Hindawi Publishing Corporation
Advances in High Energy Physics
Volume 2009, Article ID 905705, 9 pages
doi:10.1155/2009/905705

Research Article

Thermodynamics in Loop Quantum Cosmology

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Received 2 December 2008; Accepted 5 January 2009

Recommended by George Siopsis

Loop quantum cosmology (LQC) is very powerful to deal with the behavior of early universe. Moreover, the effective loop quantum cosmology gives a successful description of the universe in the semiclassical region. We consider the apparent horizon of the Friedmann-Robertson-Walker universe as a thermodynamical system and investigate the thermodynamics of LQC in the semiclassical region. The effective density and effective pressure in the modified Friedmann equation from LQC not only determine the evolution of the universe in LQC scenario but also are actually found to be the thermodynamic quantities. This result comes from the energy definition in cosmology (the Misner-Sharp gravitational energy) and is consistent with thermodynamic laws. We prove that within the framework of loop quantum cosmology, the elementary equation of equilibrium thermodynamics is still valid.

1. Introduction

Loop quantum gravity (LQG) [1–5] is a nonperturbative and background independent quantization of gravity. One of the important and successful applications of LQG is loop quantum cosmology (LQC). It has been shown that LQC resolves the problem of classical singularities both in an isotropic model [6] and in a less symmetric homogeneous model [7]. LQC also gives a quantum suppression of classical chaotic behavior near singularities in Bianchi-IX models [8, 9]. Furthermore, it has been shown that nonperturbative modification of the matter Hamiltonian leads to a generic phase of inflation [10–12]. On the other hand, we know that spacetime thermodynamic properties result from, in a sense, quantum effects of spacetime [13]. Therefore, it is very interesting and important to investigate the thermodynamics of quantum gravity. There are indeed many results on thermodynamical implications of loop quantum gravity [14, 15], but very little discussion on the thermodynamics of loop quantum cosmology, which will be the focus of the present paper.

In LQC, the phase space for spatially flat universe is spanned by coordinates $c = \gamma\dot{a}$, being the gravitational gauge connection, and $p = a^2$, being the densitized triad. γ is the

Barbero-Immirzi parameter, and a is the scale factor of the universe. In the LQC scenario, the evolution of the universe can be divided into three phases. (i) Initially, there is a truly discrete quantum phase which is described by a difference equation [16]. In this stage, the universe may be nonequilibrium due to the fast quantum evolution. (ii) As the volume of universe increases and matter density decreases, the discrete quantum effect becomes less important, the universe enters an intermediate semiclassical phase in which the evolution equations take a continuous form but with modifications due to nonperturbative quantum effects [10]. In this stage, the effective loop quantum cosmology is valid, and it is reasonable to approximately treat the universe as a thermodynamic system in equilibrium. The thermodynamic properties are subject to quantum effects, and we are most interested in this stage. (iii) Finally, there is the classical phase in which the quantum effects vanish and the usual continuous equations describing cosmological behavior are established and so is the usual thermodynamics [17, 18].

In recent years, many authors endeavor to study the thermodynamics [14, 15] of black holes in the semiclassical context and the framework of LQG. Now people have studied the effective theory, though not complete yet due to quantum back-reaction [19], in loop cosmology. Thus, there is considerable interest in the thermodynamic properties of the universe in LQC scenario. With the universe being nonstationary and evolving, the thermodynamics is different from the black hole systems. It is conceivable that some of the mechanisms involved in establishing thermal equilibrium may be modified, especially when the expansion time scale becomes comparable to that of the matter processes responsible for establishing the thermal equilibrium.

To resolve this issue, we develop a procedure to study the thermodynamic properties at the apparent horizon of the Friedmann-Robertson-Walker (FRW) universe. Our analysis is based on the effective theory of LQC and the homogeneous and isotropic cosmological settings. Fundamentally, comparing the modified Friedmann equation with the ordinary one, we derive the effective density and pressure of the perfect fluid. Then, we introduce the Misner-Sharp energy [20], which is different from the other forms of energy for its relation to the structure of the spacetime and one can relate it to the Einstein equation. From the expression of the Minser-Sharp energy, we get the physical meaning of the effective density. Furthermore, from the conservation law, we get the physical meaning of the effective pressure. To understand the intrinsic essence of the effective density and pressure, we prove that within the framework of loop quantum cosmology, the fundamental relation of thermodynamics is still valid.

This paper is organized as follows. In Section 2, we briefly review the framework of the effective LQC. We present the dynamics in terms of effective density and pressure, which will be defined there. Then in Section 3, we obtain the thermodynamic origin of the effective density and pressure. Some elementary consequences are also noted. In Section 4, we conclude this paper with some discussions on the further implications for phenomenology.

2. A Short Review of Effective Theory of LQC

In this section, we give a short review of the effective framework of LQC before we study the thermodynamics. The classical form of Hamiltonian for spatially flat universe is

$$H_{\mathrm{cl}} = -\frac{3}{8\pi\gamma^2}\sqrt{p}c^2 + H_M(p,\phi). \tag{2.1}$$

There are two kinds of important modifications in the LQC. The first one is based on the modification to the behavior of inverse scale factor below a critical scale factor (the inverse volume modification). The second one essentially comes from the discrete quantum geometric nature of spacetime (quadratic modification), as predicted by the LQG. Besides these two kinds of corrections, there is also the more generic quantum back-reaction which gives rise to effective potentials. In this paper, we only consider the corrections coming from the quadratic modification. But it is worthy to note that our result is valid for general effective potential. With the quadratic modification, the effective Hamiltonian becomes [21–23]

$$H_{\text{eff}} = -\frac{3}{8\pi\gamma^2\overline{\mu}^2}\sqrt{p}\sin^2(\overline{\mu}c) + H_M(p,\phi). \tag{2.2}$$

The variable $\overline{\mu}$ corresponds to the dimensionless length of the edge of the elementary loop and is given by

$$\overline{\mu} = \xi p^{\lambda}, \tag{2.3}$$

where $\xi > 0$ and λ depend on the particular scheme in the holonomy corrections. In this paper, we take $\overline{\mu}$-scheme, which gives

$$\xi^2 = 2\sqrt{3}\pi\gamma l_p^2 \tag{2.4}$$

and $\lambda = -1/2$, where l_p is Planck length. With this effective Hamiltonian, we have the canonical equation

$$\dot{p} = \{p, H_{\text{eff}}\} = -\frac{8\pi\gamma}{3}\frac{\partial H_{\text{eff}}}{\partial c} \tag{2.5}$$

or

$$\dot{a} = \frac{\sin(\overline{\mu}c)\cos(\overline{\mu}c)}{\gamma\overline{\mu}}. \tag{2.6}$$

We define energy density and pressure of matter [24] as

$$\rho = a^{-3}H_M, \qquad P = -\frac{1}{3}a^{-2}\frac{\partial H_M}{\partial a}. \tag{2.7}$$

Combining with the constraint on Hamiltonian, $H_{\text{eff}} = 0$, we obtain the modified Friedmann equation

$$H^2 = \frac{8\pi}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right), \tag{2.8}$$

where $H \equiv \dot{a}/a$ denotes the Hubble rate, and $\rho_c \equiv 3/8\pi\gamma^2\overline{\mu}^2 p$ is the quantum critical density. Compared with the standard Friedmann equation, we can define the effective density

$$\rho_{\text{eff}} = \rho\left(1 - \frac{\rho}{\rho_c}\right). \tag{2.9}$$

Taking derivative of (2.8) and also using the conservation equation of matter $\dot{\rho}+3H(\rho+P) = 0$, we obtain the modified Raychaudhuri equation

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{4\pi}{3}\left\{\rho\left(1 - \frac{\rho}{\rho_c}\right) + 3\left[P\left(1 - \frac{2\rho}{\rho_c}\right) - \frac{\rho^2}{\rho_c}\right]\right\}. \tag{2.10}$$

Compared with the standard Raychaudhuri equation, we can define the effective pressure

$$P_{\text{eff}} = P\left(1 - \frac{2\rho}{\rho_c}\right) - \frac{\rho^2}{\rho_c}. \tag{2.11}$$

For different quantum corrections, the ρ_{eff} and P_{eff} may have different forms. But our following statement is still valid. In terms of the effective density and the effective pressure, the modified Friedmann, Raychaudhuri, and conservation equations take the following forms:

$$H^2 = \frac{8\pi}{3}\rho_{\text{eff}}, \tag{2.12}$$

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{4\pi}{3}\left(\rho_{\text{eff}} + 3P_{\text{eff}}\right), \tag{2.13}$$

$$\dot{\rho}_{\text{eff}} + 3H\left(\rho_{\text{eff}} + P_{\text{eff}}\right) = 0. \tag{2.14}$$

Till now, ρ_{eff} and P_{eff} are nothing but mathematical symbols to denote the coupling of matter and gravity. They still lack a thermodynamic origin, as noted by the authors of [25]. In the following, we will explore their intrinsic meaning in the thermodynamic sense and discuss some elementary implications based on above effective framework of LQC. But our result is more general and independent on the form of ρ_{eff} and P_{eff} which may be different when considering different quantumcorrections and quantum back reactions.

3. Thermodynamics in LQC

Let us begin with the effective LQC description of the universe evolution. For a spatially homogenous and isotropic universe described by the FRW metric, the line element is represented by

$$ds^2 = -dt^2 + a(t)^2\left(dr^2 + r^2 d\Omega^2\right), \tag{3.1}$$

where $a(t)$ is the scale factor of the universe, $t$is the cosmic time, and $d\Omega^2$ is the metric of sphere with unit radius. Thus, it is clear that all dynamical behaviors of the universe are determined by the scale factor $a(t)$. The metric (3.1) can be rewritten as

$$ds^2 = h_{ab}dx^a dx^b + \tilde{r}^2 d\Omega, \tag{3.2}$$

where $\tilde{r} = a(t)r$ and $x^0 = t$, $x^1 = r$ and the two-dimensional metric $h_{ab} = \mathrm{diag}(-1, a^2)$.

For the FRW universe, the dynamical apparent horizon, (without the whole evolution history of the universe, one cannot know whether there is a cosmological event horizon. However, apparent horizon always exists in the FRW universe since it is a local quantity of spacetime.) defined as the sphere with vanishing expansion [18], can be determined by the relation $h^{ab}\partial_a\tilde{r}\partial_b\tilde{r} = 0$ as

$$R_A = \frac{1}{H}, \tag{3.3}$$

which coincides with the Hubble horizon in this case. According to the definition of the surface gravity

$$\kappa = \frac{1}{2\sqrt{-h}}\partial_a\left(\sqrt{-h}h^{ab}\partial_b\tilde{r}\right), \tag{3.4}$$

its explicit evaluation at the dynamical apparent horizon R_A of the FRW universe reads

$$\kappa = -\frac{1}{R_A}\left(1 - \frac{\dot{R}_A}{2HR_A}\right). \tag{3.5}$$

We now introduce the Misner-Sharp spherically symmetric gravitational energy E, or simply the MS energy, defined in natural units by [20]

$$E = \frac{r}{2}\left(1 - h^{ab}\partial_a r\partial_b r\right), \tag{3.6}$$

which is the total energy (not only the passive energy) inside the sphere with radius r. The MS energy is a pure geometric quantity and is extensively used in literatures about thermodynamics of spacetime [26–28]. Its physical meaning and the comparison to the ADM mass and Bondi-Sachs energy have been given in [29]. For spherical space-time, Brown-York energy [30] agrees with the Liu-Yau energy [31], but they both differ from the MS energy. For example, for the four-dimensional Reissner-Nordström black hole, the MS energy differs from the Brown-York or Liu-Yau mass by a term which is the energy of the electromagnetic field inside the sphere, as discussed in [29].

In terms of the apparent horizon radius (3.3), the Friedmann equation (2.12) can be rewritten as

$$\frac{1}{R_A^2} = \frac{8\pi}{3}\rho_{\mathrm{eff}}. \tag{3.7}$$

Now we consider the MS energy (3.6) within the apparent horizon $r = R_A$ of the FRW universe, given by

$$E = \frac{R_A}{2}. \tag{3.8}$$

Using (3.7), we get

$$E = \frac{4\pi R_A^3}{3}\rho_{\text{eff}} = \rho_{\text{eff}} V. \tag{3.9}$$

It shows that it is reasonable to say that ρ_{eff} is indeed the energy density, not just a mathematical symbol. Then from the conservation equation (2.14), which implies energy and momentum conservation, it is also reasonable to take P_{eff} as pressure. That is to say that the gravitational effects contribute to energy density and pressure in the thermodynamical sense. In the following, we will find that this physical meaning is consistent with the fundamental relation of thermodynamics, which in turn supports this physical interpretation.

To examine the fundamental relation of thermodynamics in the setup of LQC, we consider the apparent horizon of the FRW universe as a thermodynamical system. An ansatz is made. Assume that the apparent horizon has an associated Hawking temperature T and entropy S expressed, respectively, as

$$T = \frac{\kappa}{2\pi}, \qquad S = \frac{A}{4}, \tag{3.10}$$

where $A = 4\pi R_A^2$ is the area of the apparent horizon.

Taking derivative of the energy equation (3.9) and using conservation equation (2.14), we get

$$dE = 4\pi R_A^2 \rho_{\text{eff}} \dot{R}_A dt - 4\pi R_A^3 H(\rho_{\text{eff}} + P_{\text{eff}}) dt. \tag{3.11}$$

Beside this, by taking derivative of the Friedmann equation (3.7) and using the conservation equation (2.14), we get the differential form of the Friedmann equation

$$\frac{1}{R_A^3} dR_A = 4\pi(\rho_{\text{eff}} + P_{\text{eff}}) H dt. \tag{3.12}$$

Considering the surface gravity on the apparent horizon (3.5), we can multiply both sides of the above equation by a factor $R_A(1 - \dot{R}_A/2HR_A)$ and get

$$\frac{\kappa}{2\pi} d(\pi R_A^2) = -4\pi R_A^3 (\rho_{\text{eff}} + P_{\text{eff}}) H \left(1 - \frac{\dot{R}_A}{2HR_A}\right) dt. \tag{3.13}$$

Therefore, in virtue of the ansatz (3.10) and combining the (3.11) and (3.13), one gets

$$dE = TdS + WdV, \tag{3.14}$$

where $W = (\rho_{\text{eff}} - P_{\text{eff}})/2$ is the work density if we take ρ_{eff} and P_{eff} as the energy density and pressure physically [26]. Again, we see that taking ρ_{eff} and P_{eff} as the energy density and pressure in the thermodynamical sense is consistent with the fundamental relation of thermodynamics. However, if we take ρ and P as the thermodynamical quantities, we will find that

$$dE = TdS + W'dV + \frac{\rho}{\rho_c} PdV \tag{3.15}$$

with work density $W' = (\rho - P)/2$. This equation means that the fundamental relation of thermodynamics breaks down unless we consider that the work term now does not take the form suggested by [26]. But this complicated expression for work term seems not reasonable. In contrast, the physical interpretation that ρ_{eff} and P_{eff} are thermodynamical quantities in LQC is consistent with the fundamental relation of thermodynamics. Or to say in terms of thermodynamical quantities ρ_{eff} and P_{eff}, the fundamental relation of thermodynamics is valid in LQC too.

4. Conclusion

In Conclusion, we have investigated the thermodynamic properties of the universe in LQC scenario and found that the fundamental relation of thermodynamics is valid in the effective LQC scenario. We found that the effective density ρ_{eff} and the effective pressure P_{eff} are not only a symbol to denote the coupling between the gravity and matter, but also actually the energy density and pressure in thermodynamical sense. This result comes from the energy definition in cosmology (the Misner-Sharp spherically symmetric gravitational energy) and is consistent with fundamental relation of thermodynamics.

In the following, we briefly comment on the physical meanings from the expressions of the effective energy density and pressure. When the energy density is much smaller than the quantum critical density ($\rho \ll \rho_c$), the effective density ρ_{eff} and the effective pressure P_{eff} come back to the traditional ones, that is, ρ and P, and the classical picture is recovered. Apart from the contribution of the matter sector, the effective density and pressure also receive the contribution from the spatial curvature. Also note that while for large volumes the spatial curvature is negligible to the density and pressure, for small volumes it is important. Since the ρ_{eff} and P_{eff} have thermodynamic meanings, and nonperturbative modification to the matter field at short scales implies inflation which also means a violation of the strong energy condition [32], we can expect that the wormhole solution maybe a normal object in effective LQC. Similarly, the spectrum of fluctuation of ρ_{eff} may be more important than ρ itself which contributes to the large-scale structure of the universe. All these are interesting topics for further study.

Acknowledgments

The work was supported by the National Natural Science Foundation of China (no. 10875012). The first author is indebted to Dr. Dah-Wei Chiou for his helpful discussions.

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