

Research Article

Application of Fuzzy Case-Based Reasoning and Fuzzy Analytic Hierarchy Process for Machining Cutter Planning and Control

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Cutter planning and control are the crucial problems in machining processes. The current literature indicates that the issue of cutter planning and control problem was not adequately researched in the past in a metal-cutting process. Usually, cutter planning and control problems were addressed using different optimization, simulation, and computer-aided planning (CAP) methods. To bridge this knowledge gap, this study proposed a decision support system (DSS) that can integrate fuzzy case-based reasoning (F-CBR) and fuzzy analytic hierarchy process (F-AHP) methods. This integration was applied to determine hybrid similarity measures between new and prior cases. The study provides new insights into the integration of fuzzy set theory (FST), CBR, and AHP for solving machining cutter planning and control problems. Our proposed system retrieves the best similar prior cases to reuse and adapt them to new order arrivals. A numerical example was illustrated to validate the soundness of the researched DSS.

1. Introduction

The major requirements for modern manufacturers (in the era of Industry 4.0) are flexibility, responsiveness, improved quality, and resource utilization [1, 2]. Assigning the right cutter is one of the major factors for machining operations to meet the stated requirements [3–5]. Cutter planning and control methods are used to improve machining productivity and equipment availability [6]. Several researchers focused on managing the flow cutters in contemporary manufacturing [2, 6, 7, 8, 9–11]. Rahimifard and Newman [9], Rahimifard and Newman [8], Özbayrak and Bell [10], Petruse and Brîndaşu [6], and Kasie et al. [11] suggested that cutter management strategies should be integrated with system design, planning, and control activities to reduce operational costs.

Cutters can contribute to 30-50% savings of the total operating costs in the machining process although it costs around 2-4% of the total production cost [6]. To address this problem, several frameworks have been proposed in the past. These were reviewed in different studies

[7, 12–14]. These proposed cutter planning approaches applied linear and nonlinear optimization techniques [12, 15, 16], heuristics [13], domain knowledge-based expert systems [17, 18], and computer-aided process planning (CAPP) [19-21]. Optimization models are computationally intractable when the input parameters are large in number. Heuristic algorithms are usually trapped in a local optimum. In rule-based expert systems, it is difficult to represent the complex domain knowledge of experts in the form of rules alone [1, 22–24]. In reality, cutter assignment and control problems are unstructured and open ended. Recent studies revealed that only about 20% of information is found in structured and numeric data in organizations; the remaining 80% of information is hidden in unstructured forms [25, 26]. To accommodate these features of the current manufacturing, this study proposed an intelligent decision support system (DSS) by integrating the fuzzy versions of case-based reasoning (CBR) and analytic hierarchy process (AHP). This integration was not applied to the problems of cutter assignment and control.

Regarding the novelty of the proposed DSS and its new contribution, this study proposed a new methodological approach to solve the problems of cutter planning and control. To solve the stated problem, this study integrated the existing complex theories in artificial intelligence (AI), specifically CBR and fuzzy set theory (FST), and in multipleattribute decision-making (MADM), particularly AHP and distance from the target methods. The CBR component of the proposed framework was used to construct part orders/ cases with hybrid attributes in a fuzzy environment. The AHP was utilized to determine the optimal weights of case attributes by soliciting the knowledge and experiences of experts. In addition, distance from the target approach, which is called weighted Euclidean distance, was applied for calculating hybrid similarity between new and prior cases using as inputs the outputs of F-CBR and F-AHP. In the previous related studies, such kind of combination was not utilized to solve the problems of cutter planning and control. This indicates that the DSS framework proposed in this study can have a significant academic contribution to the existing literature in DSS research for solving the problems of cutter management. The proposed framework is highly recommended when limited prior data are available in manufacturing systems.

The proposed DSS used an F-CBR method to represent unstructured (fuzzy) information from the product and process attributes of order arrivals. It utilized an F-AHP approach to prioritize case attributes in case retrieval. The proposed DSS can retrieve prior cases that have the most similar assigned cutter sets to the current order arrivals using case similarity measures. The DSS can present attribute differences between the current and retrieved cases. Based on this difference, the retrieved cases (cutter sets) were adapted as solutions for cutter requirements for new order arrivals by implementing a set of rules. This is useful for operational managers to plan and control cutters in manufacturing systems. In addition, they can enumerate the available and missed cutters and plan the purchase of the missed cutters. This study illustrated a numerical analysis to test the proposed DSS in a simulated machining environment.

This paper incorporates six sections. Section 2 reviews related studies in cutter planning and control problems. Section 3 explains the methodological approach. Section 4 analyses the results from a simulated numerical example. Section 5 discusses the results of the study. Lastly, the conclusions are articulated in Section 6.

2. Literature Review

This section reviews different studies related to cutter management strategies in metal-cutting operations. They are shown in Table 1.

The review of related studies indicated that cutter planning and problems were articulated using optimization, MADM, heuristics, simulation, and AI (e.g., ANNs, GA, and RBR) methods. From this review, it can be understood that cutter planning and control problems are a crucial agenda in recent studies. However, the integration of F-CBR and F-AHP was not sufficiently addressed in previous research on cutter planning and control problems. This study is intended to bridge this study gap in metal-cutting operations. It provides new insights into the integration of FST, CBR, and AHP for solving machining cutter planning and control problems. The proposed DSS in this study is strongly recommended when decision-making systems suffer from a shortage of prior data.

3. Methodological Approach

3.1. Methodological Integration of CBR, FST, and AHP. This section explains the methodological integration of FST, CBR, and AHP that are applied to develop an intelligent DSS to address the problem of machining cutter planning and control.

Case-based reasoning (CBR) is one of the wellrecognized analogical and inductive reasoning approaches in AI [23]. CBR approaches mimic the human reasoning and decision-making process by referring to successful experiences of similar new problems [22]. Recently, the CBR methodology has been used in a variety of problem-solving and interpretive tasks. For instance, some of the successful applications were presented by Kasie et al. [11], Kasie and Bright [32, 33, 34], Zhang et al. [35], and Zhao et al. [36]. Furthermore, it can be easily trained by a few training datasets as compared with other machine learning technologies [1, 33, 34, 37]. The CBR methodology is useful to develop advisory systems and provide recommendations to human decision-makers in unstructured and complex situations [38]. In addition, CBR systems can progress from accumulated experience to improve the accuracy of similar case retrieval activities as shown by Kasie et al. [11], Kasie and Bright [33], and Zhang et al. [35].

According to Aamodt and Plaza [22], the CBR methodology can be explained by phases such as (a) retrieving the best similar past case, (b) reusing the knowledge and experiences in the retrieved case, (c) revising the retrieved case to adapt it as a solution to the current problem, and (d) retaining the adapted solution for future retrieval. To support the CBR process, rules from the general domain are usually incorporated into the CBR methodology [22, 24, 39]. This application is shown in recent studies of manufacturing operations [1, 2, 11, 34].

A case is contextualized knowledge and experience that can be represented using hybrids of multiple attributes [2, 35]. In real industries, some of the case attributes represent uncertainty, incompleteness, and vagueness [1, 40–42]. If at least one of the case attributes is represented in such situations that force decision-makers to describe the case attribute by fuzzy set/fuzzy number/ linguistic terms, then the constructed case is known as a fuzzy case [43]. The reasoning or decision-making process based on this kind of constructed case is called fuzzy case-based reasoning (F-CBR) by referring to several studies in different problem domains (e.g., see [11, 33, 34, 35, 36, 41, 42]). These fuzzy attributes are expressed in verbal terms that improve the agility of CBR

	Methodology	Discrete event simulation	Rule-based reasoning (RBR)	Heuristics of tool life to tool size ratio	CAPP software	Augmented reality technologies	Fuzzy logic and multiple-attribute decision-making	Genetic algorithm (GA)	Analytic hierarchy process (AHP)	Artificial neural networks (ANN), GA, and fuzzy logic	Ontological approach	Lean benchmarking	CBR, DES, and relational database management tools	Neutrosophic CBR and best-worst method (BWM)
TABLE 1: Related studies in cutter management.	Proposed system	A scheduling system for the simultaneous planning of workpieces, cutting tools, and fixtures in FMS	A knowledge-based DSS for short-term scheduling of part-cutter assignment	Cutting tool selection and allocation in FMS	A system for cutting tool planning integrated with CAPP	An augmented reality system for cutter planning and control	A fuzzy MADM system to select complaint polishing cutters	Two models for cutter delivery and cutter demand prediction in the metal-cutting process	A system to select cutter manufacturing materials	Cutters selection system from a big relational database of machining operations	An ontology-based cutter configuration system for machining process	A system for cutter planning and control at strategic, technical, and logistical aspects	A theoretical DSS model for stabilizing the flows of cutters, fixtures, and jigs	A DSS for part-cutting assignment and control in turning operations
	Authors	Rahimifard and Newman [9]	Özbayrak and Bell [10]	Buyurgan et al. [13]	Meseguer and Gonzalez [14]	Petruse and Brîndaşu [6]	Arunachalam et al. [27]	Sun et al. [28]	Li et al. [29]	Saranya et al. [30]	Zhou et al. [31]	Tomelero et al. [4]	Kasie et al. [11]	Kasie and Bright [32]
	SN	1	2	ŝ	4	5	9	4	7	8	6	10	12	13

ŧ . 4:00 rta Pr Related ÷ methodologies [2, 41, 42]. For further reading, the recent applications of an F-CBR methodology in different problem domains were reviewed by Kasie et al. [1] and Zhang et al. [35]. These cases are easily constructed using an object-oriented case representation method as shown by Bergmann et al. [44], Kasie et al. [1], and Kasie [2].

Cases are usually constructed using hybrids of multiple attributes to treat historical cases as alternative courses of action and case attributes as selection criteria [1, 2, 35, 36, 42, 45]. The importance of integrating multiple-attribute decision-making (MADM) methods in the CBR methodology for weighing case attributes and calculating hybrid similarity measures for case retrieval was studied by Kasie et al. [1], Kasie and Bright [33, 34], and Zhang et al. [35]. For weighting case attributes, AHP is a well-known expert knowledge/experience elicitation approach in MADM [46]. Its applications for case attribute weighting were presented by different recent studies [1, 2, 33, 34, 42, 47].

AHP decomposes and synthesizes hierarchically complex decision problems to determine the preference of an attribute to other case attributes using a pairwise comparison [48–50]. The recent developments of AHP and its integrated applications with other methods were reviewed by Ishizaka and Labib [51] and Ho [50]. Among them, the combinations of the AHP and CBR methodologies were studied by Kuo [52], An et al. [53], Changchien and Lin [54], Faez et al. [42], Wu et al., [55], Park and Han [56], Kasie et al. [1], Kasie [2], and Kasie and Bright [33, 34]. According to Chen and Hwang [40], fuzzy set was not incorporated in the original AHP. It was studied to include FST by Van Laarhoven and Pedrycz [57] and Buckley [58]. For further reading, the various applications of F-AHP were reviewed by Demirel et al. [59]. The advantages of F-AHP over other MADM methods to rank case attributes were studied in recent studies (see, for example, [2, 33, 34]). Furthermore, Ozdogan et al. [60] presented the advantage of F-AHP over a fuzzy technique for order preference by similarity to the ideal solution (F-TOPSIS) method for evaluating the performance of municipal services. In addition, Lesani et al. [61] applied an F-AHP to rank and select the best objectoriented (OO) programming language using different factors.

3.2. Preliminaries. The following basics of fuzzy set theory (FST) that are useful for this study are stated by referring to the studies by Chen et al. [62], Zimmermann [43], Faez et al. [42], and Kasie and Bright [33].

Definition 1. A fuzzy set (FS) \check{A} is defined in a universe of discourse X, in which its elements are designated by x; then, FS \check{A} is expressed by order pairs: $\check{A} = \{(x, \mu_{\check{A}}(x))/x \in X\}$, where $\mu_{\check{A}}(x)$ is the membership function of x to FS \check{A} and its value is a real number within the interval [0, 1].

Definition 2. An FS \check{A} is a normal FS in the universe of discourse *X* if it contains at least one element $x \in X/\mu_{\check{A}}(x) = 1$ as presented in Figure 1.



FIGURE 1: A fuzzy number Å.

Definition 3. A FS \hat{A} is a convex FS in the universe of discourse *X* if and only if for any two elements of \check{A} such that $x_1, x_2 \in X$ and any $\lambda \in [0, 1]$; then, $\mu_{\check{A}} (\lambda x_1 + (1 - \lambda) x_2) \ge \min \{\mu_{\check{A}} (x_1), \mu_{\check{A}} (x_2) \text{ (see Figure 1).} \}$

Definition 4. A fuzzy number (FN) A is an FS in the universe of X that fulfills the requirements of normality and convexity of fuzzy sets (Figure 1).

This study used specific FNs such as trapezoidal NFs (\check{A}_2) in the form of $\check{a}_1 \leq \check{a}_2 \leq \check{a}_3 \leq \check{a}_4$ and/or triangular fuzzy numbers (\check{A}_1) in the form of $\check{a}_1 \leq \check{a}_2 \leq \check{a}_3$, which are shown in Figure 2, where $\check{a}_1, \check{a}_2, \check{a}_3$, and \check{a}_4 are real numbers.

Definition 5. As presented by Kasie and Bright [33], the membership function of a triangular FN $\breve{A}, \mu_{\breve{A}}(x)$ is defined as follows:

$$\mu_{\check{A}}(x) = \begin{cases} \frac{x - \check{a}_1}{\check{a}_2 - \check{a}_1}, & \check{a}_1 \le x \le \check{a}_2, \\\\ \frac{\check{a}_3 - x}{\check{a}_3 - \check{a}_2}, & \check{a}_2 \le x \le \check{a}_3, \\\\ 0, & \text{otherwise.} \end{cases}$$
(1)

Definition 6. Similarly, the membership function of a trapezoidal FN \breve{A} is defined as follows:

$$\mu_{\check{A}}(x) = \begin{cases} \frac{x - \check{a}_{1}}{\check{a}_{2} - \check{a}_{1}}, & \check{a}_{1} \le x \le \check{a}_{2}, \\ 1, & \check{a}_{2} \le x \le \check{a}_{3}, \\ \\ \frac{\check{a}_{4} - x}{\check{a}_{4} - \check{a}_{3}}, & \check{a}_{3} \le x \le \check{a}_{4}, \\ 0, & \text{otherwise.} \end{cases}$$
(2)

Definition 7. According to Hejazi et al. [63] and Kasie and Bright [34], let $\breve{P} = (\breve{p}_1, \breve{p}_2, \breve{p}_3, \breve{p}_4)$ and $\breve{Q} = (\breve{q}_1, \breve{q}_2, \breve{q}_3, \breve{q}_4)$ be two trapezoidal fuzzy numbers in standard forms, which are



FIGURE 2: Triangular FN (A_1) and trapezoidal FN (A_2) .

normal and convex (their maximum membership values or heights, $h_{\breve{p}} = h_{\breve{Q}} = 1$). If trapezoidal fuzzy numbers are in standard forms, they can be expressed as $0 \le \breve{P} = (\breve{p}_1, \breve{p}_2,$

 $\check{p}_3, \check{p}_4) \le 1$ and $0 \le \check{Q} = (\check{q}_1, \check{q}_2, \check{q}_3, \check{q}_4) \le 1$; the distance between two trapezoidal FNs can be calculated as follows:

distance
$$(\breve{P},\breve{Q}) = \left[\left(\sum_{i=1}^{4} \frac{\left| \breve{P}_{i} - \breve{q}_{i} \right|}{4} \right) \left(\frac{\min\left(P\left(\breve{P}\right), P\left(\breve{Q}\right)\right)}{\max\left(P\left(\breve{P}\right), P\left(\breve{Q}\right)\right)} \right) \left(\frac{\min\left(A\left(\breve{P}\right), A\left(\breve{Q}\right)\right) + 1}{\max\left(A\left(\breve{P}\right), A\left(\breve{Q}\right)\right) + 1} \right) \right],$$
 (3)

where $P(\check{P})$ and $P(\check{Q})$ are the geometric perimeters of trapezoidal FN \check{P} and FN \check{Q} , respectively, $A(\check{P})$ and $A(\check{Q})$ are the geometric areas of trapezoidal FN \check{P} and FN \check{Q} , respectively, and $h_{\check{P}}$ and $h_{\check{Q}}$ are the maximum membership values of normal trapezoidal FN \check{P} and FN \check{Q} , respectively.

Definition 8. According to Hejazi et al. [63] and Chen and Sanguansat [64], let $\breve{P} = (\breve{p}_1, \breve{p}_2, \breve{p}_3, \breve{p}_4)$ and $\breve{Q} = (\breve{q}_1, \breve{q}_2, \breve{q}_3, \breve{q}_4)$ be any two positive trapezoidal FNs, which are normal and convex; the following basic arithmetic operations are applicable for convex and normal trapezoidal FNs:

$$\min(h_{\breve{P}} = 1, h_{\breve{O}} = 1) = 1.$$
(4)

(1) Addition of normal trapezoidal FNs:

$$\check{P} \oplus \check{Q} = ((\check{p}_1 + \check{q}_1, \check{p}_2 + \check{q}_2, \check{p}_3 + \check{q}_3, \check{p}_4 + \check{q}_4); (1)).$$
(5)

(2) Subtraction of normal trapezoidal FNs:

$$\breve{P} \odot \breve{Q} = \left((\breve{p}_1 - \breve{q}_1, \breve{p}_2 - \breve{q}_2, \breve{p}_3 - \breve{q}_3, \breve{p}_4 - \breve{q}_4); (1) \right).$$
(6)

(3) Multiplication of normal trapezoidal FNs:

$$\check{P} \otimes \check{Q} = \left(\left(\check{p}_1 \check{q}_1, \check{p}_2 \check{q}_2, \check{p}_3 \check{q}_3, \check{p}_4 \check{q}_4 \right); (1) \right). \tag{7}$$

(4) Division of normal trapezoidal FNs:

$$\breve{P}_{\overrightarrow{Q}} = \left(\left(\frac{\breve{P}_1}{\breve{q}_4}, \frac{\breve{P}_2}{\breve{q}_3}, \frac{\breve{P}_3}{\breve{q}_2}, \frac{\breve{P}_4}{\breve{q}_1} \right); (1) \right).$$
(8)

(5) Inverse of a normal trapezoidal FN:

$$\breve{P}^{-1} = \left(\frac{1}{\breve{P}_4}, \frac{1}{\breve{P}_3}, \frac{1}{\breve{P}_2}, \frac{1}{\breve{P}_1}\right).$$
(9)

(6) Multiplying a normal trapezoidal FN by a positive constant *c*:

$$c\breve{P} = (c\breve{p}_1, c\breve{p}_3, c\breve{p}_3, c\breve{p}_4).$$
(10)

3.3. Methodological Framework. The proposed methodological framework for this study is presented in Figure 3 for developing an intelligent DSS for cutter planning and control. It integrates CBR, AHP, and FST as stated in Section 3.1. The framework includes four major stages such as (1) the preparation and case construction stage, (2) the case retrieval stage, (3) the case adaptation stage, and (4) the case retraining stage.

3.3.1. Preparation and Case Construction Stage. Firstly, different data cleaning methods were applied to handle noisy data and outliers from the part order descriptions. Then, the data were organized for case construction. The second importance of this stage was selecting experienced experts who could identify hybrid case attributes, which are useful for finding part similarities for planning a set of required cutters. After identifying the hybrid case attributes, different target and prior fuzzy cases were constructed, using fourteen hybrid case attributes. This study used an objective-oriented (OO) case representation method using the freely available Java platform. The elements of the hybrid case attributes are numeric, nominal, symbolic, and linguistic terms.

For case construction, different linguistic terms were converted into standard triangular fuzzy numbers. They are presented in Table 2 and Figure 4. Triangular fuzzy numbers are special trapezoidal fuzzy numbers for $\breve{P} = (\breve{p}_1, \breve{p}_2,$



FIGURE 3: Methodological framework of the proposed DSS.

 \breve{p}_3, \breve{p}_4) and $\breve{p}_2 = \breve{p}_3$ (see Figure 2). Although Table 2 and Figure 4 present normal and convex triangular FNs in standard forms, the conversion scale is very flexible to create trapezoidal fuzzy numbers by joining any adjacent triangular

fuzzy numbers. For example, a trapezoidal FN P = (0.2, 0.3, 0.4, 0.5) can be formed by joining two triangular FNs, $\tilde{A}_1 = (0.2, 0.3, 0.4)$ and $\tilde{A}_2 = (0.3, 0.4, 0.5)$. This eleven-conversion scale was proposed by Kasie et al. [1]. In this

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TABLE 2:	Proposed	linguistic	terms to	r fuzzy	number	conversion	scale	[1].	

Verbal term	Fuzzy number	Triangular fuzzy number
Extremely low (EL)	$\widetilde{0.0}$	(0.0, 0.0, 0.1)
Very low (VL)	$\widetilde{0.1}$	(0.0, 0.1, 0.2)
Low (Lo)	$\widetilde{0.2}$	(0.1, 0.2, 0.3)
Fairly low (FL)	$\widetilde{0.3}$	(0.2, 0.3, 0.4)
Moderately low (ML)	$\widetilde{0.4}$	(0.3, 0.4, 0.5)
Moderate (Mo)	$\widetilde{0.5}$	(0.4, 0.5, 0.6)
Moderately high (MH)	$\widetilde{0.6}$	(0.5, 0.6, 0.7)
Fairly high (FH)	$\widetilde{0.7}$	(0.6, 0.7, 0.8)
High (Hi)	$\widetilde{0.8}$	(0.7, 0.8, 0.9)
Very high (VH)	$\widetilde{0.9}$	(0.8, 0.9, 1.0)
Extremely high (EH)	$\widetilde{1.0}$	(0.9, 1.0, 1.0)



conversion scale, the variable x is a standard fuzzy number within [0, 1] and $\mu_{\tilde{P}}(x)$ is the degree of membership of x to the verbal terms in Figure 4.

To weigh the selected hybrid case attributes, the study used a group-based fuzzy analytic hierarchy process (F-AHP). This weighing is useful while calculating a hybrid similarity measure because all case attributes will not have the same contribution to the case retrieval and decision-making process. The entire group-based F-AHP of this study is presented in Figure 5. The importance of one attribute over the other was stated using verbal terms such as "equally important," "moderately important," "extremely important," and so on in the pairwise comparison of the selected case attributes. The researchers used a combination of the F-AHP algorithm proposed by Buckley [58] and the fuzzy ranking method presented by Chen and Chen [65] for weighting case attributes. The combined algorithm stated here uses the basic mathematic operations expressed by equations (1) and (2) and from equations (4)-(10):

(1) Determine a fuzzy comparison matrix \tilde{A} for *n* case attributes, whose elements are triangular fuzzy numbers:

$$\widetilde{A} = \widetilde{a}_{ij} = \left(b_{ij}, c_{ij}, d_{ij}\right) \quad \forall i, j.$$
(11)

(2) Determine the fuzzy geometric mean of fuzzy comparison values of each criterion as

$$\widetilde{z}_{i} = \left(\prod_{j=1}^{n} \widetilde{a}_{ij}\right)^{1/n}, \quad \forall i.$$
(12)

(3) Find the fuzzy weight *w*_i for each attribute using the product of each *z*_i and inverse of the summation of fuzzy geometric means:

$$\widetilde{w}_{i} = \widetilde{z}_{i} \otimes \left(\sum_{i=1}^{n} \widetilde{z}_{i}\right)^{-1} = (\mathrm{lw}_{i}, \mathrm{mw}_{i}, \mathrm{uw}_{i}), \qquad (13)$$

where lw_i , mw_i , and uw_i are the lower, middle, and upper values of elements in \tilde{w}_i , respectively, which must be arranged in increasing order.

(4) Defuzzify the fuzzy weights into their corresponding crisp values C_i using the method proposed by Chen and Chen [65]:

$$C_i = \frac{C_{i,\text{mean}}}{1 + C_{i,\text{std}}},\tag{14}$$

where $C_{i,\text{mean}}$ and $C_{i,\text{std}}$ are the mean and standard deviation of the fuzzy weight of case attributes, respectively.



FIGURE 5: Proposed group-based case attribute weighting using F-AHP.

(5) Normalize the crisp weights:

$$w_i = \frac{C_i}{\sum_{i=1}^n C_i}.$$
(15)

Table 3 presents the relationships among fuzzy AHPbased linguistic terms, their equivalent triangular fuzzy numbers, and fuzzy reciprocals. Similar approaches were applied by Wu et al. [55], Kasie et al. [1], and Kasie and Bright [34] in other problem domains.

3.3.2. Case Retrieval Stage. This stage is useful for calculating hybrid similarities between historical and target cases. Furthermore, it was used to analyze the attribute variations between the target and prior cases, which are significant inputs for the case adaptation stage. A hybrid distance from the target approach was applied to find these similarity measures. This method usually computes the linear distance between pair values of each attribute and finally determines the cumulative hybrid distance between the two cases (see [2, 33, 34–36]). The weights of case attributes in F-AHP were normalized. The hybrid weighted Euclidean distance between a target case T and a prior case P was calculated as follows:

$$d_{\text{eul}}(T,P) = \sqrt[n]{\sum_{j=1}^{n} \left[w_j \left(d_{\text{eul}} \left(v_j^T, v_j^P \right) \right) \right]^2}$$

$$d_{\text{eul}} \left(v_j^T, v_j^P \right) \in [0, 1],$$
(16)

where *n* is the number of case attributes; W_j is the normalized weight of the *j*th case attribute; $d_{eul}(v_j^T, v_j^P)$ is the attribute value-based distance between a target case *T* and a prior case *P*, and v_j^T and v_j^P are the values of the jth attribute for cases *T* and *P*, respectively.

For calculating individual distances, $d_{eul}(v_j^T, v_j^P)$, for various categories of attributes such as numeric, categorical, symbolic, and linguistic (fuzzy) attributes, the researchers used the same approach as Kasie and Bright [33, 34]. For fuzzy case attributes, the researchers used equation (3) to find the distance between fuzzy case attributes.

For numerical case attribute,

$$d_{\text{eul}}\left(v_{j}^{T}, v_{j}^{P}\right) = \frac{\left|v_{j}^{T} - v_{j}^{P}\right|}{v_{j,\text{max}} - v_{j,\text{min}}},$$

$$v_{j,\text{min}} \leq v_{j}^{T}, v_{j}^{P} \leq v_{j,\text{max}},$$
(17)

where $v_{j,\min}$ and $v_{j,\max}$ are the minimum and maximum values of the *j*th attribute, respectively, to standardize distance measures within [0, 1] to avoid the influences of measurement units and scales.

For categorical and symbolic attributes,

$$d_{\rm eul}(v_j^T, v_j^P) = |v_j^T - v_j^P| = \begin{cases} 1 \text{ if } v_j^T \neq v_j^P, \\ 0 \text{ if } v_j^T = v_j^P. \end{cases}$$
(18)

Since the weighted hybrid distance and similarity are inversely related, the hybrid similarity measure between a target case T and a prior case P, $s_{eul}(T, P)$, was calculated Extremely important

In between

istic terms for the	thangular fuzzy fiumber cor	iversion scale (adapted in	JIII [1]).		
Equivalent fuzzy	number/reciprocal	Equivalent triangular fuzzy			
Number	Reciprocal	Number	Reciprocal		
ĩ	ĩ	(1, 1, 1)	(1, 1, 1)		
$\widetilde{2}$	ĩ/2	(1, 2, 3)	(1/3, 1/2, 1)		
$\tilde{3}$	ĩ/3	(2, 3, 4)	(1/4, 1/3, 1/2)		
$\widetilde{4}$	$\widetilde{1}/4$	(3, 4, 5)	(1/5, 1/4, 1/3)		
5	ĩ/5	(4, 5, 6)	(1/6, 1/5, 1/4)		
$\tilde{6}$	ĩ/6	(5, 6, 7)	(1/7, 1/6, 1/5)		
7	ĩ/7	(6, 7, 8)	(1/8, 1/7, 1/6)		
	Equivalent fuzzy Number Ĩ 2 3 4 5 6 7	Equivalent fuzzy number/reciprocalNumberReciprocal $\tilde{1}$ $\tilde{1}$ $\tilde{2}$ $\tilde{1}/2$ $\tilde{3}$ $\tilde{1}/3$ $\tilde{4}$ $\tilde{1}/4$ $\tilde{5}$ $\tilde{1}/5$ $\tilde{6}$ $\tilde{1}/6$ $\tilde{7}$ $\tilde{1}/7$	$\begin{tabular}{ c c c c c c c c c c c c c c c c c c c$		

 $\tilde{1}/8$

ĩ/9

TABLE 3: Proposed linguistic terms for the triangular fuzzy number conversion scale (adapted from [1])

using the reciprocal of a natural exponential function [35, 36]:

$$s_{\rm eul}(T, P) = \frac{1}{\exp (d_{\rm eul}(T, P))},$$
 (19)

 $\tilde{8}$

õ

$$s_{\text{eul}}(T, P) \in [0, 1].$$

After calculating these similarity measures, the best similar case to the target case was selected as a retrieved case for reuse and adaptation depending on this best similarity measure.

3.3.3. Case Adaptation Stage. The main task of this phase was passing decisions and recommending different activities based on the hybrid similarity of the retrieved and target cases. If the retried and target case is identical $(s_{eul}(T, P) \approx 1)$, the proposed DSS recommends the direct reuse of the retrieved case. This means that a set of cutters assigned to the retrieved part can be reused in the new part order. If they are not identical $(s_{eul}(T, P) < 1)$, the revisions of the retrieved case are recommended. With the help of attribute differences, additional cutters can be added or unnecessary ones can be removed from the retrieved cases to modify them according to the new part order requirement.

Following the recommendations for reuse or revision of the retrieved, the proposed DSS checks the physical availability of the required cutters from the relational database of cutters. If they are physically and healthily available, it plans the assignment of the required cutters for new part orders; otherwise, it recommends the purchase or manufacture of cutters to fulfill the new requirements of new part orders.

3.3.4. Case Retaining Stage. The case retaining stage is usually useful to index the proposed solution for future retrieval when a new similar part arrives. In the case of the proposed system, this part checks the acceptance of the proposed solution by human experts/users. If the proposal is accepted by its users, it can be implemented and indexed as a retained case; otherwise, it can be revised by human experts before it is applied. After the revision, the proposed solution can be implemented and retained for future retrieval.

All computations and decisions of the methodological framework were coded in the Java platform. To develop the proposed DSS, several instance and static functions were

developed and many in-built library functions were applied from the selected platform. In addition, more than eighty rules were used to support the case reasoning process. Most of the applied functions and rules are similar to those applied by Kasie and Bright [33, 34].

(7, 8, 9)

(8, 9, 9)

4. Simulation of a Numerical Example

This section applies the methodological integration and framework presented in Section 3 by simulating different part orders as target and prior cases. The proposed DSS was implemented in a simulated machining environment (turning center) that produces different rotational elements using a set of cutters.

4.1. Preparation and Case Construction Stage. Three human experts were selected by the researchers to identify essential case attributes that are useful for planning and assigning cutters. These three experts were selected by the researchers based on their technical knowledge and experiences in turning/machining operations at a shop floor level. Their knowledge and experiences were utilized to select crucial case attributes and to rate the importance of the selected attributes using F-AHP. The experts selected fourteen attributes for case construction and a hybrid similarity measure between target and historical cases/orders.

The reason for selecting fourteen case attributes was experience and knowledge from the selected experts for turning operations. The researchers learned from the experts that part-cutter assignment tasks are highly dependent on (1) part geometries such as the diameter and cut depth of the workpieces and the quality of finished products such as the precision and surface finish of products. For example, if a high-precision product is ordered, a high-quality cutter is needed for different turning operations. (2) Construction materials such as material composition, hardening, and heat treatment affect the physical properties of parts. Based on this, an identical cutter cannot be assigned for hard and soft construction materials. (3) The types of operations required to machine parts directly determine the cutter requirements. For example, the cutter required for thread-making operations is different from boring operations. In this regard, the simulated machining center for this case uses only seven operations such as turning, facing, knurling, threading, reaming, boring, and drilling.

(1/9, 1/8, 1/7)

(1/9, 1/9, 1/8)

The simulated part attributes were preprocessed to handle outliers and noise. For example, the dimensions of workpieces, which were beyond the capability of the simulated process, were filtered. The simulated prior and target cases were represented using an object-oriented approach in the Java platform. The selected fourteen attributes were hierarchically evaluated by three experts by using an F-AHP. The mean values of the three experts were calculated to determine the optimal weight of each case attribute. For this evaluation, several equations from equations (11)–(15) were applied to prioritize the case attributes.

The fourteen case features selected by the three experts were expressed with a hybrid of numeric, symbolic, nominal, and fuzzy attributes. A hybrid case representation is useful to describe different types of case attributes using more appropriate measurement scales, and it makes the case construction process more flexible [33-36]. The selected attributes are useful for assigning cutters for rotational machining operations. This study used numeric attributes to measure the diameter (Dm) and the depth of turn (Dt) of the workpieces in millimeters. That was because these two attributes could easily be measured using numeric values in millimeters. Fuzzy features were applied to describe the precision (Pr) and surface roughness (Sr) of the finished parts. Mostly, these attributes were very difficult to measure in terms of numeric values. Instead, they could be described by fuzzy/linguistic terms such as high precision and low surface roughness. The hardness (Ha) of the workpieces was expressed in terms of fuzzy features. It could not be described in any other categories of attributes except in fuzzy terms. The fuzzy terms were converted into fuzzy numbers using the conversion scales proposed in Figure 4. Symbolic attributes were used to describe the types of construction materials (Cm) and heat treatment (Ht) of workpieces. There are different classes of material composition and heat treatment types. These classes were described by symbolic/text terms as case attributes. Rotational operations were expressed using nominal attributes. This includes some of the common operations such as turning (Tr), facing (Fg), knurling (Kn), threading (Td), reaming (Re), boring (Bg), and drilling (Dg). When a part order used a specific operation, it was given a nominal attribute of "1" for that operation; otherwise, it was given a nominal attribute of "0." Similar hybrid case representations were applied by several studies in other problem domains (e.g., see Kasie [2], Kasie and Bright [33, 34], Zhang et al. [35], and Zhao et al. [36]).

These fourteen case attributes were hierarchically structured into three levels. The summary of these hierarchical evaluation results is shown in Table 4. The primary level of the hierarchy incorporates three major attributes: (a) part geometry (PG), (b) part construction (PC), and (c) machining operations (MO). Mostly, the selection of cutters highly depends on the physical geometry of workpieces, the characteristics of part construction materials, and the types of operation used to machine part orders. The three major case attributes were subdivided into their middle-level subattributes. The middle-level subattributes were also branched into the bottom-level attributes. The normalized weights of the major attributes and subattributes at their specific levels were calculated using the concepts presented in Table 3, in Figure 5, and from equations (11)-(15).

The role of experts was to rate case attributes independently using the F-AHP at three levels. The fuzzy ratings of the three experts (group-based) using triangular FNs are presented in Tables 5–10. These were used as inputs to compute the optimal weight of case attributes. From these inputs, the researchers calculated the averages of fuzzy ratings from the experts using equation (10). The averages of the fuzzy evaluations (triangular FNs) are shown in Tables 11–16. Finally, from equations (11)–(15), they were applied to determine the optimal weights of case attributes. For example, the three major case attributes were evaluated by three experts separately using the F-AHP (Table 5), and the average fuzzy values are determined as shown in Table 11.

The result from Table 11 is equivalent to the fuzzy matrix, $\tilde{A} = \tilde{a}_{ij}$, from equation (11) for n = 3. After determining this fuzzy table/matrix, from equations (12)–(15), they were applied by combining the fuzzy ranking methods proposed by Buckley [58] and Chen and Chen [65]. The weights for the three primary attributes PG, PC, and MO were found as $w_1 = 0.123$, $w_2 = 0.294$, and $w_3 = 0.583$, respectively. The same procedure was applied to the remaining attributes at the specified levels.

The optimal weights were found as $w_1 = 0.621$ and $w_2 = 0.379$ for external and internal machining operations, respectively.

The optimal weights were determined as $w_1 = 0.275$, $w_2 = 0.267$, $w_3 = 0.235$, and $w_4 = 0.223$ for PG subattributes denoted as Pr, Sr, Dm, and Dt, respectively.

The optimal weights were calculated as $w_1 = 0.371$, $w_2 = 0.297$, and $w_3 = 0.332$ for CM subattributes denoted as Cm, Ht, and Ha, respectively.

The optimal weights were determined as $w_1 = 0.172$, $w_2 = 0.104$, $w_3 = 0.465$, and $w_4 = 0.372$ for external operations denoted as Fg, Kn, Tr, and Td, respectively.

The optimal weights were determined as $w_1 = 0.310$, $w_2 = 0.318$, and $w_3 = 0.372$ for internal operations denoted as Re, Bg, and Dg, respectively.

Using the results/outputs from Tables 11–16 and applying equations from (12)–(15), the summarized results in Table 4 are found. The local weight of each attribute at its specific level is shown in (.). The global optimal weight of every attribute was proportionally calculated by multiplying the local weights from the three levels.

To create cases of order arrivals, eight new/target part orders (T1-T8) and three prior cases (P1-P3) were deliberately simulated using the combination of the OO methods from the Java platform and Microsoft Excel tools. The cases are presented in Table 17, including their hybrid attributes for calculating hybrid similarities between the target and prior cases. The three prior cases contain assigned cutter sets (CS), which were the solutions to their cutter requirements.

	Attribute level		Weight	
Primary	Secondary	Third	Global w_i calculation	Global w_i
		Dm (0.235)	(0.123) (0.235)	0.029
DC(0.122)		Dt (0.223)	(0.123) (0.223)	0.027
PG (0.125)	—	Pr (0.275)	(0.123) (0.271)	0.034
		Sr (0.267)	(0.123) (0.271)	0.032
		Cm (0.371)	(0.294) (0.371)	0.109
CM (0.294)	_	Ht (0.297)	(0.294) (0.297)	0.087
		Ha (0.332)	(0.294) (0.332)	0.098
		Fg (0.172)	(0.583) (0.621) (0.172)	0.062
	$E_{\rm return ol} (0.621)$	Kn (0.104)	(0.583) (0.621) (0.104)	0.038
	External (0.621)	Tr (0.465)	(0.583) (0.621) (0.465)	0.168
MO (0.583)		Td (0.259)	(0.583) (0.621) (0.259)	0.094
		Re (0.310)	(0.583) (0.379) (0.306)	0.068
	Internal (0.379)	Bg (0.318)	(0.583) (0.379) (0.322)	0.071
		Dg (0.372)	(0.583) (0.379) (0.372)	0.082

TABLE 4: Hierarchy of case attributes and their weights.

TABLE 5: Group-based expert evaluation of major attributes using F-AHP.

Criteria		PG			PC			МО		
	Exp1	Exp2	Exp3	Exp1	Exp2	Exp3	Exp1	Exp2	Exp3	
PG	ĩ	ĩ	ĩ	ĩ/3	$\tilde{1}/2$	$\tilde{1}/4$	ĩ/5	ĩ/3	ĩ/6	
PC	ĩ	$\widetilde{2}$	$\widetilde{4}$	ĩ	ĩ	ĩ	ĩ/3	$\tilde{1}/2$	ĩ/3	
МО	5	ĩ	$\tilde{6}$	ĩ	$\widetilde{2}$	ĩ	ĩ	ĩ	ĩ	

Note: Exp = Expert.

TABLE 6: Group-based	expert evaluation of	external and inter	nal machining	using F-AHP.
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Criteria		External		Internal				
	Exp1	Exp2	Exp3	Exp1	Exp2	Exp3		
External	ĩ	ĩ	ĩ	ĩ	ĩ	$\widetilde{2}$		
Internal	ĩ/3	ĩ/2	ĩ	ĩ	ĩ	ĩ		

Cuitouio	Pr				Sr			Dm			Dt		
Criteria	Exp1	Exp2	Exp3	Exp1	Exp2	Exp3	Exp1	Exp2	Exp3	Exp1	Exp2	Exp3	
Pr	ĩ	ĩ	ĩ	$\tilde{2}$	ĩ	ĩ	ĩ	ĩ	ĩ	ĩ	ĩ	ĩ	
Sr	$\tilde{1}/2$	ĩ	ĩ	ĩ	ĩ	ĩ	ĩ	$\tilde{2}$	$\tilde{2}$	ĩ	ĩ	ĩ	
Dm	$\tilde{1}/2$	$\tilde{1}/2$	ĩ	ĩ	$\tilde{1}/2$	$\tilde{1}/2$	ĩ	ĩ	ĩ	$\tilde{2}$	ĩ	ĩ	
Dt	$\tilde{1}/2$	$\tilde{1}/2$	ĩ	$\tilde{1}/2$	ĩ	$\tilde{1}/3$	$\tilde{1}/2$	ĩ	ĩ	ĩ	ĩ	ĩ	

TABLE 7: Group-based expert evaluation of PG attributes using F-AHP.

TABLE 8: Group-based expert evaluation of construction material (CM) using F-AHP								(
TABLE 6. Gloup-based expert evaluation of construction material (GM) using 1-ATT	TADIE 8.	Group based	ovnort	ovaluation	of	construction	material	(CM)	incing	E_AHD
	IADLE 0.	Group-Dascu	CAPCIL	cvaluation	01	construction	material	(UNI)	using	1-7111

Criteria		Cm			Ht			Ha		
	Exp1	Exp2	Exp3	Exp1	Exp2	Exp3	Exp1	Exp2	Exp3	
Cm	ĩ	ĩ	ĩ	$\tilde{2}$	ĩ	$\tilde{2}$	ĩ	$\tilde{2}$	ĩ	
Ht	$\tilde{1}/2$	ĩ/3	$\tilde{1}/2$	ĩ	ĩ	ĩ	$\tilde{1}/2$	$\tilde{1}/2$	ĩ	
На	ĩ/2	ĩ	ĩ/2	$\tilde{2}$	$\tilde{2}$	ĩ	ĩ	ĩ	ĩ	

4.2. Hybrid Similarity Measure and Case Retrieval. The hybrid similarity measure between the target and prior cases $s_{eul}(T, P)$ was calculated using equation (19). To calculate case similarities, this study measured individual distances

between corresponding case attributes using equation (3) for fuzzy attributes, equation (17) for numeric attributes, and equation (18) for nominal and symbolic attributes. Then, by integrating normalized case attributes from the F-AHP and

	E~				17			T			T 1		
Criteria	Fg				Kn			lr			Id		
Cincila	Exp1	Exp2	Exp3	Exp1	Exp2	Exp3	Exp1	Exp2	Exp3	Exp1	Exp2	Exp3	
Fg	ĩ	ĩ	ĩ	ĩ	$\tilde{2}$	ĩ	ĩ/3	$\tilde{1}/4$	$\tilde{1}/2$	$\tilde{1}/2$	$\tilde{1}/2$	ĩ	
Kn	$\tilde{1}/2$	ĩ	$\tilde{1}/2$	ĩ	ĩ	ĩ	ĩ/5	$\tilde{1}/4$	ĩ/3	$\tilde{1}/2$	ĩ	ĩ/3	
Tr	ĩ	$\widetilde{4}$	$\widetilde{2}$	$\tilde{5}$	$\widetilde{4}$	ĩ	ĩ	ĩ	ĩ	$\widetilde{2}$	ĩ	ĩ	
Td	ĩ	ĩ	ĩ	ĩ	ĩ	ĩ	$\tilde{1}/2$	ĩ/3	$\tilde{1}/2$	ĩ	ĩ	ĩ	

TABLE 9: Group-based expert evaluation of external operations using F-AHP.

TABLE 10: Group-based expert evaluation of internal operations using F-AHP.

Criteria		Re			Bg		Dg		
	Exp1	Exp2	Exp3	Exp1	Exp2	Exp3	Exp1	Exp2	Exp3
Re	ĩ	ĩ	ĩ	ĩ	ĩ/2	ĩ	ĩ/2	ĩ	$\tilde{1}/2$
Bg	ĩ	$\widetilde{2}$	ĩ	ĩ	ĩ	ĩ	ĩ	ĩ	$\tilde{1}/2$
Dg	$\widetilde{2}$	ĩ	$\widetilde{2}$	ĩ	ĩ	$\widetilde{2}$	ĩ	ĩ	ĩ

TABLE 11: The average fuzzy values of the three experts from Table 5.

Criteria	PG	PC	МО
PG	(1, 1, 1)	(47/180, 13/36, 11/18)	(47/252, 7/30, 19/60)
PC	(2, 3, 4)	(1, 1, 1)	(5/18, 7/18, 2/3)
МО	(11/3, 14/3, 17/3)	(5/3, 11/3, 11/3)	(1, 1, 1)

TABLE	12.	The	average	fuzzy	values	of	the	three	ovnorte	from	Table	6
IABLE	12:	Ine	average	ruzzy	values	OI	the	three	experts	from	Table	6.

Criteria	External	Internal
External	(1, 1, 1)	(4/3, 2, 8/3)
Internal	(19/36, 11/18, 5/6)	(1, 1, 1)

TABLE 13: The average fuzzy values of the three experts from Table 7.

Criteria	Pr	Sr	Dm	Dt
Pr	(1, 1, 1)	(1, 4/3, 5/3)	(1, 5/3, 7/3)	(1, 5/3, 7/3)
Sr	(7/9, 5/6, 1)	(1, 1, 1)	(1, 5/3, 7/3)	(4/3, 2, 8/3)
Dm	(5/9, 2/3, 1)	(5/9, 2/3, 1)	(1, 1, 1)	(1, 4/3, 5/3)
Dt	(5/9, 2/3, 1)	(19/36, 11/18, 5/6)	(7/9, 5/6, 1)	(1, 1, 1)

TABLE 14: The average fuzzy values of the three experts from Table 8.

Criteria	Cm	Ht	Ha
Cm	(1, 1, 1)	(4/3, 7/3, 10/3)	(1, 5/3, 7/3)
Ht	(11/36, 4/9, 5/6)	(1, 1, 1)	(5/9, 2/3, 1)
Ha	(5/9, 2/3, 1)	(1, 5/3, 7/3)	(1, 1, 1)

TABLE 15: The average	fuzzy values	of the three experts f	rom Table 9.
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Criteria	Fg	Kn	Tr	Td
Fg	(1, 1, 1)	(1, 5/3, 7/3)	(47/180, 13/36, 11/18)	(5/9, 2/3, 1)
Kn	(5/9, 2/3, 1)	(1, 1, 1)	(37/180, 47/180, 13/36)	(19/36, 11/18, 5/6)
Tr	(8/3, 3, 13/3)	(3, 4, 5)	(1, 1, 1)	(4/3, 7/3, 10/3)
Td	(1, 5/3, 7/3)	(4/3, 2, 8/3)	(11/36, 4/9, 5/6)	(1, 1, 1)

Criteria	Re	Bg	Dg	
Re	(1, 1, 1)	(7/9, 5/6, 1)	(5/9, 2/3, 1)	
Bg	(1, 4/3, 5/3)	(1, 1, 1)	(7/9, 5/6, 1)	
Dg	(1, 5/3, 7/3)	(1, 4/3, 5/3)	(1, 1, 1)	

TABLE 16: The average fuzzy values of the three experts from Table 10.

TABLE 17: Simulated new and prior cases with fourteen hybrid case attributes.

Casas		Case attributes													CS
Cases	Mt	Ht	Dt	Dm	Pr	Sr	Ha	Tr	Fg	Kn	Td	Dg	Bg	Re	Co
<i>T</i> 1	Carbon steel	No	37	95	$\widetilde{0.6}$	$\widetilde{0.9}$	$\widetilde{0.6}$	1	1	0	1	0	1	0	_
T2	Alloy steel	An	41	150	$\widetilde{0.8}$	$\widetilde{0.6}$	$\widetilde{0.7}$	1	0	0	0	1	1	1	_
T3	Cast iron	No	25	122	$\widetilde{0.5}$	$\widetilde{0.7}$	$\widetilde{0.9}$	1	0	1	0	1	0	1	_
T4	Alloy steel	An	40	150	$\widetilde{0.8}$	$\widetilde{0.6}$	$\widetilde{0.7}$	1	0	1	0	1	0	1	_
T5	Carbon steel	No	26	90	$\widetilde{0.7}$	$\widetilde{0.9}$	$\widetilde{0.6}$	1	1	0	1	0	1	0	_
<i>T</i> 6	Cast iron	An	32	120	$\widetilde{0.5}$	$\widetilde{0.6}$	$\widetilde{0.9}$	1	0	1	0	1	0	1	_
T7	Alloy steel	An	40	150	$\widetilde{0.8}$	$\widetilde{0.6}$	$\widetilde{0.7}$	1	0	1	0	1	0	1	
<i>T</i> 8	Carbon steel	No	35	95	$\widetilde{0.6}$	$\widetilde{0.9}$	$\widetilde{0.6}$	1	1	0	1	0	1	0	
<i>P</i> 1	Alloy steel	An	40	150	$\widetilde{0.8}$	$\widetilde{0.7}$	$\widetilde{0.7}$	1	0	1	1	1	0	1	CS1
P2	Carbon steel	No	35	90	$\widetilde{0.7}$	$\widetilde{0.9}$	$\widetilde{0.6}$	1	1	0	1	1	0	0	CS2
P3	Cast iron	An	30	120	$\widetilde{0.5}$	$\widetilde{0.6}$	$\widetilde{0.9}$	1	0	1	0	1	0	1	CS3

Note: An = annealed and No = normalized to describe the type of heat treatment.

individual distance measures, a weighted and normalized Euclidean distance between new and prior cases, $d_{eul}(T, P)$, was calculated using equation (16). This is one of the MADM methods, and it is usually known as the distance from the target (DFT) method [2, 34]. Using the inverse relationship between distance and similarity, this study applied equation (19) to measure similarities between new and prior cases. The summary of these measures is shown in Table 18. The value of $s_{eul}(T, P)$ is [0, 1] as indicated by equation (19). Its maximum value is the best similarity measure between two cases, and the corresponding prior case is called a retrieved case (*R*).

The similarity measure between the target and retrieved cases, which is the hightest similarity measure between a target and prior cases is denoted by $s_{eul}(T, R)$. The values are shown in bold in Table 18. The assigned cutters of retrieved cases are used as solutions for the cutter requirements of target/new cases. However, the retrieved cases should be revised depending on differences in similarity measures between the two cases. The prior cases served as alternative solutions for the target cases in this study. The retrieved case is the best alternative. In addition, Table 19 presents the retrieved cases (R) for target cases (the best alternative) and the retrieved cutter set (CS) to be adapted as the solution for target cases to fulfill their cutter requirement. The number of alternative solutions for the arrival of each target case is also presented in Table 19. The number of these alternatives increases as several part orders are processed since the processed orders are retained as learned cases for future order arrivals. This happened when the target cases T4, T7, and T8 arrived in the system. The previous target cases T1, T4, and T5 were retrieved to serve as solutions to T4, T7, and T8, respectively.

4.3. Recommended Revisions between Target and Retrieved Cases. As the result summary in Table 19 indicates, most of the retrieved cutter sets must be revised to serve as a solution to the corresponding target cases. This is because there is a significant difference between the retrieved and target cases. For example, the best hybrid similarity between the target T1 and the retrieved P2 is $s_{\rm eul}(T,R) = 0.96$. In this situation, revision of the retrieved cutter set is strongly recommended. Slight differences are observed in the cut depth (1mm) and diameter (5 mm) of the workpieces. Furthermore, a very low (0.1) variation is shown in the precision of the finished products of the two cases. The variations shown in these three attributes can be accepted because they may not be significant in real situations. However, significant variations are indicated in cutting operations. In this regard, because the target case does not require the drilling operation, the drilling cutter has to be removed from the retrieved cutter set (CS2). On the other hand, it needs a boring cutter, which was not included in CS2. Based on this evidence, at least a boring cutter should be added and the drilling cutter has to be removed from the retrieved solution (CS2) to adapt it as a solution for the target problem T1.

Similar revisions can be recommended for target cases *T*2, *T*3, *T*4, and *T*5 depending on the attribute value variations between them and the corresponding retrieved cases shown in Table 19. However, for target cases, *T*6, *T*7, and *T*8, the hybrid similarity between the target and retrieved cases is extremely high, i.e., $s_{eul}(T, R) = 1.0$. In this situation, the retrieved cutters CS3, CS7, and CS8 can be reused without any revisions. The target and retrieved cases are almost identical for these three cases (see Table 17). In addition, the proposed

TABLE 18: Hybrid similarity measures between target and prior cases, $s_{eul}(T, P)$.

			,			e	1	cui		
	<i>P</i> 1	P2	P3	P4/T1	P5/T2	P6/T3	P7/T4	P8/T5	P9/T6	P10/T7
<i>T</i> 1	0.83	0.96	0.80							
<i>T</i> 2	0.94	0.81	0.87	0.81						
Τ3	0.84	0.83	0.93	0.82	0.85					
T4	0.90	0.82	0.89	0.80	0.94	0.87				
T5	0.81	0.90	0.80	0.95	0.80	0.81	0.80			
<i>T</i> 6	0.86	0.81	0.99	0.80	0.88	0.92	0.89	0.79		
T7	0.90	0.82	0.90	0.81	0.93	0.87	1.00	0.79	0.89	
T8	0.81	0.90	0.79	0.93	0.80	0.81	0.79	0.99	0.79	0.80

The bold values indicate the best similarity measures between the target and prior cases, i.e., the similarity measure between the target and the retrieved cases. Retrieved cases are special kinds of prior cases that are with the highest similarity value with the target cases.

inder in outinitially of the cuse retrieval stage	TABLE	19:	Summary	of	the	case	retrieval	stage
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Target	Best alternative (R)	$s_{eul}(T,R)$	Retrieved CS for adaptation	Number of alternatives	Remark
<i>T</i> 1	P2	0.96	CS2	3	
<i>T</i> 2	<i>P</i> 1	0.94	CS1	4	
Τ3	P3	0.93	CS3	5	
T4	T1/P4	0.94	CS4	6	Learned case of T1
<i>T</i> 5	T2/P5	0.95	CS5	7	Learned case of T2
<i>T</i> 6	P3	0.99	CS3	8	
T7	T4/P7	1.00	CS7	9	Learned case of T4
T8	T5/P8	0.99	CS8	10	Learned case of T5

system uses relational databases to check the current state and availability of the required cutters. If they are not available in a functional state, it recommends the fabrication/purchase of them.

4.4. Case Retaining Stage. All revised and reused solutions were retained and indexed for future retrieval of similar cutter assignment problems. This was illustrated for the target cases T4, T5, T7, and T8. For these four target problems, the cutter sets assigned for the previous targets T1, T2, T4, and T5 were retained and retrieved for adaptation and reuse when T4, T5, T7, and, T8 arrived at the system as new problems (see Table 19).

5. Discussion

This section briefly explains the academic contributions, qualitative comparison with previous studies, managerial implications, and limitations of the proposed DSS in this study.

5.1. Academic Contribution. This study reviewed several proposed frameworks/models that were used to solve cutter planning and control problems in manufacturing systems. Most of the previous proposals utilized various mathematical, multiple criteria decision-making, artificial intelligence (ANN, CBR, RBR, and FST), and heuristics (such as GA) methods (Table 1). However, the integration of fuzzy CBR and fuzzy AHP was not applied by previous studies in machining cutter planning and control problems. To bridge this study gap, the paper proposed a novel solution approach in cutter management problems using the integration of these two approaches

for a group-based decision-making process. This integration was applied to cutter management problems for the first time in this study. The results of this combination are presented in Tables 18 and 19. The proposed DSS is very useful in situations when limited prior data are available in manufacturing systems. This indicated that the DSS framework proposed by this study could have a significant academic contribution to the existing literature in DSS research for solving the problems of cutter planning and control.

5.2. Comparison with Existing Studies. When the proposed DSS is compared with the existing studies and similar systems, the proposed system in this study is very flexible to address various environments. The proposed DSS is strongly recommended when manufacturing systems are challenged with a shortage of prior data as compared with frameworks proposed using ANN like the one proposed by Saranya et al. [30]. As shown in Section 4 (Table 18), the proposed DSS was initialized with only three prior cases and progressed over time as more order arrivals were served by the system. This is one of the advantages of the proposed DSS to solve the problems of cutter planning and control. This kind of advantage can never be achieved by ANN methods since ANN methods require a large amount of prior datasets for training and testing ANN-based systems [32-34, 37, 45]. In addition, applying GA methods (e.g., [28]) requires creating a huge number of random solutions for fitness tests, crossover, and mutation operations. The stated operation uses several complex iterations to achieve an optimal solution. This indicates that the proposed DSS in this study is very simple and agile for implementation specifically when manufacturers have a limited amount of prior data due to various reasons. Usually, sufficient data may not be available when manufacturing systems are in a design phase or prior data collection is expensive.

5.3. Managerial Implication. For managerial implications, operational managers can plan and control their machining cutters. They can retrieve previously used cutter sets and adapt/reuse them for the current part orders based on the feature variations between the target and prior part orders as shown in Table 19. Cutter planning and control personnel can enumerate their cutters and investigate the state of the available cutters. Based on this evidence, the manager can plan the purchase or fabrication of missing cutters. This can minimize the unnecessary holding and downtime costs of cutters by stabilizing their flow during planned production windows. This is a useful opportunity to improve the utilization of available resources in machining operations.

5.4. Research Limitation. This study implemented the proposed DSS in a simulated manufacturing environment to illustrate the applicability of the system. To enhance the applicability of the system, the researchers will work to test the proposed system in actual manufacturing environments.

6. Conclusion

Previous studies used different analytical mathematical, multiple criteria decision-making, artificial intelligence, and heuristics methods to solve cutter management problems. However, the integration of fuzzy CBR and fuzzy AHP was applied in machining cutter planning and control problems. To bridge this study gap, the paper proposed a novel solution approach in cutter management problems using the integration of these two approaches for a group-based decisionmaking process. The proposed approach utilized a machinelearning paradigm in artificial intelligence (AI). These characteristics of AI were very useful to improve the performance of the proposed approach over time. For example, referring to Tables 18 and 19, initially, the proposed system started with three alternative solutions; however, for the last target case, ten alternative prior cases were presented as alternative solutions. In machine learning, when various alternatives exist for a given problem, it decreases the probability of the solution being trapped by local optimal solutions.

For hybrid case construction and hybrid similarity measure, an OO method was applied to represent the fuzzy target and prior cases using hybrids of numerical, nominal, symbolic, and fuzzy values. This was important to make a flexible case representation of the proposed solution approach. In addition, the fuzzy CBR methodology is useful to address uncertainty and vagueness in human decisionmaking as shown in Table 17. On the other hand, a group-based fuzzy AHP was implemented to elicit the judgments of three experts to rank the importance of case attributes (see Tables 5–16). The outputs of these two fuzzy case construction and fuzzy case attribute weighing approaches serve as the input for hybrid similarity measures between target and prior cases.

Data Availability

No underlying data were collected or produced in this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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