Hindawi
Advances in Fuzzy Systems
Volume 2022, Article ID 1477098, 14 pages
https://doi.org/10.1155/2022/1477098

Research Article

Extension of the TOPSIS Method for Decision Making Problems under Complex Fuzzy Data Based on the Central Point Index

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Received 4 April 2022; Revised 28 June 2022; Accepted 20 July 2022; Published 27 October 2022

Academic Editor: Reza Lotfi

This paper presents the CP-TOPSIS Model in group decision-making using complex Fuzzy Data. Complex numbers were employed in this model, and the central point index was used to define both the negative and positive ideals as well as the distance between each option. In this approach, the options are graded using complex data (due to replacing linguistic variables). One of the advantages of this model in decision-making is the capability that creates a complex fuzzy technique for investigating, grading, and selecting the best option related to complex fuzzy data. The results show that this model effectively rates and grades the complex fuzzy data through an alternative period. Quantum mechanics wave functions could not be analyzed, nor could signals or time series or stock exchange transactions predict factors of a multiperiod alternation, nor could predictions be made about any of these variables. As a result, there are numerical results in rating with high precision.

1. Introduction

In everyday life, different criteria are usually measured implicitly. Such decisions may be made solely based on intuition and judgment. There are several crises that must be dealt with; therefore, it is critical to recognize the situation accurately. The site of a nuclear power facility, for instance, is a highly complex subject involving numerous parameters. Therefore, multicriteria decision-making (MCDM) is related to constructing and solving decision-making and planning problems with various criteria. MCDM was developed as a part of operation research in designing computational and mathematical tools to support the subjective assessment of the criteria performance by decision-makers [1].

In the last few years, MCDM tools and their applications have been used in different studies to solve problems associated with the industry [2], science [3], and engineering [4]. MCDM offers the best choice for decision-making out of a finite number of options [5]. Besides, due to the changes and developments over the past decades, MCDM is one of the fastest tools and methods for selecting the best option in the decision-making business. The decision-makers need help deciding between the fast and superior options. Decision-making methods are well accepted in all areas of the decision-making process with the help of computers. MCDM is vital in operations research and management science and is the basis for making and developing various computer-based methods [6]. MCDM issues can be classified into two categories: Multiple Attribute Decision-Making (MADM) and Multiple Objective Decision-Making (MODM). MADM is the most well-known branch of decision-making and a general class of operations research models dealing with decision-making problems under some criteria.

MADM approach requires selecting among decision alternatives described by their attributes. MADM problems are assumed to have a predetermined and limited number of decision alternatives. Solving a MADM problem involves sorting and ranking. As an alternative to the decision-making matrix and additional information provided by the decision-maker, MADM methodologies can be used to rank, screen, or select among the options. Moreover, MODM is a powerful tool to help find decisions that will best satisfy several conflicting objectives. Contrary to the MADM, the decision alternatives are not given in the MODM approach. Instead, MODM provides a mathematical framework for designing a set of decision alternatives. A solution's suitability for implementation is determined by how closely it approaches a given set of goals. The MADM can be classified into two categories of compensatory and noncompensatory methods. Compensatory MADM methods incorporate trade-offs between high and low performance in the analysis. Those methods that do not perform such incorporation are termed noncompensatory. Hwang and Yoon [5] presented several MADM methods. Two methods used in the present work are briefly explained as follows:

TOPSIS: the principle behind TOPSIS is simple; the chosen alternative should be as close to the ideal and far from the negative ideal solution as possible.

Distance from Target: this method and its results are also straightforward for graphical description. First, target values for each attribute are chosen, not requiring to be exhibited by any available alternative.

The TOPSIS method has some advantages, including its simplicity, comprehensibility, ease of calculation, and ability to measure the relative performance of each alternative using a simple mathematical form. Fuzzy TOPSIS is one of the most effective techniques in MCDM fuzzy, which was introduced by Wang Yoon [5]. The TOPSIS method is based on the premise that the alternative should simultaneously have the shortest distance with the (PIS) positive ideal solution and the farthest distance with the (NIS) negative ideal solution. In the TOPSIS method, for each criterion, the relative weighted distances between the various options are listed in relation to PIS and NIS. The relative distances represent efficiency. Therefore, the efficiency of each option against each criterion can be operated in this way. Torfi et al. [7] proposed a hierarchical (AHP) process to determine the relative weighting of evaluation criteria and a fuzzy TOPSIS to rank alternatives. Liao and Cao [8] presented an integrated fuzzy TOPSIS approach and multichoice goal programming (MCGP) to select support in supply chain management. The problem of selecting multicriteria group support was simulated by Zouggari and Benyoucef [9] based on the fuzzy TOPSIS approach. Doukas et al. [10] developed a formula for separating weights to rank alternatives. A ranking formula was proposed by Kuo [11] based on the method of Dukas et al. by normalizing the performance measure of each alternative relative to the cost and profit criteria, that is, d_i^+ and d_i^-. In both methods, the weights assigned to the separations have crisp values, and the use of crisp weights limits the range of applications, as linguistic weights are commonly used in fuzzy TOPSIS. In this paper, we used complex language weights to weight the complex data described in sections.

This paper is devoted to the discussion of complex fuzzy number ranking and is expected to have wide applications in other areas of complex fuzzy data ranking. The following is how this article is organized: Section 2 discusses the basic concept of complex fuzzy numbers and their other applications. In Section 3, a detailed discussion of the concepts of complex fuzzy numbers (including definitions and fuzzy distances) is devoted. In this section, a new method for ranking complex fuzzy numbers with a central point index is introduced, and finally, an example of this ranking method is introduced. In Section 4, the results of the new ranking method are presented. In Section 5, managerial insights are presented, and finally in Section 6, the work's conclusion is fundamentally summarized, and the next field of research is offered.

2. Literature Review

The concept of the type-1 fuzzy set was introduced by LotfiZadeh [12]. In 1975, he introduced the type-2 fuzzy set, in which its membership function was a fuzzy number [13]. A complex fuzzy set was presented by Ramot et al. [14]. In recent years, big data has become a growing research concern. High and low data have often been associated with fluctuations and uncertainty. Uncertainty may be because of factors such as corruption or loss of data components, and a cyclical and repetitive pattern may cause fluctuations in data. For example, there is more temperature in summer and less in winter or more traffic during rush hours. Fuzzy sets have been recently used for ranking the fuzzy numbers with many applications in numerous sectors of production, industry, and management. Some applications of fuzzy sets in industry and management include flow workshop scheduling (FSS), interactive fuzzy solution method, self-adapting artificial fish swarm algorithm (SAAFSA) [15], hybrid fuzzy and data-driven robust optimization for resilience and sustainable health care supply chain [16], viable medical waste chain network design [17], two-level solid planning for the location of renewable energy in conditions of uncertainty [18], sustainable supply chain network design [19], a robust optimization model for sustainable and resilient closed-loop supply chain network design [20], a robust time-cost-quality-energy-environment trade-off with resource-constrained due to uncertainty [21], Viable Supply Chain Network Design by considering Blockchain Technology and Cryptocurrency [22], robust optimization of risk-aware, resilient and sustainable closed-loop supply chain network design [23], an extended robust mathematical model to project the course of COVID-19 epidemic in Iran [24], linear programming of complex integers (CILP) to handle location decisions [25], developing a sustainable operational management system [26], a Covering Tour Approach for Disaster Relief Locating and Routing with Fuzzy Demand [27], a parallel machine sequence-dependent group scheduling problem with the goal of minimizing total weighted [28], hybrid artificial intelligence and robust optimization [55], and Location Improved Harmony Search Algorithm [29]. However, despite their many applications, prioritizing the existing options was not considered in ranking the complex fuzzy numbers. Therefore, in the present paper, we examined the available options for ranking complex fuzzy numbers by the TOPSIS decision method in polar conditions while considering the periodicity of these numbers.

The recent complex fuzzy sets are innovative with the increasing number of applications. Among the applications of complex fuzzy sets is the analysis of solar activity based on the number of recorded sunspots [14], signal processing [14], stock trading in the New York Stock Exchange [30], voter turnout forecast [30], multiperiodic factor prediction [31], prediction of voter turnout in elections [32], complex fuzzy logic systems [30, 33–37], image restoration [38], an integrated approach based on artificial intelligence, and novel metaheuristic algorithms to predict demand for dairy products [39].

The complex fuzzy set modifies the fuzzy membership function's initial premise. In certain instances, however, it is argued that a second dimension must be added to the concept of membership function. The fundamental principle of the phase is unaffected by the second characteristic of membership. In both the ordinary and the complex fuzzy sets, the membership function is the same fuzzy function. In the apparent dimension of the membership function, the novelty of complex fuzzy sets is the membership function, the power factor, or the fuzzy period. According to the definition of a complex fuzzy set, each grade of membership is defined by an amplitude term and a phase term. Therefore, the difference between the two complex fuzzy sets can be simply calculated by combining the difference between the amplitude terms and the phase terms. In this regard, Zhang et al. [40] proposed a distance measure to determine the equality of complex fuzzy sets. Alkouri and Salleh [41] introduced several distance measures for complex fuzzy sets. Nonetheless, in the distance measure determined by Zhang et al. [40, 41], the periodicity of the phase term of a complex fuzzy set as a function was disregarded. The cyclic representation of the complex membership in a complex fuzzy set is ignored. We modified the distance measure for complex fuzzy sets.

In this research, we defined and presented a new strategy for prioritizing a model of decision-making, in which complex fuzzy numbers play a role. A rating algorithm was constructed based on the central point index and the TOPSIS-oriented technique. We also employ linguistic terminology to analyze possibilities. In this study, we labeled this method, the so-called CP-TOPSIS. For ranking and picking the best options based on complex fuzzy data, a complex fuzzy technique is developed. These options may have many applications, including analyzing wave functions in quantum mechanics and many physical quantities. This paper presents a new rating method through a central point index based on the center of mass in complex fuzzy numbers used in the TOPSIS method. The modified distance function, which is applicable for the complex fuzzy numbers based on the complex membership function, is described in the TOPSIS method and is also shown with a practical instance in the TOPSIS complex method.

3. Problem Statement

Definition 1. Let U be a universe of discourse. A complex fuzzy set A on U with a membership function $\mu(x)$ for every $x \in U$ specifies that each element is assigned a complex membership degree in the complex set of numbers $\mathbb{C}$. The complex fuzzy set A can be represented as a set of ordered pairs

$$A = \{(x, \mu_A(x)) \colon x \in U\}, \tag{1}$$

where the membership function has a complex form $\mu_A(x) = r_A(x).e^{i\omega_A(x)}$, the amplitude term $r_A(x)$ and the phase term $w_A(x)$ are both real-valued, and $r_A(x) \in [0, 1], i = \sqrt{-1}$. The function $\mu_A(x)$ maps U into unit disk of the complex plane, and the function $e^{i\omega_A(x)}$ is a periodic function with periodicity 2π.

$$\begin{aligned} \mathrm{Arg}_A(x) &= \omega_A(x) + 2k\pi, \\ k &= 0, \pm 1, \pm 2, \ldots, \end{aligned} \tag{2}$$

in which $0 \le \omega_A(x) < 2\pi$ is the main argument of the function. Consider distance measure for complex fuzzy sets:

$$\begin{aligned} d(A, B) = &\frac{1}{2}\left[\sum_{i=1}^{n} \left| r_A(x_i) - r_B(x_i) \right|\right] \\ &+ \frac{1}{2\pi}\sum_{i=1}^{n}\left[\left|\omega_A(x_i) - \omega_B(x_i)\right|\right], \end{aligned} \tag{3}$$

where

$$\begin{aligned} A &= \left\{(x, \mu_A(x)) \colon x \in U, \mu_A(x) = r_A(x).e^{i\omega_A(x)}\right\}, \\ B &= \left\{(x, \mu_B(x)) \colon x \in U \mu_B = r_B(x).e^{i\omega_B(x)}\right\}. \end{aligned} \tag{4}$$

Complex fuzzy sets are a generalization of ordinary fuzzy sets. Any ordinary fuzzy set can be represented as a complex fuzzy set by simply setting the argument component of the membership function for each element, $\omega(x)$, to zero. As a result, the membership function argument is the distinction between ordinary and complex fuzzy sets. Indeed, without an argument for the membership function, the complex fuzzy set becomes a regular fuzzy set, and the membership function range is [0, 1].

Example 1. Let $A = \{-1, 0, 1, 2\}$ and its complex membership function are

$$A = \frac{0.6e^{i1.2\pi}}{-1} + \frac{1.0e^{i2\pi}}{0} + \frac{0.8e^{i1.6\pi}}{1} + \frac{0.5e^{i\pi}}{2}. \tag{5}$$

Definition 2. Let A, B be two complex fuzzy sets on universal set U with membership function $\mu_A(x) = r_A(x)e^{i\omega_A(x)}$ and $\mu_B(x) = r_B(x)e^{i\omega_B(x)}$, and then,

(i) $\mu_{A\cup B}(x) = r_{A\cup B}(x)e^{i\omega_{A\cup B}(x)} = \max\{(r_A(x), r_B(x))\}e^{i\max\{\omega_A(x),\omega_B(x)\}}$

(ii) $\mu_{\overline{A}}(x) = r_{\overline{A}}(x)e^{i\omega_{\overline{A}}(x)} = (1 - r_A(x))e^{i(2\pi - \omega_A(x))}$

(iii) $\mu_{A\cup B}(x) = r_{A\cup B}(x)e^{i\omega_{A\cup B}(x)} = \min\{(r_A(x), r_B(x))\}e^{i\min\{\omega_A(x),\omega_B(x)\}}$

(iv) $\mu_{A\circ B}(x) = r_{A\circ B}(x)e^{i\omega_{A\circ B}(x)} = (r_A(x).r_B(x))\}\, e^{i2\pi(\omega_A(x)/2\pi.\omega_B(x)/2\pi)}$, complex fuzzy product

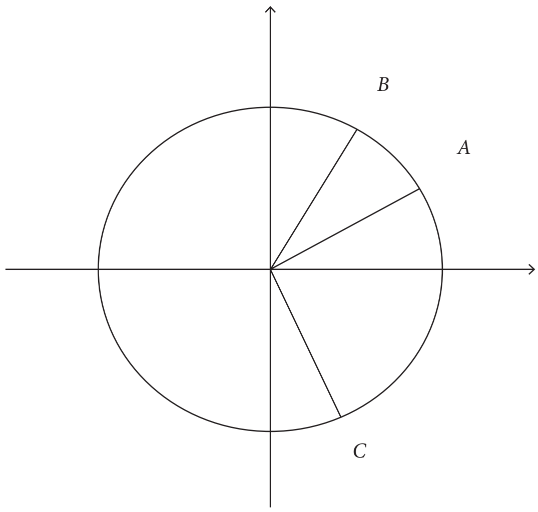

Figure 1

(v) $\mu_{\widehat{A+B}} = r_{\widehat{A+B}}(x)e^{i\omega_{\widehat{A+B}}} = (r_A(x) + r_B(x) - r_A(x)r_B(x))e^{i2\pi(\omega_A(x)/2\pi+\omega_A(x)/2\pi-\omega_A(x)/2\pi.\omega_A(x)/2\pi)}$. , complex fuzzy sum.

Zhang et al. [40] defined the distance function as follows:

Definition 3. Let $U = \{x_1, x_2, \cdots, x_n\}$ and A, B be two complex fuzzy numbers:

$$A = \{(x, \mu_A(x)) : x \in U\},\quad B = \{(x, \mu_B(x)) : x \in U\}, \tag{6}$$

where the membership functions are

$$\mu_A(x) = r_A(x).e^{i\omega_A}(x) \text{and} \mu_B(x) = r_B(x).e^{i\omega_B}(x), \tag{7}$$

where $r_A(x) r_B(x), \omega_A(x), \omega_B(x)$ are real-valued and $i = \sqrt{-1}$. Let $d_z\colon U \times U \longrightarrow \mathbb{R}$ be distance the function such that

$$d_z(A, B) = \max\left(\sup_{x\in U}|r_A(x) - r_B(x)|, \frac{1}{2\pi}\sup_{x\in U}|\omega_A(x) - \omega_B(x)|\right). \tag{8}$$

The following is a counterexample of distance measure 2:

Example 2. Let $A = e^{i0.2\pi}, B = e^{i2.3\pi}, C = e^{i1.7\pi}$, and then, by using relation 2, we have

$$d(A, B) = 0.525\ d(A, C) = 0.375. \tag{9}$$

So, $d(A, B) > d(A, C)$. Based on Figure 1 (the coordinates of points A, B, and C are obtained using the polar form of complex numbers), we have

$$d(A, B) > d(A, C). \tag{10}$$

As shown in Figure 1, this inequality is not correct [42]. Therefore, we can use the following definition to correct the inaccuracy of relations 2 and 3.

Definition 4. Let A be a complex fuzzy number with a membership function $\mu_A(x) = r_A(x).e^{i\omega_A(x)}$, and then, we have

$$\omega_A^r(x) = \begin{cases} \omega_A(x_i), & \text{if}|\omega_A(x_i)| \le 2\pi, \\ \omega_A(x_i)\text{MOD}(2\pi), & \text{if}|\omega_A(x_i)| > 2\pi, \end{cases} \tag{11}$$

$\omega_A(x_i)\text{MOD}(2\pi)$ is the residue modulo 2π. Thus, the modified distance functions 2 and 3 for complex fuzzy sets are as follows:

$$d(A, B) = \frac{1}{2}\left[\sum_{i=1}^{n}|r_A(x_i) - r_B(x_i)|\right] + \frac{1}{2\pi}\left[\sum_{i=1}^{n}|\omega_A^r(x_i) - \omega_B^r(x_i)|\right], \tag{12}$$

$$d_z(A, B) = \max\left(\sup_{x\in U}|r_A(x) - r_B(x)|, \frac{1}{2\pi}\sup_{x\in U}|\omega_A^r(x) - \omega_B^r(x)|\right). \tag{13}$$

Also, the normalized form 5 is as follows:

$$d_n(A, B) = \frac{1}{2n}\left[\sum_{i=1}^{n}|r_A(x_i) - r_B(x_i)|\right] + \frac{1}{2\pi}\left[\sum_{i=1}^{n}|\omega_A^r(x_i) - \omega_B^r(x_i)|\right]. \tag{14}$$

Then, the function of the normalized Euclidean distance is

$$d_{nE}(A, B) = \left[\frac{1}{2n}\sum_{i=1}^{n}(r_A(x_i) - r_B(x_i))^2 + \frac{1}{4\pi^2}(\omega_A^r(x_i) - \omega_B^r(x_i))^2)\right]^{1/2}, \tag{15}$$

so, we have

$$0 \le d_n(A, B) \le 1,\quad 0 \le d_z(A, B) \le 1,\quad 0 \le d_{nE}(A, B) \le 1. \tag{16}$$

3.1. Distances for Complex Fuzzy Sets. Let $U = \{x_1, x_2, \ldots, x_n\}$ be a universe of discourse, and the complex fuzzy set A may be represented as the set of ordered pairs:

$$A = \left\{\left(x, \hat{\text{I}}_A^{1/4}(x)\right)\middle| x \in U\right\}, \tag{17}$$

where the membership function $\hat{\text{I}}_A^{1/4}(x)$ is of the form $r_A(x)\hat{\text{A}} \cdot e^{i\omega_A(x)}, i = \sqrt{-1}$, and the amplitude term $r_A(x)$ and the phase term $\omega_A(x)$ are both real-valued. We know that the complex exponential function $e^{i\omega_A(x)}$ is a periodic function with periodicity 2π. Therefore, we consider $\omega_A(x) \in [0, 2\pi)$; otherwise, we take the residue modulo 2π into account.

Let A, B be two complex fuzzy sets in U and let p be a parameter satisfying $1 \le p \le \infty$. The Minkowski distances [43]:

$$d_p(A,B) = \left[\frac{1}{2}\sum_{i=1}^{n}\left(\left|r_A(x_i) - r_B(x_i)\right|^p + \frac{1}{\pi^p}\left(\min\begin{pmatrix}\left|\omega_A(x_i) - \omega_B(x_i)\right|, \\ 2\pi - \left|\omega_A(x_i) - \omega_B(x_i)\right|\end{pmatrix}^p\right)\right)\right]^{1/p}. \quad (18)$$

The normalized Minkowski distances [43]:

$$l_p(A,B) = \left[\frac{1}{2}\sum_{i=1}^{n}\left(\left|r_A(x_i) - r_B(x_i)\right|^p + \frac{1}{\pi^p}\left(\min\begin{pmatrix}\left|\omega_A(x_i) - \omega_B(x_i)\right|, \\ 2\pi - \left|\omega_A(x_i) - \omega_B(x_i)\right|\end{pmatrix}^p\right)\right)\right]^{1/p}. \quad (19)$$

If $p = 1$, then d is called the Hamming distance of complex fuzzy sets [43]:

$$d_1(A,B) = \frac{1}{2}\sum_{i=1}^{n}\left(\left|r_A(x_i) - r_B(x_i)\right| + \frac{1}{\pi}\left(\min\begin{pmatrix}\left|\omega_A(x_i) - \omega_B(x_i)\right|, \\ 2\pi - \left|\omega_A(x_i) - \omega_B(x_i)\right|\end{pmatrix}\right)\right), \quad (20)$$

and the normalized Hamming distance of complex fuzzy sets is [43]

$$l_1(A,B) = \frac{1}{2n}\sum_{i=1}^{n}\left(\left|r_A(x_i) - r_B(x_i)\right| + \frac{1}{\pi}\left(\min\begin{pmatrix}\left|\omega_A(x_i) - \omega_B(x_i)\right|, \\ 2\pi - \left|\omega_A(x_i) - \omega_B(x_i)\right|\end{pmatrix}\right)\right). \quad (21)$$

If $p = 2$, then d is termed the Euclidean distance of complex fuzzy sets:

$$d_2(A,B) = \left[\frac{1}{2}\sum_{i=1}^{n}\left(\left|r_A(x_i) - r_B(x_i)\right|^2 + \frac{1}{\pi^2}\left(\min\begin{pmatrix}\left|\omega_A(x_i) - \omega_B(x_i)\right|, \\ 2\pi - \left|\omega_A(x_i) - \omega_B(x_i)\right|\end{pmatrix}^2\right)\right)\right]^{1/2}, \quad (22)$$

and the normalized Euclidean distance of complex fuzzy sets is [43]

$$l_2(A,B) = \left[\frac{1}{2n}\sum_{i=1}^{n}\left(\left|r_A(x_i) - r_B(x_i)\right|^2 + \frac{1}{\pi^2}\left(\min\begin{pmatrix}\left|\omega_A(x_i) - \omega_B(x_i)\right|, \\ 2\pi - \left|\omega_A(x_i) - \omega_B(x_i)\right|\end{pmatrix}^2\right)\right)\right]^{1/2}. \quad (23)$$

3.2. Centroid-Index Ranking Method of Complex Fuzzy Numbers. The ranking of fuzzy numbers has been a concern in fuzzy Multiple Attribute Decision-Making (MADM) since its inception. More than 20 fuzzy ranking indices have been proposed since 1976. Various techniques are applied to compare the fuzzy numbers. The fuzzy number ranking, including complex fuzzy numbers to choose the best option from different options, is completely based on ranking or comparing the data. Various fuzzy ranking methods have been proposed ranging from one to several fuzzy number properties. In ranking fuzzy numbers with a center point index, the geometric center of a fuzzy number is found. Each geometric center has two components $(\tilde{x}, \tilde{y})$, where $\tilde{x}$ is on the horizontal axis, and $\tilde{y}$ is on the vertical coordinate axis.

Ranking fuzzy numbers are considered either by values $\tilde{x}$ alone or with the values $\tilde{x}$ and $\tilde{y}$. Several methods have been proposed in this regard, including Yager's method to rank fuzzy numbers based on$\tilde{x}$. Yager [44] proposed a centroid-index ranking method to calculate the value $\tilde{x}$ for a fuzzy number A as follows:

$$\tilde{x} = \int_0^1 w(x) f_A(x) \mathrm{d}x \int_0^1 f_A(x) \mathrm{d}x, \quad (24)$$

where $w(x)$ is a weighting function measuring the importance of the value x, and f_A denotes the membership function of the fuzzy number A and f_A: $X \longrightarrow [0, 1]$. Murakami et al. (1983) proposed a centroid-index ranking method for calculating the center of gravity (COG) point (x^*, y^*) for each fuzzy number. The larger the value of x^*, the better the ranking of fuzzy numbers. We proposed a centroid-index ranking method to calculate the centroid point of complex fuzzy numbers. Since any complex fuzzy number contains a polar representation, in polar coordinates R and θ, respectively, the directional distance is used from the pole and the angle between this half-line and the polar axis. Therefore, we consider ranking based on both components. In polar coordinates, the relationship between Cartesian (x, y) and polar (R, θ) coordinates is as follows:

$$\begin{aligned} x &= R\cos(\theta), \\ y &= R\sin(\theta). \end{aligned} \quad (25)$$

Consider a segment with the radius of R and polar angle of θ, given that the center of mass is located $2/3R$ from the pole, as depicted below. The area of a polar sector with a radius R and the polar angle θ equals $A = 1/2R^2\theta$. Thus, the element area is equal to $dA = 1/2R^2 d\theta$. In the triangle, the centroid is located 2/3 of the distance from the vertex. Hence, the centroid of this segment is at 2/3 of the distance from the pole. Considering the coordinates of the center of mass (x_c, y_c), we can calculate the centroid of this element (Figure 2):

$$\begin{aligned} x_c &= \frac{2}{3} R\cos(\theta), \\ y_c &= \frac{2}{3} R\sin(\theta). \end{aligned} \quad (26)$$

As a result, the center of mass of the whole sector with radius R and angle θ is

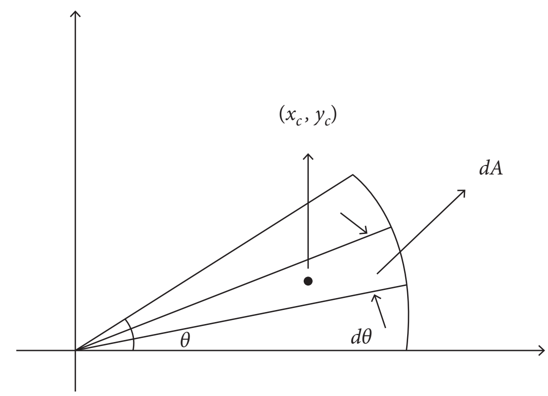

Figure 2

Table 1

Complex number	Centroid point $(\tilde{x}_i, \tilde{y}_i)$	Distance $D(A) = \sqrt{\tilde{x}^2 + \tilde{y}^2}$
A_1	$(-0.0623, 0.1919)$	0.2018
A_2	$(0, 0)$	0
A_3	$(-0.1009, 0.0733)$	0.1247
A_4	$(0, 0.2122)$	0.2122

$$\begin{aligned} \tilde{x} &= \int_0^\theta x dA \int_0^\theta \mathrm{d}A = \int_0^\theta (2/3) R\cos(\theta) 1/2R^2 \mathrm{d}\theta \int_0^\theta (1/2) R^2 \mathrm{d}\theta, \\ &= \frac{1/3R^3 \int_0^\theta \cos(\theta)\mathrm{d}\theta}{1/2R^2 \int_0^\theta \mathrm{d}\theta} = \frac{1/3R^3 \sin(\theta)}{1/2R^2\theta} = 2/3R\frac{\sin(\theta)}{\theta}. \end{aligned} \quad (27)$$

Also,

$$\begin{aligned} \tilde{y} &= \int_0^\theta y\, dA \int_0^\theta dA, \\ &= \int_0^\theta 2/3R\sin(\theta) 1/2R^2 d\theta \int_0^\theta 1/2R^2 d\theta, \\ &= \frac{1/3R^3 \int_0^\theta \sin(\theta) d\theta}{1/2R^2 \int_0^\theta d\theta}, \\ &= \frac{1/3R^3 [-\cos(\theta)]_0^\theta}{1/2R^2\theta} = \frac{2}{3} R \frac{1-\cos(\theta)}{\theta}. \end{aligned} \quad (28)$$

So,

$$\tilde{x} = \frac{2}{3} R \frac{\sin(\theta)}{\theta}, \tilde{y} = \frac{2}{3} R \frac{1-\cos(\theta)}{\theta}. \quad (29)$$

To rank complex fuzzy numbers, we can use the distance function between the center points $(\tilde{x}, \tilde{y})$ and the origin as shown in Table 1:

$$D(A) = \sqrt{\tilde{x}^2 + \tilde{y}^2}. \quad (30)$$

Let $A_1, A_2, \cdots, A_n$ be complex numbers then for every A_i, A_j, and then, we have

(i) If $D(A_i) < D(A_j), 1 \le i \le n, 1 \le j \le n$ then $A_i < A_j$
(ii) If $D(A_i) = D(A_j), 1 \le i \le n, 1 \le j \le n$ then $A_i = A_j$

(iii) If $D(A_i) > D(A_j), 1 \le i \le n, 1 \le j \le n$ then $A_i > A_j$.

Example 3. Let $A_1 = 0.6e^{i1.2\pi}/-1, A_2 = 1.0e^{i2\pi}/0, A_3 = 0.8e^{i1.6\pi}/1, A_4 = 0.5e^{i\pi}/2$, and then

3.3. Linguistic Variable of Complex Fuzzy Sets. An original element in human knowledge representation is the linguistic variables, which is important when using indicators to measure variables and get numbers. Besides, when we asked human experts to describe these variables, they gave us words such as good, slightly, more, or less to describe the variables.

Definition 5. Let A be a complex fuzzy set in U, and then, very A, very-very A, indeed A, a little A, slightly A, more or less A, and extremely A are defined as a complex fuzzy set in U with membership functions [41]:

(i) $\text{very}A = \{(x, \mu_{\text{very}A}(x)\}, \mu_{\text{very}A}(x) = [r_A(x)]^2 e^{i2\omega_A(x)}$

(ii) $\text{very}-\text{very}A = \{(x, \mu_{\text{very}-\text{very}A}(x))\}, \mu_{\text{very}-\text{very}A}(x) = [r_A(x)]^4 e^{i4\omega_A(x)}$

(iii) $\text{indeed}\,A = \{(x, \mu_{\text{indeed}A}(x))\}$ where

$$\mu_{\text{indeed}A}(x) = \begin{cases} 2|\mu_A(x)|^2 e^{i\omega_A(x)}, & 0 \le |\mu_A(x)| \le 0.5, \\ 1 - 2[1 - |\mu_A(x)|]^2 e^{i\omega_A(x)}, & 0.5 \le |\mu_A(x)| \le 1, \end{cases} \tag{31}$$

(iv) $\text{alittle}A = \{(x, \mu_{\text{alittle}A}(x))\}, \mu_{\text{alittle}A}(x) = [r_A(x)]^{1.3} e^{i1.3\omega_A(x)}$

(v) $\text{slightly}A = \{(x, \mu_{\text{slightly}A}(x))\}, \mu_{\text{slightly}A}(x) = [r_A(x)]^{1.7} e^{i1.7\omega_A(x)}$

(vi) $\text{extremely}A = \{(x, \mu_{\text{extremly}A}(x))\}, \mu_{\text{extremly}A}(x) = [r_A(x)]^3 e^{i3\omega_A(x)}$

(vii) $\text{moreorless}A = \{(x, \mu_{\text{moreorless}A}(x))\}, \mu_{\text{moreorless}A}(x) = [r_A(x)]^{1/2} e^{i1/2\omega_A(x)}$

By representing the relative performance using the expressed complex fuzzy numbers, the fuzzy quantity set will be considered {Worst, VL, L, M, H, VH, Best}. The complex fuzzy rankings for the language variables are presented in Table 2.

Table 2: The complex fuzzy rankings for linguistic variables.

Fuzzy numbers	Option evaluation	Weights
$(\mu_{\text{very-very}})$	Very − Very $(V-V)$	Worst (W)
(μ_{extremly})	extremly (ext)	verylow (VL)
(μ_{very})	Very (V)	low (L)
μ_{indeed}	indeed (ind)	medium (M)
(μ_{slightly})	slightly (S)	high (H)
(μ_{alittle})	alittle (alit)	veryhigh (VH)
$(\mu_{\text{moreorless}})$	moreorless (mol)	Best (B)

3.4. CP-TOPSIS Method. We proposed a systematic method for ranking complex fuzzy numbers based on TOPSIS in this section after briefly discussing the underlying ideal of TOPSIS. This is termed the developed CP-TOPSIS method. Geometrically speaking, the rank of the option selected as the best from a set of multiple alternatives should have the lowest distance from the ideal solution and the greatest distance from the negative ideal solution, according to the TOPSIS algorithm's fundamental principle. The TOPSIS method evaluates the following decision matrix, which refers to m alternatives based on n criteria (Table 3).

Table 3

	C_1	C_2		C_n
A_1	z_{11}^k	z_{12}^k	$\cdots$	z_{12}^k
A_2	z_{21}^k	z_{22}^k	$\cdots$	z_{2n}^k
$\vdots$	$\cdots$	$\cdots$	$\cdots$	$\ddots$
A_m	z_{m1}^k	z_{m2}^k	$\cdots$	z_{mn}^k

(i) Let there be K decision-makers $D^1, D^2, \cdots, D^K$ responsible for evaluating m alternatives $(A_i, i = 1, 2, \ldots, m)$ under n qualitative criteria $(C_j, j = 1, 2, \ldots, n)$.

(a) $A_1, A_2, \cdots, A_m$, m possible alternatives for decision-makers.

(b) $C_1, C_2, \cdots, C_n$, n criteria for measuring the effectiveness of options.

(c) Let $z_{ij}^k, i = 1, 2, \cdots m, j = 1, 2, \ldots, n1, k = 1, 2, \cdots, K$, be the rating assigned to alternative A_i under criterion C_j by decision-maker D^k.

To show the importance of each criterion, we used the weight vectors $W^k = [w_1^k, w_2^k, \cdots, w_n^k]$ assigned by the decision-maker D^k to criterion C_j such that

$$|w_1^k| + |w_2^k| + \cdots + |w_n^k| = 1. \tag{32}$$

Now, we calculate the real and imaginary parts of these components.

(ii) Let $Z_{ij}^k = (x_{ij}^k, y_{ij}^k), i = 1, \cdots, m, j = 1, \cdots, n, k = 1, \cdots, K$ be a rank assigned to A_i under C_j by decision-maker D^k. Linguistic variables and their corresponding complex fuzzy numbers are given in Definition 5. The overall rating of $Z_{ij} = (x_{ij}, y_{ij})$ is as follows:

$$Z_{ij} = \frac{1}{K} \circ \left(Z_{ij}^1 \hat{+} Z_{ij}^2 \hat{+} \cdots \hat{+} Z_{ij}^K\right), \tag{33}$$

where

$$x_{ij} = \sum_{k=1}^{K} \frac{x_{ij}^k}{k}, \qquad y_{ij} = \sum_{k=1}^{K} \frac{y_{ij}^k}{k}. \tag{34}$$

(iii) Let $w_j^k(u_j^k, v_j^k), j = 1 \ldots, k, k = 1, \ldots, K$ be the important weights assigned by the decision-maker

D^k to criterion C_j. The importance weights are quantified by linguistic values represented by complex fuzzy numbers, as stated in Definition 5. The averaged weight of each criterion $w_j = (u_j, v_j)$ can be evaluated as

$$w_j = \frac{1}{K} \circ \left(w_j^1 \hat{+} w_j^2 \hat{+} \cdots \hat{+} w_j^k\right), \quad (35)$$

where $u_j = \sum_{k=1}^{K} u_j^k / k, v_j = \sum_{k=1}^{K} v_j^k / k, j = 1, \cdots n$. Moreover, we converted w_j^k to its polar state.

(iv) We normalized the decision matrix by the following equations. Let $Z_{ij} = (x_{ij}, y_{ij})$ be the decision matrix components ordered in pairs. We obtained the maximum and minimum of each column for each component and normalized them using the following equations. The components $\tilde{x}_{ij}$ and $\tilde{y}_{ij}$ are obtained as follows:

$$\begin{aligned}\tilde{x}_{ij} &= \frac{x_{ij} - \min_i\{x_{ij}\}}{\max_i\{x_{ij}\} - \min_i\{x_{ij}\}},\\ \tilde{x}_{ij} &= \frac{x_{ij} - \min_i\{x_{ij}\}}{\max_i\{x_{ij}\} - \min_i\{x_{ij}\}},\\ \tilde{y}_{ij} &= \frac{y_{ij} - \min_i\{y_{ij}\}}{\max_i\{y_{ij}\} - \min_i\{y_{ij}\}}.\end{aligned} \quad (36)$$

Thus, the normalized components are $\tilde{Z}_{ij} = (\tilde{x}_{ij}, \tilde{y}_{ij})$. Then, the complex form of the normalized ranking matrix was calculated. The weighted normalized decision matrix was denoted as follows (by Definition 2):

$$\begin{aligned}\tilde{Q}_{ij} &= \tilde{Z}_{ij} \circ w_j,\\ i &= 1, 2, \ldots, m,\\ j &= 1, 2, \ldots, n.\end{aligned} \quad (37)$$

where Q_{ij} s are weighted normalized values.

(v) Here, in the centroid method proposed by 4, the complex fuzzy weighted ratings of alternatives were defuzzified because formulas of the defuzzification procedure could improve the execution of decision-making. Let Q_{ij} be the complex fuzzified value of $\tilde{Q}_{ij}$ with the real and imaginary components p_{ij} and q_{ij} by the centroid using equations (29) and (30):

$$\begin{aligned}Q_{ij} &= \left(p_{ij}, q_{ij}\right),\\ p_{ij} &= \frac{2}{3} R \frac{\sin(\theta)}{\theta},\\ q_{ij} &= \frac{2}{3} R \frac{1 - \cos(\theta)}{\theta}.\end{aligned} \quad (38)$$

Now, we display a complex fuzzy form of a matrix to calculate the values of Q_j^+, Q_j^-.

It should be noted that, in each step, we used the following equations to convert the polar coordinates of complex numbers to a regular pair of that numbers:

When

$$\begin{aligned}&z = re^{i\theta} \text{then} x = r\cos\theta,\\ &y = r\sin\theta.\end{aligned} \quad (39)$$

But, when $z = (x, y)$, then

$$\theta = \arg(z) = \begin{cases} \arctan\left(\frac{y}{x}\right) + \pi & \text{if } x > 0, y \geq 0,\\ \arctan\left(\frac{y}{x}\right) + \pi & \text{if } x < 0, y \geq 0,\\ \arctan\left(\frac{y}{x}\right) - \pi & \text{if } x < 0, y < 0,\\ \frac{\pi}{2} & \text{if } x = 0, y > 0,\\ -\frac{\pi}{2} & \text{if } x = 0, y < 0,\\ \infty & \text{if } x = 0, y = 0,\end{cases}$$
$$r = |z| = \sqrt{x^2 + y^2}. \quad (40)$$

Now $z = re^{i\theta}$.

(vi) The positive ideal solution (PIS), A^+, and negative ideal solution (NIS), A^-, can be obtained as follows. To obtain these answers, we used the polar area of the factor Q_{ij}. For this purpose, suppose $Q_{ij} = r_{ij} e^{i w_{ij}}$, and then, the polar area is obtained from the following formula:

$$A_{Q_{ij}} = \int_0^{\omega_{ij}} \frac{1}{2} r^2 \mathrm{d}\theta. \quad (41)$$

If ω_{ij} is negative, we change the boundary of integral. The positive ideal solution is as follows:

$$\begin{aligned}A^+ &= \left(Q_1^+, Q_2^+, \cdots, Q_n^+\right)\\ &= \left(\left(\max_i A_{Q_{ij}} | j \in I\right), \left(\min_i A_{Q_{ij}} | j \in J\right)\right),\end{aligned} \quad (42)$$

The negative ideal solution is as follows:

$$\begin{aligned}A^- &= \left(Q_1^-, Q_2^-, \cdots, Q_n^-\right)\\ &= \left(\left(\min_i A_{Q_{ij}} | j \in I\right), \left(\max_i A_{Q_{ij}} | j \in J\right)\right),\end{aligned} \quad (43)$$

(vii) The distance of each alternative from (PIS) is as follows:

$$d_i^+ = \sqrt{\sum_{j=1}^{n} \left(D\left(Q_{ij} - Q_j^+\right)\right)^2}, \quad i = 1, 2, \ldots, m. \tag{44}$$

The distance of each alternative from (NIS) is as follows:

$$d_i^- = \sqrt{\sum_{j=1}^{n} \left(D\left(Q_{ij} - Q_j^-\right)\right)^2}, \quad i = 1, 2, \ldots, m. \tag{45}$$

(viii) The ranking order of all alternatives from the highest-ranking index to the lowest is determined by calculating the closeness coefficients of alternatives by the following equation. However, as the option A_i gets closer to A^+ and the further away from A^-, the $\widetilde{R}_i$ value tends to be 1.

$$R_i = \frac{d_i^-}{d_i^+ + d_i^-}, \quad i = 1, 2, \ldots, m. \tag{46}$$

Now, all the steps of this new ranking method are shown in Figure 3.

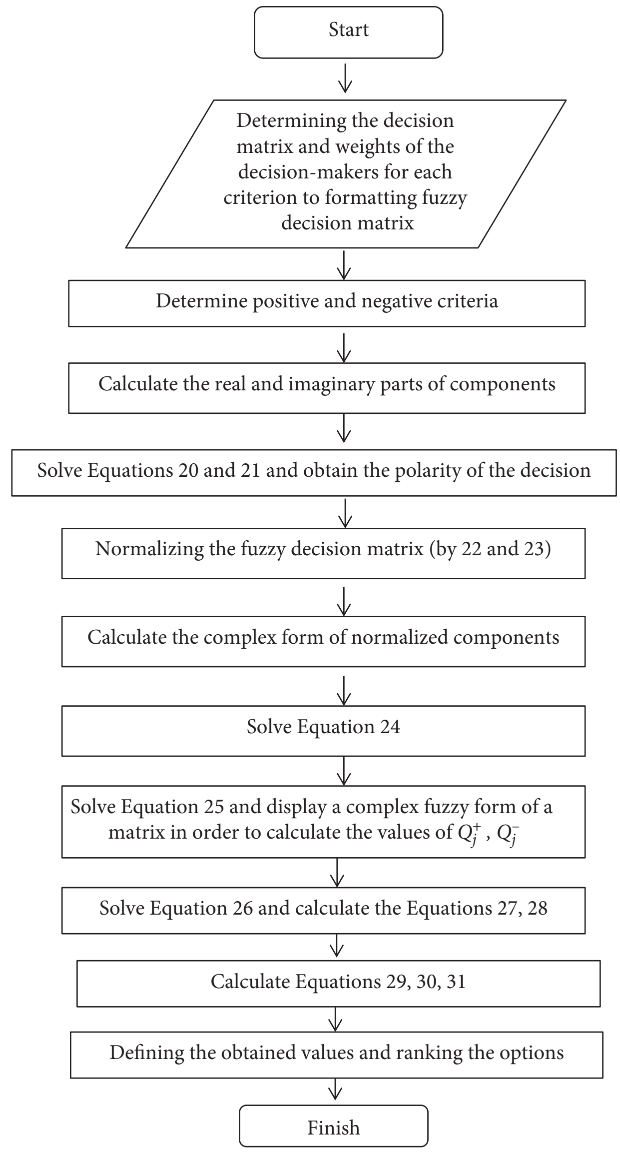

FIGURE 3: The flow chart with steps of the proposed algorithm.

Example 4. Let $U = \{1, 2, 3\}$ and M, N be two complex fuzzy sets:

$$M = \frac{0.6e^{i0.2\pi}}{1} + \frac{0.4e^{i0.5\pi}}{2} + \frac{0.5e^{i1.7\pi}}{3},$$
$$N = \frac{0.45e^{i0.5\pi}}{4} + \frac{0.66e^{i1.3\pi}}{5} + \frac{0.37e^{i0.3\pi}}{6}. \tag{47}$$

The set of decision makers is $D = \{D_1, D_2\}$. The following is a table of alternative ranking by Tables 4–16 decision-maker D_1:

TABLE 4

Deceision maker D_1			
	C_1	C_2	C_3
A_1	v	alit	ext
A_2	alit	ext	mol
A_3	s	ind	v
A_4	mol	v	s
A_5	$v - v$	ext	alit
A_6	mol	alit	s

The following is a table of alternative ranking by decision-maker D_2:

The next table is the weighting of criteria by decision makers:

The following table shows the complex fuzzy values assigned to the alternatives for D_1:

The following table shows the complex fuzzy values assigned to the alternatives for D_2:

Now, we get the real and imaginary parts of these elements:

And, we show also the values of D_2:

Now, we get the weighted values of the decision makers:

The ordered form of weighted values is as follows:

According to 21, the complex form of weighted average is equal to the following:

Now, by using 20, we get the ranking matrix as follows:

Now, normalize the ranking matrix using relations 22 and 23:

To normalize aggregated matrix, we require the polar form of values because it is difficult to normalize complex

Table 5

Deceision maker D_2			
	C_1	C_2	C_3
A_1	$v-v$	ind	ext
A_2	v	s	mol
A_3	ind	alit	ext
A_4	mol	$v-v$	alit
A_5	s	v	$v-v$
A_6	ind	mol	v

Table 6

	D_1	D_2
C_1	L	W
C_2	VL	B
C_3	H	M

Table 7

	C_1	C_2	C_3
A_1	$(0.36e^{i0.4\pi})$	$(0.51e^{i0.26\pi})$	$(0.21e^{i0.6\pi})$
A_2	$(0.30e^{i0.65\pi})$	$(0.06e^{i1.5\pi})$	$(0.63e^{i0.25\pi})$
A_3	$(0.3e^{i2.89\pi})$	$(0.5e^{i1.7\pi})$	$(0.25e^{i3.4\pi})$
A_4	$(0.67e^{i0.25\pi})$	$(0.2e^{i\pi})$	$(0.25e^{i0.85\pi})$
A_5	$(0.18e^{i5.2\pi})$	$(0.28e^{i3.9\pi})$	$(0.58e^{i1.69\pi})$
A_6	$(0.6e^{i\pi})$	$(0.27e^{i2.6\pi})$	$(0.18e^{i0.51\pi})$

Table 8

	C_1	C_2	C_3
A_1	$(0.12e^{i0.8\pi})$	$(0.68e^{i0.2\pi})$	$(0.21e^{i0.6\pi})$
A_2	$(0.16e^{i\pi})$	$(0.21e^{i0.85\pi})$	$(0.63e^{i0.25\pi})$
A_3	$(0.5e^{i1.7\pi})$	$(0.4e^{i2.21\pi})$	$(0.12e^{i5.1\pi})$
A_4	$(0.67e^{i0.25\pi})$	$(0.04e^{i2\pi})$	$(0.35e^{i0.65\pi})$
A_5	$(0.49e^{i2.21\pi})$	$(0.43e^{i2.6\pi})$	$(0.18e^{i5.2\pi})$
A_6	$(0.27e^{i2\pi})$	$(0.6e^{i\pi})$	$(0.13e^{i0.6\pi})$

Table 9: Values of D_1.

	C_1	C_2	C_3
A_1	$(0.1112, 0.3423)$	$(0.3491, 0.3717)$	$(-0.0648, 0.1997)$
A_2	$-0.1361, 0.2673)$	$(0.0, -0.06)$	$(0.4454, 0.4454)$
A_3	$(-0.2822, -0.1016)$	$(0.2938, -0.4045)$	$(-0.0772, -0.2377)$
A_4	$(0.4737, 0.4737)$	$(-0.2, 0.0)$	$(-0.2022, 0.1469)$
A_5	$(-0.1456, -0.1058)$	$(0.2662, -0.0865)$	$(0.3260, -0.4797)$
A_6	$(-0.6, 0.0)$	$(-0.0834, 0.2567)$	$(-0.0056, -0.1799)$

Table 10: Values of D_2.

	C_1	C_2	C_3
A_1	$(-0.0970, 0.0705)$	$(-0.3214, 0.5065)$	$(-0.0648, 0.1997)$
A_2	$(-0.16, 0.0)$	$(-0.1871, 0.0953)$	$(0.4454, 0.4454)$
A_3	$(0.2938, -0.4045)$	$(0.3160, 0.2451)$	$(-0.1141, -0.0370)$
A_4	$(0.4737, 0.4737)$	$(0.04, 0.0)$	$(-0.1588, 0.3118)$
A_5	$(0.3871, 0.3003)$	$(-0.1328, 0.4089)$	$(-0.1456, -0.1058)$
A_6	$(0.27, 0.0)$	$(-0.6, 0.0)$	$(-0.0401, 0.1236)$

Table 11

	D_1	D_2
C_1	$L = (0.0257^{i3.39\pi})$	$W = (0.0006e^{i6.78\pi})$
C_2	$VL = (0.0041e^{i5.08\pi})$	$B = (0.4007e^{i0.84\pi})$
C_3	$H = (0.0446e^{i2.88\pi})$	$M = (0.0551e^{i1.69\pi})$

Table 12

	D_1	D_2
C_1	$(0.00870, -0.0241)$	$(-0.00046, 0.00038)$
C_2	$(0.0039, -0.0010)$	$(-0.3511, 0.1930)$
C_3	$(-0.0414, 0.0164)$	$(0.0316, -0.0450)$

Table 13

	Complex form of weighted average
C_1	$(0.0262e^{-i1.32\pi})$
C_2	$(0.4031e^{i3.78\pi})$
C_3	$(0.0972e^{i2.13\pi})$

Table 14: Ranking matrix.

Aggregated ranking matrix			
A_1	$(0.43e^{i1.04\pi})$	$(0.84e^{i0.43\pi})$	$(0.37e^{1.02\pi})$
A_2	$(0.41e^{i1.3\pi})$	$(0.25e^{i1.71\pi})$	$(0.86e^{i1.06\pi})$
A_3	$(0.65e^{i2.33\pi})$	$(0.7e^{i2.03\pi})$	$(0.34e^{-i0.17\pi})$
A_4	$(0.89e^{i0.46\pi})$	$(0.23e^{i2\pi})$	$(0.51e^{i1.22\pi})$
A_5	$(0.58e^{i1.66\pi})$	$(0.58e^{i1.43\pi})$	$(0.65e^{i0.7\pi})$
A_6	$(0.7e^{i2\pi})$	$(0.70e^{i2.3\pi})$	$(0.28e^{i0.95\pi})$

Table 15: The aggregated matrix of D_1 and D_2 experts' opinions.

A_1	$(0.0, 0.3266)$	$(0.3761, 1.0000)$	$(0.4180, 0.3547)$
A_2	$(0.1648, 0.1269)$	$(0.3397, 0.2659)$	$(0.0, 0.1926)$
A_3	$(0.6722, 0.7674)$	$(1.0000, 0.4559)$	$(1.0, 0.1786)$
A_4	$(0.4776, 1.0000)$	$(0.4330, 0.4084)$	$(0.3972, 0.0)$
A_5	$(0.6266, 0.0)$	$(0.0, 0.0)$	$(0.4068, 1.0)$
A_6	$(1.000, 0.3653)$	$(0.6533, 0.8171)$	$(0.4996, 0.4334)$

Table 16

complex form of normalized ranking matrix			
A_1	$(0.3266e^{i0.5\pi})$	$(1.0683e^{i0.38\pi})$	$(0.5482e^{i0.22\pi})$
A_2	$(0.2079e^{i0.2\pi})$	$(0.4313e^{i0.21\pi})$	$(0.1926e^{i0.5\pi})$
A_3	$(1.0201e^{i0.27\pi})$	$(1.099e^{i0.13\pi})$	$(1.0158e^{i0.05\pi})$
A_4	$(1.1081e^{i0.35\pi})$	$(0.5952e^{i0.24\pi})$	$(0.3972e^{i0.0\pi})$
A_5	$(0.6266e^{i0.0\pi})$	$(0.0e^{i0.0\pi})$	$(1.0795e^{i0.37\pi})$
A_6	$(1.0646e^{i0.11\pi})$	$(1.0461e^{i0.28\pi})$	$(0.6613e^{i0.22\pi})$

Table 17

$\widetilde{Q}_{ij} = \widetilde{z}_{ij} \circ w_j$ normalized weighted matrix			
A_1	$(0.00855e^{-i0.33\pi})$	$(0.4306e^{i0.71\pi})$	$(0.0531e^{i0.23\pi})$
A_2	$(0.0054e^{-i0.13\pi})$	$(0.1738e^{i0.39\pi})$	$(0.0186e^{i0.53\pi})$
A_3	$(0.0267e^{-i0.17\pi})$	$(0.4430e^{i0.24\pi})$	$(0.0985e^{i0.05\pi})$
A_4	$(0.0290e^{-i0.23\pi})$	$(0.2399e^{i0.45\pi})$	$(0.0385e^{i0.0\pi})$
A_5	$(0.0164e^{i0.0\pi})$	$(0.0e^{i0.0\pi})$	$(0.1047e^{i0.39\pi})$
A_6	$(0.0278e^{-i0.07\pi})$	$(0.4216e^{i0.52\pi})$	$(0.0641e^{i0.23\pi})$

Table 18

Q_{ij}deffuzification matrix			
A_1	(0.00473, −0.00268)	(0.10296, 0.20757)	(0.03239, 0.01224)
A_2	(0.00352, −0.00072)	(0.08897, 0.06253)	(0.00741, 0.00814)
A_3	(0.01696, −0.00464)	(0.26813, 0.10616)	(0.06539, 0.00514)
A_4	(0.01769, −0.00668)	(0.11173, 0.09543)	(0.02566, 0.0)
A_5	(0.01093, 0.0)	(0.0, 0.0)	(0.05360, 0.03767)
A_6	(0.01838, −0.00202)	(0.17171, 0.18285)	(0.03911, 0.01477)

Table 19

Q_{ij}polar matrix			
A_1	$(0.00543e^{-i0.16\pi})$	$(0.2317e^{i0.35\pi})$	$(0.03462e^{i0.11\pi})$
A_2	$(0.00359e^{-i0.06\pi})$	$(0.10874e^{i0.19\pi})$	$(0.011e^{i0.26\pi})$
A_3	$(0.01758e^{-i0.08\pi})$	$(0.2883e^{i0.11\pi})$	$(0.0655e^{i0.02\pi})$
A_4	$(0.0189^{-i0.011\pi})$	$(0.14693e^{i0.22\pi})$	$(0.02566e^{i0.0\pi})$
A_5	$(0.01093e^{i0.0\pi})$	$(0.0e^{i0.0\pi})$	$(0.0655e^{i0.19\pi})$
A_6	$(0.01849e^{-i0.03\pi})$	$(0.2508e^{i0.25\pi})$	$(0.0418e^{i0.11\pi})$

Table 20

A_{Qij}PolarArea			
A_1	(7.410×10^{-6})	(0.029514)	(0.000207)
A_2	(1.214×10^{-6})	(0.003529)	(0.0000494)
A_3	(38.83×10^{-6})	(0.014369)	(0.000134)
A_4	(61.72×10^{-6})	(0.00746)	(0.0)
A_5	(0.0×10^{-6})	(0.0)	(0.00128)
A_6	(16.11×10^{-6})	(0.024701)	(0.000301)

Table 21

d_1^+	d_2^+	d_3^+	d_4^+	d_5^+	d_6^+
0.036416	0.157561	0.193840	0.119496	0.234398	0.078236

Table 22

d_1^-	d_2^-	d_3^-	d_4^-	d_5^-	d_6^-
0.232221	0.110817	0.291249	0.147244	0.046900	0.251748

Table 23

R_1	R_2	R_3	R_4	R_5	R_6
0.864441	0.412913	0.600403	0.552013	0.166727	0.762909

forms directly. Thus, we use the polar form and then normalize values. In order to use 24, we obtain the complex form of the normalized ranking matrix:

Now, we return to the complex form because we should build the weighed matrix since we can easily multiply complex numbers. Note that multiplication of the complex numbers was defined in previous equations. We use the complex form of the weights and construct a weighted matrix using complex multiplication. The weighted matrix in Table 17 is in complex form by using 24, and we obtain the normalized weighted matrix:

We determine positive and negative ideals. For this purpose, we should perform a defuzzification process because it helps determine the maximum and minimum of each column. Due to the difficulty of determining the maximum and minimum values of complex numbers, we use two formulas. First, data are written as ordered pairs, as presented in Table 18. This is the first defuzzification step, which is not the main step. We do the phase defuzzification process using centroid method 25:

In the second defuzzification step, we try to calculate the polar area of factors. Therefore, we should transform Table 18 into the complex form as presented in Table 19. Now, we display complex fuzzy form of matrix in order to calculate the values of Q_j^+, Q_j^-:

Table 19 is used to determine the polar area of the factors after the matrices had been converted to polar form. Accordingly, Table 20 is obtained, which provides a criterion to select positive and negative ideals in complex forms. The following areas are obtained in micro:

Now, (PIS) is as follows:

$$A^+ = \left(16.11 * 10^{-6}, 0.29514, 0.001280\right), \tag{48}$$

so, we have

$$\begin{aligned} Q_1^+ &= 0.01849e^{-i0.03\pi}, \\ Q_2^+ &= 0.2317e^{i0.35\pi}, \\ Q_3^+ &= 0.0655e^{i0.19\pi}. \end{aligned} \tag{49}$$

(NIS) is as follows:

$$A^- = (0.0, 0.0, 0.0), \tag{50}$$

so, we have

$$\begin{aligned} Q_1^- &= 0.01093e^{i0.0\pi}, \\ Q_2^- &= 0.0e^{i0.0\pi}, \\ Q_3^- &= 0.02566e^{i0.0\pi}. \end{aligned} \tag{51}$$

The distance between each alternative and positive and negative ideals should be calculated. Since the distance formula is based on the complex form of numbers, and ideals are complex as shown in Table 21, we use the table of complex data and calculate the differences, as given in Tables 22 and 23. The values of d_i^+ are as follows:

The values of d_i^- are as follows:

We get the closeness coefficients of each alternative according to relation 31:

Ultimately, we use the closeness coefficient formula and rank the alternatives, as presented in Table 24. So, the rating alternatives are as follows:

4. Results

In this paper, by defining a new method, we presented a new method to prioritize a model of decision-making, in which data are of complex fuzzy numbers, and finally, a rating algorithm was designed based on the central point index and TOPSIS-oriented method. This paper aims to establish a

Table 24

	A_1	A_2	A_3	A_4	A_5	A_6
R_i	0.864441	0.412913	0.600403	0.552013	0.166727	0.762909
Ranking	1	5	3	4	6	2

complex fuzzy technique for rating and choosing the best options for the complex fuzzy data that have many uses including analyzing wave functions in quantum mechanics and a lot of physical quantities.

5. Managerial Insights

As the uncertainty conditions play a vital role in the decision-making process by managers, the complex fuzzy sets have many utilizations in the industry, management, and financial sector. Given the importance of this topic, this paper will grade complex fuzzy options through CP-TOPSIS developmental method. Recently, the complex fuzzy set contains a range of values, in which its membership function may be obtained. In contrast to the traditional fuzzy membership function, this range is not limited [0, 1] because it will be expanded into the unit circle on the complex page. Therefore, the complex fuzzy set presents a mathematical framework for describing a membership on the set in terms of a complex number. The possible applications, which provide a new concept, include the complex fuzzy display, solar activity (by measuring the number of sunspots), signal process usage, problems in forecasting time series and comparing two national economies, identifying and measuring signals, analysis, wave functions within quantum mechanics, and many physical quantities, through the complex fuzzy relation. Today, there is no device such as home appliances created based on its technical structure without using the complex fuzzy sets. We often use and enjoy the complex fuzzy logic in our routine life. Also, it can be referred to as the motor systems (elevator), automotive industry (design of ABS braking system), designing controllers of home appliances (washing machine and refrigerator), and cameras, and artificial intelligence is one of the most interesting applications in the computer games like special cinematic sights.

6. Conclusions and Outlook

Grading the complex fuzzy numbers has a prominent role in management, engineering, and industry. The complex fuzzy numbers are shown by a complex form membership function. Due to the presence of real numbers that can be arranged as linearity, the complex fuzzy numbers may have overlapped each other. Therefore, it is difficult to grade all of them. Currently, a novel CP-TOPSIS grading method is proposed in this study that is related to the complex fuzzy numbers. The proposed fuzzy set is a new approach that can complex the membership functions. It is vital to have a profound understanding of the aspects of complex fuzzy sets in order to make effective use of the possibilities those sets offer. As a result, we allocated several sections for the properties and features of the complex membership function, such as complex subscription, multiplication, and collection. One of the advantages of the new method, in some cases, is the difficulty of identifying exact values of properties, and we considered their values as the complex fuzzy data. Accordingly, we proposed grading data based on the central point index. In this regard, we explained the distance functions for the complex fuzzy numbers based on fuzzy period and domain. Finally, we introduced a developmental method, CT-TOPSIS for rating complex fuzzy data. In this research, CT-TOPSIS is created as complex fuzzy sets, and a method for determining the most desirable choice is proposed. While the fuzzy data is complex, the best and the most effective option is presented between all possible options. The fuzzy decision matrix is calculated using the results` concept. In this method, despite the distance considerations, an option is considered regarding the solution of a fuzzy positive ideal and its distance, far from the solution related to a fuzzy negative ideal. It means that the lower the distance of an option evaluated through the solution of a fuzzy positive ideal is, and the more the distance from the solution of a fuzzy negative ideal is, the better the ranking is. This sort of ranking, which is introduced in this study for the first time, corresponds to the complex fuzzy numbers. We hope that a new way is paved for future works upon the complex fuzzy data investigation due to many applications in the industry, management, and financial sectors.

Data Availability

Data are available within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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