Research Article

On Application of Ordered Fuzzy Numbers in Ranking Linguistically Evaluated Negotiation Offers

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The main purpose of this paper is to investigate the application potential of ordered fuzzy numbers (OFN) to support evaluation of negotiation offers. The Simple Additive Weighting (SAW) and the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) methods are extended to the case when linguistic evaluations are represented by OFN. We study the applicability of OFN for linguistic evaluation negotiation options and also provide the theoretical foundations of SAW and TOPSIS for constructing a scoring function for negotiation offers. We show that the proposed framework allows us to represent the negotiation information in a more direct and adequate way, especially in ill-structured negotiation problems, allows for holistic evaluation of negotiation offers, and produces consistent rankings, even though new packages are added or removed. An example is presented in order to demonstrate the usefulness of presented fuzzy numerical approach in evaluation of negotiation offers.

1. Introduction

The process of defining, evaluating, and building a negotiation template is an important part of negotiation analysis, as well as constructing a scoring function, which is realized in the prenegotiation phase [1–3]. The negotiation template specifies a negotiation space by defining negotiation issues and acceptable resolution levels (options). The scoring function helps evaluating negotiation packages which take into account negotiator's preferences with respect to all given issues, as well as their relative importance. Because negotiation packages are often characterized by several contradictory criteria, the multicriteria techniques are useful for evaluating negotiation offers and building negotiation-scoring functions [4]. The most popular techniques used for supporting a negotiation process are

- (i) Simple Additive Weighting method (SAW)/The Simple Multi Attribute Rating Technique [5, 6].
- (ii) The Analytic Hierarchy Process [7].
- (iii) Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) [8, 9].

There are also studies showing applicability of other techniques for building scoring functions such as

- (i) Measuring Attractiveness by a Categorical Based Evaluation Technique [10].
- (ii) Utility Additive Method [11].
- (iii) Measuring Attractiveness near Reference Situations [10, 11].
- (iv) Generalized Regression with Intensities of Preference [11].

Each of those methods has its advantaged and disadvantages; thus selecting the “best” method for a particular problem is a really difficult task. The choice between mentioned techniques depends on the negotiation problem, types of criteria, available information, decision maker's cognitive abilities, and properties of the multicriteria technique. In real negotiation situations, the options cannot be assessed precisely in a quantitative form, but still they may be in a qualitative one. This implies the usability of the linguistic approach for evaluating negotiation offers. The approximate

technique may represent qualitative/quantitative options verbally by means of linguistic variable, i.e., variable whose values are not numbers but words or sentences in a natural or artificial language. One possibility of modelling linguistic values is the application of fuzzy sets and fuzzy numbers. The application of fuzzy sets in negotiations was competently discussed in [12].

The linguistic approach in negotiation support was considered in [10]. In this paper, we focus on the problem of extending the scale of values used in evaluating the negotiation options to include the linguistic scale that is, for example, expressions: very bad, bad, average, good, and very good together with intermediate values such as “at least good” or “at most good”. To deal with a problem defined in this way and involving uncertain information given in linguistic terms *at least* or *at most* we used ordered fuzzy numbers (OFN). The main contribution of the paper is the discussion about applicability of Oriented Fuzzy SAW (OF-SAW) and Oriented Fuzzy TOPSIS (OF-TOPSIS) procedure based on OFNs in scoring negotiation offers.

The main goal is to investigate the effectivity of ordered fuzzy numbers (OFN) application to support evaluation of negotiation offers. The paper is organized as follows. Section 2 outlines the mathematical formulation OFNs. The general linguistic approach based on OFNs is outlined in Section 3. The OF-SAW and OF-TOPSIS are presented in Section 4. The example of application of the OF-SAW and OF-TOPSIS methods for negotiation-scoring system is presented in the Section 5. The numerical example of using OF-SAW and OF-TOPSIS is discussed in Section 6. Finally, Section 7 concludes the article, summarizes the main findings of this research, and proposes some future research directions.

2. Ordered Fuzzy Number Concept

The ordered fuzzy numbers (OFNs) were intuitively introduced by Kosiński and his cowriters [13] as an extension of the concept of fuzzy numbers introduced by Dubois and Prade [14]. OFNs are also called Kosiński's numbers [15]. The Kosiński's theory was revised in [16]. In this paper, we restrict our considerations to the case of trapezoidal OFN (TrOFN) defined in the following way.

Definition 1. For any monotonic sequence $\{a, b, c, d\} \subset \mathbb{R}$, the trapezoidal ordered fuzzy number (TrOFN) $\vec{Tr}(a, b, c, d)$ is determined explicitly by its membership functions $\mu_{Tr}(\cdot | a, b, c, d) \in [0, 1]^{\mathbb{R}}$ as follows:

$$\mu_{Tr}(x | a, b, c, d) = \begin{cases} 0, & x \notin [a, d] = [d, a], \\ \frac{x-a}{b-a}, & x \in [a, b] = [b, a], \\ 1, & x \in [b, c] = [c, b], \\ \frac{x-d}{c-d}, & x \in]c, d] = [d, c[. \end{cases} \quad (1)$$

Let us note that this identity describes additionally extended notation of numerical intervals which are used in this work.

The condition $a < d$ fulfilment determines the positive orientation of TrOFN $\vec{Tr}(a, b, c, d)$. Any positively oriented TrOFN is interpreted as such imprecise number, which may increase. The condition $a > d$ fulfilment determines the negative orientation of TrOFN $\vec{Tr}(a, b, c, d)$. Negatively oriented TrOFN is interpreted as such imprecise number, which may decrease. For the case $a = d$, TrOFN $\vec{Tr}(a, a, a, a)$ represents crisp number $a \in \mathbb{R}$, which is not oriented. The space of all TrOFNs we will denote by the symbol $\mathbb{K}_{Tr}$. The following definition fits well with the colloquial understanding of the concept of fuzziness.

Definition 2. A functional $\Psi : \mathbb{K}_{Tr} \rightarrow \mathbb{R}$ is called significantly fuzzy iff it is dependent on all parameters of its argument.

Kosiński has introduced the arithmetic operators of dot product $\odot$ of K-sum $\oplus$ for TrOFN in following way:

$$\beta \odot \vec{Tr}(a, b, c, d) = \vec{Tr}(\beta \cdot a, \beta \cdot b, \beta \cdot c, \beta \cdot d), \quad (2)$$

$$\begin{aligned} \vec{Tr}(a, b, c, d) \oplus \vec{Tr}(p - a, q - b, r - c, s - d) \\ = \vec{Tr}(p, q, r, s). \end{aligned} \quad (3)$$

Kosiński has shown that there exists TrOFNs that their K-sum is not TrOFN [17]. For this reason, in [16] K-sum $\oplus$ is replaced by the sum $\boxplus$ determined as follows:

$$\begin{aligned} \vec{Tr}(a, b, c, d) \boxplus \vec{Tr}(p - a, q - b, r - c, s - d) \\ = \begin{cases} \vec{Tr}(\min\{p, q\}, q, r, \max\{r, s\}) & (q < r) \vee (q = r \wedge p \leq s) \\ \vec{Tr}(\max\{p, q\}, q, r, \min\{r, s\}) & (q > r) \vee (q = r \wedge p > s) \end{cases} \end{aligned} \quad (4)$$

The distance between TrOFNs is calculated in the following way:

$$\begin{aligned} d(\vec{Tr}(a, b, c, d), \vec{Tr}(p, q, r, s)) \\ = \sqrt{(a-p)^2 + (b-q)^2 + (c-r)^2 + (d-s)^2} \end{aligned} \quad (5)$$

Ranking OFN plays a very important role in fuzzy decision-making. Despite many ranking methods proposed in literature, there is no universal technique. Decision makers have to rank fuzzy numbers with their own intuition, preferences, and consistently with considered problem. In this paper we use concept of defuzzification technique extended for TrOFN.

Definition 3. Defuzzification functional is the map $\phi : \mathbb{K}_{Tr} \rightarrow \mathbb{R}$ satisfying for any monotonic sequence $\{a, b, c, d\} \subset \mathbb{R}$ the following conditions:

$$\min\{a, b, c, d\} \leq \phi(\vec{Tr}(a, b, c, d)) \leq \max\{a, b, c, d\} \quad (6)$$

$$\forall_{r \in \mathbb{R}}:$$

$$\phi(\vec{Tr}(a, b, c, d) \boxplus \vec{Tr}(r, r, r, r)) = \phi(\vec{Tr}(a, b, c, d)) \quad (7)$$

$$+ r,$$

$\forall_{r \in \mathbb{R}}:$

$$\phi\left(r \odot \overleftrightarrow{Tr}(a, b, c, d)\right) = r \bullet \phi\left(\overleftrightarrow{Tr}(a, b, c, d)\right). \quad (8)$$

In [18] we find following defuzzification methods:

(i) the weighted maximum functional

$$\phi_{WM}\left(\overleftrightarrow{Tr}(a, b, c, d) \mid \lambda\right) = \lambda \bullet b + (1 - \lambda) \bullet c, \quad (9)$$

$$\lambda \in [0; 1],$$

(ii) the first maximum functional

$$\phi_{FM}\left(\overleftrightarrow{Tr}(a, b, c, d)\right) = \phi_{WM}\left(\overleftrightarrow{Tr}(a, b, c, d) \mid 1\right) = b, \quad (10)$$

(iii) the last maximum functional

$$\phi_{LM}\left(\overleftrightarrow{Tr}(a, b, c, d)\right) = \phi_{WM}\left(\overleftrightarrow{Tr}(a, b, c, d) \mid 0\right) = c, \quad (11)$$

(iv) the middle maximum functional

$$\phi_{MM}\left(\overleftrightarrow{Tr}(a, b, c, d)\right) = \phi_{WM}\left(\overleftrightarrow{Tr}(a, b, c, d) \mid \frac{1}{2}\right) \\ = \frac{1}{2} \bullet (b + c), \quad (12)$$

(v) the gravity center functional

$$\phi_{CG}\left(\overleftrightarrow{Tr}(a, b, c, d)\right) \\ = \begin{cases} \frac{a^2 + a \bullet b + b^2 - c^2 - c \bullet d - d^2}{3(a + b - c - d)} & a \neq d \\ a & a = d \end{cases} \quad (13)$$

(vi) the geometrical mean functional

$$\phi_{GM}\left(\overleftrightarrow{Tr}(a, b, c, d)\right) = \begin{cases} \frac{a \bullet b - c \bullet d}{a + b - c - d} & a \neq d \\ a & a = d. \end{cases} \quad (14)$$

We see that only the gravity center $\phi_{CG}(\bullet)$ and the geometrical mean $\phi_{GM}(\bullet)$ functionals are significantly fuzzy ones. Only these functionals will be applied for determining benchmarks for research described in the Section 5. Let us note that all defuzzification formulas do not imply much computational effort and does not require a priori knowledge of the set of all alternatives.

3. The Linguistic Approach in Decision-Making

Information in a quantitative setting is usually expressed by means of numerical values. However, there are situations dealing with uncertainty or vague information in which the use of linguistic assessments instead of numerical values may

be more useful. Linguistic decision analysis is based on the use of a linguistic approach and it is applied for solving decision-making problems under linguistic information [19]. Herrera-Viedma pointed out also that *"its application in the development of the theory and methods in decision analysis is very beneficial because it introduces a more flexible framework which allows us to represent the information in a more direct and adequate way when we are unable to express it precisely. In this way, the burden of quantifying a qualitative concept is eliminated"* [19].

In information sciences, natural language word is considered as linguistic variable defined as fuzzy subset in the predefined space $\mathbb{X}$. Then these linguistic variables may be transformed with the use of fuzzy set theory [20–22]. From decision-making point view, the linguistic variable transformation methodologies are reviewed in [1, 13, 19]. In the literature, there are many applications of linguistic decision analysis to solve real-world problems such as, e.g., group decision-making, multicriteria decision-making, consensus, marketing, software development, education, material selection, and personnel management as well as negotiation support. For review variety of application linguistic models in decision-making, see, for example, [19].

In [19] the steps of fuzzy linguistic analysis are systematically described. In general, any linguistic value is characterized by means of a label with semantic value. The label is an expression belonging to given linguistic term set. Finally, a mechanism for generating the linguistic descriptors is provided.

The semantic value meaning may be imprecise. Thus each label from applied linguistic term set is represented by fuzzy subset in the real line $\mathbb{R}$.

An important parameter to be determined at the first step is the granularity of uncertainty, i.e., the cardinality of the linguistic term set used for showing the information. The uncertainty granularity indicates the capacity of distinction that may be expressed. The knowledge value is increasing with the increase in granularity. The typical values of cardinality used in the linguistic models are odd ones, usually between 5 and 13. It is worth noting that the idea of granular computing goes from Zadeh [23] who wrote *"fuzzy information granulation underlies the remarkable human ability to make rational decisions in an environment of imprecision, partial knowledge, partial certainty and partial truth."* Also Yao [24] pointed up that *"the consideration of granularity is motivated by the practical needs for simplification, clarity, low cost, approximation, and the tolerance of uncertainty"*.

In our model all linguistic terms are linked with Tentative Order Scale

$$TOS = \{V_1; V_2; \dots; V_r\} \quad (15)$$

which is previously determined as discrete set of linear ordered linguistic values. Each element of Tentative Order Scale is called reference point. Reference points are ordered from the worst one to the best one. In our paper, we shall use following Tentative Order Scale:

$$TOS = \left\{ \begin{array}{l} V_1 = \text{Very Bad} = VB \\ V_2 = \text{Bad} = B \\ V_3 = \text{Average} = A \\ V_4 = \text{Good} = G \\ V_5 = \text{Very Good} = VG \end{array} \right\}. \quad (16)$$

It is obvious that each reference point V_j is equivalent to the numerical diagnosis $j \in \mathbb{N}$.

Now, we focus on the problem of Tentative Order Scale enlargement of intermediate values. For this purpose we use the phrases “at least” described by the symbol $\mathcal{L}$ and “at most” described by the symbol $\mathcal{M}$.

The expression $\mathcal{L}.V_j$ means “no worse than V_j and worse than V_{j+1} ”. The expression $\mathcal{M}.V_j$ means “no better than V_j and better than V_{j-1} ”. Moreover, we assume that the $\mathcal{L}.V_j$ “is better than” $\mathcal{M}.V_j$.

In this way we obtain extended Order Scale

$$OS = \{VB; \mathcal{L}.VB; \mathcal{M}.B; B; \mathcal{L}.B; \mathcal{M}.A; A; \mathcal{L}.A; \mathcal{M}.G; G; \mathcal{L}.G; \mathcal{M}.VG; VG\}. \quad (17)$$

Let us note that expressions $\mathcal{M}.VB$ and $\mathcal{L}.VG$ do not belong to above extended order scale.

The numerical representation of the phrases “at least” is the relation “great or equal” denoted by the GE. Thus, expression $\mathcal{L}.V_j$ is equivalent to numerical diagnosis $GE.j$.

The numerical representation of the phrases “at most” is the relation “less or equal” denoted by the LE. Thus, expression $\mathcal{M}.V_j$ is equivalent to numerical diagnosis $LE.j$.

In this way we obtain the Numerical Diagnosis Set

$$ND = \{1; GE.2; LE.2; 2; GE.2; LE.3; 3; GE.3; LE.4; 4; GE.4; LE.5; 5\}. \quad (18)$$

All numerical diagnoses $GE.j$ and $LE.j$ are imprecise ones. Therefore according to suggestion given in [25], numerical diagnosis will be represented for any $a \in \mathbb{N}$ by TrONF in following way:

$$a \longrightarrow \overleftarrow{S}_1 = \overleftarrow{Tr}(a, a, a, a); \quad (19)$$

$$GE.a \longrightarrow \overleftarrow{S}_2 = \overleftarrow{Tr}\left(a, a, a + \frac{1}{2}, a + 1\right); \quad (20)$$

$$LE.a \longrightarrow \overleftarrow{S}_3 = \overleftarrow{Tr}\left(a, a, a - \frac{1}{2}, a - 1\right). \quad (21)$$

Let us note that the sum and dot product of the numbers $\overleftarrow{S}_1$, $\overleftarrow{S}_2$, and $\overleftarrow{S}_3$ are also TrOFN. Taking into account (19) together with (20) and (21), we determine the converting system, which transforms numerical diagnoses into performance ratings described by TrOFN in the following way:

$$j \longrightarrow \overleftarrow{X}_j = \overleftarrow{Tr}(j, j, j, j) \quad \text{for } j = 1, 2, \dots, 5 \quad (22)$$

$$LE.j \longrightarrow \overleftarrow{X}_{Lj} = \overleftarrow{Tr}\left(j, j, j - \frac{1}{2}, j - 1\right) \quad (23)$$

for $j = 2, 3, 4, 5$

$$GE.j \longrightarrow \overleftarrow{X}_{Gj} = \overleftarrow{Tr}\left(j, j, j + \frac{1}{2}, j + 1\right) \quad (24)$$

for $j = 1, 2, 3, 4$.

It is worth noting that the orientation OFN is used for representing relation LE (negative orientation) and GE (positive orientation).

In this way, we obtain following numerical order scale:

$$NOS = \left\{ \overleftarrow{X}_j : j = 1, 2, \dots, 5 \right\} \cup \left\{ \overleftarrow{X}_{Lj} : j = 2, 3, 4, 5 \right\} \cup \left\{ \overleftarrow{X}_{Gj} : j = 1, 2, 3, 4 \right\} \quad (25)$$

determined by trapezoidal OFN.

The consolidated mechanism of transformation of considered linguistic values into performance ratings is presented in Table 1.

4. The Oriented Fuzzy SAW and Oriented Fuzzy TOPSIS

In this section we describe the SAW and TOPSIS algorithm based on TrOFN. We consider here a multicriteria decision-making problem with m alternatives $A_1, A_2, \dots, A_m \in A$ and n decision criteria $C_1, C_2, \dots, C_n$ with which the alternative performances are measured. For n criteria, we have the weight vector

$$w = (w_1, w_2, \dots, w_n) \in (\mathbb{R}_0^+)^n \quad (26)$$

where

$$w_1 + w_2 + \dots + w_n = 1 \quad (27)$$

and w_j is the weight of j -th criterion denoting the importance of this criterion in the evaluation of the alternatives.

The Simple Additive Weighting (SAW) method is a scoring method based on the concept of a weighted average of performance ratings. This method is also known as the Simple Multi Attribute Rating Technique (SMART) [5, 6]. For the case of performance ratings described by TrOFN, the SAW was first generalized in [25]. In this paper, this SAW generalization is converted in the way that the K-sum (3) is replaced by sum (4). Generalized in this way SAW algorithm we will be called Oriented Fuzzy SAW (OF-SAW). The OF-SAW can be described in the following steps.

Step 1. Define the set of evaluation criteria C_j ($j = 1, \dots, n$) and the set of feasible alternatives A_i ($i = 1, \dots, m$).

Step 2. Define by (17) the Order Scale OS and by (25) the numerical order scale NOS with performance ratings represented by TrOFN.

Step 3. Define the criterion weights w_j which describe the importance of criterion C_j ($j = 1, \dots, n$).

TABLE 1: Transformation of linguistic values into numerical order scale* NOS.

Linguistic values	Order Scale	Numerical Diagnosis Set	Numerical Order Scale NOS
Very Bad	VB	1	$\overrightarrow{Tr}(1, 1, 1, 1)$
at least Very Bad	$\mathcal{L}.VB$	GE.1	$\overrightarrow{Tr}(1, 1, 1.5, 2)$
at most Bad	$\mathcal{M}.B$	LE.2	$\overrightarrow{Tr}(2, 2, 1.5, 1)$
Bad	B	2	$\overrightarrow{Tr}(2, 2, 2, 2)$
at least Bad	$\mathcal{L}.B$	GE.2	$\overrightarrow{Tr}(2, 2, 2.5, 3)$
at most Average	$\mathcal{M}.AV$	LE.3	$\overrightarrow{Tr}(3, 3, 2.5, 2)$
Average	AV	3	$\overrightarrow{Tr}(3, 3, 3, 3)$
at least Average	$\mathcal{L}.AV$	GE.3	$\overrightarrow{Tr}(3, 3, 3.5, 4)$
at most Good	$\mathcal{M}.G$	LE.4	$\overrightarrow{Tr}(4, 4, 3.5, 3)$
Good	G	4	$\overrightarrow{Tr}(4, 4, 4, 4)$
at least Good	$\mathcal{L}.G$	GE.4	$\overrightarrow{Tr}(4, 4, 4.5, 5)$
at most Very Good	$\mathcal{M}.VG$	LE.5	$\overrightarrow{Tr}(5, 5, 4.5, 4)$
Very Good	VG	5	$\overrightarrow{Tr}(5, 5, 5, 5)$

* The Tentative Order Scale is written in bold.

Source: own elaboration.

Step 4. Evaluate each alternative $A_i \in A$ by the vector $(\overrightarrow{X}_{i,1}, \overrightarrow{X}_{i,2}, \dots, \overrightarrow{X}_{i,n}) \in (NOS)^n$, where $\overrightarrow{X}_{i,j}$ is the performance rating of i -th alternative with respect to j -th criterion.

Step 5a. Aggregate the performance ratings with respect to all the criteria for each alternative using the criterion functional given as aggregated evaluation coefficient

$$\overrightarrow{SAW}(A_i) = w_1 \odot \overrightarrow{X}_{i,1} \boxplus w_2 \odot \overrightarrow{X}_{i,2} \boxplus \dots \boxplus w_n \odot \overrightarrow{X}_{i,n} \quad (28)$$

Step 6a. Transform the obtained result $\overrightarrow{SAW}(A_i)$ to scoring function $\phi(\overrightarrow{SAW}(A_i))$ where the function ϕ is one of defuzzi-fication formulae (9)-(14).

Step 7a. Rank all alternatives A_i according to decreasing values $\phi(\overrightarrow{SAW}(A_i))$.

Higher value of $\phi(\overrightarrow{SAW}(A_i))$ implies higher rank of alternative A_i . Therefore, higher value $\phi(\overrightarrow{SAW}(A_i))$ means that alternative A_i is better. Only scoring functions $\phi_{CG}(\overrightarrow{SAW}(\bullet))$ and the geometrical mean $\phi_{GM}(\overrightarrow{SAW}(\bullet))$ are significantly fuzzy SAW methods which are considered in this paper.

Hwang and Yoon [26] have developed TOPSIS for solving MCDM problems. The basic TOPSIS principle is that the chosen alternative should have the “shortest distance” to the Positive Ideal Solution (PIS) and the “longest distance” to the Negative Ideal Solution (NIS). The PIS is the solution maximizing the benefit criteria and minimizing the cost ones. The NIS is the solution maximizing the cost criteria and minimizing the benefit ones.

Chen and Hwang [27] have proposed Fuzzy TOPSIS, which is a generalization of TOPSIS to the case when fuzzy number describes imprecise performance ratings. Next, Fuzzy TOPSIS is generalized to the case when performance ratings are represented by TrOFN. Generalized in this way TOPSIS algorithm will be called Oriented Fuzzy TOPSIS (OF-TOPSIS).

The first four steps of the OF-TOPSIS algorithm are similar to the corresponding steps of OF-SAW. The fifth and the next steps are as follows [25].

Step 5b. Identify the PIS and the NIS, which are

$$PIS = (\overrightarrow{X}_5, \overrightarrow{X}_5, \dots, \overrightarrow{X}_5) \in (NOS)^n \quad (29)$$

$$NIS = (\overrightarrow{X}_1, \overrightarrow{X}_1, \dots, \overrightarrow{X}_1) \in (NOS)^n \quad (30)$$

where $\overrightarrow{X}_1 = \overrightarrow{Tr}(1, 1, 1, 1)$ and $\overrightarrow{X}_5 = \overrightarrow{Tr}(5, 5, 5, 5)$.

Step 6b. For each alternative A_i ($i = 1, 2, \dots, m$), calculate its distances from PIS and NIS.

$$d(A_i, PIS) = \sum_{j=1}^n w_j \cdot d(\overrightarrow{X}_{i,j}, \overrightarrow{X}_5) \quad (31)$$

$$d(A_i, NIS) = \sum_{j=1}^n w_j \cdot d(\overrightarrow{X}_{i,j}, \overrightarrow{X}_1) \quad (32)$$

where the mapping $d(\bullet, \bullet)$ is the distance defined by (5).

Step 7a. For each alternative A_i , compute value of criterion functional given as coefficient of relative nearness to PIS

$$CN(A_i) = \frac{d(A_i, NIS)}{d(A_i, NIS) + d(A_i, PIS)} \quad (33)$$

Step 8a. Rank all alternatives A_i according to decreasing values $CN(A_i)$.

The OF-TOPSIS is significantly fuzzy scoring function. It is worth noting that, in general, the ideal solution consists of the maximum values of benefit criteria and of minimal values of cost criteria. At the same time, in this case, the anti-ideal solution consists of the maximum values of benefit criteria and of minimal values of cost criteria. In the proposed OF-TOPSIS, we identify the NIS as a solution consisting of the maximum values from the scale. Simultaneously, we identify the PIS as a solution consisting of the minimum values from the scale. This implies that when including a new alternative or modifying the data of one of the alternatives, decision maker does not need to reevaluate the previously evaluated alternative and scores of all those alternatives remain stable. This technique also avoids rank reversals [19, 22, 24].

5. Application Ordered Fuzzy Number for Evaluation of Negotiation Offers

The scale of linguistic evaluations depends on decision maker. In general, words are less precise than numbers. In the prenegotiation phase the negotiation template is evaluated. It specifies the negotiation space by defining the negotiation issues with their importance and feasible options of these issues [2]. The template is used next to support the negotiators in evaluating the offers, analyzing the negotiation progress, scale of concessions, among many others [11]. In the case of big negotiation problems or continuous options, the wide ranges of options are reduced to salient ones only to discretize the negotiation problem and make it easier to analyze. Next, the negotiation offer scoring system is built to support negotiator in their decisions.

In this paper we propose linguistic approach in evaluation negotiation offers based on TrOFN. The human understanding and human problem solving involve perception, abstraction, representation, and understanding of real-world problems, as well as their solutions, at a different levels of granularity. The linguistic approach in decision-making can deal with the problem of mentioned granularity of information.

In the proposed model we assume that the problem issues are evaluated by linguistics terms instead of numerical values. This approach can be more appropriate and realistic in many negotiation problems for different reasons. First of all, there are situations in which the information cannot be assessed precisely in a quantitative form but may be in the qualitative one (e.g., when the warranty condition may be evaluated by terms like “good”, “average”, and “bad”). In some cases, we cannot use precise quantitative information because either it is unavailable or its computation is too expensive. Then an “approximate value” represented by the linguistic term may be used instead of numerical one. Also, taking into account the human perception, people are often led to describe their preferences in natural language instead of numerical values

(for instance, evaluate price between 25 and 28 as good, price between 29 and 35 as bad, etc.).

Thus, the linguistic variable concept may be especially useful in describing situations which are too complex or ill-defined for evaluation in numerical way. Moreover, the linguistic variable approach is necessary for holistic evaluation of negotiation offers.

To formalize linguistic decision analysis, we start with the following definitions:

- (i) negotiation package is an alternative determined as offer, which negotiators may send to or receive from their opponent,
- (ii) negotiation issue is a criterion of evaluation of negotiation package,
- (iii) negotiation template is the set of all evaluated options.

Then let us use the following notation for evaluating negotiation offers:

- (i) $F = \{f_1, f_2, \dots, f_n\}$ is the set of negotiation issues the negotiator uses to evaluate the offers,
- (ii) Y_j is the scope of the issue f_j ,
- (iii) the Cartesian product $Y = \prod_{i=1}^n Y_i$ is the set of all feasible negotiation packages,
- (iv) $P = \{P_1, P_2, \dots, P_i, \dots, P_m\} \subseteq Y$ is the negotiation template,
- (v) OS is the linguistic order scale defined by (17) for evaluation negotiation issues,
- (vi) the functions $Z_j : Y_j \rightarrow OS$ evaluate options of the issue f_j by linguistic order scale OS,
- (vii) NOS is the numerical order scale defined by (25),
- (viii) the vector $(\vec{X}_{i,1}, \vec{X}_{i,2}, \dots, \vec{X}_{i,n})$ represents negotiation package P_i , where $\vec{X}_{i,j} \in NOS$ is the performance rating of alternative P_i with respect to the criterion f_j .

The evaluation of negotiation offers can be accomplished in the following steps:

- (i) determine the set of negotiation issues F , the scopes Y_j for all issue f_j , and the weight vector w ;
- (ii) determine the negotiation template P ;
- (iii) define the linguistic order scale OS for evaluation of negotiation issues and transform it into the numerical order scale NOS represented by trapezoidal OFN;
- (iv) for each criterion f_j define function Z_j evaluating options by linguistic terms from scale OS;
- (v) provide the linguistic evaluation offers from the set P and find the representation of negotiation packages by trapezoidal OFN;
- (vi) decide for the OF-SAW with defuzzification formula or for OF-TOPSIS procedure for evaluating offers;
- (vii) obtain the scoring system for packages from P and ranking them.

TABLE 2: The mechanism for Seller' evaluating negotiation templates by linguistic order scale.

Unit Price	Order scale	Complaint conditions	Order scale	Time of payment	Order scale
20-21-22	VB	E	VB	21-22-23-24	VB
23	$\mathcal{L}.VB$	E+	$\mathcal{L}.VB$	20	$\mathcal{L}.VB$
24	$\mathcal{M}.B$	D-	$\mathcal{M}.B$	19	$\mathcal{M}.B$
25-26-27	B	D	B	16-17-18	B
28	$\mathcal{L}.B$	D+	$\mathcal{L}.B$	15	$\mathcal{L}.B$
29	$\mathcal{M}.AV$	C-	$\mathcal{M}.AV$	14	$\mathcal{M}.AV$
30-31-32	AV	C	AV	11-12-13	AV
33	$\mathcal{L}.AV$	C+	$\mathcal{L}.AV$	10	$\mathcal{L}.AV$
34	$\mathcal{M}.G$	B-	$\mathcal{M}.G$	9	$\mathcal{M}.G$
35-36-37	G	B	G	6-7-8	G
38	$\mathcal{L}.G$	B+	$\mathcal{L}.G$	5	$\mathcal{L}.G$
39	$\mathcal{M}.VG$	A-	$\mathcal{M}.VG$	4	$\mathcal{M}.VG$
40-41-42	VG	A	VG	1-2-3	VG

Source: own elaboration.

6. Illustrative Example and Discussion

In this section, we present the numerical example to illustrate the application trapezoidal OFN for evaluation negotiation packages. We test practically all steps of negotiator linguistic preference analysis to show the usefulness of proposed approach. Let as assume that Seller and Buyer negotiate the conditions of the potential business contract. Then the support for Seller's negotiations may be prepared as follows.

At the beginning negotiator defines the negotiation problem, identifies the objectives, transforms them into the negotiation issues, and defines the negotiation space. All packages are measured with regard to every issue, using a related measurement scale. We assume that the negotiators may choose the way of describing the resolution levels of the issues. These evaluations may be based on different types of data (numerical values; linguistic or mixed values) and subjective judgments. Formally, negotiator determines the set of issue F , set of options for each issue Y_i , and the weight vector w .

In our example the following negotiation issues are discussed:

- (i) f_1 is unit price expressed in €,
- (ii) f_2 is complaint conditions described verbally,
- (iii) f_3 is time of payment determined in days.

Next, negotiator defines the negotiation template by the numerical value for criteria f_1 and f_3 and verbally for f_2 .

- (i) f_1 : unit price: from 20€ to 42€ for both sides;
- (ii) f_2 : complaint conditions: five options described verbally:

- A: "5% defects and 2% penalty",
- B: "5% defects and 4% penalty",
- C: "7% defects and 4% penalty",

D: "3% defects and no penalty"

E: "4% defects and no penalty".

- (iii) f_3 : time of payment (days): from 1 to 24 days for both sides.

From the Seller's point view, the criterion f_1 is a benefit issue, and the criterion f_3 is a cost issue. In agree with the Seller's preferences, we order the verbal options of the criterion f_2 alphabetically in the way that the option $A \in F_2$ is the best. Moreover, the Seller can accept some complaint conditions near the verbally options described above, for instance, which differ 0.5% in defects or in penalty. Thus we ought to distinguish phrases and evaluated them as

- (i) almost as good as the complaint condition Q denoted by the symbol Q-,
- (ii) a bit better than the complaint condition Q denoted by the symbol Q+.

Now we potentially have $23 \cdot 13 \cdot 24 = 7176$ different packages The Seller proposes following weight vector:

$$w = [0.6, 0.2, 0.2]. \quad (34)$$

The scoring system for all negotiation offers can be obtained out of various combinations of the possible options.

In the next step the Seller decides to evaluate all issues using linguistic order scale OS determined by (17). The mechanisms for linguistic evaluating of a negotiation template are described for each issue f_j in Table 2.

In our example, the Seller considers 15 negotiation packages described in Table 3. These packages are evaluated with use by linguistic order scale in way described in Table 2. Obtained linguistic evaluations are presented in Table 3.

In the next step, the Seller numerically evaluates negotiations packages by means of numerical order scale NOS described in Table 1. Obtained numerical evaluations are

TABLE 3: Negotiations packages and their linguistic evaluation.

Package	Unit price		Complaint conditions		Time of payment	
	Option	Linguistic evaluation	Option	Linguistic evaluation	Option	Linguistic evaluation
P1	21	VB	B+	$\mathcal{L}.B$	9	$\mathcal{M}.G$
P2	27	B	E-	$\mathcal{M}.VG$	7	G
P3	23	$\mathcal{L}.VB$	D	G	10	$\mathcal{L}.AV$
P4	24	$\mathcal{M}.B$	C	AV	10	$\mathcal{L}.AV$
P5	25	B	C+	$\mathcal{L}.AV$	8	G
P6	28	$\mathcal{L}.B$	D-	$\mathcal{M}.G$	4	$\mathcal{M}.VG$
P7	32	AV	D-	$\mathcal{M}.G$	10	$\mathcal{L}.AV$
P8	28	$\mathcal{L}.B$	C	AV	4	$\mathcal{M}.VG$
P9	29	$\mathcal{M}.AV$	A+	$\mathcal{L}.VB$	23	VB
P10	31	AV	B-	$\mathcal{M}.B$	20	$\mathcal{L}.VB$
P11	33	$\mathcal{L}.AV$	B	B	18	B
P12	33	$\mathcal{L}.AV$	B+	$\mathcal{L}.B$	17	B
P13	37	G	C	AV	14	$\mathcal{M}.AV$
P14	33	$\mathcal{L}.AV$	D-	$\mathcal{M}.G$	14	$\mathcal{M}.AV$
P15	41	VG	B	B	14	$\mathcal{M}.AV$

Source: own elaboration.

described in Table 4. These numerical evaluations are applied for determining by (28) the aggregated evaluation coefficients $\overleftrightarrow{\text{SAW}}(P_i)$, which are presented in Table 4.

Now we juxtapose defuzzificated values of OF-SAW coefficient with SAW coefficient values determined with the use of numerical order scale represented by crisp real numbers. We will consider the numerical order scales NOS1, NOS2, and NOS3 described in Table 5.

The OF-SAW method determines aggregated evaluation coefficients which are TrOFN. TrOFNs comparison may be ambiguous. Thus we have to use one of the defuzzification formulas to obtain rank ordering of negotiation packages. The results of defuzzification formulas are shown in Table 6. The WM defuzzification formula is used with the coefficient $\lambda = 0.1$.

In the next step, for each numerical order scale described in Table 5, all negotiations packages are evaluated by values of SAW coefficients. Evaluations obtained in this way are presented in Table 6. It is obvious that, in this case, defuzzification of obtained results is not needed. Finally, Table 6 lists 9 different evaluating methods with evaluations represented by crisp real numbers. All these methods differ in used defuzzification method or numerical order scale. Evaluations obtained by the various evaluating methods are shown in the successive columns of Table 6.

Any column of Table 6 outlines some raking of negotiation packages. We observe that these rankings determine partial or strict orders. Thus, we describe observed rankings by tired ranks [28]. All ranks are presented in Table 7. Any row of this table lists different ranks of chosen negotiation package. The ranking outlined by supposition $\phi_{GC}(\text{FSAV}(\bullet))$ is the unique strict order which is significantly fuzzy. For this reason, we choose this ranking as the benchmark. For better

clarity of our considerations, the rows of Table 7 are ordered by means of the above benchmark.

We consider a family of rankings linked to the same criterion functional. Considered rankings differ in the method of determining the criterion functional value. Then we evaluate this family in the way that the best ranking is one, which is the most similar to other evaluated rankings. The use of this criterion stems from the belief that the ranking of one most similar to the other is their best substitute. We will measure the similarity using the Spearman's rho (the Spearman's multiple rank correlation coefficient) determined for the case of tired ranks [28] in following way:

$$\rho_s = \frac{12}{m \cdot (m^2 - 1) \cdot (l - 1)} \sum_{k=1}^l \sum_{i=1}^m r_{k,i} \cdot r_{s,i} - \frac{3 \cdot (m + 1)}{m - 1}, \quad (35)$$

where the individual variables denote the following:

$r_{k,i}$ is i -th package negotiation rank outlined by k -th method of determining the criterion functional value,
 s is index of evaluated ranking,
 m is the number of ordered package negotiations,
 l is the number of considered rankings.

The values of Spearman's rho computed for each evaluated rankings are presented in the last row of Table 7. We see that the best rankings are such one that it is significantly fuzzy. Moreover, there does not exist such not significantly fuzzy ranking, which is better than any significantly fuzzy one.

Now we will consider the case when the Seller applies the OF-TOPSIS criterion functional for evaluating negotiation packages. The OF-TOPSIS is significantly fuzzy method of

TABLE 4: Numerical evaluation of negotiation packages.

Package	Unit price	Complaint conditions	Time of payment	$\overleftrightarrow{SAW}(P_i)$
P1	$\overleftrightarrow{Tr}(1, 1, 1, 1)$	$\overleftrightarrow{Tr}(2, 2, 2.5, 3)$	$\overleftrightarrow{Tr}(4, 4, 3.5, 3)$	$\overleftrightarrow{Tr}(1.8, 1.8, 1.8, 1.8)$
P2	$\overleftrightarrow{Tr}(2, 2, 2, 2)$	$\overleftrightarrow{Tr}(5, 5, 4.5, 4)$	$\overleftrightarrow{Tr}(4, 4, 4, 4)$	$\overleftrightarrow{Tr}(3.0, 3.0, 2.9, 2.8)$
P3	$\overleftrightarrow{Tr}(1, 1, 1.5, 2)$	$\overleftrightarrow{Tr}(4, 4, 4, 4)$	$\overleftrightarrow{Tr}(3, 3, 3.5, 4)$	$\overleftrightarrow{Tr}(2.0, 2.0, 2.4, 2.8)$
P4	$\overleftrightarrow{Tr}(2, 2, 1.5, 1)$	$\overleftrightarrow{Tr}(3, 3, 3, 3)$	$\overleftrightarrow{Tr}(3, 3, 3.5, 4)$	$\overleftrightarrow{Tr}(2.4, 2.4, 2.2, 2.0)$
P5	$\overleftrightarrow{Tr}(2, 2, 2, 2)$	$\overleftrightarrow{Tr}(3, 3, 3.5, 4)$	$\overleftrightarrow{Tr}(4, 4, 4, 4)$	$\overleftrightarrow{Tr}(2.6, 2.6, 2.7, 2.8)$
P6	$\overleftrightarrow{Tr}(2, 2, 2.5, 3)$	$\overleftrightarrow{Tr}(4, 4, 3.5, 3)$	$\overleftrightarrow{Tr}(5, 5, 4.5, 4)$	$\overleftrightarrow{Tr}(3.0, 3.0, 3.1, 3.2)$
P7	$\overleftrightarrow{Tr}(3, 3, 3, 3)$	$\overleftrightarrow{Tr}(4, 4, 3.5, 3)$	$\overleftrightarrow{Tr}(3, 3, 3.5, 4)$	$\overleftrightarrow{Tr}(3.2, 3.2, 3.2, 3.2)$
P8	$\overleftrightarrow{Tr}(2, 2, 2.5, 3)$	$\overleftrightarrow{Tr}(3, 3, 3, 3)$	$\overleftrightarrow{Tr}(5, 5, 4.5, 4)$	$\overleftrightarrow{Tr}(2.8, 2.8, 3.0, 3.2)$
P9	$\overleftrightarrow{Tr}(3, 3, 2.5, 2)$	$\overleftrightarrow{Tr}(1, 1, 1.5, 2)$	$\overleftrightarrow{Tr}(1, 1, 1, 1)$	$\overleftrightarrow{Tr}(2.2, 2.2, 2.0, 1.8)$
P10	$\overleftrightarrow{Tr}(3, 3, 3, 3)$	$\overleftrightarrow{Tr}(2, 2, 1.5, 1)$	$\overleftrightarrow{Tr}(1, 1, 1.5, 2)$	$\overleftrightarrow{Tr}(2.4, 2.4, 2.4, 2.4)$
P11	$\overleftrightarrow{Tr}(3, 3, 3.5, 4)$	$\overleftrightarrow{Tr}(2, 2, 2, 2)$	$\overleftrightarrow{Tr}(2, 2, 2, 2)$	$\overleftrightarrow{Tr}(2.6, 2.6, 2.9, 3.2)$
P12	$\overleftrightarrow{Tr}(3, 3, 3.5, 4)$	$\overleftrightarrow{Tr}(2, 2, 2.5, 3)$	$\overleftrightarrow{Tr}(2, 2, 2, 2)$	$\overleftrightarrow{Tr}(2.6, 2.6, 3.0, 3.4)$
P13	$\overleftrightarrow{Tr}(4, 4, 4, 4)$	$\overleftrightarrow{Tr}(3, 3, 3, 3)$	$\overleftrightarrow{Tr}(3, 3, 2.5, 2)$	$\overleftrightarrow{Tr}(3.6, 3.6, 3.5, 3.4)$
P14	$\overleftrightarrow{Tr}(3, 3, 3.5, 4)$	$\overleftrightarrow{Tr}(4, 4, 3.5, 3)$	$\overleftrightarrow{Tr}(3, 3, 2.5, 2)$	$\overleftrightarrow{Tr}(3.2, 3.2, 3.3, 3.4)$
P15	$\overleftrightarrow{Tr}(5, 5, 5, 5)$	$\overleftrightarrow{Tr}(2, 2, 2, 2)$	$\overleftrightarrow{Tr}(3, 3, 2.5, 2)$	$\overleftrightarrow{Tr}(4.0, 4.0, 3.9, 3.8)$

Source: own elaboration.

TABLE 5: The mechanisms of transforming an order scale into a numerical order scale.

Order Scale	Numerical Diagnosis Set	Numerical Order Scale		
		NOS1	NOS2	NOS3
VB	1	1	1	1
$\mathcal{L}.VB$	GE.1	1	1.5	1.25
$\mathcal{M}.B$	LE.2	2	1.5	1.75
B	2	2	2	2
$\mathcal{L}.B$	GE.2	2	2.5	2.25
$\mathcal{M}.AV$	LE.3	3	2.5	2.75
AV	3	3	3	3
$\mathcal{L}.AV$	GE.3	3	3.5	3.25
$\mathcal{M}.G$	LE.4	4	3.5	3.75
G	4	4	4	4
$\mathcal{L}.G$	GE.4	4	4.5	4.25
$\mathcal{M}.VG$	LE.5	5	4.5	4.75
VG	5	5	5	5

Source: own elaboration.

evaluation. The results obtained using OF-TOPSIS will be analyzed in the same manner as the above results obtained using the SAW were analyzed.

At the first, the coefficients of relative nearness to PIS $CN(P_i)$ are computed with use of numerical order scale NOS for the numerical evaluations presented in Table 4 which are applied for determining by (33) coefficients $CN(P_i)$, which are presented in Table 8. In currently considered case, the ranking outlined by values $CN(P_i)$ computed for numerical order scale NOS is a unique ranking determining significantly fuzzy strict order. For this reason, we choose this ranking

as a benchmark. For better clarity of our considerations, the rows of Table 8 are ordered by means of the above benchmark. Moreover, we compute the values $CN(P_i)$ for numerical order scales NOS1, NOS2, and NOS3 described in Table 5.

Each considered negotiation package ranking increases with decreasing in values of $CN(\bullet)$ coefficient. We describe observed rankings by tired ranks [28]. All ranks are presented in Table 8. We measure the rankings' similarity using the Spearman's rho determined by (35). The values of Spearman's rho computed for each evaluated rankings are presented in

TABLE 6: Package evaluations obtained by SAW methods.

Numerical Order Scale	NOS						NOS1	NOS2	NOS3
SAW method	OF-SAW						SAW		
Defuzzification method	FM	LM	MM	WM*	CG	GM	-	-	-
Packages	Package evaluation								
P1	1.80	1.80	1.80	1.80	1.80	1.80	1.80	1.80	1.80
P2	3.00	2.90	2.95	2.91	2.92	2.93	3.00	2.90	2.95
P3	2.00	2.40	2.20	2.36	2.31	2.27	2.00	2.40	2.20
P4	2.40	2.20	2.30	2.22	2.24	2.27	2.40	2.20	2.30
P5	2.60	2.70	2.65	2.69	2.68	2.67	2.60	2.70	2.65
P6	3.00	3.10	3.05	3.09	3.08	3.07	3.00	3.20	3.05
P7	3.20	3.20	3.20	3.20	3.20	3.20	3.20	3.20	3.20
P8	2.80	3.00	2.90	2.98	2.96	2.93	2.80	3.00	2.90
P9	2.20	2.00	2.10	2.02	2.04	2.07	2.20	2.00	2.10
P10	2.40	2.40	2.40	2.40	2.40	2.40	2.40	2.40	2.40
P11	2.60	2.90	2.75	2.87	2.83	2.80	2.60	2.90	2.75
P12	2.60	3.00	2.80	2.96	2.91	2.87	2.60	3.00	2.80
P13	3.60	3.50	3.55	3.51	3.52	3.53	3.60	3.50	3.55
P14	3.20	3.30	3.25	3.29	3.28	3.27	3.20	3.30	3.35
P15	4.00	3.90	3.95	3.91	3.92	3.93	4.00	3.90	3.95

Source: own elaboration.

TABLE 7: The negotiation packages' rankings determined by SAW methods.

Numerical Order Scale	NOS						NOS1	NOS2	NOS3
SAW method	OF-SAW						Crisp		
Defuzzification method	FM	LM	MM	WM	CG	GM	-	-	-
Packages	Tired ranks								
P15	1	1	1	1	1	1	1	1	1
P13	2	2	2	2	2	2	2	2	2
P14	3.5	3	3	3	3	3	3.5	3	3
P7	3.5	4	4	4	4	4	3.5	4.5	4
P6	5.5	5	5	5	5	5	5.5	4.5	5
P8	7	6.5	7	6	6	6.5	7	6.5	7
P2	5.5	8.5	6	8	7	6.5	5.5	8.5	6
P12	9	6.5	8	7	8	8	9	6.5	8
P11	9	8.5	9	9	9	9	9	8.5	9
P10	11.5	11.5	11	11	11	11	9	10	10
P5	9	10	10	10	10	10	11.5	11.5	11
P3	14	11.5	13	12	12	12.5	14	11.5	13
P4	11.5	13	12	13	13	12.5	11.5	13	12
P9	13	14	14	14	14	14	13	14	14
P1	15	15	15	15	15	15	15	15	15
Spearman's rho	0,962	0,970	0,984	0,980	0,984	0,984	0,958	0,966	0,982

Source: own elaboration.

TABLE 8: The negotiation packages' rankings determined by OF-TOPSIS method.

Numerical Order Scale	NOS		NOS1		NOS2		NOS3	
TOPSIS method	OF-TOPSIS				TOPSIS			
Packages		Rank		Rank		Rank		Rank
P15	0.731	1	0,750	1	0,725	1	0,738	1
P13	0.631	2	0,650	2	0,625	2	0,638	2
P14	0.566	3	0,550	3.5	0,575	3	0,563	3
P7	0.548	4	0,550	3.5	0,550	4	0,550	4
P6	0.516	5	0,500	5.5	0,521	5	0,513	5
P8	0.486	6	0,450	7	0,500	6.5	0,475	7
P2	0.477	7	0,500	5.5	0,475	8.5	0,488	6
P12	0.475	8	0,400	9	0,500	6.5	0,450	8
P11	0.456	9	0,400	9	0,475	8.5	0,438	9
P5	0.419	10	0,400	9	0,425	10	0,413	10
P10	0.358	11	0,350	11.5	0,350	11.5	0,350	11
P3	0.342	12	0,312	13	0,350	11.5	0,288	13
P4	0.324	13	0,350	11.5	0,300	13	0,325	12
P9	0.273	14	0,267	14	0,250	14	0,275	14
P1	0.203	15	0,200	15	0,200	15	0,200	15
Spearman's rho		0,985		0,972		0,971		0,985

Source: own elaboration.

the last row of Table 8. We see that the best ranking is order outlined by significantly fuzzy OF-TOPSIS.

We observe that the OF-SAW and OF-TOPSIS methods are linked to methods different criterion functional. It implies that the ranking obtained by using the OF-TOPSIS may differ from the ranking obtained using OF-SAW. The negotiator can evaluate subjectively which of the rankings fit his or her preferences better.

7. Conclusions

In this paper we propose a linguistic approach based on ordered fuzzy numbers which can be implemented to evaluate the negotiations packages. In our approach the negotiator's preferences over the issues are represented by TrOFN and then the OF-SAW or OF-TOPSIS method is adopted to determine the global rating of each package. We show that ranking outlined by significantly fuzzy methods may be better than ranking outlined by conventional methods.

One of the key advantages of the approach proposed is its usefulness for building a general scoring function in the ill-structured negotiation problem, namely, the situation in which the problem itself as well as the negotiator's preferences cannot be precisely defined or the available information is subjective and imprecise. Secondly, both variants of scoring functions produce consistent rankings, even though the new packages are added (or removed) and do not result in rank reversal. The main key advantages of the proposed approach are the following:

- (1) Presented techniques allow for linguistic preference attractiveness elicitation of issue options. The linguistic preference analysis based on OFN is useful for building scoring function in the ill-structured negotiation problem in the context of qualitative issues, in satiation holistic evaluation negotiation offers.
- (2) The computation processes of determining the scoring function take into account negotiation space of each issue as well the concepts of reservation and aspiration levels.
- (3) The procedure makes it possible to expand the negotiation template by introducing new package after the preference elicitation has been conducted (within the actual negotiation space) without modifying ranking preliminary estimated packages. That means that proposed *scoring function* produces consistent ranking even after new packages are added (or removed) and does not lead to rank reversal.

The future work will focus on verifying the usefulness of a linguistic approach in negotiation experiments taking into account different recommendation linguistic sets and usefulness linguistic approach for holistic evaluation negotiation offers. To achieve these goals we should examine possibilities of effective enlargement of thesaurus of imprecise phrases extending Tentative Order Scale.

Moreover, we should test other multicriteria techniques based on OFN for evaluation negotiation packages. Let us note here that compared values of the scoring function are

fuzzy. Therefore, in future we ought to study application of fuzzy order to problems described in this paper.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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