Hindawi Publishing Corporation
Advances in High Energy Physics
Volume 2008, Article ID 630414, 16 pages
doi:10.1155/2008/630414

Research Article

Intermediate Inflation or Late Time Acceleration?

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Received 29 June 2008; Accepted 29 August 2008

Recommended by Terry Sloan

The expansion rate of "intermediate inflation" lies between the exponential and power law expansion but corresponding accelerated expansion does not start at the onset of cosmological evolution. Present study of "intermediate inflation" reveals that it admits scaling solution and has got a natural exit form it at a later epoch of cosmic evolution, leading to late time acceleration. The corresponding scalar field responsible for such feature is also found to behave as a tracker field for gravity with canonical kinetic term.

1. Introduction

It is now almost certain that the universe contains 70% of dark energy, which is evolving slowly in such a manner that at present the value of the equation of state parameter is $w < -1/3$, or more precisely, $w = -1$, so that the universe is presently accelerating (see, e.g., [1] for a recent review). Λ CDM-model is the simplest one that can explain the present observable features of the universe. However, in order to comply the vacuum energy density, $\rho_{\text{vac}} \approx 10^{74}\,\text{GeV}^4$ (calculated from quantum field theory as–$\rho_{\text{vac}} \approx m_{pl}^4/16\pi^2$) with the critical density, $\rho_\Lambda \approx 10^{-47}\,\text{GeV}^4$ (related to the cosmological constant as–$\rho_\Lambda = \Lambda m_{pl}^2/8\pi$), it requires to set up yet another energy scale in particle physics. This problem is known as the "coincidence problem" when stated as "why Λ took 15 billion years to dominate over other kinds of matter present in the universe?" A scalar field with dynamical equation of state w_ϕ, dubbed as quintessence field [2], appears to get rid of this problem, which during "slow roll" over the potential, acquires negative pressure and finally acts as effective cosmological constant (Λ_{eff}). Nevertheless, this quintessence field requires to be fine tuned for the energy density of the scalar field (ρ_ϕ) or the corresponding effective cosmological constant (Λ_{eff}), to be comparable with the present energy density of the universe. Tracker fields [3–8] are introduced to overcome the fine tuning problem. Tracker fields have attractor-like solutions in the sense that a wide range of initial conditions (namely, a wide range of initial values of ρ_ϕ) rapidly converges to a common cosmic evolutionary track with ρ_Λ, and finally settles down to the present observable universe with $\rho_\phi \approx \rho_\Lambda$. Thus, tracker solutions avoid both the coincidence problem and the fine tuning problem, of course for certain type of potentials, without any need for defining a new energy scale.

The important parameter required to check for the existence of the tracker solutions is $\Gamma = V''(\phi)V(\phi)/V'(\phi)^2$, $V(\phi)$ being the scalar potential. For quintessence, the condition for the existence of the tracker solution with $w_\phi < w_B$, (where w_ϕ and w_B are the state parameters of the scalar field and the background field, resp.) is $\Gamma > 1$, or equivalently, $|\lambda| = |V'(\phi)/\kappa V(\phi)| \approx |H/\dot{\phi}|$ decreasing as $V(\phi)$ decreases. Tacker solution further requires a nearly constant Γ, which is satisfied if $|d(\Gamma - 1)/Hdt| \ll |\Gamma - 1|$, or equivalently, $|\Gamma^{-1}(d(\Gamma - 1)/Hdt)| \approx |\Gamma'/\Gamma(V'/V)| \equiv \overline{\Gamma} \ll 1$ [3–5]. The condition $w_\phi < w_B$ is required for the present day acceleration of the universe. So, eventually, the slope of the potential becomes sufficiently flat ensuring accelerated expansion at late times. The same condition for k-essence models, having noncanonical form of kinetic energy, requires $\Gamma > 3/2$ and is slowly varying [9], where $\Gamma = g''(\phi)g(\phi)/g'(\phi)^2$, $g(\phi)$ being the coupling parameter, in the absence of the potential. For noncanonical, Dirac, Born, and Infeld (DBI) tachyonic action the tracking behaviour has been investigated in [10]. Such type of scalar fields remain subdominant until recently.

General theory of relativity with a minimally coupled scalar field admits a solution in the form $a = a_0 \exp(At^f)$, (where a is the scale factor and $a_0 > 0, A > 0$, and $0 < f < 1$ are constants) which was dubbed as intermediate inflation in the nineties [11–13]. The expansion rate in the intermediate inflation is faster than power law and slower than exponential ones. Some aspects of intermediate inflation have been studied in the past years [14, 15]. Particularly, in a recent work [16] it has been shown that such inflationary model can encounter the observational features of the three year Wilkinson Microwave Anisotropy Probe (WMAP) data [17] with spectral index $n_s = 1$, considering nonzero tensor-to-scalar ratio r. Such solutions [11–13] may also appear in other theories of gravitation [18, 19]. In fact, the action obtained under modification of Einstein's theory by the introduction of higher order curvature invariant terms (which is essentially four dimensional effective action of higher dimensional string theories) has been found to be reasonably good candidate to explain the presently observed cosmological phenomena. In particular, in a recent work it has been found [20] that Gauss-Bonnet interaction in four dimensions with dynamic dilatonic scalar coupling leads to a solution ($a = a_0 \exp(At^f)$) in the above form, where the universe starts evolving with a decelerated exponential expansion. Such solutions encompasses the cosmological evolution, as the dilatonic scalar during evolution behaves as stiff fluid, radiation, and pressureless dust. Solutions of these type are known as scaling solutions [21, 22] in which the energy density of the scalar field (ρ_ϕ) mimics the background matter energy density. It then comes out of the scaling regime [21, 22] and eventually the Universe starts accelerating. Asymptotically, the scalar behaves as effective cosmological constant. The deceleration parameter corresponding to such solution is given by $q = -1 + (1 - f)/Aft^f$. Thus, unlike usual inflationary models with exponential or power law expansion, accelerated expansion of the scale factor corresponding to intermediate inflation [11–13] does not start at the onset of the cosmological evolution, rather it starts after the lapse of quite some time.

Inflation should have started at the Planck epoch so that it can solve the initial conditions, namely, the horizon and the flatness problems of the standard model and can lead to almost a scale invariant spectrum of density perturbation. As such, the epoch at which the accelerated expansion of the scale factor in intermediate inflation starts, it has also been arbitrarily taken as the Planck's era. But it is not true. Because, as observed in the context of Gauss-Bonnet gravity [20], such solutions admit synchronize scaling between ρ_ϕ

and ρ_B, which can happen long after the Planck's era. Thus it is required to study the so-called intermediate inflation in some more detail.

A comprehensive study in the present work reveals that (1) under different choices of the superpotential $H(\phi)$ solutions in the form $a = a_0 \exp(At^f)$ are realized which lead to late time acceleration and, therefore, should not be treated as inflationary model of early universe. (2) It has also been observed that even for a noncanonical form of kinetic energy, the same result is reproduced with the same form of potential. (3) Finally, it has been shown that in the presence of background matter such solutions are again admissible. The nature of such solution also reveals that the equation of state of the scalar field (w_ϕ) follows that of the background matter (w_B) closely, and finally the scalar field comes out of the scaling regime [21, 22], leading to accelerated expansion of the universe. For the standard form of kinetic energy the tracking behaviour of the scalar field requires to constrain the present matter density parameter $\Omega_{mo} \leq 0.2$.

In Section 2, we have started with a k-essence action [23–26] in its simplest form, keeping only a coupling parameter $g(\phi)$ in the kinetic energy term and writing down the field equations. In Section 3, instead of choosing the form of the potential, we have chosen different forms of the super-potential $H(\phi)$ [27, 28], and presented explicit solutions in the form $a = a_0 \exp(At^f)$, discussed above, for standard $g = 1/2$ and nonstandard form of kinetic energy $g = g(\phi)$. Finally in Section 4, similar solutions in the presence of background matter featuring tracking behaviour for canonical kinetic energy term have been presented.

2. Action and the field equations

The generalized k-essence [23–26] noncanonical Lagrangian,

$$L = g(\phi)F(X) - V(\phi), \tag{2.1}$$

where $X = (1/2)\partial_\mu\phi\partial^\mu\phi$, when coupled to gravity may be expressed in the following simplest form:

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa^2} - g(\phi)\phi_{,\mu}\phi^{,\mu} - V(\phi)\right], \tag{2.2}$$

where a coupling parameter, $g(\phi)$, is coupled with the kinetic energy term. $g(\phi)$ has got a Brans-Dicke origin, $g = \omega/\phi$ too, $\omega(\phi)$ being the Brans-Dicke parameter. This is the simplest form of an action in which both canonical and noncanonical forms of kinetic energies can be treated. For the spatially flat Robertson-Walker space-time

$$ds^2 = -dt^2 + a^2(t)\left[dr^2 + r^2\{d\theta^2 + \sin^2(\theta)d\phi^2\}\right], \tag{2.3}$$

the field equations are

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = 2\dot{H} + 3H^2 = -\kappa^2[g\dot{\phi}^2 - V(\phi)] = -8\pi G p, \tag{2.4}$$

$$3\frac{\dot{a}^2}{a^2} = 3H^2 = \kappa^2[g\dot{\phi}^2 + V(\phi)] = 8\pi G\rho, \tag{2.5}$$

where $H = \dot{a}/a$ is the Hubble parameter. In addition, we have got the ϕ variation equation

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{1}{2}\frac{g'}{g}\dot{\phi}^2 + \frac{V'}{2g} = 0, \tag{2.6}$$

which is not an independent equation, rather it is derivable from (2.4) and (2.5). In the above, over-dot and dash ($'$) stand for differentiations with respect to time and ϕ, respectively. Instead of using (2.4) and (2.5), it is always useful to parametrize the motion in terms of the field variable ϕ [13, 29–31]. Thus, with $\kappa^2 = 8\pi G$, the above set of equations can be expressed as

$$\dot{\phi} = -\left(\frac{H'(\phi)}{\kappa^2 g(\phi)}\right), \tag{2.7}$$

and in the Hamilton-Jacobi form

$$[H'(\phi)]^2 - 3\kappa^2 H^2(\phi)g(\phi) + \kappa^4 V(\phi)g(\phi) = 0. \tag{2.8}$$

The two important parameters of the theory, namely, the equation of state w_ϕ and the deceleration q parameters, are expressed as

$$w_\phi = -1 - \frac{2\dot{H}}{3H^2}; \qquad q = -1 - \frac{\dot{H}}{H^2}. \tag{2.9}$$

Now, we are to solve for a (in view of H), ϕ, $g(\phi)$, and $V(\phi)$ from (2.7) and (2.8), and so, two additional assumptions are required. It is found that some specific forms of the super-potential $H(\phi)$ [27, 28] lead to the so-called intermediate inflationary solutions.

3. Solution in the form $a = a_0 e^{(At^f)}$ and its dynamics

This section is devoted in presenting the solution of the scale factor in the form $a = a_0 e^{(At^f)}$, which was dubbed as intermediate inflation earlier.

3.1. Case-I

Let us choose the following form of the super-potential $H(\phi(t))$:

$$H = \frac{h}{\phi^n}, \tag{3.1}$$

where $h > 0$ and $n > 0$ are constants. In view of this assumption, (2.7) becomes

$$g\dot{\phi} = \frac{nh}{\kappa^2}\phi^{-(n+1)}. \tag{3.2}$$

We can now solve the set of (2.8), (3.1), and (3.2), provided that we choose a particular form of $g(\phi)$ or $V(\phi)$. In the following, we choose the standard form of kinetic energy, that is, $g = 1/2$. For the most natural choice $g = 1/2$, the field variables are found from (3.2) and (3.1) as

$$\phi = \left[\frac{2hn(n+2)}{\kappa^2}t\right]^{1/(n+2)},$$
$$H = mt^{-(n/(n+2))}, \tag{3.3}$$
$$a = a_0 \exp\left[\left(\frac{m(n+2)}{2}\right)t^{(2/(n+2))}\right],$$

where $m = h^{(2/(n+2))}[\kappa^2/2n(n+2)]^{(n/(n+2))}$. The above form of the scale factor, the Hubble parameter, and the scalar field ϕ can be expressed, respectively, as

$$a = a_0 \exp\left[At^f\right], \qquad H = \frac{Af}{t^{(1-f)}}, \qquad \phi = \left[\frac{8h(1-f)}{\kappa^2 f^2}t\right]^{f/2}, \tag{3.4}$$

where $A = m(n+2)/2 > 0$ and $0 < f = 2/(n+2) < 1$ are related to the constants h and n. In view of (2.9), the state parameter and the deceleration parameter evolve as

$$w_\phi = -1 + \frac{n}{3A}t^{-f}; \qquad q = -1 + \frac{n}{2A}t^{-f}. \tag{3.5}$$

The form of the potential, in view of (2.8), for such a solution is restricted to

$$V(\phi) = \frac{h^2}{\kappa^4}\left(\frac{3\kappa^2}{\phi^{2n}} - \frac{2n^2}{\phi^{2(n+1)}}\right) \tag{3.6}$$

which has the form of double inverse power. Thus, we have just reproduced the solutions obtained by Barrow [11, 12] and Muslimov [13], starting from a particular choice of the superpotential. As previously mentioned, it was found in the nineties and was dubbed as intermediate inflation, since the expansion rate of the scale factor is greater than the power law but less than standard exponential law. For such an expansion rate,

$$\rho + p = \frac{4n^2h^2}{\kappa^4\phi^{2(n+1)}} > 0, \tag{3.7}$$

that is, the weak energy condition is always satisfied and so $w_\phi \geq -1$. However,

$$\rho + 3p = \frac{6}{\kappa^2}\left(-\dot{H} - H^2\right) = \frac{6Af}{\kappa^2}\left[(1-f) - Aft^f\right]t^{-(2-f)} \tag{3.8}$$

implies that the strong energy condition is violated at

$$t_a > \left(\frac{1-f}{Af}\right)^{1/f}, \tag{3.9}$$

when $w_\phi < -1/3$, and t_a corresponds to the time at which the acceleration starts.

It is to be noted that the necessary condition for inflation, namely, $\ddot{a} > 0$ or more precisely, $(d/dt)(H^{-1}/a) < 0$, is satisfied under the same above condition (3.9). Eventually, for large ϕ, namely, $\phi \gg \sqrt{2}n/\kappa$ the potential energy starts dominating over the kinetic energy, that is, $\dot{\phi}^2 \ll V(\phi)$, and thus the slow roll condition, $\epsilon = -\dot{H}/H^2 < 1$, is also satisfied under condition (3.9). However, inflation is supposed to have started at Planck's era, so that it can solve the initial problems of the standard model, namely, horizon and flatness problems and the structure formation problem and lead nearly to a scale invariant spectrum. Here we observe that accelerated expansion of the scale factor in the so-called intermediate inflation does not start at the onset of cosmic evolution. So, now we can ask the following question: "what happened prior to the onset of the accelerated expansion in the so-called intermediate inflationary era?"

The solution dictates that the universe starts evolving from an infinitely decelerated exponential expansion with $w_\phi > 1$. It might appear that we are considering a highly unorthodox cosmological model involving an ultrahard equation of state and superluminal speed of sound. This is true in some sense, since as mentioned earlier, in order to study the situations under which such solutions emerge and its dynamics, we have not considered the presence of any form of background matter explicitly. Nevertheless, this situation is quite similar to the phantom models where super negative pressure gives rise to ultranegative equation of state, indicating that the effective velocity of sound in the medium, $v = \sqrt{|dp/d\rho|}$, might become larger than the velocity of light. Likewise, here the universe starts evolving with such a situation which actually demonstrates that the corresponding era is classically forbidden and it is required to invoke quantum cosmology at that era.

Now, during the evolution, the universe passes transiently through the stiff fluid era $w_\phi = 1$, the radiation dominated era $w_\phi = 1/3$, the pressureless dust era $w_\phi = 0$, the transition (from deceleration to acceleration) era $w_\phi = -1/3$, and asymptotically tends to the magic line, that is, the vacuum energy dominated inflationary era $w_\phi \approx -1$, ensuring asymptotic de-Sitter expansion.

3.2. Case-II

In this subsection, we show that the same set of solutions obtained in case-I can be reproduced even for a noncanonical kinetic term. To show this, we observe that the above set of solutions does not necessarily require to fix up $g = 1/2$, rather they are found even for a functional form of $g = g(\phi)$, that is, with a noncanonical kinetic term. This can be checked easily by assuming solution (3.4) of the scale factor along with assumption (3.1) for the Hubble parameter. The field ϕ, the potential $V(\phi)$, and the coupling parameter $g(\phi)$ are then found as

$$\phi = \left(\frac{h}{Af}\right)^{1/n} t^{((1-f)/n)};$$

$$V = \frac{h^2}{\kappa^2}\left[\frac{3}{\phi^{2n}} - \left(\frac{h^f}{Af}\right)^{1/(1-f)} \frac{(1-f)}{\phi^{n((2-f)/(1-f))}}\right];$$
$$g = \frac{hn^2}{\kappa^2(1-f)}\left(\frac{Af}{h}\right)^{1/(1-f)} \phi^{[nf/(1-f)-2]}. \tag{3.10}$$

Since $f > 0$ requires $((2-f)/(1-f)) > 2$, the potential is also found in the same above form (3.6).

Thus we find that even a noncanonical kinetic term reproduces the same set of solutions obtained with a canonical kinetic term. So, in principle it is possible to find a field theory with a noncanonical kinetic term, such that the cosmological solution is exactly the same as the solution of the said theory with canonical kinetic term. This proves our second claim which is of course a new result.

It is to be mentioned that for $f = 2/(n+2)$, the noncanonical kinetic term turns out to be canonical and the results arrived at in case-I are recovered.

3.3. Case-III

In this subsection we show that even a different form of the superpotential in the presence of a nonstandard form of kinetic energy leads to similar form of the scale factor obtained in the previous subsections. Let us choose the form of the superpotential $H(\phi(t))$ as

$$H = \kappa^2 e^{-l\phi} \tag{3.11}$$

with $l > 0$. So,

$$g\dot{\phi} = le^{-l\phi}; \qquad \frac{\dot{H}}{H^2} = -\frac{l^2}{\kappa^2 g}; \qquad V = -le^{-l\phi}\dot{\phi} + 3\kappa^2 e^{-2l\phi}. \tag{3.12}$$

In view of (2.9) and (3.11), it is clear that for a constant g, w_ϕ becomes nondynamical. So, to find the solutions explicitly, let us further make the following choice:

$$\dot{\phi} = e^{-ml\phi}, \tag{3.13}$$

where m is a constant. Under this choice,

$$g = le^{(m-1)l\phi}, \tag{3.14}$$

and the potential can be expressed as the algebraic sum of two inverse exponents as

$$V(\phi) = 3\kappa^2 e^{-2l\phi} - le^{-(m+1)l\phi}. \tag{3.15}$$

This form of the potential [22, 32–35] is found as a result of compactifications in superstring models and is usually considered to exit from the scaling regime in the presence of background matter. Solutions for the scalar field and the scale factor are obtained as

$$\phi = \frac{1}{ml}\ln(mlt); \qquad a = a_0 \exp\left[\frac{m\kappa^2}{(m-1)(ml)^{1/m}} t^{(m-1)/m}\right]. \tag{3.16}$$

Thus the scale factor can be expressed in the same form (3.4) of the so-called intermediate inflation for $m > 1$. Further, the equation of state and the deceleration parameters (2.9) are obtained as

$$w_\phi = -1 + \frac{2l}{3\kappa^2}(mlt)^{(1-m)/m}; \qquad q = -1 + \frac{l}{\kappa^2}(mlt)^{(1-m)/m}. \tag{3.17}$$

Now, for $m > 1$,

$$\rho + p = 2l \exp[-(m-1)l\phi] > 0, \tag{3.18}$$

that is, the weak energy condition is always satisfied while

$$\rho + 3p = 6\left(l \exp[-(m-1)l\phi] - \kappa^2 \exp[-2l\phi]\right), \tag{3.19}$$

that is, the strong energy condition is violated at $t \geq (1/ml)(1/\kappa^2)^{m/(m-1)}$, which corresponds to $\exp(m-1)l\phi \geq l/\kappa^2$, the epoch of transition from decelerating to the accelerating phase. Further, the necessary condition for inflation and the slow roll condition are satisfied at the same epoch. The universe expands exponentially but decelerates from a infinitely large value. The equation of state w_ϕ starts from indefinitely large value at the beginning. $w_\phi \to \infty$ implies a greater effective velocity of sound than that of light in the corresponding medium, which also appears in phantom models having super negative pressure. So classically it has got no meaning at all. Such result only dictates the importance of invoking quantum cosmology before the equation of state reaches stiff fluid era.

Here again, during evolution the universe passes transiently through the stiff fluid era $w_\phi = 1$, the radiation dominated era $w_\phi = 1/3$, the pressureless dust era $w_\phi = 0$, the transition (from deceleration to acceleration) era $w_\phi = -1/3$, and asymptotically tends to de-Sitter expansion. Thus, this solution also has got the same features as the previous one.

4. Presence of background matter

Observations suggest that our universe is presently filled with at least 70% of dark energy, 26% or less amount of cold dark matter, about 4% of Baryons, and 0.005% of radiation [36]. So, to consider a realistic model, presence of background distribution of all types of Baryonic and non-Baryonic matter should be accounted for explicitly. In this section, our motivation is to check if the above form of the scale factor admits viable cosmological solution in the presence of background matter. The field equations now can be arranged as

$$\dot{H} = -\kappa^2\left[g\dot{\phi}^2 + \sum\frac{\rho_{B_i} + p_{B_i}}{2}\right], \tag{4.1}$$

$$\dot{H} + 3H^2 = \kappa^2\left[V(\phi) + \sum\frac{\rho_{B_i} - p_{B_i}}{2}\right], \tag{4.2}$$

where ρ_{B_i} and p_{B_i} are the energy density and pressure of the background matter, respectively, and the sum over i stands for the sum of different types of background matter present in the universe (radiation + cold dark matter + baryonic matter). Further, since the scalar field is minimally coupled to the background, so continuity equations for the background matter and the scalar field hold independently. Hence, we can write

$$\dot{\rho}_{B_i} + 3H(1 + w_{B_i})\rho_{B_i} = 0 = \dot{\rho}_\phi + 3H(1 + w_\phi)\rho_\phi. \tag{4.3}$$

Thus we have

$$\rho_{B_i} = {\rho_{Bi}}^i a^{-3(1+w_{Bi})}, \tag{4.4}$$

$$\rho_\phi = {\rho_\phi}^i a^{-3(1+w_\phi)}, \tag{4.5}$$

where ${\rho_{Bi}}^i$ and ${\rho_\phi}^i$ are the initial values of the background and the scalar field energy densities. If we now plug in the solution of the scale factor in the form $a = a_0 \exp At^f$, with $a_0 > 0$, $A > 0$, and $0 < f < 1$, then the potential is found in view of (4.2) as

$$V = \frac{1}{\kappa^2}\left[\frac{3A^2 f^2}{t^{2(1-f)}} - \frac{Af(1-f)}{t^{(2-f)}}\right] - \sum\left(\frac{(1 - w_{B_i})\rho_{B_i}}{2}\right), \tag{4.6}$$

while the kinetic term can be obtained in view of (4.1) as

$$g\dot{\phi}^2 = \frac{Af(1-f)}{\kappa^2 t^{(2-f)}} - \sum\left(\frac{(1 - w_{B_i})\rho_{B_i}}{2}\right). \tag{4.7}$$

In (4.6) and (4.7), we have used the equation of state $p_{B_i} = w_{B_i}\rho_{B_i}$, for the background matter. An expression of w_ϕ is given as a function of t:

$$w_\phi = \frac{\left[2a_0^4 Af(1-f) - 3a_0^4 A^2 f^2 t^f\right] e^{4At^f} - \kappa^2\left(\rho_r^i/3\right)t^{2-f}}{3a_0^4 A^2 f^2 t^f e^{4At^f} - \kappa^2\left[\rho_r^i + a_0\rho_m^i e^{At^f}\right]t^{2-f}}. \tag{4.8}$$

Now, as already discussed in Section 3, it is not difficult to see that during evolution the universe passes transiently through the stiff fluid era $w_\phi = 1$, the radiation dominated era $w_\phi = 1/3$, the pressureless dust era $w_\phi = 0$, the transition (from deceleration to acceleration) era $w_\phi = -1/3$, and asymptotically tends to the vacuum energy-dominated inflationary era $w_\phi \approx -1$. Thus, the equation of state w_ϕ follows the matter equation of state w_B closely and so it corresponds to the scaling solution [21, 22]. It finally comes out of the scaling regime and enters into the transition era.

4.1. Canonical Lagrangian

For the canonical Lagrangian with the standard form of kinetic energy ($g = 1/2$), it is not possible to find a solution of ϕ vide equation (4.7) in closed form and so one cannot express $t = t(\phi)$. Hence, the form of the potential $V = V(\phi)$ remains obscure. However, it is still possible to check if the field is tracker. As mentioned in the introduction, the condition to check the existence of tracking solution is

$$\Gamma = \frac{V''V}{V'^2} > 1 \Longrightarrow \Gamma = \frac{\ddot{V}V}{\dot{V}^2} - \frac{V\ddot{\phi}}{\dot{V}\dot{\phi}} > 1, \tag{4.9}$$

along with the fact that it is nearly constant which is true provided

$$|\bar{\Gamma}| = \left|\frac{\Gamma'}{\Gamma V'/V}\right| \ll 1 \Longrightarrow |\bar{\Gamma}| = \left|\frac{\dot{\Gamma}}{\Gamma \dot{V}/V}\right| \ll 1. \tag{4.10}$$

Using the scalar field equation (2.6) with $g = 1/2$, the first condition translates to

$$\Gamma = \frac{\ddot{V}V}{\dot{V}^2} + 3H\frac{V}{\dot{V}} + \frac{V}{\dot{\phi}^2}. \tag{4.11}$$

Now, in the unit $\kappa^2 = 8\pi G = 1$ and taking $A = 0.5$, $f = 0.5$, as before, for which $a = e^{\sqrt{t}/2}$ and $H = 1/4\sqrt{t}$, where we have chosen $a_0 = 1$ without any loss of generality, Γ is finally expressed in view of the solutions (4.6) and (4.7) as

$$\begin{aligned}
\Gamma = {} & \frac{\Big(3e^{2\sqrt{t}}\left(-2+3\sqrt{t}\right) - 24e^{\sqrt{t}/2}t^{3/2}\rho_m^i - 16t^{3/2}\rho_r^i}{12e^{2\sqrt{t}} - 48e^{\sqrt{t}/2}t^{3/2}\rho_m^i - 64t^{3/2}\rho_r^i} \\
& + \frac{9e^{2\sqrt{t}}\left(-2\sqrt{t}+3t\right) - 72e^{\sqrt{t}/2}t^2\rho_m^i - 48t^2\rho_r^i}{4\left(-9e^{2\sqrt{t}}\left(-1+\sqrt{t}\right) + 18e^{\sqrt{t}/2}t^2\rho_m^i + 16t^2\rho_r^i\right)} \\
& + \left(\left(3e^{2\sqrt{t}}\left(-2+3\sqrt{t}\right) - 24e^{\sqrt{t}/2}t^{3/2}\rho_m^i\right) - 16t^{3/2}\rho_r^i\right) \\
& \quad \times \left(9e^{2\sqrt{t}}\left(-5+4\sqrt{t}\right) - 9e^{\sqrt{t}/2}\left(2+3\sqrt{t}\right)t^2\rho_m^i - 16\left(t^2+2t^{5/2}\right)\rho_r^i\right)\right) \\
& \quad \times \left(2\left(\left(-9e^{2\sqrt{t}}\right)\left(-1+\sqrt{t}\right) + 18e^{\sqrt{t/2}}t^2\rho_m^i + 16t^2\rho_r^i\right)^2\right)^{-1}.
\end{aligned} \tag{4.12}$$

One can express ρ_r^i, the initial amount of radiation density, in terms of its present value ρ_r^0, in view of (4.4), with $w_{Bi} = w_r = 1/3$, as

$$\rho_r^i = e^{4At_0^f}\rho_r^0, \tag{4.13}$$

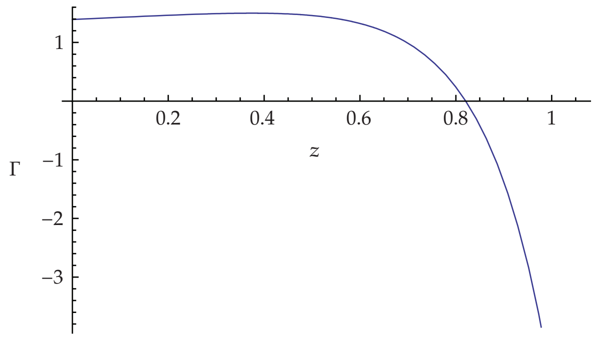

Figure 1: The plot of Γ against z for $\Omega_{m0} = 0.18$ shows that $\Gamma > 1$ for $z < 0.7$, and is nearly constant for $z < 0.55$. Thus, the field starts tracking for $z \leq 0.5$.

which is related to the present value of the density parameter Ω_{r0} and the critical density $\rho_c^0 = 3H_0^2$ through the relation

$$\frac{\rho_r^0}{\rho_c^0} = \Omega_{r0}. \tag{4.14}$$

Hence, for $H_0^{-1} = 14.4\,\text{Gyr.}$, that is, $h = 0.68$, which yields a comfortable age of the universe $t_0 = 13\,\text{Gyr}$ as before, and taking $\Omega_{r0} = 0.00005$, one finds $\rho_r^i = 9.77 \times 10^{-4}$. Likewise, ρ_m^i, the initial amount of matter density (Baryonic and CDM), may be fixed in view of (4.4) with $w_{Bi} = w_m = 0$, knowing Ω_{m0} as

$$\rho_m^i = e^{3At_0^f}\rho_m^0 = e^{3At_0^f}\rho_c^0\Omega_{m0} = 3H_0^2 e^{3At_0^f}\Omega_{m0}. \tag{4.15}$$

However, we will not fix up Ω_{m0} apriori, rather we use the manipulation programme of Mathematica to set up Ω_{m0} corresponding to a tracker field. Using the relation between the redshift parameter z and the proper time t, namely,

$$1 + z = \frac{a(t_0)}{a(t)} = \exp\left[A\left(t_0^f - t^f\right)\right], \tag{4.16}$$

where $a(t_0)$ is the present value of the scale factor, while $a(t)$ is the value of the same at any arbitrary time t, we plot Γ against redshift, in Figure 1, for $\rho_m^i = 0.5796$, which corresponds to $\Omega_{m0} = 0.18$. It shows that $\Gamma > 1$ for $z < 0.7$ and remains nearly constant for $z < 0.6$, implying that the field starts tracking from $z < 0.6$.

$\overline{\Gamma}$ can be also calculated in a straightforward manner, which appears in a very cumbersome form and therefore has not been presented here. However, the plot of $\overline{\Gamma}$ versus the redshift z, in Figure 2, shows that $|\overline{\Gamma}| < 1$, for $z < 0.45$. In fact, the usage of ”Manipulate Plot” of “Mathematica 6” shows that the tracking behaviour of the field starts for $\Omega_{m0} < 0.2$. Thus, the tracking behaviour requires somewhat lesser amount of Baryonic and CDM, which has not been ruled out by the presently estimated amount of matter through different observations. It is also interesting to note that to keep Γ nearly constant, it is sufficient to consider $|\overline{\Gamma}| < 1$, rather than $|\overline{\Gamma}| \ll 1$.

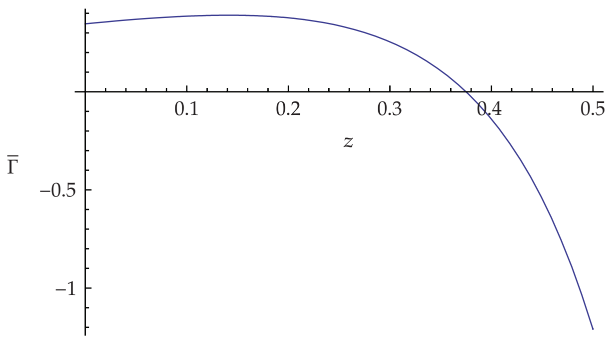

Figure 2: The plot of $\overline{\Gamma}$ against z for $\Omega_{m0} = 0.18$ shows that $|\overline{\Gamma}| < 1$, for $z < 0.45$ and so the field starts tracking from that epoch.

It is also possible to give an estimate of the value of the redshift parameter z at which the universe started accelerating, which corresponds to $w_\phi \leq -1/3$. In view of (4.8) and the values of the parameters of the theory already taken up, namely, $f = A = 0.5$, $a_0 = 1$, $t_0 = 13\,\text{Gyr}$, $\rho_r^i = 9.77 \times 10^{-4}$, and $\rho_m^i = 0.5796$, the acceleration is found to start at $t_a \geq 2.14\,\text{Gyr}$. Hence using (4.16), the redshift z_a at which acceleration started is given by

$$z_a = \exp\left[A\left(t_0^f - t_a^f\right)\right] - 1 = 1.9. \tag{4.17}$$

Thus in the present model, we observe that the acceleration has started somewhat earlier than usually estimated in view of ΛCDM and other existing models. Finally, the present value of the state parameter under the above parametric choices is found from (4.8) to be $w_{\phi 0} = -0.77$. It of course increases with Ω_{m0}, but then one has to sacrifice the tracking behaviour.

4.2. Noncanonical Lagrangian

In the Section 4.1, we observed that for canonical Lagrangian, explicit forms of the potential $V(\phi)$ remains obscure since it was not possible to find an expression for $t = t(\phi)$ in view of (4.7). However, choosing some particular form of the scalar field ϕ, it is possible to express the potential $V(\phi)$ and the coupling parameter $g(\phi)$ as functions of ϕ. It is also noticed that the first term of the potential is predominantly dominating as the universe evolves. Thus the potential remains positive asymptotically, though it starts from an indefinitely large negative value. As an example, let us choose ϕ as a monotonically increasing function of time,

$$\phi = t. \tag{4.18}$$

So the potential and the kinetic energy of the scalar field take the following forms, respectively,

$$\begin{aligned} V &= \frac{1}{\kappa^2}\left[\frac{3A^2 f^2}{\phi^{2(1-f)}} - \frac{Af(1-f)}{\phi^{(2-f)}}\right] - \sum\left(\frac{(1-w_{B_i})\rho_{B_i}{}^i}{2\left[a_0 \exp\left\{A\phi^f\right\}\right]^{3(1+w_{B_i})}}\right), \\ g\dot{\phi}^2 &= g(\phi) = \frac{Af(1-f)}{\kappa^2\phi^{(2-f)}} - \sum\left(\frac{(1+w_{B_i})\rho_{B_i}{}^i}{2\left[a_0 \exp\left\{A\phi^f\right\}\right]^{3(1+w_{B_i})}}\right). \end{aligned} \tag{4.19}$$

The energy density and the pressure of the scalar field are expressed as

$$\begin{aligned}\rho_\phi &= \frac{3A^2 f^2}{\kappa^2 \phi^{2(1-f)}} - \sum\left(\frac{{\rho_{B_i}}^i}{\left[a_0 \exp\left\{A\phi^f\right\}\right]^{3(1+w_{B_i})}}\right),\\ p_\phi &= \frac{1}{\kappa^2}\left[\frac{2Af(1-f)}{\phi^{(2-f)}} - \frac{3A^2 f^2}{\phi^{2(1-f)}}\right] - \sum\left(\frac{w_{B_i}{\rho_{B_i}}^i}{\left[a_0 \exp\left\{A\phi^f\right\}\right]^{3(1+w_{B_i})}}\right).\end{aligned} \tag{4.20}$$

The scale factor admits scaling solution as already mentioned and the scaling of ρ_ϕ becomes sloth as $V(\phi)$ starts dominating over the kinetic energy. However, as mentioned in the introduction, the condition given by Chiba [9] to check the tracking behaviour of the scalar field in k-essence model requires absence of the potential $V(\phi) = 0$. Since there is no method to check such behaviour in the presence of the potential so, it is not possible to see if the scalar field is a tracker field. However, we can make certain estimate relevant with the present observations. In the presence of both the radiation and matter in the form of dust, the potential and the state parameters have the following expression:

$$V = \frac{1}{\kappa^2}\left[\frac{3A^2 f^2}{\phi^{2(1-f)}} - \frac{Af(1-f)}{\phi^{(2-f)}}\right] - \frac{{\rho_r}^{(0)}}{3a_0^4 e^{4A\phi^f}} - \frac{{\rho_m}^{(0)}}{2a_0^3 e^{3A\phi^f}}, \tag{4.21}$$

which contains algebraic sum of two inverse powers and two inverse exponents and

$$w_\phi = \frac{p_\phi}{\rho_\phi} = \frac{\left[2Af(1-f) - 3A^2 f^2 \phi^f\right] e^{4A\phi^f} - (\rho_r^i/3)\phi^{2-f}}{3A^2 f^2 \phi^f e^{4A\phi^f} - \left[\rho_r^i + \rho_m^i e^{A\phi^f}\right]\phi^{2-f}}. \tag{4.22}$$

As before, to have an idea of the redshift value (z_a) of transition from deceleration to acceleration of the universe and the present value of the state parameter ($w_{\phi 0}$), we make the same comfortable choice as before, that is, $f = 0.5$, $A = 0.5$, and $H_0^{-1} = 14.42\,\text{Gyr}$, corresponds to $h = 0.68$, which set the age of the universe to $t_0 = 13\,\text{Gyr}$. Further, taking $\rho_r^i = 9.77 \times 10^{-4}$ as calculated in the previous subsection and $\rho_m^i = 0.969$, corresponding to $\Omega_{m0} = 0.3$, the time of transition to accelerated expansion from deceleration is calculated from (4.8) to be $t_a = 1.6\,\text{Gyr}$. Corresponding redshift value is found in view of (4.16) to be $z_a = 2.2$. Finally, one can find the present value of the state parameter in view of (4.8) to be $w_{\phi 0} = -0.9$. In this case the acceleration appears to start even earlier, but the present value of the state parameter is pretty close to the values calculated in other models. It is to be mentioned that we have chosen $A = f = 0.5$ just to give a somewhat good looking appearance to the scale factor and particularly the potential. However, a judicious choice of the parameters A and f can give much smaller value of z_a and $w_{\phi 0} \approx -1$.

Finally, we have plotted $V(\phi)$ and $g(\phi)$ against ϕ in Figures 3 and 4, respectively. It is interesting to note that $g(\phi)$ remains almost flat at the later epoch, ensuring ever accelerating and asymptotic de-Sitter universe. On the other hand, a cusp in $V(\phi)$ separates early deceleration and late time acceleration.

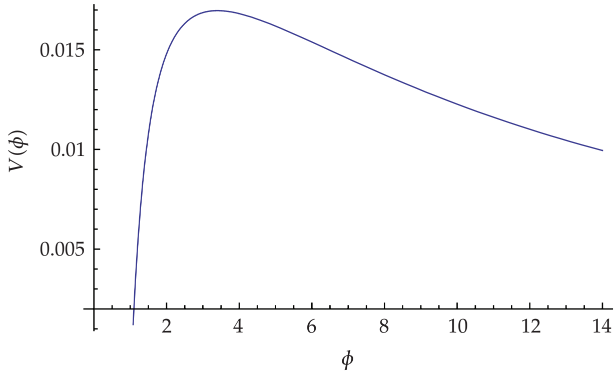

Figure 3: The form of the potential $V(\phi)$ has been plotted against ϕ (with $f = 0.5$, $h = 0.68$, $t_0 = 13\,\text{Gyr}$, $\Omega_{mo} = 0.3$). Early deceleration and late time acceleration are clearly distinguished by a cusp.

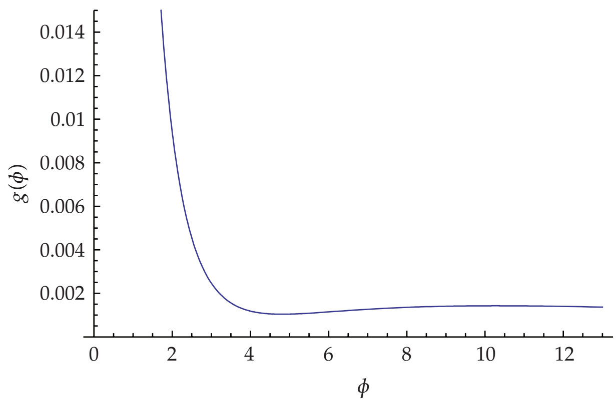

Figure 4: The form of the coupling parameter $g(\phi)$ has been plotted against ϕ (with $f = 0.5$, $h = 0.68$, $t_0 = 13\,\text{Gyr}$, $\Omega_{mo} = 0.3$). It remains almost flat at the later epoch of cosmic evolution, ensuring late time acceleration.

5. Concluding remarks

Several interesting features have been revealed in the present model. Firstly, it has been found that the cosmological solution in the form $a = a_0 e^{(At^f)}$ with $a_0 > 0, A > 0$, and $0 < f < 1$ may be treated as dark energy model leading to late time cosmic acceleration, rather than intermediate inflation at early universe. The second interesting result is that the gravitational field equations with both canonical $g = 1/2$ and noncanonical kinetic term $g = g(\phi)$ produce the same set of solutions with the same form (sum of double inverse power) of potential. Thirdly, the sum of double inverse exponential potential has also been found to produce the same form of solution for noncanonical kinetic term $g = g(\phi)$. Although, in view of such solution $a = a_0 e^{(At^f)}$, in the presence of background matter, it is not possible to express the potential V as a function of the scalar field $V(\phi)$, for a canonical kinetic term with $g = 1/2$, however, tracking behaviour, that is, $\Gamma > 1$ and its slowly varying nature have been expatiated in Figures 1 and 2. Such behaviour constraints the present value of density parameter to $\Omega_{m0} < 0.2$. For the noncanonical form of kinetic energy, the potential is in the form of the sum of double inverse powers and double inverse exponents. The present value of the equation of state w_ϕ parameter has been found to be $w_{\phi 0} = -0.9$, which is close to the present observational result obtained from other models. Thus, the particular choice of the scale factor and the form of the potential lead to a viable dark energy model with late time cosmic acceleration which has been depicted in both the plots of the potential $V(\phi)$ and the coupling parameter $g(\phi)$. A different choice of parameters A and f might lead to acceleration even at a lower redshift value z with $w_{\phi 0} \approx -1$. Thus, we conclude that a minimally coupled scalar field admitting the above form of solution of the scale factor $a = a_0 e^{(At^f)}$ has all the nice features to account for the dark energy of the present universe.

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