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Research Article

Fuzzy Nano *z*-locally Closed Sets, Extremally Disconnected Spaces, Normal Spaces, and Their Application

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In this paper, we introduce fuzzy nano (resp. δ, δS, P and Z) locally closed set and fuzzy nano (resp. δ, δS, P and Z) extremally disconnected spaces in fuzzy nano topological spaces. Also, we introduce some new spaces called fuzzy nano (resp. δ, δS, P and Z) normal spaces and strongly fuzzy nano (resp. δ, δS, P and Z) normal spaces with the help of fuzzy nano (resp. δ, δS, P and Z)-open sets in fuzzy nano topological space. Numerical data is used to quantify the provided features. Furthermore, using fuzzy nano topological spaces, an algorithm for multiple attribute decision-making (MADM) with an application in medical diagnosis is devised.

1. Introduction

Through his significant theory on fuzzy sets, Zadeh [1] made the first effective attempt in mathematical modeling to contain non-probabilistic uncertainty, i.e. uncertainty that is not caused by randomness of an event. The study of fuzzy calculus plays a vital role in the field of mathematics due to its useful applications in variety of scientific domains including statistics, applied mathematics, dynamics and mathematical biology. Many applications of fuzzy mathematics can be found in engineering, bio-mathematics and basic sciences. A novel technique to solve the fuzzy system of equations has been presented by Mikaeilvand et al. [2]. Also many applications of fuzzy integral equations have been presented by various authors [3, 4]. A fuzzy set is one in which each element of the universe belongs to it, but with a value or degree of belongingness that falls between 0 and 1, and these values are referred to as the membership value of each element in that set. Chang [5] was the first to propose the concept of fuzzy topology later on.

Pawlak [6] introduces Rough set theory in 1992 as a substitute mathematical tool for describing reasoning and deciding how to handle vagueness and uncertainty. This theory uses equivalence relations to approximate sets, and it is used in conjunction with the principal non-statistical techniques to data analysis. Lower and upper approximations are two definite sets that commonly characterise a rough set. The greatest definable set included inside the given collection of objects is the lower approximation, whereas the smallest definable set that contains the provided set is the upper approximation. Rough set concepts are frequently stated in broad terms using topological operations such as interior and closure, which are referred to as approximations.

Lellis Thivagar [7] introduced a new topology called nano topology in 2013, which is an extension of rough set theory. He also created Nano topological spaces, which are defined in terms of approximations and the boundary region of a subset of the universe using an equivalence relation. The Nano open sets are the constituents of a Nano topological space, while the Nano closed sets are their complements. The term "nano" refers to anything extremely small. Nano topology, then, is the study of extremely small surfaces. Nano topology is based on the concepts of approximations and indiscernibility relations. In addition, in [8], nano delta open sets in nano topological space were investigated.

This paper follows the definition of Lellis Thivagar et al. [9]. Generalizations of (fuzzy nano) open sets are a major topic in (fuzzy nano) topology. One of the important generalizations is a Z-open sets [10] which was studied in classical topology by El-Magharabi and Mubarki. Later on, many studies which investigated a nano topologies have been done such as nano M-open sets [11], nano Z-open sets [12], Z-closed sets in double fuzzy topological spaces [13, 14] and Z-open sets in a fuzzy nano topological spaces by Thangammal et al. [15].

Kuratowski and Sierpinski [16] explored the difference of two closed subsets of a n-dimensional Euclidean space in 1921, and the notion of a locally closed subset of a topological space was a key instrument in their work. Ganster and Reilly [17] defined LC-continuity in a topological space using locally closed sets in 1989.

Multiple attribute decision-making (MADM) is a decision-making process that takes into account the best possible options. Decisions were taken in mediaeval times without taking into account data uncertainties, which could lead to a potential outcome. Inadequate outcomes have real-life consequences. If we deduced the consequence of obtained data without hesitancy, the results would be ambiguous, indeterminate, or incorrect. Without hesitation, I determined the result of the obtained data. MADM had a significant impact on Management, disease diagnosis, economics, and industry are examples of real-world problems. Each decision maker makes hundreds of decisions each time to carry out the key component. It should be a logical assessment of his or her job. MADM is a programme that helps you tackle difficult problems. For this, there are complex problems with a variety of parameters. The problem must be identified in MADM by defining viable alternatives, assessing each alternative against the criteria established by the decision-maker or community of decision-makers, and finally selecting the optimal alternative. To deal with the complications and complexity of MADM problems, a range of useful mathematical methods such as fuzzy sets, neutrosophic sets, and soft sets were developed.

Zafer et al. [18] introduced and developed the MADM method based on rough fuzzy information. Several mathematicians have worked on correlation coefficients, similarity measurements, aggregation operators, topological spaces, and decision-making applications in this area. These structures feature better decision-making solutions and provide distinct formulas for diverse sets. It has a wide range of applications in domains such as medical diagnosis, pattern identification, social sciences, artificial intelligence, business, and multi-attribute decision making. The problems associated with these cases are interesting, and developing a hypothesis for them has prompted many scholars [19–21] to pay attention to them *Motivation and objective*. No investigation on fuzzy nano Z locally closed set, fuzzy nano Z extremally disconnected spaces, fuzzy nano Z normal spaces and strongly fuzzy nano Z normal spaces in fuzzy nano topological space has been reported in the fuzzy literature. We present this innovative notion of fuzzy nano topological space and apply it to the MADM issue based on the concepts of fuzzy sets [1], nano topological spaces [7], and neutrosophic nano topological space [9]. The enlarged and hybrid motivation and goal work is described in detail throughout the article. Under certain conditions, we ensure that other FS hybrid systems are special FNts. Our proposed model and techniques are discussed in terms of their robustness, durability, superiority, and simplicity. This is the most prevalent model, and it is used to collect vast amounts of data in AI, engineering, and medical applications. Similar research can simply be duplicated in the future using alternative methodologies and hybrid structures.

The following is how this article is organised: Section 2 is devoted to discussing various fuzzy set theory and fuzzy nano topology definitions and results. In Section 3, we introduce the notion of fuzzy nano Z locally closed set and establish some of characterizes. The concept of fuzzy nano Z extremally disconnected spaces is introduced in fuzzy nano topological spaces and also gives some properties and theorems of such concepts in Section 4. In Sections 5 and 6, fuzzy nano Z normal space and strongly fuzzy nano Z normal spaces are introduced and proved many theorems. As a numerical example, in Sections 7 & 8, we devised a method for solving the MADM issue related to Medical Diagnosis utilising $\mathscr{F}\mathscr{N}ts$. We also discussed the algorithms' efficiency, advantage, consistency, and validity. In Section 9, the work's conclusion is fundamentally summarised, and the next field of research is offered.

2. Preliminaries

This part explains the concepts and findings that we need to know in order to comprehend the manuscript.

Definition 1 (see [1]). A function f from X into the unit interval I is called a fuzzy set (briefly, $\mathscr{F}s$) in X.

Definition 2 (see [1]). If G and H are any two fuzzy subsets (briefly, $\mathscr{F}subs$) of a set X, then

(i) $G \leq H$ iff $\mu_G(l) \leq \mu_H(l)$, $\forall\, l$ in X. (ii) $G = H$, if $G(l) = H(l)\, \forall\, l$ in X. (iii) $(G \vee H)(l) = \max\{G(l), H(l)\}$, $\forall\, l$ in X. (iv) $(G \wedge H)(l) = \min\{G(l), H(l)\}$, $\forall\, l$ in X.

Definition 3 (see [1]). The complement of a $\mathcal{F}subs\, G$ in X, denoted by $1-G$, is the $\mathcal{F}subs$ of X defined by $1-G(l)$, $\forall\, l$ in X.

Definition 4 (see [9]). Let U be a non-empty set and R be an equivalence relation on U. Let F be a $\mathcal{F}s$ in U with the membership function μ_F. The fuzzy nano lower (upper) approximations and fuzzy nano boundary of F in the approximation (U,R) denoted by $\underline{\mathcal{F}\mathcal{N}}(F), \overline{\mathcal{F}\mathcal{N}}(F)$ and $B_{\mathcal{F}\mathcal{N}}(F)$ are respectively defined as follows: (i) $\underline{\mathcal{F}\mathcal{N}}(F) = \{\langle l, \mu_{\underline{R}(A)}(L)\rangle / Y \in [l]_R, l \in U\}$ (ii) $\overline{\mathcal{F}\mathcal{N}}(F) = \{\langle l, \mu_{\overline{R}(A)}(l)\rangle / y \in [l]_R, l \in U\}$ (iii) $B_{\mathcal{F}\mathcal{N}}(F) = \overline{\mathcal{F}\mathcal{N}}(F) - \underline{\mathcal{F}\mathcal{N}}(F)$ where $\mu_{\underline{R}(A)}(l) = \bigwedge_{y\in[l]_R} \mu_A(y)$. $\mu_{\overline{R}(A)}(l) = \bigvee_{y\in[l]_R} \mu_A(y)$. The collection $\tau_{\mathcal{F}}(F) = \{0_{\mathcal{F}}, 1_{\mathcal{F}}, \underline{\mathcal{F}\mathcal{N}}(F), \overline{\mathcal{F}\mathcal{N}}(F), B_{\mathcal{F}\mathcal{N}}(F)\}$ forms a topology called as fuzzy nano topology and $(U, \tau_{\mathcal{F}}(F))$ as fuzzy nano topological space (briefly, $\mathcal{F}\mathcal{N}ts$). The elements of $\tau_{\mathcal{F}}(F)$ are called fuzzy nano open (briefly, $\mathcal{F}\mathcal{N}o$) sets. Elements of $[\tau_{\mathcal{F}}(F)]^c$ are called fuzzy nano closed (briefly, $\mathcal{F}\mathcal{N}c$) sets.

3. Fuzzy nano Z locally closed sets

The idea of fuzzy nano Z locally closed sets, which represents a class of generalisations of fuzzy nano Z open sets, is introduced in this section. The main features of fuzzy nano Z closed sets are established, as well as certain characterizations.

Definition 5. Let $(U, \tau_{\mathcal{F}}(F))$ be a $\mathcal{F}\mathcal{N}ts$ with respect to F where F is a fuzzy subset of U. Let S be a fuzzy subset of U. Then fuzzy nano

(i) interior of S (briefly, $\mathcal{F}\mathcal{N}\text{int}(S)$) is represented as $\mathcal{F}\mathcal{N}\text{int}(S) = \vee\{O\colon O \le S \& O \text{ is a } \mathcal{F}\mathcal{N}o \text{ set in} U\}$.

(ii) closure of S (briefly, $\mathcal{F}\mathcal{N}cl(S)$) is represented as $\mathcal{F}\mathcal{N}cl(S) = \wedge\{L\colon S \le L \& L \text{ is a } \mathcal{F}\mathcal{N}c \text{ set in } U\}$.

(iii) regular open (briefly, $\mathcal{F}\mathcal{N}ro$) set if $S = \mathcal{F}\mathcal{N}\text{int}(\mathcal{F}\mathcal{N}cl(S))$.

(iv) regular closed (briefly, $\mathcal{F}\mathcal{N}rc$) set if $S = \mathcal{F}\mathcal{N}cl(\mathcal{F}\mathcal{N}\text{int}(S))$.

(v) δ interior of S (briefly, $\mathcal{F}\mathcal{N}\delta\text{int}(S)$) is represented as $\mathcal{F}\mathcal{N}\delta\text{int}(S) = \vee\{O\colon O \le S \& O \text{ is a } \mathcal{F}\mathcal{N}ro \text{ set in } U\}$.

(vi) δ closure of S (briefly, $\mathcal{F}\mathcal{N}\delta cl(S)$) is represented as $\mathcal{F}\mathcal{N}\delta cl(S) = \wedge\{L\colon S \le L \& L \text{ is a } \mathcal{F}\mathcal{N}rc \text{ set in } U\}$.

(vii) δ-open (briefly, $\mathcal{F}\mathcal{N}\delta o$) set if $S = \mathcal{F}\mathcal{N}\delta\text{int}(S)$.

(viii) δ-semi open (briefly, $\mathcal{F}\mathcal{N}\delta\mathcal{S}o$) set if $S \le \mathcal{F}\mathcal{N}cl(\mathcal{F}\mathcal{N}\delta\text{int}(S))$.

(ix) pre open (briefly, *FNPo*) set if $S \le \mathcal{F}\mathcal{N}\text{int}(\mathcal{F}\mathcal{N}cl(S))$.

(x) δ semi interior of S (briefly, $\mathcal{F}\mathcal{N}\delta\mathcal{S}\text{int}(S)$) is represented as $\mathcal{F}\mathcal{N}\delta\mathcal{S}\text{int}(S) = \vee\{O\colon O \le S \& O \text{ is a } \mathcal{F}\mathcal{N}\delta\mathcal{S}o \text{ set in } U\}$.

(xi) δ semi closure of S (briefly, $\mathcal{F}\mathcal{N}\delta\mathcal{S}cl(S)$) is represented as $\mathcal{F}\mathcal{N}\delta\mathcal{S}cl(S) = \wedge\{L\colon S \le L \& L \text{ is a } \mathcal{F}\mathcal{N}\delta\mathcal{S} \text{ set in } U\}$.

(xii) pre interior of S (briefly, FNPint(S)) is represented as $\mathcal{F}\mathcal{N}\delta\mathcal{S}\text{int}(S) = \vee\{O\colon O \le S \& O \text{ is a } \mathcal{F}\mathcal{N}\delta\mathcal{S}o \text{ set in } U\}$.

(xiii) pre closure of S (briefly, FNPint(S)) is represented as $\mathcal{F}\mathcal{N}\delta\mathcal{S}\text{int}(S) = \vee\{O\colon O \le S \& O \text{ is a } \mathcal{F}\mathcal{N}\delta\mathcal{S}o \text{ set in } U\}$.

The complement of an $\mathcal{F}\mathcal{N}\delta\mathcal{S}cl(S)$ (resp. $\mathcal{F}\aleph ZO(U,A)$) set is called a fuzzy nano δ (resp. fuzzy nano δ-semi & fuzzy nano pre) closed (briefly, $\mathcal{F}\mathcal{N}\delta c$ (resp. $\mathcal{F}\aleph ZO(U,A)$) in U. Definition 6. Let $(U, \tau_{\mathcal{F}}(F))$ be a $\mathcal{F}\mathcal{N}ts$. Then a fuzzy subset S in U is said to be a fuzzy nano

(i) Z-open (briefly, $\mathcal{F}\aleph Zo$) set if $S \le \mathcal{F}\aleph cl(\mathcal{F}\aleph\delta\text{int}(S)) \vee \mathcal{F}\aleph\text{int}(\mathcal{F}\aleph cl(S))$,

(ii) Z-closed (briefly, $\mathcal{F}\aleph Zc$) set if $\mathcal{F}\aleph\text{int}(\mathcal{F}\aleph\delta cl(S)) \wedge \mathcal{F}\aleph cl(\mathcal{F}\aleph\text{int}(S)) \le S$.

(iii) Z-interior (resp. closure) of O is the union (resp. intersection) of all $\mathcal{F}\aleph Zo$ (resp. $\mathcal{F}\aleph Zc$) sets contained in O and denoted by $\mathcal{F}\aleph Z\text{int}(O)$ (resp. $\mathcal{F}\aleph Zcl(O)$).

All $\mathcal{F}\aleph Zo$ (resp. $\mathcal{F}\aleph Zc$) sets of a space $(U, \tau_{\mathcal{F}}(F))$ will be denoted by $\mathcal{F}\aleph ZO(U,A)$ (resp. $\mathcal{F}\aleph ZC(U,A)$).

Remark 1. The following diagram shows the relationship between any set in $\mathcal{F}\aleph$anots*ts* of $\mathcal{F}\mathcal{N}Zos$'s (resp. $\mathcal{F}\mathcal{N}Zcs$'s).

Definition 7. A function $h\colon (U_1, \tau_{\mathcal{F}}(F_1)) \longrightarrow (U_2, \tau_{\mathcal{F}}(F_2))$ is said to be fuzzy

(i) nano (resp. Z) continuous (briefly, $\mathcal{F}\aleph Cts$ (resp. $\mathcal{F}\aleph ZCts$)), if $\forall\, \mathcal{F}\aleph o$ set M of U_2, the set $h^{-1}(M)$ is $\mathcal{F}\aleph o$ (resp. $\mathcal{F}\aleph Zo$) set of U_1.

(ii) nano Z irresolute (briefly, $\mathcal{F}\aleph ZIrr$) function, if $\forall\, \mathcal{F}\aleph Zo$ subset M of U_2, the set $h^{-1}(M)$ is $\mathcal{F}\aleph Zo$ subset of U_1.

(iii) nano (resp. δ, δS, P and Z) open map (briefly, $\mathcal{F}\mathcal{N}O$ and $\mathcal{F}\mathcal{N}ZO$)) if the image of each $\mathcal{F}\mathcal{N}o$ set in U_1 is $\mathcal{F}\mathcal{N}o$ and $\mathcal{F}\mathcal{N}Zo$) in U_2.

(iv) nano (res δ, δS, P and Z) closed map (briefly, $\mathcal{F}\mathcal{N}C$ and $\mathcal{F}\mathcal{N}ZC$)) if the image of each $\mathcal{F}\mathcal{N}c$ set in U_1 is $\mathcal{F}\mathcal{N}c$ and $\mathcal{F}\mathcal{N}Zc$) in U_2.

Definition 8. Let A and B be any two fuzzy subsets of a $\mathcal{F}\mathcal{N}ts$'s. Then A is fuzzy nan δ, δS, P and Z) q-neighbourhood (briefly, $\mathcal{F}\mathcal{N}q\text{-}nbh\ d$ (resp. $\mathcal{F}\mathcal{N}\delta q$, $\mathcal{F}\mathcal{N}\delta\mathcal{S}$, $\mathcal{F}\mathcal{N}\delta So$, $\mathcal{F}\mathcal{N}Zo$ a*nbh d* and $\mathcal{F}\mathcal{N}Zq\text{-}nbh\ d$)) with B if there exists a $\mathcal{F}\mathcal{N}o$ and $\mathcal{F}\mathcal{N}Zo$) set O with $AqO \le B$.

Definition 9. Let $(U, \tau_{\mathcal{F}}(F))$ be a $\mathcal{F}\mathcal{N}ts$ is called fuzzy nano (resp. δ, δS, P and Z) locally closed (briefly, $\mathcal{F}\mathcal{N}LC$

(resp$\mathscr{FN}\delta\mathscr{S}$, $\mathscr{FN}\delta So$, $\mathscr{FN}Zo$ a and $\mathscr{FN}ZLC$)) set if $A = B \wedge C$ where B is a $\mathscr{FN}o$ (resp. $\mathscr{FN}\delta\mathscr{S}$, $\mathscr{FN}\delta So$, $\mathscr{FN}Zo$ and $\mathscr{FN}Zo$) set and C is a $\mathscr{FN}c$ (resp. $\mathscr{FN}\delta\mathscr{S}$, $\mathscr{FN}\delta So$, $\mathscr{FN}Zo$ a and $\mathscr{FN}Zc$) set.

Example 1. Assume $U = \{s_1, s_2, s_3, s_4\}$ and $U/R = \{\{s_1, s_4\}, \{s_2\}, \{s_3\}\}$.

Let $S = \{\langle s_1/0.2\rangle, \langle s_2/0.3\rangle, \langle s_3/0.4\rangle, \langle s_4/0.1\rangle\}$ be a $\mathscr{F}subs$ of U.

$$\begin{aligned} \underline{\mathscr{FN}}(S) &= \left\{\langle\frac{s_1, s_4}{0.1}\rangle, \langle\frac{s_2}{0.3}\rangle, \langle\frac{s_3}{0.4}\rangle\right\}, \\ \overline{\mathscr{FN}}(S) &= \left\{\langle\frac{s_1, s_4}{0.2}\rangle, \langle\frac{s_2}{0.3}\rangle, \langle\frac{s_3}{0.4}\rangle\right\}, \\ B_{\mathscr{FN}}(S) &= \left\{\langle\frac{s_1, s_4}{0.2}\rangle, \langle\frac{s_2}{0.3}\rangle, \langle\frac{s_3}{0.4}\rangle\right\}. \end{aligned} \tag{1}$$

Thus $\tau_{\mathscr{F}}(S) = \{0_{\mathscr{F}}, 1_{\mathscr{F}}, \underline{\mathscr{FN}}(S), \overline{\mathscr{FN}}(S) = B_{\mathscr{FN}}(S)\}$. Then

(i) $\mathscr{FN}LC \Rightarrow C = A \wedge B = \{\langle s_1, s_4/0.1\rangle, \langle s_2/0.3\rangle, \langle s_3/0.4\rangle\} \wedge \{\langle s_1, s_4/0.8\rangle, \langle s_2/0.7\rangle, \langle s_3/0.6\rangle\} == \{\langle s_1, s_4/0.1\rangle, \langle s_2/0.3\rangle, \langle s_3/0.4\rangle\}$

(ii) $\mathscr{FN}\delta\mathscr{S}LC \Rightarrow C = A \wedge B = \{\langle s_1, s_4/0.4\rangle, \langle s_2/0.6\rangle, \langle s_3/0.5\rangle\} \wedge \{\langle s_1, s_4/0.3\rangle, \langle s_2/0.7\rangle, \langle s_3/0.4\rangle\} = \{\langle s_1, s_4/0.3\rangle, \langle s_2/0.6\rangle, \langle s_3/0.4\rangle\}$

(iii) $\mathscr{FN}ZLC \Rightarrow C = A \wedge B = \{\langle s_1, s_4/0.4\rangle, \langle s_2/0.6\rangle, \langle s_3/0.5\rangle\} \wedge \{\langle s_1, s_4/0.3\rangle, \langle s_2/0.7\rangle, \langle s_3/0.4\rangle\} = \{\langle s_1, s_4/0.3\rangle, \langle s_2/0.6\rangle, \langle s_3/0.4\rangle\}$.

(iv) $\mathscr{FN}\delta\mathscr{S}LC \Rightarrow C = A \wedge B = \{\langle s_1, s_4/0.4\rangle, \langle s_2/0.6\rangle, \langle s_3/0.5\rangle\} \wedge \{\langle s_1, s_4/0.3\rangle, \langle s_2/0.7\rangle, \langle s_3/0.4\rangle\} = \{\langle s_1, s_4/0.3\rangle, \langle s_2/0.6\rangle, \langle s_3/0.4\rangle\}$.

(v) $\mathscr{FN}ZLC \Rightarrow C = A \wedge B = \{\langle s_1, s_4/0.4\rangle, \langle s_2/0.6\rangle, \langle s_3/0.5\rangle\} \wedge \{\langle s_1, s_4/0.3\rangle, \langle s_2/0.7\rangle, \langle s_3/0.4\rangle\} = \{\langle s_1, s_4/0.3\rangle, \langle s_2/0.6\rangle, \langle s_3/0.4\rangle\}$.

Proposition 1. *Let $(U, \tau_{\mathscr{F}}(F))$ be a $\mathscr{FN}ts$. (i) Every $\mathscr{FN}Zc$ (resp. $\mathscr{FN}Zo$) set is $\mathscr{FN}ZLC$ set. (ii) Every $\mathscr{FN}c$ (resp. $\mathscr{FN}o$) set is $\mathscr{FN}LC$ set. (iii) Every $\mathscr{FN}\delta c$ (resp. $\mathscr{FN}\delta o$) set is $\mathscr{FN}\delta LC$ set. (iv) Every $\mathscr{FN}\delta\mathscr{S}c$ (resp. $\mathscr{FN}\delta\mathscr{S}o$) set is $\mathscr{FN}\delta\mathscr{S}LC$ set. (v) Every $\mathscr{FN}\delta\mathscr{S}c$ (resp. $\mathscr{FN}\delta\mathscr{S}o$) set is $\mathscr{FN}\delta\mathscr{S}LC$ set.*

Proof. (i) Let A be a $\mathscr{FN}Zc$ set in U. Then A can be written as $A = A \wedge 1_N$, where A is a $\mathscr{FN}ZLC$ set and 1_N is a $\mathscr{FN}Zo$ set. Therefore A is a $\mathscr{FN}Zc$ set. The rest of the cases are the same. □

Proposition 2. *Let $(U, \tau_{\mathscr{F}}(F))$ be a $\mathscr{FN}ts$. Every $\mathscr{FN}LC$ set is $\mathscr{FN}ZLC$ (resp. $\mathscr{FN}LC$ and $\mathscr{FN}ZLC$) set.*

Proof. Let A be a $\mathscr{FN}LC$ set in U. Then A can be written as $A = U \wedge V$, where U is $\mathscr{FN}o$ set and V is $\mathscr{FN}c$ set. Since every $\mathscr{FN}o$ ($\mathscr{FN}c$) set is $\mathscr{FN}Zo$ ($\mathscr{FN}Zc$), A is the intersection of $\mathscr{FN}Zo$ set and $\mathscr{FN}Zc$ set and hence A is $\mathscr{FN}ZLC$ set. The rest of the cases are the same. □

Remark 2. The converse of the preceding proposition does not have to be true, as the following example demonstrates.

Example 2. In Example 1, $C = A \wedge B = \{\langle s_1, s_4/0.4\rangle, \langle s_2/0.6\rangle, \langle s_3/0.5\rangle\} \wedge \{\langle s_1, s_4/0.3\rangle, \langle s_2/0.7\rangle, \langle s_3/0.4\rangle\} = \{\langle s_1, s_4/0.3\rangle, \langle s_2/0.6\rangle, \langle s_3/0.4\rangle\}$. Then C is $\mathscr{FN}ZLC$ but not $\mathscr{FN}LC$.

Theorem 1. *Let $(U, \tau_{\mathscr{F}}(F))$ be a $\mathscr{FN}ts$. Then O is $\mathscr{FN}ZLC$ (resp. $\mathscr{FN}LC$,Every $\mathscr{FN}\delta\mathscr{S}c$ (resp. $\mathscr{FN}\delta\mathscr{S}o$) set is $\mathscr{FN}\delta\mathscr{S}LC$) if and only if $O = B \wedge \mathscr{FN}Zcl(O)$ (resp. $O = B \wedge \mathscr{FN}cl(O)$, $O \leq B \wedge \mathscr{FN}Zcl(O)$ and therefore $O = B \wedge \mathscr{FN}Zcl(O)$ for some $\mathscr{FN}Zo$) for some $\mathscr{FN}Zo$ (resp. $\mathscr{FN}o$,$O \leq \mathscr{FN}Zcl(O)$ implies $O \leq B \wedge \mathscr{FN}Zcl(O)$) set B.*

Proof. Let O be a $\mathscr{FN}ZLC$ set. Then $O = B \wedge C$, where B is $\mathscr{FN}Zo$ set and C is $\mathscr{FN}Zc$ set in X. Since $O \leq C$, $\mathscr{FN}Zcl(O) \leq \mathscr{FN}Zcl(C)$ and so $B \wedge \mathscr{FN}Zcl(O) \leq O$. Olso $O \leq B$ and $O \leq \mathscr{FN}Zcl(O)$ implies $O \leq B \wedge \mathscr{FN}Zcl(O)$ and therefore $O = B \wedge \mathscr{FN}Zcl(O)$ for some $\mathscr{FN}Zo$ set B. Conversely, assume $O = B \wedge \mathscr{FN}Zcl(O)$. Since $\mathscr{FN}Zcl(O)$ is $\mathscr{FN}Zc$ set and B is $\mathscr{FN}Zo$ set, O is $\mathscr{FN}Zc$ set. □

The rest of the cases are the same.

4. Fuzzy nano Z extremally disconnected space

In this section, we introduce fuzzy nano Z extremally disconnected space and we obtain several characterizations based on fuzzy set.

Definition 10. Let $(U, \tau_{\mathscr{F}}(F))$ be a $\mathscr{FN}ts$ is called fuzzy nano (resp δ, δS, P and Z) extremally disconnected (briefly, $\mathscr{FN}Ex\ DC\ on$ (resp. $\mathscr{FN}Ex\ DC\ on$, $\mathscr{FN}Ex\ DC\ on$, $\mathscr{FN}ZEx\ DC\ on$ and $\mathscr{FN}ZEx\ DC\ on$)) space if the $\mathscr{FN}$ (resp. $\mathscr{FN}ts$, $\mathscr{FN}ZEx\ DC\ on$, $\mathscr{FN}Zo$ and $\mathscr{FN}Z$) closure of every $\mathscr{FN}o$ (resp. $\mathscr{FN}ts$, $\mathscr{FN}ZEx\ DC\ on$, $\mathscr{FN}Zo$ and $\mathscr{FN}Zo$) set in U is $\mathscr{FN}o$ (resp.$\mathscr{FN}ts$, $\mathscr{FN}ZEx\ DC\ on$, $\mathscr{FN}Zo$ and $\mathscr{FN}Zo$) set in U, or equivalently, if the $\mathscr{FN}$ (resp. $\mathscr{FN}ZEx\ DC\ on$, $\mathscr{FN}Zo$ and $\mathscr{FN}Z$) interior of every $\mathscr{FN}c$ (resp. $\mathscr{FN}ZEx\ DC\ on$, $\mathscr{FN}Zo$ and $\mathscr{FN}Zc$) set of U is $\mathscr{FN}c$ (resp$\mathscr{FN}ts$, $\mathscr{FN}ZEx\ DC\ on$, $\mathscr{FN}Zo$ and $\mathscr{FN}Zc$) set in U.

Example 3. In Example 1, $\mathscr{FN}Z$ closure of every $\mathscr{FN}Zo$ set in U is $\mathscr{FN}Zo$ set in U.

Remark 3. Every $\mathscr{FN}Ex\ DC\ on$ space is $\mathscr{FN}ZEx\ DC\ on$ (resp. $\mathscr{FN}ZEx\ DC\ on$, $\mathscr{FN}Zo$) space.

Theorem 2. *Let $(U, \tau_{\mathscr{F}}(F))$ be a $\mathscr{FN}ts$. Then the following are similar.*

(i) U is $\mathscr{FN}ZEx\ DC\ on$ space.

(ii) $\mathscr{FN}Z\text{int}(A)$ is $\mathscr{FN}Zc$ set, for each $\mathscr{FN}Zc$ set of U.

(iii) $\mathscr{FN}Zcl(\mathscr{FN}Zcl(A))^c = (\mathscr{FN}Zcl(A))^c$ for each $\mathscr{FN}Zo$ set of U.

(iv) $B = (\mathcal{F}\mathcal{N}Zcl(A))^c$ *implies* $\mathcal{F}\mathcal{N}Zcl(B) = (\mathcal{F}\mathcal{N}Zcl(A))^c$ *for each pair of* $\mathcal{F}\mathcal{N}Zo$ *set* A & B *of* U.

Proof. (i) ⇒ (ii) Let A be a $\mathcal{F}\mathcal{N}Zc$ set of U. Then A^c is $\mathcal{F}\mathcal{N}Zo$ set of U. Since U is $\mathcal{F}\mathcal{N}ZEx\ DC\ on$ space, $\mathcal{F}\mathcal{N}Zcl(A^c)$ is $\mathcal{F}\mathcal{N}Zo$ set. But $\mathcal{F}\mathcal{N}Zo$ set $\mathcal{F}\mathcal{N}Zcl(A^c) = (\mathcal{F}\mathcal{N}Z\text{int}(A))^c$. Therefore $\mathcal{F}\mathcal{N}Z\text{int}(A)$ is $\mathcal{F}\mathcal{N}Zc$ set. (ii) ⇒ (iii) Suppose that A is a $\mathcal{F}\mathcal{N}Zo$ set of U. Then $\mathcal{F}\mathcal{N}Zcl(\mathcal{F}\mathcal{N}Zcl(A))^c = \mathcal{F}\mathcal{N}Zcl(\mathcal{F}\mathcal{N}\ Z\text{int}(A)^c)$. By assumption, $\mathcal{F}\mathcal{N}Z\text{int}(A)^c$ is a $\mathcal{F}\mathcal{N}Zc$ set of U.So, $\mathcal{F}\mathcal{N}Zcl(\mathcal{F}\mathcal{N}Z\text{int}(A)^c) = \mathcal{F}\mathcal{N}Z\text{int}(A)^c = (\mathcal{F}\mathcal{N}Zcl(A))^c$. (iii) ⇒ (iv) Let A and B be $\mathcal{F}\mathcal{N}Zo$ sets of U. We put $B = (\mathcal{F}\mathcal{N}Zcl(A))^c$. From the assumption, $\mathcal{F}\mathcal{N}Zcl(B) = (\mathcal{F}\mathcal{N}Zcl(\mathcal{F}\mathcal{N}Zcl(A)))^c = (\mathcal{F}\mathcal{N}Zcl(A))^c$. (iv) ⇒ (i) Let A be a $\mathcal{F}\mathcal{N}Zo$ set of U. Let $B = (\mathcal{F}\mathcal{N}Zcl(A))^c$. From the assumption, we obtain that $\mathcal{F}\mathcal{N}Zcl(B) = (\mathcal{F}\mathcal{N}Zcl(A))^c$. So, $(\mathcal{F}\mathcal{N}Zcl(B))^c = \mathcal{F}\mathcal{N}Zcl(A)$. Hence $\mathcal{F}\mathcal{N}Z\text{int}(B^c) = \mathcal{F}\mathcal{N}Z\ cl(A)$. Thus $\mathcal{F}\mathcal{N}Zcl(A)$ is a $\mathcal{F}\mathcal{N}Zo$ set A of U. Then U is $\mathcal{F}\mathcal{N}ZEx\ DC\ on$ space. □

Remark 4. The Theorem 2 also holds for $\mathcal{F}\mathcal{N}ZNor$ and $\mathcal{F}\mathcal{N}ZC$ sets.

Theorem 3. *Let* $(U, \tau_{\mathcal{F}}(F))$ *be a* $\mathcal{F}\mathcal{N}ts$ *is* $\mathcal{F}\mathcal{N}ZEx\ DC\ on$ *(resp.* $\mathcal{F}\mathcal{N}Ex\ DC\ on$, $\mathcal{F}\mathcal{N}Ex\ DC\ on$, $\mathcal{F}\mathcal{N}Ex\ DC\ on$ *and* $\mathcal{F}\mathcal{N}Ex\ DC\ on$*) space if and only if* $\mathcal{F}\mathcal{N}Zcl(A) = \mathcal{F}\mathcal{N}Z\text{int}(\mathcal{F}\mathcal{N}Zcl(A))$ *(resp.* $\mathcal{F}\mathcal{N}Zcl(B) = (\mathcal{F}\mathcal{N}Zcl(\mathcal{F}\mathcal{N}Zcl(A)))^c = (\mathcal{F}\mathcal{N}Zcl(A))^c$, $\mathcal{F}\mathcal{N}Zcl(\mathcal{F}\mathcal{N}Z\text{int}(A)^c) = \mathcal{F}\mathcal{N}Z\text{int}(A)^c = (\mathcal{F}\mathcal{N}Zcl(A))^c$ *and* $\mathcal{F}\mathcal{N}Zcl(B) = (\mathcal{F}\mathcal{N}Zcl(\mathcal{F}\mathcal{N}\ Zcl(A)))^c = (\mathcal{F}\mathcal{N}Zcl(A))^c$*) for every* $\mathcal{F}\mathcal{N}Zo$ *(resp.* $\mathcal{F}\mathcal{N}o$, $\mathcal{F}\mathcal{N}ZNor$ *and* $\mathcal{F}\mathcal{N}ZC$*) set* A *of* U.

Proof. Let A be a $\mathcal{F}\mathcal{N}Zo$ set in a $\mathcal{F}\mathcal{N}ZEx\ DC\ on$ space. Then $\mathcal{F}\mathcal{N}Zcl(A)$ is $\mathcal{F}\mathcal{N}Zo$ set. This implies $\mathcal{F}\mathcal{N}Zcl(A) = \mathcal{F}\mathcal{N}Z\text{int}(\mathcal{F}\mathcal{N}Zcl(A))$. Conversely, Let A be a $\mathcal{F}\mathcal{N}Zo$ set and $\mathcal{F}\mathcal{N}Zcl(A) = \mathcal{F}\mathcal{N}Z\text{int}(\mathcal{F}\mathcal{N}Zcl(A))$. Hence $\mathcal{F}\mathcal{N}Z\,cl(A)$ is $\mathcal{F}\mathcal{N}Zo$ set. Therefore U is $\mathcal{F}\mathcal{N}ZEx\ DC\ on$ space. □

The rest of the cases are the same.

5. Fuzzy nano Z normal spaces

In this section, we first present fuzzy nano Z normal spaces and scrutinize their essential properties.

Definition 11. Let $(U, \tau_{\mathcal{F}}(F))$ be a $\mathcal{F}\mathcal{N}ts$ is said to be fuzzy nano (resp. δ, δS, P and Z) normal (briefly, $\mathcal{F}\mathcal{N}Nor$ (resp.$\mathcal{F}\mathcal{N}ZNor$ and $\mathcal{F}\mathcal{N}ZC$ and $\mathcal{F}\mathcal{N}Zc$ and $\mathcal{F}\mathcal{N}ZNor$)) normal if for any two disjoint $\mathcal{F}\mathcal{N}c$ (resp. $\mathcal{F}\mathcal{N}ZNor$ and $\mathcal{F}\mathcal{N}ZC$ and $\mathcal{F}\mathcal{N}Zc$) sets A and B, ∃ disjoint $\mathcal{F}\mathcal{N}o$ (resp$\mathcal{F}\mathcal{N}ZNor$ and $\mathcal{F}\mathcal{N}ZC$ and $\mathcal{F}\mathcal{N}Zc$ and $\mathcal{F}\mathcal{N}Zo$) sets L & $M \ni A \leq L$ and $B \leq M$.

Proposition 3. *Every* $\mathcal{F}\mathcal{N}Nor$ *is* $\mathcal{F}\mathcal{N}ZNor$.

Proof. Let U be a $\mathcal{F}\mathcal{N}Nor$. Then for any two disjoint $\mathcal{F}\mathcal{N}c$ sets A and B respectively, then there exists disjoint $\mathcal{F}\mathcal{N}o$ sets

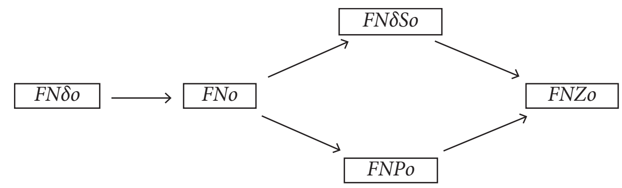

Figure 1: $\mathcal{F}\mathcal{N}Zos$'s in $\mathcal{F}\mathcal{N}ts$.

L & M such that $A \leq L$ and $B \leq M$. Since every $\mathcal{F}\mathcal{N}o$ sets are $\mathcal{F}\mathcal{N}Zo$ sets by Figure 1. Hence, U is $\mathcal{F}\mathcal{N}ZNor$. □

Theorem 4. *In a* $\mathcal{F}\mathcal{N}ts\ (U, \tau_{\mathcal{F}}(F))$*, the following are comparable: (i)* U *is* $\mathcal{F}\mathcal{N}ZNor$*. (ii)* $\forall\, \mathcal{F}\mathcal{N}Zc$ *set* A *in* U *and every* $\mathcal{F}\mathcal{N}Zo$ *set* L *containing* A*, ∃ a* $\mathcal{F}\mathcal{N}Zo$ *set* M *containing* $A \ni \mathcal{F}\mathcal{N}cl(M) \leq L$*. (iii) For each pair of disjoint* $\mathcal{F}\mathcal{N}Zc$ *sets* A & B *in* U*, ∃ a* $\mathcal{F}\mathcal{N}Zo$ *set* L *containing* $A \sum \mathcal{F}\mathcal{N}cl(L) \wedge B = 0_N$*. (iv) For each pair of disjoint* $\mathcal{F}\mathcal{N}Zc$ *sets* $A \ni B$ *in* U*,* $\exists\, \mathcal{F}\mathcal{N}Zo$ *sets* L & M *containing* A *and* B *respectively* $\sum \mathcal{F}\mathcal{N}cl(L) \wedge \mathcal{F}\mathcal{N}cl(M) = 0_N$.

Proof. (i) ⇒ (ii): Let L be a $\mathcal{F}\mathcal{N}Zo$ set containing the $\mathcal{F}\mathcal{N}Zc$ set A. Then $B = L^c$ is a $\mathcal{F}\mathcal{N}Zc$ set disjoint from A. Since U is $\mathcal{F}\mathcal{N}ZNor$, ∃ disjoint $\mathcal{F}\mathcal{N}Zo$ sets M and W containing A and B respectively. Then $\mathcal{F}\mathcal{N}cl(M)$ is disjoint from B. Since if $y_\beta \in B$, the set W is a $\mathcal{F}\mathcal{N}Zo$ set containing y_β disjoint from M. Hence $\mathcal{F}\mathcal{N}cl(M) \leq L$. (ii) ⇒ (iii): Let A and B be disjoint $\mathcal{F}\mathcal{N}Zc$ sets in U. Then B^c is a $\mathcal{F}\mathcal{N}Zo$ set containing A. By (ii), there exists a $\mathcal{F}\mathcal{N}Zo$ set L containing $A \sum \mathcal{F}\mathcal{N}cl(L) \leq B^c$. Hence $\mathcal{F}\mathcal{N}cl(L) \wedge B = 0_N$. This proves (iii). □

(iii) ⇒ (iv): Let A and B be disjoint $\mathcal{F}\mathcal{N}Zc$ sets in U. Then, by (iii), there exists a $\mathcal{F}\mathcal{N}Zo$ set L containing $A \sum \mathcal{F}\mathcal{N}cl(L) \leq B^c$. Since $\mathcal{F}\mathcal{N}cl(L)$ is $\mathcal{F}\mathcal{N}Zc$, B and $\mathcal{F}\mathcal{N}cl(L)$ are disjoint $\mathcal{F}\mathcal{N}Zc$ sets in U. Again by (iii), there exists a $\mathcal{F}\mathcal{N}Zo$ set M containing $\sum \mathcal{F}\mathcal{N}cl(L) \wedge \mathcal{F}\mathcal{N}cl(M) = 0_N$. This proves (iv).

(iv) ⇒ (i): Let A and B be the disjoint $\mathcal{F}\mathcal{N}Zc$ sets in U. By (iv), $\exists\, \mathcal{F}\mathcal{N}Zo$ sets L and M containing A and B respectively $\sum \mathcal{F}\mathcal{N}cl(L) \wedge \mathcal{F}\mathcal{N}cl(M) = 0_N$. Since $L \wedge M \leq \mathcal{F}\mathcal{N}cl(L) \wedge \mathcal{F}\mathcal{N}cl(M)$, L and M are disjoint $\mathcal{F}\mathcal{N}Zo$ sets containing A and B respectively. Thus U is $\mathcal{F}\mathcal{N}ZNor$.

Theorem 5. *Let* $(U, \tau_{\mathcal{F}}(F))$ *be a* $\mathcal{F}\mathcal{N}ts$ *is* $\mathcal{F}\mathcal{N}ZNor$ *if and only if* $\forall\, \mathcal{F}\mathcal{N}Zc$ *set* F & $\mathcal{F}\mathcal{N}Zo$ *set* G *containing* F*, there exists a* $\mathcal{F}\mathcal{N}Zo$ *set* $M \sum F \leq M \leq \mathcal{F}\mathcal{N}cl(M) \leq G$.

Proof. Let $(U, \tau_{\mathcal{F}}(F))$ be $\mathcal{F}\mathcal{N}ZNor$. Let F be a $\mathcal{F}\mathcal{N}Zc$ set and let G be a $\mathcal{F}\mathcal{N}Zo$ set containing F. Then F & G^c are disjoint $\mathcal{F}\mathcal{N}Zc$ sets. Since U is $\mathcal{F}\mathcal{N}ZNor$, ∃ disjoint $\mathcal{F}\mathcal{N}Zo$ sets M_1 & $M_2 \ni F \leq M_1$ and $G^c \leq M_2$. Thus $F \leq M_1 \leq M_2^c \leq G$. Since M_2^c is $\mathcal{F}\mathcal{N}Zc$, so $\mathcal{F}\mathcal{N}cl(M) \leq \mathcal{F}\mathcal{N}cl\ (M_2^c) = M_2^c \leq G$. Take $M = M_1$.This implies that $F \leq M \leq \mathcal{F}\mathcal{N}cl(M) \leq G$. Conversely, assume the situation remains the same. Let H_1 & H_2 be two disjoint $\mathcal{F}\mathcal{N}Zc$ sets in U. Then H_2^c is a $\mathcal{F}\mathcal{N}Zo$ set containing H_1. By assumption, there exists a $\mathcal{F}\mathcal{N}Zo$ set $M \sum H_1 \leq M \leq \mathcal{F}\mathcal{N}cl(M) \leq H_2^c$ Since M is $\mathcal{F}\mathcal{N}Zo$ and $\mathcal{F}\mathcal{N}cl(M)$ is $\mathcal{F}\mathcal{N}Zc$. Then $(\mathcal{F}\mathcal{N}cl(M))^c$ is

$\mathcal{FN}Zo$. Now $\mathcal{FN}cl(M) \leq H_2^c$ implies that $H_2 \leq (\mathcal{FN}cl(M))^c$ Also $M \wedge (\mathcal{FN}cl(M))^c \leq \mathcal{FN}cl(M) \wedge (\mathcal{FN}cl(M))^c = 0_N$. That is M and $(\mathcal{FN}cl(M))^c$ are disjoint $\mathcal{FN}Zo$ sets containing H_1 and H_2 respectively. This shows that $(U, \tau_{\mathcal{F}}(F))$ is $\mathcal{FN}ZNor$. □

Theorem 6. *For a $\mathcal{FN}ts$ $(U, \tau_{\mathcal{F}}(F))$, then the following are comparable: (i) U is $\mathcal{FN}ZNor$. (ii) For any two $\mathcal{FN}Zo$ sets L & M whose union is 1_N, $\exists \mathcal{FN}Zc$ subsets A of L & B of M whose union is also U.*

Proof. (i) ⇒ (ii): Let L & M be two $\mathcal{FN}Zo$ sets in a $\mathcal{FN}ZNor$ space $U \ni 1_N = L \vee M$. Then L^c, M^c are disjoint $\mathcal{FN}Zc$ sets. Since U is $\mathcal{FN}ZNor$, then ∃ disjoint $\mathcal{FN}Zo$ sets G_1 & $G_2 \ni L^c \leq G_1$ and $M^c \leq G_2$. Let $A = G_1^c$ and $B = G_2^c$. Then A & B are $\mathcal{FN}Zc$ subsets of L & M respectively $\ni A \vee B = 1_N$. This proves (ii). (ii) ⇒ (i): Let A & B be disjoint $\mathcal{FN}Zc$ sets in U. Then A^c and B^c are $\mathcal{FN}Zo$ sets whose union is 1_N. By (ii), there exists $\mathcal{FN}Zc$ sets F_1 & $F_2 \ni F_1 \leq A^c, F_2 \leq B^c$ and $F_1 \vee F_2 = 1_N$. Then F_1^c & F_2^c are disjoint $\mathcal{FN}Zo$ sets containing A and B respectively. Therefore U is $\mathcal{FN}ZNor$. □

Theorem 7. *Let f: $(U_1, \tau_{\mathcal{F}}(F_1)) \longrightarrow (U_2, \tau_{\mathcal{F}}(F_2))$ be a function. (i) If f is injective, $\mathcal{FN}ZIrr$, $\mathcal{FN}Zo$ & U_1 is $\mathcal{FN}ZNor$ then U_2 is $\mathcal{FN}ZNor$. (ii) If f is $\mathcal{FN}ZIrr$, $\mathcal{FN}Zc$ and U_2 is $\mathcal{FN}ZNor$ then U_1 is $\mathcal{FN}ZNor$.*

Theorem 8. *If given a pair of disjoint $\mathcal{FN}Zc$ sets A, B of $(U, \tau_{\mathcal{F}}(F))$, there is $\mathcal{FN}ZCts$ function $f \sum f(A) = 0_N$ and $f(B) = 1_N$, then $(U, \tau_{\mathcal{F}}(F))$ is $\mathcal{FN}ZNor$.*

Theorem 9. *Let f: $(U_1, \tau_{\mathcal{F}}(F_1)) \longrightarrow (U_2, \tau_{\mathcal{F}}(F_2))$ be a function. If f is a $\mathcal{FN}Cts$, $\mathcal{FN}ZO$ bijection of a $\mathcal{FN}Nor$ space U_1 into a space U_2 and if every $\mathcal{FN}Zc$ set in U_2 is $\mathcal{FN}c$, then U_2 is $\mathcal{FN}ZReg$.*

Proof. Let M_1 and M_2 be $\mathcal{FN}Zc$ sets in U_2. Then by assumption, M_2 is $\mathcal{FN}c$ in U_2. Since f is a $\mathcal{FN}Cts$ bijection, $f^{-1}(M_1)$ and $f^{-1}(M_2)$ is a $\mathcal{FN}c$ set in U_1. Since U_1 is $\mathcal{FN}Nor$, there exist disjoint $\mathcal{FN}o$ sets L_1 and L_2 in $U_1 \sum f^{-1}(M_1) \leq L_1$ and $f^{-1}(M_2) \leq L_2$. Since f is $\mathcal{FN}ZO$, $f(L_1)$ and $f(L_2)$ are disjoint $\mathcal{FN}Zo$ sets in U_2 containing M_1 and M_2 respectively. Hence U_2 is $\mathcal{FN}ZNor$. □

Remark 5. Theorems 4, 5, 6, 7, 8 & 9 are also holds for $\mathcal{FN}ZNor$ and $\mathcal{FN}ZC$ sets.

6. Strongly fuzzy nano Z normal spaces

The principles of strongly fuzzy nano Z normal spaces are introduced in this section. We describe each of these notions and show how they are related to one another.

Definition 12. A $\mathcal{FN}ts(U, \tau_{\mathcal{F}}(F))$ is said to be strongly fuzzy nano Z (resp δ, δS, P) normal (briefly, $St\mathcal{FN}ZNor$ (resp. $\mathcal{FN}ZNor$ and $\mathcal{FN}ZC$ and $\mathcal{FN}Zc$)) if for every pair of disjoint $\mathcal{FN}c$ sets A & B in U, there are disjoint $\mathcal{FN}Zo$ (resp. $\mathcal{FN}ZNor$ and $\mathcal{FN}ZC$) sets L and M in U containing A & B respectively.

Example 4. In Example 1, $A = \{\langle s_1, s_4/0.8\rangle, \langle s_2/0.7\rangle, \langle s_3/0.6\rangle\}$, $B = \{\langle s_1, s_4/0.9\rangle, \langle s_2/0.7\rangle, \langle s_3/0.6\rangle\}$ are $\mathcal{FN}c$ sets. $L = \{\langle s_1, s_4/0.8\rangle, \langle s_2/0.7\rangle, \langle s_3/0.7\rangle\}$, $M = \{\langle s_1, s_4/0.9\rangle, \langle s_2/0.7\rangle, \langle s_3/0.7\rangle\}$ are $\mathcal{FN}Zc$ sets in U containing A & B respectively.

Theorem 10. *Let $(U, \tau_{\mathcal{F}}(F)) - b \pm \sqrt{b^2 - 4ac}/2a$ be a $\mathcal{FN}ts$. Every $\mathcal{FN}ZNor$ space is $St\mathcal{FN}ZNor$.*

Proof. Suppose U is $\mathcal{FN}ZNor$. Let A & B be disjoint $\mathcal{FN}c$ sets in U. Then A & B are $\mathcal{FN}Zc$ in U. Since U is $\mathcal{FN}ZNor$, ∃ disjoint $\mathcal{FN}o$ sets L & M containing A and B respectively. Since, every $\mathcal{FN}o$ is $\mathcal{FN}Zo$, L and M are $\mathcal{FN}Zo$ in U. This implies that U is $St\mathcal{FN}ZNor$. □

Theorem 11. *In a $\mathcal{FN}ts$ $(U, \tau_{\mathcal{F}}(F))$, the following are comparable: (i) U is $St\mathcal{FN}ZNor$. (ii) ∀ $\mathcal{FN}c$ set F in U and every $\mathcal{FN}o$ set L containing F, there exists a $\mathcal{FN}Zo$ set M containing F $\sum \mathcal{FN}Zcl(M) \leq L$. (iii) For each pair of disjoint $\mathcal{FN}c$ sets M_1 & M_2 in U, there exists a $\mathcal{FN}Zo$ set L containing $M_1 \sum \mathcal{FN}Zcl(L) \wedge M_2 = 0_N$.*

Proof. (i) ⇒ (ii): Let L be a $\mathcal{FN}o$ set containing the $\mathcal{FN}c$ set F. Then $H = L^c$ is a $\mathcal{FN}c$ set disjoint from F. Since U is $St\mathcal{FN}ZNor$, ∃ disjoint $\mathcal{FN}Zo$ sets M and W containing F & H respectively. Then $\mathcal{FN}Zcl(M)$ is disjoint from H, since if $y_\beta \in H$, the set W is a $\mathcal{FN}Zo$ set containing y_β disjoint from M. Hence $\mathcal{FN}Zcl(M) \leq L$. (ii) ⇒ (iii): Let M_1 & M_2 be disjoint $\mathcal{FN}c$ sets in U. Then M_2^c is a $\mathcal{FN}o$ set containing M_1. By (ii), there exists a $\mathcal{FN}Zo$ set L containing $M_1 \sum \mathcal{FN}Zcl(L) \leq M_2^c$. Hence $\mathcal{FN}Zcl(L) \wedge M_2 = 0_N$. This proves (iii). (iii) ⇒ (i): Let M_1 and M_2 be the disjoint $\mathcal{FN}Zc$ sets in U. By (iii), there exists a $\mathcal{FN}Zo$ set L containing $M_1 \sum \mathcal{FN}Zcl(L) \wedge M_2 = 0_N$. Take $M = \mathcal{FN}Zcl(L)^c$. Then L and M are disjoint $\mathcal{FN}Zo$ sets containing M_1 and M_2 respectively. Thus U is $St\mathcal{FN}ZNor$. □

Theorem 12. *For a $\mathcal{FN}ts$ $(U, \tau_{\mathcal{F}}(F))$, then the following are comparable: (i) U is $St\mathcal{FN}ZNor$. (ii) For any two $\mathcal{FN}o$ sets L & M whose union is 1_N, $\exists \mathcal{FN}Zc$ subsets M_1 of L and M_2 of M whose union is also 1_N.*

Proof. (i) ⇒ (ii): Let L & M be two $\mathcal{FN}o$ sets in a $St\mathcal{FN}ZNor$ space $U \ni 1_N = L \vee M$. Then L^c, M^c are disjoint $\mathcal{FN}c$ sets. Since U is $St\mathcal{FN}ZNor$, then ∃ disjoint $\mathcal{FN}Zo$ sets G_1 and $G_2 \ni L^c \leq G_1$ and $M^c \leq G_2$. Let $M_1 = G_1^c$ and $M_2 = G_2^c$. Then M_1 & M_2 are $\mathcal{FN}Zc$ subsets of L and M respectively $\ni M_1 \vee M_2 = 1_N$. This proves (ii). (ii) ⇒ (i): Let M_1 & M_2 be disjoint $\mathcal{FN}c$ sets in U. Then M_1^c and M_2^c are $\mathcal{FN}o$ sets whose union is U. By (ii), there exists $\mathcal{FN}Zc$ sets F_1 and $F_2 \ni F_1 \leq M_1^c, F_2 \leq M_2^c$ and $F_1 \vee F_2 = 1_N$. Then F_1^c and F_2^c are disjoint $\mathcal{FN}Zo$ sets containing M_1 and M_2 respectively. Therefore U is $St\mathcal{FN}ZNor$. □

Theorem 13. *Let h: $(U_1, \tau_{\mathcal{F}}(F_1)) \longrightarrow (U_2, \tau_{\mathcal{F}}(F_2))$ be a function. (i) If h is injective, $\mathcal{FN}Cts$, $\mathcal{FN}ZO$ and U_1 is*

Table 1: Fuzzy values for patients.

Symptoms/Patient	Pat [1]	Pat [2]	Pat [3]	Pat [4]	Pat [5]
Weight gain	0.9	0.8	0.0	0.3	0.3
Tiredness	0.0	0.1	0.8	0.1	0.6
Myalgia	0.3	0.8	0.3	0.2	0.3
Swelling of legs	0.9	0.4	0.2	0.4	0.4
Mensus Problem	0.2	0.3	0.4	0.9	0.7

$St\mathcal{FN}ZNor$ then U_2 is $St\mathcal{FN}ZNor$. (ii) If h is $\mathcal{FN}ZIrr$, $\mathcal{FN}ZO$ and U_2 is $St\mathcal{FN}ZNor$ then U_1 is $St\mathcal{FN}ZNor$.

Proof. (i) Suppose U_1 is $St\mathcal{FN}ZNor$. Let M_1 and M_2 be disjoint $\mathcal{FN}c$ sets in U_2. Since h is $\mathcal{FN}Cts$, $h^{-1}(M_1)$ and $h^{-1}(M_2)$ are $\mathcal{FN}c$ in U_1. Since U_1 is $St\mathcal{FN}ZNor$, $\exists$ disjoint $\mathcal{FN}Zo$ sets L and M in $U_1 \sum h^{-1}(M_1) \leq L$ and $h^{-1}(M_2) \leq M$. Now $h^{-1}(M_1) \leq L \Rightarrow M_1 \leq h(L)$ and $h^{-1}(M_2) \leq M \Rightarrow M_2 \leq h(M)$. Since h is a $\mathcal{FN}ZO$ map, $h(L)$ and $h(M)$ are $\mathcal{FN}Zo$ in U_2. Also $L \wedge M = 0_N \Rightarrow h(L \wedge M) = 0_N$ and h is injective, then $h(L) \wedge h(M) = 0_N$. Thus $h(L)$ and $h(M)$ are disjoint $\mathcal{FN}Zo$ sets in U_2 containing M_1 and M_2 respectively. Thus, U_2 is $St\mathcal{FN}ZNor$. (ii) Suppose U_2 is $\mathcal{FN}ZNor$. Let M_1 and M_2 be disjoint $\mathcal{FN}c$ sets in U_1. Since h is $\mathcal{FN}ZIrr$ and $\mathcal{FN}ZC$, $h(M_1)$ and $h(M_2)$ are $\mathcal{FN}Zc$ in U_2. Since U_2 is $\mathcal{FN}ZNor$, there exist disjoint $\mathcal{FN}Zo$ sets L and M in $U_2 \sum h(M_1) \leq L$ and $h(M_2) \leq M$. That is $M_1 \leq h^{-1}(L)$ and $M_2 \leq h^{-1}(M)$. Since h is $\mathcal{FN}ZIrr$, $h^{-1}(L)$ and $h^{-1}(M)$ are disjoint $\mathcal{FN}Zo \sum M_1 \leq h^{-1}(L)$ and $M_2 \leq h^{-1}(M)$. Thus U_1 is $\mathcal{FN}ZNor$. □

Remark 6. Theorems 10, 11, 12 & 13 are also holds for $\mathcal{FN}ZNor$ and $\mathcal{FN}ZC$ sets.

7. Fuzzy score function

We provide a fuzzy scoring function for decision-making problems using fuzzy information in this part, which is based on a methodical approach.

Definition 13. Let S: $M \longrightarrow [0, 1]$. The Fuzzy score function (in short, $\mathcal{F}SF$) is $S(M) = 1/k \sum_{i=1}^{k} \mu_{M_i}$ that represents the average of positiveness of the fuzzy component μ_M.

The specific technique to deal with selecting the correct qualities and alternatives in a decision-making situation utilising fuzzy sets is proposed in the following fundamental steps.

Step 1: Problem field selection: Consider the universe of discourse (set of objects) m, the set of alternatives n, the set of decision attributes p.

Step 2: Construct a fuzzy matrix of alternative verses objects and object verses decision attributes. Calculation Part:

Step 3: Frame the in-discernibility relation R on m.

Step 4: Construct the fuzzy nano topologies τ_j and ν_k.

Step 5: Find the score values by Definition 1 each of the entries of the $\mathcal{FN}ts$. Conclusion part:

Step 6: Organize the fuzzy score values of the alternatives $\tau_1 \leq \tau_2 \leq \cdots \leq \tau_n$ and the attributes $\nu_1 \leq \nu_2 \leq \cdots \leq \nu_p$. Choose the attribute ν_p for the alternative τ_1 and ν_{p-1} for the alternative τ_2 etc. If $n < p$, then ignore ν_k, where $k = 1, 2, \ldots, n - p$.

7.1. Numerical example. New medical breakthroughs have expanded the number of data available to clinicians, which includes vulnerabilities. The process of grouping multiple sets of symptoms under a single term of illness is extremely challenging in medical diagnosis. In this section, we use a medical diagnosis problem to demonstrate the usefulness and applicability of the above-mentioned approach.

Step 1: Problem field selection: Consider the following tables, which provide information from five patients who were consulted by physicians, Patient 1 (Pat [5]), Patient 2 (Pat [5]), Patient 3 (Pat [5]), Patient 4 (Pat [5]), Patient 5 (Pat [5]) and symptoms are Weight gain (Wg), Tiredness (Td), Myalgia (Ml), Swelling of legs (Sl), Mensus Problem (Mp). We need to find the patient and to find the disease such as Lymphedema, Insomnia, Hypothyroidism, Menarche, Arthritis of the patient. The data in Tables 1 and 2 are explained by the membership, the indeterminacy and the non-membership functions of the patients and diseases respectively.

Step 2: Construct the in-discernibility relation for the correlation between the symptoms is given as $U/R = \{\{Wg\}, \{Ml\}, \{T\ d\}, \{Sl\}, \{Mp\}\},$.

Step 3: From fuzzy nano topologies for (τ_j) and (ν_k):

(i) $\tau_1^* = \{0_{\mathcal{F}}, 1_{\mathcal{F}}, 0.9, 0.3, 0.2\}$.
(ii) $\tau_2^* = \{0_{\mathcal{F}}, 1_{\mathcal{F}}, 0.8, 0.1, 0.4, 0.3\}$.
(iii) $\tau_3^* = \{0_{\mathcal{F}}, 1_{\mathcal{F}}, 0.8, 0.3, 0.2, 0.4\}$.
(iv) $\tau_4^* = \{0_{\mathcal{F}}, 1_{\mathcal{F}}, 0.3, 0.1, 0.2, 0.4, 0.9\}$.
(v) $\tau_5^* = \{0_{\mathcal{F}}, 1_{\mathcal{F}}, 0.3, 0.6, 0.4, 0.7\}$.

(i) $\nu_1^* = \{0_{\mathcal{F}}, 1_{\mathcal{F}}, 0.2, 0.7, 0.9\}$.
(ii) $\nu_2^* = \{0_{\mathcal{F}}, 1_{\mathcal{F}}, 0.9, 0.2\}$.
(iii) $\nu_3^* = \{0_{\mathcal{F}}, 1_{\mathcal{F}}, 0.9, 0.1, 0.2\}$.
(iv) $\nu_4^* = \{0_{\mathcal{F}}, 1_{\mathcal{F}}, 0.6, 0.1, 0.2, 0.9\}$.
(v) $\nu_5^* = \{0_{\mathcal{F}}, 1_{\mathcal{F}}, 0.1, 0.9, 0.4, 0.3\}$.

Step 5: Find fuzzy score functions: (i) $\mathcal{F}SF(\tau_1) = 0.48$. (ii) $\mathcal{F}SF(\tau_2) = 0.4333$. (iii) $\mathcal{F}SF(\tau_3) = 0.45$. (iv) $\mathcal{F}SF(\tau_4) = 0.4143$. (v) $\mathcal{F}SF(\tau_5) = 0.5$.

(i) $\mathcal{F}SF(\nu_1) = 0.56$. (ii) $\mathcal{F}SF(\nu_2) = 0.525$. (iii) $\mathcal{F}SF(\nu_3) = 0.44$. (iv) $\mathcal{F}SF(\nu_4) = 0.4667$. (v) $\mathcal{F}SF(\nu_5) = 0.45$.

Step 6: Final decision: Arrange fuzzy nano score values for the alternatives $\tau_1, \tau_2, \tau_3, \tau_4, \tau_5$ and the attributes ν_1,

Table 2: Fuzzy values for disease.

Symptoms/patients	Weight gain	Tiredness	Myalgia	Swelling of legs	Mensus problem
Lymphedema	0.0	0.2	0.7	0.9	0.2
Insomnia	0.0	0.9	0.2	0.2	0.2
Hypothyroidism	0.9	0.1	0.0	0.1	0.2
Menarche	0.6	0.1	0.2	0.2	0.9
Arthritis	0.0	0.1	0.9	0.4	0.3

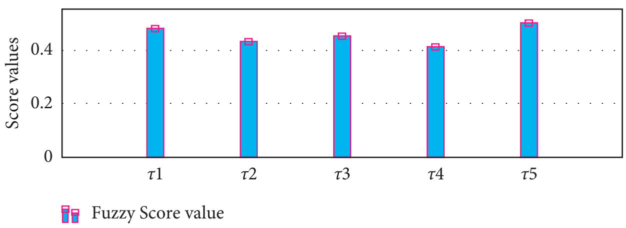

Figure 2: Fuzzy score values for diseases.

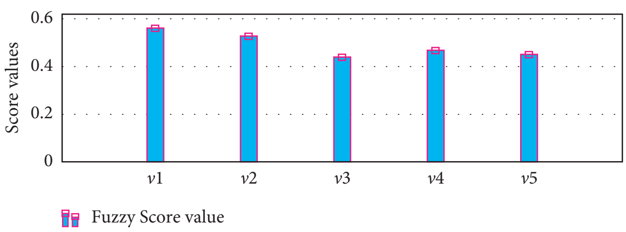

Figure 3: Fuzzy Score values for Patients.

ν_2, ν_3, ν_4, ν_5 in ascending order. We get the following sequences $\tau_5 \leq \tau_1 \leq \tau_3 \leq \tau_2 \leq \tau_4$ and $\nu_1 \leq \nu_2 \leq \nu_4 \leq \nu_5 \leq \nu_3$. Thus the patient Pat ^{5} suffers from Hypothyroidism, the patient Pat^5 suffers from Arthritis, the patient Pat^5 suffers from Menarche, the patient Pat^5 suffers from Lymphedema and the patient Pat^5 suffers from Insomnia. The results are presented in Figures 2 and 3.

8. Final thoughts and future work

This paper adds to the growing body of knowledge about fuzzy nano topological spaces. The obtained results show that most of the offered concepts' nano topological features are kept in the framework of fuzzy nano topologies, implying that some topological prerequisites are unnecessary. Because the study's limitations are relaxed, exploring nano topological notions using fuzzy nano topologies has a benefit. On the other hand, by extending fuzzy nano Z locally closed sets, a few characteristics of particular topological concepts are partially lost. We will finish introducing the main fuzzy nano topological concepts using fuzzy nano Z open sets, such as fuzzy nano Z locally continuous, respective mappings and homomorphisms, separation axioms, compactness and connectedness in fuzzy nano topological spaces, in this work. Our study plan also includes testing the concepts and results presented here with various generalisations of fuzzy nano Z open sets, such as fuzzy nano e open and fuzzy nano Z^* open sets. Furthermore, we will use these expansions of fuzzy nano Z open sets to present new types of rough approximations and apply them to improve set accuracy metrics.

Data Availability

Data used to support this study are included within this paper.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

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