R}$;

(ii) normal; that is, there exists $x \in \mathbb{R}$ such that $\mu_{\widetilde{A}}(x) = 1$ and $\nu_{\widetilde{A}}(x) = 0$;

(iii) convex for the membership function $\mu_{\widetilde{A}}(x)$; that is, $\mu_{\widetilde{A}}(\lambda x_1 + (1-\lambda)x_2) \ge \min\{\mu_{\widetilde{A}}(x_1), \mu_{\widetilde{A}}(x_2)\}$ for all $x_1, x_2 \in \mathbb{R}$, $0 \le \lambda \le 1$;

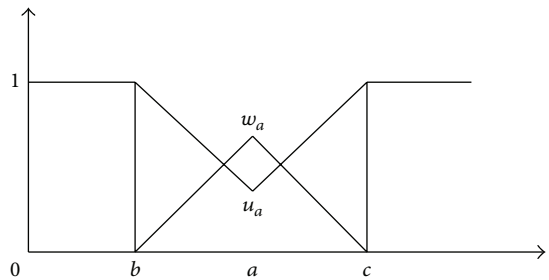

Figure 2: TIFN.

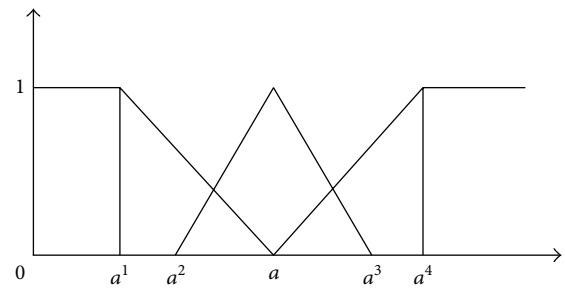

Figure 3: Triangular normal IFN.

(iv) concave for the nonmembership function $\nu_{\widetilde{A}}(x)$; that is, $\nu_{\widetilde{A}}(\lambda x_1 + (1-\lambda)x_2) \le \max\{\nu_{\widetilde{A}}(x_1), \nu_{\widetilde{A}}(x_2)\}$ for all $x_1, x_2 \in \mathbb{R}$, $0 \le \lambda \le 1$.

2.4. TIFN [30]. Let $\widetilde{A} = \{(b,a,c), w_a, u_a\}$ be a TIFN. Then the membership and the nonmembership function of $\widetilde{A}$ are expressed as

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{(x-b)}{(a-b)} w_a & \text{if } b \le x \le a \\ \dfrac{(c-x)}{(c-a)} w_a & \text{if } a \le x \le c \\ 0 & \text{if } x < b \text{ or } x > c, \end{cases}$$
$$\nu_{\widetilde{A}}(x) = \begin{cases} \dfrac{(a-x)}{(a-d)} u_a & \text{if } b \le x \le a \\ \dfrac{(x-a)}{(e-a)} u_a & \text{if } a \le x \le c \\ 1 & \text{if } x < b \text{ or } x > c. \end{cases} \tag{2}$$

That TIFN is expressed by Figure 2.

A TIFN is called normal if $w_a = 1$ and $u_a = 0$ for at least one x.

3. Proposed Ranking Technique for Normal TIFN

Let $\widetilde{A} = \{(a^2, a, a^3), (a^1, a, a^4)\}$ be a normal TIFN. Then the membership and the nonmembership functions of $\widetilde{A}$ are expressed as

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{\left(x-a^2\right)}{\left(a-a^2\right)} & \text{if } a^2 \le x \le a \\ \dfrac{\left(a^3-x\right)}{\left(a^3-a\right)} & \text{if } a \le x \le a^3 \\ 0 & \text{if } x < a^2 \text{ or } x > a^3, \end{cases}$$
$$\nu_{\widetilde{A}}(x) = \begin{cases} \dfrac{(a-x)}{\left(a-a^1\right)} & \text{if } a^1 \le x \le a \\ \dfrac{(x-a)}{\left(a^4-a\right)} & \text{if } a \le x \le a^4 \\ 1 & \text{if } x < a^1 \text{ or } x > a^4, \end{cases} \tag{3}$$

which is presented in Figure 3.

It is to be noted here that if the nonmembership value decreases, acceptance of the IFN increases. As a consequence, if the value of complement of the nonmembership function increases, the acceptance possibility of the IFN increases. Considering those concepts, membership functions as well as the complement of the nonmembership functions are taken into account to rank the IFNs. The complement of the nonmembership function of a normal TIFN $\widetilde{A}$ is given by

$$1 - \nu_{\widetilde{A}}(x) = \begin{cases} \dfrac{\left(x-a^1\right)}{\left(a-a^1\right)} & \text{if } a^1 \le x \le a \\ \dfrac{\left(a^4-x\right)}{\left(a^4-a\right)} & \text{if } a \le x \le a^4 \\ 0 & \text{if } x < a^1 \text{ or } x > a^4. \end{cases} \tag{4}$$

Let us define two probability density functions $f_1(x)$ and $f_2(x)$ corresponding to the membership function $\mu_{\widetilde{A}}(x)$ and the complement of the nonmembership function $1 - \nu_{\widetilde{A}}(x)$, respectively.

Let $f_1(x) = k_1 \mu_{\widetilde{A}}(x)$; then $\int_{-\infty}^{\infty} f_1(x)dx = 1$ implies $k_1 \int_{-\infty}^{\infty} \mu_{\widetilde{A}}(x)dx = 1$; that is,

$$k_1 \left[\int_{a^2}^{a} \frac{\left(x-a^2\right)}{\left(a-a^2\right)} dx + \int_{a}^{a^3} \frac{\left(a^3-x\right)}{\left(a^3-a\right)} dx \right] = 1; \tag{5}$$

that is,

$$k_1 = \frac{2}{\left(a^3-a^2\right)}. \tag{6}$$

Thus

$$f_1(x) = \begin{cases} \dfrac{2\left(x-a^2\right)}{\left(a^3-a^2\right)\left(a-a^2\right)} & \text{if } a^2 \le x \le a \\ \dfrac{2\left(a^3-x\right)}{\left(a^3-a^2\right)\left(a^3-a\right)} & \text{if } a \le x \le a^3 \\ 0 & \text{if } x < a^2 \text{ or } x > a^3. \end{cases} \tag{7}$$

Similarly, defining $f_2(x) = k_2(1 - \nu_{\widetilde{A}}(x))$, the constant k_2 and the function $f_2(x)$ are obtained as

$$k_2 = \frac{2}{(a^4 - a^1)},$$
$$f_2(x) = \begin{cases} \dfrac{2(x - a^1)}{(a^4 - a^1)(a - a^1)} & \text{if } a^1 \le x \le a \\ \dfrac{2(a^4 - x)}{(a^4 - a^1)(a^4 - a)} & \text{if } a \le x \le a^4 \\ 0 & \text{if } x < a^1 \text{ or } x > a^4. \end{cases} \tag{8}$$

Let $f(x)$ be the probability density function corresponding to the normal TIFN $\widetilde{A}$ which is defined as

$$f(x) = \lambda f_1(x) + (1 - \lambda) f_2(x), \quad (0 \le \lambda \le 1). \tag{9}$$

A technique for defuzzification of fuzzy number using Mellin's transform was developed by R. Saneifard and R. Saneifard [36]. There are lots of successful applications [45–49] of this technique. From that viewpoint, Mellin's transformation has been applied to find the defuzzified value of IFNs. Mellin's transform is given by

$$M_X(t) = \int_0^\infty x^{t-1} f(x)\, dx. \tag{10}$$

Here X denotes the random variable corresponding to the normal TIFN $\widetilde{A}$. Thus

$$\begin{aligned} M_X(t) &= \int_0^\infty x^{t-1} (\lambda f_1(x) + (1 - \lambda) f_2(x))\, dx \\ &= 2\lambda \left[\int_{a^2}^{a} x^{t-1} \frac{(x - a^2)}{(a - a^2)(a^3 - a^2)} dx \right. \\ &\quad \left. + \int_a^{a^3} x^{t-1} \frac{(a^3 - x)}{(a^3 - a)(a^3 - a^2)} dx \right] + 2(1 - \lambda) \\ &\quad \cdot \left[\int_{a^1}^{a} x^{t-1} \frac{(x - a^1)}{(a - a^1)(a^4 - a^1)} dx \right. \\ &\quad \left. + \int_a^{a^4} x^{t-1} \frac{(a^4 - x)}{(a^4 - a)(a^4 - a^1)} dx \right] \\ &= \frac{2\lambda}{(a - a^2)(a^3 - a^2)} \left[\frac{\left(a^{t+1} - (a^2)^{t+1}\right)}{(t+1)} \right. \\ &\quad \left. - \frac{a^2\left(a^t - (a^2)^t\right)}{t} \right] \\ &\quad + \frac{2\lambda}{(a^3 - a)(a^3 - a^2)} \left[\frac{a^3\left((a^3)^t - a^t\right)}{t} \right. \\ &\quad \left. - \frac{\left((a^3)^{t+1} - a^{t+1}\right)}{(t+1)} \right] \\ &\quad + \frac{2(1 - \lambda)}{(a - a^1)(a^4 - a^1)} \left[\frac{\left(a^{t+1} - (a^1)^{t+1}\right)}{(t+1)} \right. \\ &\quad \left. - \frac{a^1\left(a^t - (a^1)^t\right)}{t} \right] \\ &\quad + \frac{2(1 - \lambda)}{(a^4 - a)(a^4 - a^1)} \left[\frac{a^4\left((a^4)^t - a^t\right)}{t} \right. \\ &\quad \left. - \frac{\left((a^4)^{t+1} - a^{t+1}\right)}{(t+1)} \right]. \end{aligned} \tag{11}$$

For $t = 2$, Mellin's transform converted to the definition of expectation of a random variable. Since the target is to find the expected or defuzzified value of TIFNs, $t = 2$ has been considered.

Thus the crisp equivalent value of the normal TIFN is found as

$$\begin{aligned} V(\widetilde{A}) = M_X(2) &= \frac{2\lambda}{(a^3 - a^2)} \left[\frac{\left((a^2)^2 + aa^2 + (a)^2\right)}{3} \right. \\ &\quad \left. - \frac{a^2(a^2 + a)}{2} \right] + \frac{2\lambda}{(a^3 - a^2)} \left[\frac{a^3(a^3 + a)}{2} \right. \\ &\quad \left. - \frac{\left((a^3)^2 + a^3 a + (a)^2\right)}{3} \right] \end{aligned}$$

$$+\frac{2(1-\lambda)}{(a^4-a^1)}\left[\frac{\left((a)^2+aa^1+\left(a^1\right)^2\right)}{3}\right.$$
$$\left.-\frac{a^1\left(a+a^1\right)}{2}\right]+\frac{2(1-\lambda)}{(a^4-a^1)}\left[\frac{a^4\left(a^4+a\right)}{2}\right.$$
$$\left.-\frac{\left(\left(a^4\right)^2+a^4a+(a)^2\right)}{3}\right],$$
$$V\left(\widetilde{A}\right)=\frac{\lambda\left(a^2+a^3-a^1-a^4\right)+\left(a^1+a^4+a\right)}{3}. \tag{12}$$

For any two normal TIFNs $\widetilde{A}$ and $\widetilde{B}$ if $V(\widetilde{A})$ and $V(\widetilde{B})$ represent their equivalent crisp values, then,

(1) $V(\widetilde{A}) < V(\widetilde{B})$ if and only if $\widetilde{A} \lesssim \widetilde{B}$;

(2) $V(\widetilde{A}) > V(\widetilde{B})$ if and only if $\widetilde{A} \gtrsim \widetilde{B}$;

(3) $V(\widetilde{A}) = V(\widetilde{B})$ if and only if $\widetilde{A} \cong \widetilde{B}$.

It is worthy to mention here that Wang and Kerre [50] proposed seven axioms as the reasonable properties of ordering fuzzy quantities for an ordering approach. The properties of IFNs depend on two FSs, namely, membership function and nonmembership function; and it can easily be shown that all the seven axioms have been satisfied by the proposed ranking technique as described above.

The derived process of ranking of normal TIFNs is to be summarized through the following algorithm.

3.1. Solution Algorithm

Step 1. Write the membership function $\mu_{\widetilde{A}}(x)$ and nonmembership function $\nu_{\widetilde{A}}(x)$ of the normal TIFN $\widetilde{A}$ (refer to (3)).

Step 2. Calculate the complement $1-\nu_{\widetilde{A}}(x)$ of the nonmembership function $\nu_{\widetilde{A}}(x)$ of the normal TIFN $\widetilde{A}$ (refer to (4)).

Step 3. Construct the probability density function $f_1(x)$ for the membership function $\mu_{\widetilde{A}}(x)$ and $f_2(x)$ for the complement of the nonmembership function $\nu_{\widetilde{A}}(x)$ of the normal TIFN $\widetilde{A}$ (refer to (7) and (8)).

Step 4. Take the convex combination of $f_1(x)$ and $f_2(x)$ to form the probability density function of the normal TIFN $\widetilde{A}$ (refer to (9)).

Step 5. Use Mellin's transform to calculate the crisp value of the normal TIFN $\widetilde{A}$ (refer to (12)).

Step 6. Stop.

3.2. Illustrative Example. The following three TIFNs are considered to find their equivalent crisp value by the proposed technique.

Let $\widetilde{A} = \{(1.5, 2, 2.5), (1, 2, 3)\}$, $\widetilde{B} = \{(0.6, 1, 1.4), (0.2, 1, 1.8)\}$, and $\widetilde{C} = \{(2.2, 3, 3.8), (1.8, 3, 4.2)\}$ be three TIFNs.

The membership function of the TIFN $\widetilde{A}$ is given by

$$\mu_{\widetilde{A}}(x)=\begin{cases}\dfrac{x-1.5}{0.5} & \text{if } 1.5\le x\le 2\\ \dfrac{2.5-x}{0.5} & \text{if } 2\le x\le 2.5\\ 0 & \text{otherwise.}\end{cases} \tag{13}$$

The nonmembership function of $\widetilde{A}$ is presented as

$$\nu_{\widetilde{A}}(x)=\begin{cases}2-x & \text{if } 1\le x\le 2\\ x-2 & \text{if } 2\le x\le 3\\ 1 & \text{otherwise.}\end{cases} \tag{14}$$

The complement of the non-membership function $\nu_{\widetilde{A}}(x)$ is calculated as

$$1-\nu_{\widetilde{A}}(x)=\begin{cases}x-1 & \text{if } 1\le x\le 2\\ 3-x & \text{if } 2\le x\le 3\\ 0 & \text{otherwise.}\end{cases} \tag{15}$$

Let $f_1(x)$ and $f_2(x)$ be the respective density functions of the membership function and the complement of nonmembership function which are calculated as defined in the proposed methodology. Then the density function of the TIFN $\widetilde{A}$ is considered as $(x) = \lambda f_1(x) + (1-\lambda) f_2(x)$, $(0 \le \lambda \le 1)$.

Using Mellin's transform the crisp value of TIFN $\widetilde{A}$ is calculated as $V(\widetilde{A}) = 2$. Similarly the crisp value of TIFNs $\widetilde{B}$ and $\widetilde{C}$ is evaluated as $V(\widetilde{B}) = 1$, $V(\widetilde{C}) = 3$, $(0 \le \lambda \le 1)$.

Thus the ordering in the proposed ranking technique is $\widetilde{B} \lesssim \widetilde{A} \lesssim \widetilde{C}$.

3.3. Comparison of the Proposed Ranking Method with the Predefined Methods. The proposed ranking technique is compared with other predefined ranking methods [29–34] to explore the consistency of the proposed ranking methodology through Table 1.

From Table 1 it is evident that the ordering obtained using the proposed methodology is identical in comparison with other techniques. This indicates that the proposed methodology is consistent with the other predefined techniques which can be considered as an alternative technique for ranking IFNs. However, the superiority of the proposed approach would be reflected in the context of solving BLPPs with TIFNs.

Using the proposed ranking method of normal TIFN a fuzzy stochastic linear bilevel programming (FSLBLP) model is developed and solved in the following section.

Table 1: Comparison between ranking techniques.

Methodology	IFN	Ranking value	Result
Dubey and Mehra [29]	$\widetilde{A} = \{(1.5, 2, 2.5), (1, 2, 3)\}$	$R(\widetilde{A}, \lambda) = 1.33 + 0.34\lambda$	$\widetilde{B} \lesssim \widetilde{A} \lesssim \widetilde{C}$
	$\widetilde{B} = \{(0.6, 1, 1.4), (0.2, 1, 1.8)\}$	$R(\widetilde{B}, \lambda) = 0.47 + 0.26\lambda$	
	$\widetilde{C} = \{(2.2, 3, 3.8), (1.8, 3, 4.2)\}$	$R(\widetilde{C}, \lambda) = 2.2 + 0.27\lambda$	
Li [30]	$\widetilde{A} = \{(1.5, 2, 2.5), (1, 2, 3)\}$	$R(\widetilde{A}, \lambda) = \frac{2}{1.67 - 0.34\lambda}$	$\widetilde{B} \lesssim \widetilde{A} \lesssim \widetilde{C}$
	$\widetilde{B} = \{(0.6, 1, 1.4), (0.2, 1, 1.8)\}$	$R(\widetilde{B}, \lambda) = \frac{1}{1.53 - 0.26\lambda}$	
	$\widetilde{C} = \{(2.2, 3, 3.8), (1.8, 3, 4.2)\}$	$R(\widetilde{C}, \lambda) = \frac{3}{1.8 - 0.27\lambda}$	
Nehi [31]	$\widetilde{A} = \{(1.5, 2, 2.5), (1, 2, 3)\}$	$C_\mu^k(\widetilde{A}) = 2$	$\widetilde{B} \lesssim \widetilde{A} \lesssim \widetilde{C}$
	$\widetilde{B} = \{(0.6, 1, 1.4), (0.2, 1, 1.8)\}$	$C_\mu^k(\widetilde{B}) = 1$	
	$\widetilde{C} = \{(2.2, 3, 3.8), (1.8, 3, 4.2)\}$	$C_\mu^k(\widetilde{C}) = 3$	
Wan [32]	$\widetilde{A} = \{(1.5, 2, 2.5), (1, 2, 3)\}$	$VC(\widetilde{A}_\alpha) = 0.10$, $VC(\widetilde{A}_\beta) = 0.35$	$\widetilde{B} \lesssim \widetilde{A} \lesssim \widetilde{C}$
	$\widetilde{B} = \{(0.6, 1, 1.4), (0.2, 1, 1.8)\}$	$VC(\widetilde{B}_\alpha) = 0.16$	
	$\widetilde{C} = \{(2.2, 3, 3.8), (1.8, 3, 4.2)\}$	$VC(\widetilde{C}_\alpha) = 0.10$, $VC(\widetilde{C}_\beta) = 0.28$	
Wan and Yi [34]	$\widetilde{A} = \{(1.5, 2, 2.5), (1, 2, 3)\}$	$m_\mu(\widetilde{A}) = 2$	$\widetilde{B} \lesssim \widetilde{A} \lesssim \widetilde{C}$
	$\widetilde{B} = \{(0.6, 1, 1.4), (0.2, 1, 1.8)\}$	$m_\mu(\widetilde{B}) = 1$	
	$\widetilde{C} = \{(2.2, 3, 3.8), (1.8, 3, 4.2)\}$	$m_\mu(\widetilde{C}) = 3$	
Wan et al. [33]	$\widetilde{A} = \{(1.5, 2, 2.5), (1, 2, 3)\}$	$h(\widetilde{A}) = 2$	$\widetilde{B} \lesssim \widetilde{A} \lesssim \widetilde{C}$
	$\widetilde{B} = \{(0.6, 1, 1.4), (0.2, 1, 1.8)\}$	$h(\widetilde{B}) = 1$	
	$\widetilde{C} = \{(2.2, 3, 3.8), (1.8, 3, 4.2)\}$	$h(\widetilde{C}) = 3$	
Proposed method	$\widetilde{A} = \{(1.5, 2, 2.5), (1, 2, 3)\}$	$V(\widetilde{A}) = 2$	$\widetilde{B} \lesssim \widetilde{A} \lesssim \widetilde{C}$
	$\widetilde{B} = \{(0.6, 1, 1.4), (0.2, 1, 1.8)\}$	$V(\widetilde{B}) = 1$	
	$\widetilde{C} = \{(2.2, 3, 3.8), (1.8, 3, 4.2)\}$	$V(\widetilde{C}) = 3$	

4. FSLBLP Model Formulation

An LFSBLP problem with normal TIFNs as coefficients is presented as follows:

$$\begin{aligned}
&\text{Find} \quad X(x_1, x_2, \ldots, x_n) \text{ so as to} \\
&\max_{X_1} \quad \widetilde{Z}_1 \cong \sum_{j=1}^{n} \widetilde{c}_{1j} x_j, \\
&\qquad \text{where for given } X_1; X_2 \text{ solves} \\
&\max_{X_2} \quad \widetilde{Z}_2 \cong \sum_{j=1}^{n} \widetilde{c}_{2j} x_j \qquad (16) \\
&\text{Subject to} \quad r\left(\sum_{j=1}^{n} \widetilde{a}_{ij} x_j \lesssim \widetilde{b}_i\right) \geq 1 - \gamma_i; \\
&\qquad\qquad i = 1, 2, \ldots, m \\
&\qquad x_j \geq 0; \quad j = 1, 2, \ldots, n,
\end{aligned}$$

where $\widetilde{c}_{kj} = \{(c_{kj}^2, c_{kj}, c_{kj}^3), (c_{kj}^1, c_{kj}, c_{kj}^4)\}$ and $\widetilde{a}_{ij} = \{(a_{ij}^2, a_{ij}, a_{ij}^3), (a_{ij}^1, a_{ij}, a_{ij}^4)\}$ $(k = 1, 2;\ j = 1, 2, \ldots, n;\ i = 1, 2, \ldots, m)$ are normal TIFNs and γ_i is any real number that lies within $[0, 1]$. Here $\widetilde{b}_i (i = 1, 2, \ldots, m)$ denotes fuzzy random variable following normal distribution whose mean $m_{\widetilde{b}_i} = \{(m_{b_i}{}^2, m_{b_i}, m_{b_i}{}^3), (m_{b_i}{}^1, m_{b_i}, m_{b_i}{}^4)\}$ and standard deviation $\sigma_{\widetilde{b}_i} = \{(\sigma_{b_i}{}^2, \sigma_{b_i}, \sigma_{b_i}{}^3), (\sigma_{b_i}{}^1, \sigma_{b_i}, \sigma_{b_i}{}^4)\}$ are normal TIFNs and $\cong$, $\lesssim$, and $\gtrsim$ denote fuzzily equal, less than or equal, and greater than or equal, respectively. The decision vector $X_1 = (x_{11}, x_{12}, \ldots, x_{1n_1})$ is controlled by the upper level DM and $X_2 = (x_{21}, x_{22}, \ldots, x_{2n_2})$ is controlled by the lower level DM; $X = X_1 \cup X_2$ and $n_1 + n_2 = n$.

4.1. FP Model Formulation [5]. Applying CCP technique, the probabilistic constraints $\Pr(\sum_{j=1}^{n} \widetilde{a}_{ij} x_j \lesssim \widetilde{b}_i) \geq 1 - \gamma_i$ $(i = 1, 2, \ldots, m)$ are modified as follows:

$$\Pr\left(\sum_{j=1}^{n} \widetilde{a}_{ij} x_j \lesssim \widetilde{b}_i\right) \geq 1 - \gamma_i; \text{ that is,}$$

$$\Pr\left(\widetilde{b}_i \lesssim \sum_{j=1}^{n} \widetilde{a}_{ij} x_j\right) \leq \gamma_i; \text{ that is,}$$

$$\Pr\left(\frac{\widetilde{b}_i - m_{\widetilde{b}_i}}{\sigma_{\widetilde{b}_i}} \lesssim \frac{\sum_{j=1}^{n} \widetilde{a}_{ij} x_j - m_{\widetilde{b}_i}}{\sigma_{\widetilde{b}_i}}\right) \leq \gamma_i; \text{ that is,}$$

$$\Phi\left(\frac{\sum_{j=1}^{n}\tilde{a}_{ij}x_j - m_{\tilde{b}_i}}{\sigma_{\tilde{b}_i}}\right) \le \gamma_i \text{ or}$$
$$\sum_{j=1}^{n}\tilde{a}_{ij}x_j \le m_{\tilde{b}_i} + \Phi^{-1}(\gamma_i)\,\sigma_{\tilde{b}_i}, \tag{17}$$

where the function $\Phi(\cdot)$ represents the cumulative distribution function of the standard normal fuzzy random variate.

Hence the linear fuzzy BLPP is written in the following form as

$$\begin{aligned}
&\text{Find} \quad X(x_1, x_2, \ldots, x_n) \text{ so as to}\\
&\max_{X_1} \quad \widetilde{Z}_1 \cong \sum_{j=1}^{n}\tilde{c}_{1j}x_j,\\
&\qquad \text{where for given } X_1; X_2 \text{ solves}\\
&\max_{X_2} \quad \widetilde{Z}_2 \cong \sum_{j=1}^{n}\tilde{c}_{2j}x_j\\
&\text{Subject to} \quad \sum_{j=1}^{n}\tilde{a}_{ij}x_j \le m_{\tilde{b}_i} + \Phi^{-1}(\gamma_i)\,\sigma_{\tilde{b}_i};\\
&\qquad\qquad (i = 1, 2, \ldots, m)\\
&\qquad x_j \ge 0; \quad j = 1, 2, \ldots, n.
\end{aligned} \tag{18}$$

4.2. Construction of Linear Bilevel Programming Model Using Proposed Ranking Method. In this subsection the proposed ranking function is applied to convert the FP model into a deterministic model. Using the linearity of ranking function, the above model is written as

$$\begin{aligned}
&\text{Find} \quad X(x_1, x_2, \ldots, x_n) \text{ so as to}\\
&\max_{X_1} \quad V(\widetilde{Z}_1) = \sum_{j=1}^{n}V(\tilde{c}_{1j})\,x_j,\\
&\qquad \text{where for given } X_1; X_2 \text{ solves}\\
&\max_{X_2} \quad V(\widetilde{Z}_2) = \sum_{j=1}^{n}V(\tilde{c}_{2j})\,x_j\\
&\text{Subject to} \quad \sum_{j=1}^{n}V(\tilde{a}_{ij})\,x_j\\
&\qquad \le V(m_{\tilde{b}_i}) + \Phi^{-1}(\gamma_i)\,V(\sigma_{\tilde{b}_i});\\
&\qquad\qquad i = 1, 2, \ldots, m\\
&\qquad x_j \ge 0; \quad j = 1, 2, \ldots, n.
\end{aligned} \tag{19}$$

Using the parameters of TIFNs, model (19) is expressed as

$$\begin{aligned}
&\text{Find} \quad X(x_1, x_2, \ldots, x_n) \text{ so as to}\\
&\max_{X_1} \quad V(\widetilde{Z}_1) = \sum_{j=1}^{n}\frac{\left(\lambda\left({c_{1j}}^2 + {c_{1j}}^3 - {c_{1j}}^1 - {c_{1j}}^4\right) + \left({c_{1j}}^1 + {c_{1j}}^4 + c_{1j}\right)\right)}{3}x_j,\\
&\qquad \text{where for given } X_1; X_2 \text{ solves}\\
&\max_{X_2} \quad V(\widetilde{Z}_2) = \sum_{j=1}^{n}\frac{\left(\lambda\left({c_{2j}}^2 + {c_{2j}}^3 - {c_{2j}}^1 - {c_{2j}}^4\right) + \left({c_{2j}}^1 + {c_{2j}}^4 + c_{2j}\right)\right)}{3}x_j\\
&\text{Subject to} \quad \sum_{j=1}^{n}\frac{\left(\lambda\left({a_{ij}}^2 + {a_{ij}}^3 - {a_{ij}}^1 - {a_{ij}}^4\right) + \left({a_{ij}}^1 + {a_{ij}}^4 + a_{ij}\right)\right)}{3}x_j\\
&\qquad \le \frac{\left(\lambda\left({m_{b_i}}^2 + {m_{b_i}}^3 - {m_{b_i}}^1 - {m_{b_i}}^4\right) + \left({m_{b_i}}^1 + m_{b_i} + {m_{b_i}}^4\right)\right)}{3}\\
&\qquad + \Phi^{-1}(\gamma_i)\,\frac{\left(\lambda\left({\sigma_{b_i}}^2 + {\sigma_{b_i}}^3 - {\sigma_{b_i}}^1 - {\sigma_{b_i}}^4\right) + \left({\sigma_{b_i}}^1 + \sigma_{b_i} + {\sigma_{b_i}}^4\right)\right)}{3}; \quad i = 1, 2, \ldots, m\\
&\qquad x_j \ge 0; \quad j = 1, 2, \ldots, n; \ 0 \le \lambda \le 1.
\end{aligned} \tag{20}$$

Now, the DMs are trying to optimize their objective independently under the system constraints defined above. Let $V(\widetilde{Z}_1)^b$ and $V(\widetilde{Z}_2)^b$ be the best value of the objective of the upper and lower level DMs, respectively, obtained by

solving each objective independently. Since the DMs are trying to maximize their objective and they also want that the maximum value of the objective of the other level DM cannot exceed his minimum value of the objective, therefore, the worst values $V(\widetilde{Z}_1)^w$ and $V(\widetilde{Z}_2)^w$ are calculated by considering the best solution point of the other level DM. That is,

$$\begin{aligned} V\left(\widetilde{Z}_1\right)^w &= V\left(\widetilde{Z}_1\right) \\ &\quad \left(\text{at best solution point of 2nd level DM}\right), \\ V\left(\widetilde{Z}_2\right)^w &= V\left(\widetilde{Z}_2\right) \\ &\quad \left(\text{at best solution point of 1st level DM}\right). \end{aligned} \tag{21}$$

Hence the fuzzy goal for each objective is expressed as

$$V\left(\widetilde{Z}_k\right) \gtrsim V\left(\widetilde{Z}_k\right)^b; \quad k = 1, 2. \tag{22}$$

Thus the membership function of the upper and lower level objective is constructed as

$$\mu_{V(\widetilde{Z}_k)} = \begin{cases} 0 & \text{if } V\left(\widetilde{Z}_k\right) \le V\left(\widetilde{Z}_k\right)^w \\ \dfrac{V\left(\left(\widetilde{Z}_k\right) - V\left(\widetilde{Z}_k\right)^w\right)}{\left(V\left(\widetilde{Z}_k\right)^b - V\left(\widetilde{Z}_k\right)^w\right)} & \text{if } V\left(\widetilde{Z}_k\right)^w \le V\left(\widetilde{Z}_k\right) \le V\left(\widetilde{Z}_k\right)^b \\ 1 & \text{if } V\left(\widetilde{Z}_k\right) \ge V\left(\widetilde{Z}_k\right)^b \end{cases} \quad k = 1, 2. \tag{23}$$

With the help of the above membership functions, the FGP model is defined in the next section.

4.3. Weighted FGP Model Formulation. In FGP model formulation, the membership functions are first converted into flexible membership goals by introducing under- and overdeviational variables to each of them and thereby assigning the highest membership value (unity) as the aspiration level to each of them. Also it is evident that full achievement of all the membership goals is not possible in MODM context. So the underdeviational variables are minimized to achieve the best goal values of objectives in the decision making environment.

Thus a FGP model is formulated as

$$\begin{aligned} &\text{Find} \quad X\left(x_1, x_2, \ldots, x_n\right) \text{ so as to} \\ &\text{Minimize} \quad D = \sum_{k=1}^{K} w_k d_k^- \\ &\text{Subject to} \quad \mu_{V(\widetilde{Z}_k)} + d_k^- - d_k^+ = 1; \quad k = 1, 2, \\ &\sum_{j=1}^{n} \frac{\left(\lambda\left(a_{ij}{}^2 + a_{ij}{}^3 - a_{ij}{}^1 - a_{ij}{}^4\right) + \left(a_{ij}{}^1 + a_{ij}{}^4 + a_{ij}\right)\right)}{3} x_j \le \frac{\left(\lambda\left(m_{b_i}{}^2 + m_{b_i}{}^3 - m_{b_i}{}^1 - m_{b_i}{}^4\right) + \left(m_{b_i}{}^1 + m_{b_i} + m_{b_i}{}^4\right)\right)}{3} \\ &\qquad + \Phi^{-1}\left(\gamma_i\right) \frac{\left(\lambda\left(\sigma_{b_i}{}^2 + \sigma_{b_i}{}^3 - \sigma_{b_i}{}^1 - \sigma_{b_i}{}^4\right) + \left(\sigma_{b_i}{}^1 + \sigma_{b_i} + \sigma_{b_i}{}^4\right)\right)}{3}; \\ &\qquad i = 1, 2, \ldots, m \\ &x_j \ge 0; \quad j = 1, 2, \ldots, n; \ 0 \le \lambda \le 1, \end{aligned} \tag{24}$$

where $w_k \ge 0$ represents the numerical weights of the goals which are determined as

$$w_k = \frac{p_k}{\left(V\left(\widetilde{Z}_k\right)^b - V\left(\widetilde{Z}_k\right)^w\right)}, \quad k = 1, 2; \ p_k > 0. \tag{25}$$

The developed model (24) is solved to find the most satisfactory solution in the decision making environment.

4.4. Solution Algorithm. The methodology for developing the intuitionistic FSLBLP model is presented in the form of an algorithm as follows.

Step 1. Apply CCP technique to all the fuzzy probabilistic constraints, to form a fuzzy linear bilevel programming model (refer to (18)).

Step 2. Using the proposed ranking technique of IFNs, the deterministic linear bilevel model is developed (refer to (19)).

Step 3. The individual optimal value of the objective of each DM is found in isolation.

Step 4. The worst value of the objective of the DMs is evaluated at the best solution point of the other level DMs (refer to (21)).

Step 5. Construct the fuzzy membership goals of the objective of each DM (refer to (22)).

Step 6. FGP approach is used to achieve maximum degree of each of the membership goals (refer to (24)).

Step 7. Stop.

5. Numerical Example

To illustrate the proposed methodology the following FSLBLP problem is considered with coefficients of objectives and system constraints are taken as normal TIFNs:

$$\begin{aligned} &\text{Find} && X(x_1, x_2) \text{ so as to} \\ &\text{Max} && \widetilde{Z}_1 \cong \widetilde{8}x_1 + \widetilde{17}x_2 \\ &\text{Max} && \widetilde{Z}_2 \cong \widetilde{15}x_1 + \widetilde{4}x_2 \\ &\text{Subject to} && \Pr\left(\widetilde{10}x_1 + \widetilde{6}x_2 \lesssim \widetilde{b}_1\right) \geq 0.15 \\ & && \Pr\left(\widetilde{7}x_1 + \widetilde{12}x_2 \lesssim \widetilde{b}_2\right) \geq 0.09 \\ & && x_1, x_2 \geq 0, \end{aligned} \tag{26}$$

where $\widetilde{b}_1$ and $\widetilde{b}_2$ denote fuzzy random variables following normal distribution. Here $\widetilde{8}$, $\widetilde{17}$, $\widetilde{15}$, $\widetilde{4}$, $\widetilde{10}$, $\widetilde{6}$, $\widetilde{7}$, and $\widetilde{12}$ are all normal TIFNs. Then the TIFNs can be expressed as

$$\begin{aligned} \widetilde{8} &= \{(7.6, 8, 8.5), (7, 8, 9)\}, \\ \widetilde{17} &= \{(16, 17, 18), (15, 17, 18.6)\}, \\ \widetilde{15} &= \{(14.4, 15, 15.6), (14, 15, 16.5)\}, \\ \widetilde{4} &= \{(3, 4, 4.8), (2.6, 4, 5)\}, \\ \widetilde{10} &= \{(9.4, 10, 10.8), (8.5, 10, 11.5)\}, \\ \widetilde{6} &= \{(5.6, 6, 6.5), (5, 6, 7)\}, \\ \widetilde{7} &= \{(6, 7, 8), (5.6, 7, 8.9)\}, \\ \widetilde{12} &= \{(11.2, 12, 13), (10.5, 12, 13.5)\}. \end{aligned} \tag{27}$$

The mean $m_{\widetilde{b}_i}$ and standard deviation $\sigma_{\widetilde{b}_i}$ $(i = 1, 2)$ of the fuzzy random variables $\widetilde{b}_i$ following normal distribution are given as

$$\begin{aligned} m_{\widetilde{b}_1} &= \widetilde{20} = \{(19.4, 20, 21), (19, 20, 21.2)\}, \\ \sigma_{\widetilde{b}_1} &= \widetilde{5} = \{(4, 5, 5.9), (3.8, 5, 6)\}, \\ m_{\widetilde{b}_2} &= \widetilde{21} = \{(19.8, 21, 21.7), (19.5, 21, 23)\}, \\ \sigma_{\widetilde{b}_2} &= \widetilde{4} = \{(3.6, 4, 5.5), (3, 4, 5.84)\}. \end{aligned} \tag{28}$$

Applying CCP methodology the above linear fuzzy stochastic BLPP is converted into linear fuzzy BLPP as

$$\begin{aligned} &\text{Find} && X(x_1, x_2) \text{ so as to} \\ &\text{Max} && \widetilde{Z}_1 \cong \widetilde{8}x_1 + \widetilde{17}x_2 \\ &\text{Max} && \widetilde{Z}_2 \cong \widetilde{15}x_1 + \widetilde{4}x_2 \\ &\text{Subject to} && \widetilde{10}x_1 + \widetilde{6}x_2 \lesssim \widetilde{20} + 1.04 \times \widetilde{5} \\ & && \widetilde{7}x_1 + \widetilde{12}x_2 \lesssim \widetilde{21} + 1.37 \times \widetilde{4} \\ & && x_1, x_2 \geq 0. \end{aligned} \tag{29}$$

By applying the proposed ranking technique of normal TIFNs, the above IFNs are converted into the equivalent crisp values as

$$\begin{aligned} V(\widetilde{8}) &= (8 + 0.033\lambda), \\ V(\widetilde{17}) &= (16.87 + 0.133\lambda), \\ V(\widetilde{15}) &= (15.17 - 0.17\lambda), \\ V(\widetilde{4}) &= (3.87 + 0.07\lambda), \\ V(\widetilde{10}) &= (10 + 0.07\lambda), \\ V(\widetilde{6}) &= (6 + 0.03\lambda), \\ V(\widetilde{20}) &= (20.07 + 0.07\lambda), \\ V(\widetilde{5}) &= (4.93 + 0.03\lambda), \\ V(\widetilde{7}) &= (7.17 - 0.17\lambda), \\ V(\widetilde{12}) &= (12 + 0.07\lambda), \\ V(\widetilde{21}) &= (21.17 - 0.33\lambda), \\ V(\widetilde{4}) &= (4.28 + 0.09\lambda). \end{aligned} \tag{30}$$

Hence the deterministic model of (29) is represented as

$$\begin{aligned} &\text{Max} && V(\widetilde{Z}_1) \\ & && = (8 + 0.033\lambda)\, x_1 + (16.87 + 0.133\lambda)\, x_2 \\ &\text{Max} && V(\widetilde{Z}_2) \\ & && = (15.17 - 0.17\lambda)\, x_1 \\ & && \quad + (3.87 + 0.07\lambda)\, x_2 \\ &\text{Subject to} && (10 + 0.07\lambda)\, x_1 + (6 + 0.03\lambda)\, x_2 \\ & && \leq (25.2 + 0.1\lambda) \\ & && (7.17 - 0.17\lambda)\, x_1 + (12 + 0.07\lambda)\, x_2 \\ & && \leq (27.03 - 0.21\lambda) \\ & && x_1, x_2 \geq 0. \end{aligned} \tag{31}$$

Table 2: Compromise solution obtained by proposed methodology.

Objective	Solution point	Ranking value of the objective	Membership value
$\widetilde{Z}_1$	$x_1 = 1.81, x_2 = 1.17$	$V(\widetilde{Z}_1) = 34.48$	$\mu_{V(\widetilde{Z}_1)} = 0.80$
$\widetilde{Z}_2$		$V(\widetilde{Z}_2) = 31.78$	$\mu_{V(\widetilde{Z}_2)} = 0.78$

Table 3: Comparison of solutions between existing and proposed techniques.

Ranking technique	Solution point	Objective values	Membership value
Dubey and Mehra [29]	$x_1 = 1.81$	$V(\widetilde{Z}_1) = 33.98$	$\mu_{V(\widetilde{Z}_1)} = 0.79$
	$x_2 = 1.29$	$V(\widetilde{Z}_2) = 30.86$	$\mu_{V(\widetilde{Z}_2)} = 0.76$
Proposed technique	$x_1 = 1.81$	$V(\widetilde{Z}_1) = 34.48$	$\mu_{V(\widetilde{Z}_1)} = 0.80$
	$x_2 = 1.17$	$V(\widetilde{Z}_2) = 31.78$	$\mu_{V(\widetilde{Z}_2)} = 0.78$

Now, each objective is considered independently and is solved with respect to the system of constraints in (31) to find the individual optimal values of the objectives. The results are obtained as

$$\begin{aligned} &\left(V\left(\widetilde{Z}_1\right)\right)^b = 38.00, \text{ at the point } (0, 2.25), \\ &\left(V\left(\widetilde{Z}_2\right)\right)^b = 38.23, \text{ at the point } (2.52, 0). \end{aligned} \tag{32}$$

The worst values of $V(\widetilde{Z}_1)$ and $V(\widetilde{Z}_2)$ are obtained by putting the best solution point of the 2nd and 1st objective, respectively. Thus $(V(\widetilde{Z}_1))^w = 20.16$ and $(V(\widetilde{Z}_2))^w = 8.71$.

Then the fuzzy goals of the objectives are found as $(\widetilde{Z}_1) \gtrsim 38.00, V(\widetilde{Z}_2) \gtrsim 38.23$.

The membership function of the objectives is written as

$$\begin{aligned} \mu_{V(\widetilde{Z}_1)} &= 0.056\left((8 + 0.033\lambda)\, x_1 + (16.87 + 0.133\lambda)\, x_2 \right. \\ &\quad \left. - 20.16\right), \\ \mu_{V(\widetilde{Z}_2)} &= 0.034\left((15.17 - 0.17\lambda)\, x_1 \right. \\ &\quad \left. + (3.87 + 0.07\lambda)\, x_2 - 8.71\right). \end{aligned} \tag{33}$$

Hence the FGP model can be presented by eliciting the membership goals as

$$\begin{aligned} \text{Find} \quad & X\left(x_1, x_2\right) \\ \text{So as to Min} \quad & D = 0.040 d_1^- + 0.034 d_2^- \\ \text{Subject to} \quad & \mu_{V(\widetilde{Z}_1)} + d_1^- - d_1^+ = 1 \\ & \mu_{V(\widetilde{Z}_2)} + d_2^- - d_2^+ = 1 \\ & (10 + 0.07\lambda)\, x_1 + (6 + 0.03\lambda)\, x_2 \\ & \le (25.2 + 0.1\lambda) \\ & (7.17 - 0.17\lambda)\, x_1 + (12 + 0.07\lambda)\, x_2 \\ & \le (27.03 - 0.21\lambda) \\ & x_1, x_2 \ge 0, \end{aligned} \tag{34}$$

where d_1^-, d_1^+, d_2^-, and $d_2^+ \ge 0$ with $d_1^- \cdot d_1^+, d_2^- \cdot d_2^+ = 0$.

Now solving the above model using *software* LINGO (*ver. 11*) the solutions are found as $x_1 = 1.81$, $x_2 = 1.17$ with objective values $V(\widetilde{Z}_1) = 34.48$ and $V(\widetilde{Z}_2) = 31.78$.

The solutions achieved through FGP technique are summarized in Table 2.

If the numerical example presented in this section is solved by applying the ranking technique of IFN developed by Dubey and Mehra [29], then the objective values are obtained as $V(\widetilde{Z}_1) = 33.98$ and $V(\widetilde{Z}_2) = 30.86$. The solutions achieved by two ranking techniques are summarized in Table 3.

This comparison can also be shown graphically as in Figure 4.

From the comparison table and from Figure 4 it is realized that better objective values are obtained if the example is solved by the proposed ranking technique. So the proposed ranking technique is more acceptable to the DMs in real life decision making context.

6. Conclusions

This paper presents a new ranking technique for normal TIFNs. The proposed methodology can be used to find the defuzzified value of intuitionistic trapezoidal FNs. Based on the proposed ranking technique an intuitionistic LFSBLP model is developed. This methodology covers a wider range

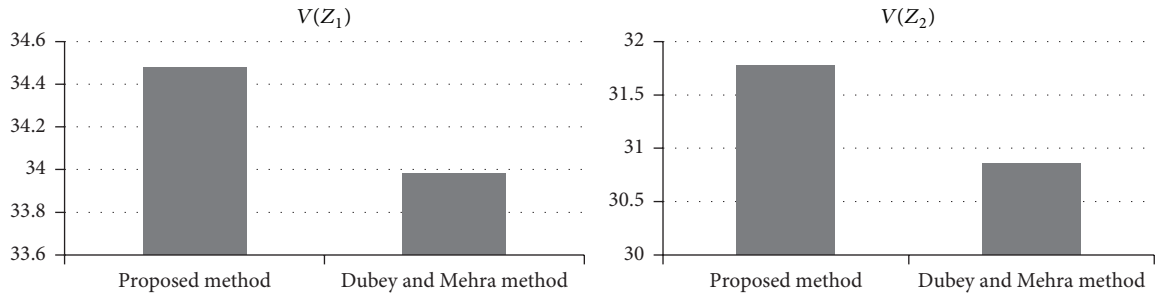

Figure 4: Comparison of solutions.

of applications with enriched solutions than earlier techniques. The proposed model can also be applied to solve many real life decision making problems in which parameters are expressed in the form of IFNs. Further the proposed methodology can be used efficiently to solve multiobjective fractional programming problems, multilevel optimization problems, and other associated problems in an intuitionistic fuzzy stochastic decision making arena. However, it is hoped that the proposed methodology may open up new vistas into the way of making decision in an imprecisely defined decision making arena.

Competing Interests

The authors declare that they have no competing interests.

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