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Research Article

Final State Interaction Effects on the $B^+ \to J/\psi\rho^+$ Decay

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The exclusive decay of $B^+ \to J/\psi\rho^+$ is studied in the framework of the QCD factorization (QCDF) method and final state interaction (FSI). A direct $B^+ \to J/\psi\rho^+$ decay is only occurred via a tree and a penguin based on the quark diagram analysis. The result that is found by using the QCDF method is less than the experimental result, so, the role of FSI is considered. The intermediate states $D^+\overline{D}^0$, $D^{+*}\overline{D}^{0*}$, $D^{+*}\overline{D}^0$, and $D^+\overline{D}^{0*}$ via the exchange of D^- and D^{-*} are contributed to the $B^+ \to J/\psi\rho^+$ decay. The above intermediate states is calculated by using the QCDF method. In the FSI effects the results of our calculations depend on "η" as the phenomenological parameter. The range of this parameter are selected from 1 to 2. For the exchanged particles D^- and D^{-*}, it is found that if $\eta = 1.58 \sim 1.83$ is selected the numbers of the branching ratio are placed in the experimental range. The experimental branching ratio of $B^+ \to J/\psi\rho^+$ decay is $(5 \pm 0.8) \times 10^{-5}$, and our prediction number is $(1.42 \pm 0.36) \times 10^{-5}$ in the absence of FSI effects, and it becomes $(4.2 \sim 5.8) \times 10^{-5}$ when FSI contributions are taken into account.

1. Introduction

The importance of FSI in weak nonleptonic B meson decays is investigated by using a relativistic chiral unitary approach based on coupled channels [1–3]. The chiral Lagrangian approach is proved to be reliable for evaluating hadronic processes, but there are too many free parameters which are determined by fitting data, so that its applications are much constrained. Therefore, we have tried to look for some simplified models which can give rise to reasonable estimation of FSI [4, 5]. The FSI can be considered as a rescattering process of some intermediate two-body states with one particle exchange in the t-channel and computed via the absorptive part of the hadronic loop level (HLL) diagrams. The calculation with the single-meson-exchange scenario is obviously much simpler and straightforward. Moreover, some theoretical uncertainties are included in an off-shell form factor which

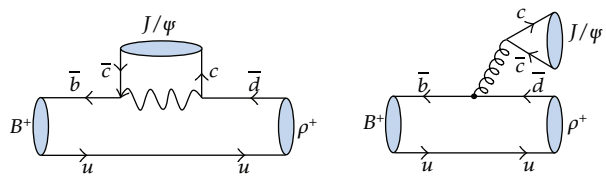

Figure 1: The Feynman diagrams contributing to the $B^+ \to J/\psi\rho^+$ decay.

modifies the effective vertices. Since the particle exchanged in the t-channel is off shell and since final state particles are hard, form factors or cutoffs must be introduced to the strong vertices to render the calculation meaningful in perturbation theory. If the intermediate two body mesons are hard enough, so that the perturbative calculation can make sense and work perfectly well, but the FSI can be modelled as the soft rescattering of the intermediate mesons. When one or two intermediate meson can reach a low-energy region where they are not sufficiently hard, one can be convinced that at this region the perturbative QCD approach fails or cannot result in reasonable values. If the intermediate mesons are soft, one can conjecture that at this region the nonperturbative QCD would dominate, and it could be attributed into the FSI effects. Because all FSI processes are concerning nonperturbative QCD [6], we have to rely on phenomenological models to analyze the FSI effects in certain reactions. In fact, after weak decays of heavy mesons, the particles produced can rescatter into other particle states through nonperturbative strong interaction. In the FSI the decay $B^+ \to J/\psi\rho^+$ is a very interesting mode [7]. We calculated the $B^+ \to J/\psi\rho^+$ decay according to QCDF method and selected the leading order Wilson coefficients at the scale m_b and obtained the $\mathrm{BR}(B^+ \to J/\psi\rho^+) = (1.42 \pm 0.36) \times 10^{-5}$. The FSI can give sizable corrections, and we can utilize it [8]. Rescattering amplitude can be derived by calculating the absorptive part of triangle diagrams. In this case, intermediate states are $D^+\overline{D}^0$, $D^{+*}\overline{D}^{0*}$, $D^{+*}\overline{D}^0$, and $D^+\overline{D}^{0*}$. Then, we calculated the $B^+ \to J/\psi\rho^+$ decay according to HLL method. Taken FSI corrections into account the branching ratio of $B^+ \to J/\psi\rho^+$ becomes $(4.2 \sim 5.8) \times 10^{-5}$ and the experimental result of this decay is $(5 \pm 0.8) \times 10^{-5}$ [9].

This paper is organized as follows. We present the calculation of QCDF for $B^+ \to J/\psi\rho^+$ decay in Section 2. In Section 3, we calculate the amplitudes of the intermediate states of $B^+ \to D^{+(*)}\overline{D}^{0(*)}$ decays. Then, we present the calculation of HLL for $B^+ \to J/\psi\rho^+$ decay in Section 4. In Section 5, we give the numerical results, and in the last section, we have a short conclusion.

2. $B^+ \to J/\psi\rho^+$ Decay in QCD Factorization

A detailed discussion of the QCDF approach can be found in [10, 11]. Factorization is a property of the heavy-quark limit, in which we assume that the b quark mass is parametrically large. The QCDF formalism allows us to compute systematically the matrix elements of the effective weak Hamiltonian in the heavy-quark limit for certain two-body final states $J/\psi\rho^+$. In this section, we obtain the amplitude of $B^+ \to J/\psi\rho^+$ decay using QCDF method. In factorization approach, there are color-suppressed tree and allowed penguin diagrams to $B^+ \to J/\psi\rho^+$ decay. We adopt leading order Wilson coefficients at the scale m_b for QCDF

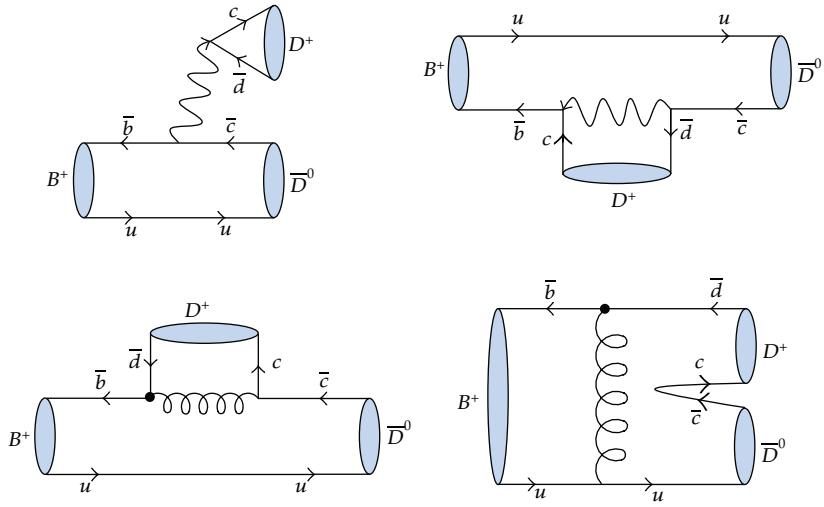

Figure 2: $B^+ \to D^+\overline{D}^0$ decay diagrams.

approach. The diagrams describing this decay are shown in Figure 1. According to the QCDF, the amplitude of $B^+ \to J/\psi\rho^+$ decay is given by

$$\begin{aligned} A_{\mathrm{QCDF}}\left(B^+ \longrightarrow \frac{J}{\psi\rho^+}\right) = & -i\frac{G_F}{\sqrt{2}} f_{J/\psi} m_{J/\psi} \left\{ (\epsilon_{J/\psi}\cdot\epsilon_\rho)(m_B+m_\rho) A_1^{B\rho}\left(m_{J/\psi}^2\right) \right. \\ & - (\epsilon_{J/\psi}\cdot p_B)(\epsilon_\rho\cdot p_B)\frac{2A_2^{B\rho}\left(m_{J/\psi}^2\right)}{m_B+m_\rho} \\ & \left. -i\varepsilon_{\mu\nu\alpha\beta}\epsilon_{J/\psi}^{\mu}\epsilon_\rho^{\nu}p_B^{\alpha}p_1^{\beta} \times \frac{2V^{B\rho}\left(m_{J/\psi}^2\right)}{m_B+m_\rho} \right\} \\ & \times \left\{ V_{cb}V_{cd}^* a_2 - V_{tb}V_{td}^*\left[a_3 + r_\chi^{J/\psi}(a_5+a_7+a_9)\right]\right\}, \end{aligned} \tag{2.1}$$

where

$$r_\chi^{J/\psi} = \frac{2m_{J/\psi}}{m_b}\frac{f_{J/\psi}^{\perp}}{f_{J/\psi}}. \tag{2.2}$$

The effective coefficients a_i, which are specific to the factorization approach, and defined as

$$\begin{aligned} a_i &= c_i^{\mathrm{eff}} + \frac{1}{N_c}c_{i+1}^{\mathrm{eff}} \quad (i = \text{odd}), \\ a_i &= c_i^{\mathrm{eff}} + \frac{1}{N_c}c_{i-1}^{\mathrm{eff}} \quad (i = \text{even}), \end{aligned} \tag{2.3}$$

where the quantities of c_i^{eff} are effective Wilson coefficients at the renormalization scale μ for the $\overline{b} \to \overline{d}$ transition. In the above amplitude the determination of a_2 in the $b \to c$ current-current transitions has received a lot of attention, the quantities of a_3, a_5 and a_7, a_9 are the QCD-penguin and electroweak-penguin coefficients, respectively. Numerical values of a_i ($i = 1, \ldots, 10$) for representative value of the phenomenological parameter N_c are displayed in Section 5.

3. Amplitudes of Intermediate State

To estimate the $B^+ \to D^{+(*)}\overline{D}^{0(*)}$ decays amplitudes of intermediate states [12], the QCDF method is again used. For $B^+ \to D^+\overline{D}^0$ decay, Feynman diagrams are shown in Figure 2, and the amplitude comes

$$\begin{aligned} A\left(B^+ \longrightarrow D^+\overline{D}^0\right) = i\frac{G_F}{\sqrt{2}} f_D \Big\{ & F_0^{BD}\left(m_B^2 - m_D^2\right) \\ & \times \Big\{ (a_1 + a_2) V_{cb} V_{cd}^* \\ & \quad + \left[a_4 + a_{10} + r_\chi^D (a_6 + a_8)\right] \lambda_p \Big\} \\ & + f_B f_D \left(b_3 + \frac{1}{2} b_{3,EW}\right) \lambda_p \Big\}, \end{aligned} \tag{3.1}$$

where

$$\begin{aligned} \lambda_p &= \sum_{p=u,c} V_{pb} V_{pd}^*, \\ b_3 &= \frac{C_F}{N_c^2}\left[c_3 A_1^i + c_5\left(A_3^i + A_3^f\right) + N_c c_6 A_3^f\right], \\ b_{3,EW} &= \frac{C_F}{N_c^2}\left[c_9 A_1^i + c_7\left(A_3^i + A_3^f\right) + N_c c_8 A_3^f\right], \\ A_1^i &\approx 2\pi\alpha_s\left[9\left(X_A - 4 + \frac{\pi^2}{3}\right) + r_\chi^{D^+} r_\chi^{D^-} X_A^2\right], \\ A_3^i &= 0, \\ A_3^f &\approx 6\pi\alpha_s\left(r_\chi^{D^+} + r_\chi^{D^-}\right)\left(2X_A^2 - X_A\right), \\ X_A &= \left(1 + \rho e^{i\phi}\right) \ln \frac{m_B}{\Lambda_{\text{QCD}}}. \end{aligned} \tag{3.2}$$

There are also some similar diagrams such as $B^+ \to D^{+*}\overline{D}^{0*}$ and $B^+ \to D^{+*}\overline{D}^0 (D^+\overline{D}^{0*})$ decays, which have polarization vectors. In their amplitude, just contribution of a_1 is

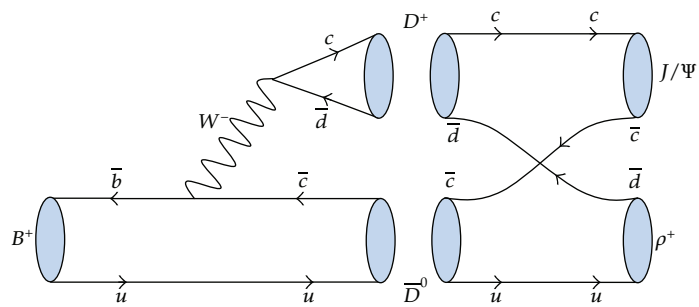

Figure 3: Quark level diagram for $B^+ \to D^{+(*)}\overline{D}^{0(*)} \to J/\psi\rho^+$.

considered, which has the dominant contribution. So the amplitude of $B^+ \to D^{+*}\overline{D}^{0*}$ decay, read as

$$A\left(B^+ \longrightarrow D^{+*}\overline{D}^{0*}\right) = -i\frac{G_F}{\sqrt{2}} f_{D^*} m_{D^*} V_{cb} V_{cd}^* \times \left\{(\epsilon_1^* \cdot \epsilon_2^*)(m_B + m_{D^*}) A_1^{BD^*}\left(m_{D^*}^2\right) - (\epsilon_1 \cdot p_B)(\epsilon_2 \cdot p_B)\frac{2A_2^{BD^*}(m_{D^*}^2)}{m_B + m_{D^*}}\right\}, \tag{3.3}$$

where $A_1^{BD^*}$ and $A_2^{BD^*}$ are form factors for $B \to D^*$ [13], and the amplitude of $B^+ \to D^{+*}\overline{D}^0$ is given by

$$A\left(B^+ \longrightarrow D^{+*}\overline{D}^0\right) = \sqrt{2} G_F m_{D^*} V_{cb} V_{cd}^* (\epsilon_{D^*} \cdot p_D)\left[f_{D^*} F_1^{BD}\left(m_{D^*}^2\right) + f_D A_0^{BD^*}\left(m_D^2\right)\right]. \tag{3.4}$$

4. Final State Interaction of $B^+ \to J/\psi\rho^+$ Decay

In our approach, we consider only leading contributions in charmless FSI, when the FSI for decay is calculated, two-body intermediate states, such as, $D^+\overline{D}^0$, $D^{+*}\overline{D}^{0*}$, $D^{+*}\overline{D}^0$, and $D^+\overline{D}^{0*}$ via the exchange of D^- and D^{-*} are contributed to the $B^+ \to J/\psi\rho^+$ decay. The quark model for $B^+ \to D^{+(*)}\overline{D}^{0(*)} \to J/\psi\rho^+$ diagram is shown in Figure 3. This diagram just shows tree level (a_1 contribution), while there are other contributions that were calculated in the last section. The diagrams which determine the HLL effects on the rate of $B^+ \to J/\psi\rho^+$ decay are shown in Figure 4. At the HLL, the charge exchange reaction between the J/ψ and ρ can proceed either by exchange of a D or D^* meson. In order to calculate the various Feynman diagrams for the reactions $D + \overline{D} \to J/\psi + \rho$. And so forth (were calculated in the nonrelativistic quark model [14]) we need the construct the effective three mesons vertices, such as, ψDD and ρDD. We begin with an SU(4) symmetry so as to include charm, and we introduce pseudoscalar ($P = \varphi_a \lambda_a$) and vector ($V^\mu = v_a^\mu \lambda_a$) meson matrices, where φ_a and

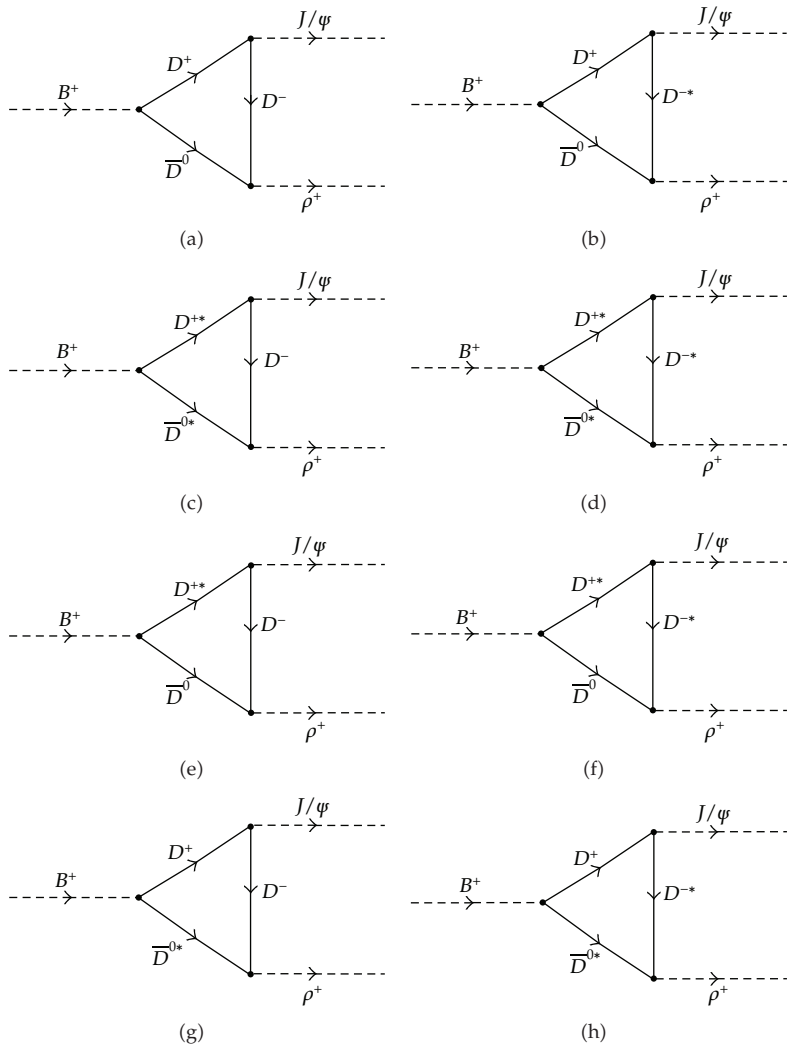

Figure 4: HLL diagrams for long-distance t-channel contribution to $B^+ \to J/\psi\rho^+$.

v_a^μ are pseudoscalar and vector multiplets and the λ_s are SU(4) generators. The free meson lagrangian is read [15]

$$\pounds_0 = \mathrm{Tr}\left(\partial^\mu P^\dagger \partial_\mu P\right) - \frac{1}{2}\,\mathrm{Tr}\left(\partial^\mu V^{\dagger,\nu} - \partial^\nu V^{\dagger,\mu}\right)\left(\partial_\mu V_\nu - \partial_\nu V_\mu\right). \tag{4.1}$$

Interaction Lagrangians are generated by replacing the space-time derivative with a gauge covariant one $\partial_\mu f \to D_\mu f = \partial_\mu f + [A_\mu, f]$, where $A_\mu = -ig/2V_\mu$. The vector mesons are

recognizable as playing roles of gauge bosons. As is usual in effective field theory strategies, to keep gauge consistency, we must collect terms of order g^2. The effective lagrangian is read

$$\begin{aligned}\pounds_{\text{int}} = {} & ig\,\mathrm{Tr}\left(PV^{\mu}\partial_{\mu}P - \partial^{\mu}PV_{\mu}P\right) \\ & + \frac{1}{2}g^2\,\mathrm{Tr}\left(PV^{\mu}V_{\mu}P - PV^{\mu}PV_{\mu}\right) \\ & + ig\,\mathrm{Tr}\left(\partial^{\mu}V^{\nu}\left[V_{\mu},V_{\nu}\right] + \left[V^{\mu},V^{\nu}\right]\partial_{\mu}V_{\nu}\right) \\ & + g^2\,\mathrm{Tr}\left(V^{\mu}V^{\nu}\left[V_{\mu},V_{\nu}\right]\right).\end{aligned} \tag{4.2}$$

The order g terms deliver three-point vertices and the order g^2 terms are responsible for so-called contact terms or four-point couplings which are necessary for a gauge invariant theory. The interactions of interest for this study are the following [5]:

$$\begin{aligned}\pounds_{DD\rho} &= ig_{DD\rho}\left(D\rho^{\mu}\partial_{\mu}\overline{D} - \partial_{\mu}D\rho^{\mu}\overline{D}\right), \\ \pounds_{D^*D\rho} &= -g_{D^*D\rho}\epsilon^{\mu\nu\alpha\beta}\left(D\partial_{\mu}\rho_{\nu}\partial_{\alpha}\overline{D}^{*}_{\beta} + \partial_{\mu}D^{*}_{\nu}\partial_{\alpha}\rho_{\beta}\overline{D}\right), \\ \pounds_{D^*D^*\rho} &= ig_{D^*D^*\rho}\Big\{\partial_{\mu}D^{*}_{\nu}\rho^{\mu}\overline{D}^{*\nu} - D^{*}_{\nu}\rho_{\mu}\partial^{\mu}\overline{D}^{*\nu} \\ &\qquad + \left(D^{*\nu}\partial_{\mu}\rho_{\nu} - \partial_{\mu}D^{*}_{\nu}\rho^{\nu}\right)\overline{D}^{*\mu} + D^{*\mu}\left(\rho^{\nu}\partial_{\mu}\overline{D}^{*}_{\nu} - \partial_{\mu}\rho_{\nu}\overline{D}^{*\nu}\right)\Big\}, \\ \pounds_{\psi DD} &= ig_{\psi DD}\psi_{\mu}\left(\partial^{\mu}D\overline{D} - D\partial^{\mu}\overline{D}\right), \\ \pounds_{\psi D^*D} &= -g_{\psi D^*D}\epsilon^{\mu\nu\alpha\beta}\partial_{\mu}\psi_{\nu}\left(\partial_{\alpha}D^{*}_{\beta}\overline{D} + D\partial_{\alpha}\overline{D}^{*}_{\beta}\right), \\ \pounds_{\psi D^*D^*} &= -ig_{\psi D^*D^*}\Big\{\psi^{\mu}\left(\partial_{\mu}D^{*\nu}\overline{D}^{*}_{\nu} - D^{*\nu}\partial_{\mu}\overline{D}^{*}_{\nu}\right) \\ &\qquad + \left(\partial_{\mu}\psi_{\nu}D^{*\nu} - \psi_{\nu}\partial_{\mu}D^{*\nu}\right)\overline{D}^{*\mu} + D^{*\mu}\left(\psi^{\nu}\partial_{\mu}\overline{D}^{*}_{\nu} - \partial_{\mu}\psi_{\nu}\overline{D}^{*\nu}\right)\Big\}, \\ \pounds_{\psi\rho DD} &= -g_{\rho DD}g_{\psi DD}\psi^{\mu}D\rho_{\mu}\overline{D}, \\ \pounds_{\psi\rho D^*D^*} &= g_{\psi D^*D^*}g_{\rho D^*D^*}\left(2D^{*\nu}\rho^{\mu}\overline{D}^{*}_{\nu} - D^{*\nu}\rho_{\nu}\overline{D}^{*\mu} - D^{*\mu}\rho^{\nu}\overline{D}^{*}_{\nu}\right).\end{aligned} \tag{4.3}$$

Here $\epsilon_{0123} = +1$ and we define the charm meson iso-doublets as

$$\begin{aligned}\overline{D}^{T} &= \left(\overline{D}^{0}, D^{-}\right), \qquad D = \left(D^{0}, D^{+}\right), \\ \overline{D}^{*T} &= \left(\overline{D}^{*0}, D^{*-}\right), \qquad D^{*} = \left(D^{*0}, D^{*+}\right).\end{aligned} \tag{4.4}$$

In this approach there are several coupling constants in (4.3). Methods for constraining them will be discussed below. The $O(g^2)$ terms carry one power of coupling constant for each three-point vertices from which the contact term collapses. The coupling constants $g_{\psi DD}$, $g_{\rho DD}$,

$g_{\psi D^* D^*}$, and $g_{\rho D^* D^*}$ can be derived in the standard framework of the vector meson dominance model (VDM) [16–18]. By using VDM to the coupling of γDD we have

$$\gamma_\rho g_{\rho DD} = \sqrt{\pi \alpha_{em}}, \qquad \gamma_{J/\psi} g_{\psi DD} = \frac{4}{3}\sqrt{\pi \alpha_{em}}, \tag{4.5}$$

where

$$\gamma_\rho = \sqrt{\frac{3\Gamma_{\rho \to e^+e^-}}{m_\rho \alpha_{em}}}, \qquad \gamma_{J/\psi} = \sqrt{\frac{3\Gamma_{J/\psi \to e^+e^-}}{m_{J/\psi} \alpha_{em}}}. \tag{4.6}$$

The same method can be used for the $\gamma D^* D^*$ coupling. Vector dominance gives

$$g_{\rho D^* D^*} = g_{\rho DD}, \qquad g_{\psi D^* D^*} = g_{\psi DD}. \tag{4.7}$$

For the $\rho D^* D$ and $\psi D^* D$ couplings the VDM can be applied to the radiative decays of D^* into D, that is, $D^* \to D\gamma$. By using the same technique we have

$$\gamma_\rho g_{\rho D^* D} = \frac{1}{3}\sqrt{\pi \alpha_{em}}\left(g_V^{\pm} + 2g_V^0\right), \qquad \gamma_{J/\psi} g_{\psi DD} = \frac{4}{3}\sqrt{\pi \alpha_{em}} g_V^{\pm} = \frac{4}{3}\sqrt{\pi \alpha_{em}} g_V^0, \tag{4.8}$$

where $g_V^{\pm,0}$ are calculated in the model prediction based on relativistic potential model that we follow [19]. With the above preparation we can write out the decay amplitude involving HLL contributions with the following formula:

$$\begin{aligned} A(B(p_B) \longrightarrow M(p_1)M(p_2) \longrightarrow M(p_3)M(p_4)) = \frac{1}{2}\int \frac{d^3\vec{p_1}}{2E_1(2\pi)^3}\frac{d^3\vec{p_2}}{2E_2(2\pi)^3} \\ \times (2\pi)^4 \delta^4(p_B - p_1 - p_2) M(B \longrightarrow M_1 M_2) \\ \times G(M_1 M_2 \longrightarrow M_3 M_4), \end{aligned} \tag{4.9}$$

for which both intermediate mesons (M_1, M_2) are pseudoscalar. And

$$\begin{aligned} A(B(p_B) \longrightarrow M(p_1)M(p_2) \longrightarrow M(p_3)M(p_4)) \\ = -i\frac{G_F}{2\sqrt{2}}\int \frac{d^3\vec{p_1}}{2E_1(2\pi)^3}\frac{d^3\vec{p_2}}{2E_2(2\pi)^3} \\ \times (2\pi)^4 \delta^4(p_B - p_1 - p_2) f_{D^*} m_{D^*} V_{cb} V_{cd}^* \end{aligned}$$

$$\times \left[(\epsilon_1^* \cdot \epsilon_2^*)(m_B + m_1) A_1^{BM_1}\left(m_2^2\right) - (\epsilon_1^* \cdot p_B)(\epsilon_2^* \cdot p_B) \frac{2A_2^{BM_1}(m_2^2)}{m_B + m_1} \right] \times G(M_1 M_2 \longrightarrow M_3 M_4), \tag{4.10}$$

in which both meson are vector. Also $G(M_1 M_2 \rightarrow M_3 M_4)$ involves hadronic vertices factor, which is defined as

$$\begin{aligned}
\langle D(p_3)\rho(\epsilon_2, p_2)|i\pounds|D(p_1)\rangle &= -ig_{DD\rho}\epsilon_2 \cdot (p_1 + p_3), \\
\langle D^*(\epsilon_3, p_3)\rho(\epsilon_2, p_2)|i\pounds|D(p_1)\rangle &= -i\sqrt{2}g_{D^*D\rho}\epsilon_{\mu\nu\alpha\beta}\epsilon_2^\mu \epsilon_3^{*\nu} p_1^\alpha p_2^\beta, \\
\langle D^*(\epsilon_3, p_3)\rho(\epsilon_2, p_2)|i\pounds|D^*(\epsilon_1, p_1)\rangle &= -i\epsilon_1^\beta \epsilon_2^\nu \epsilon_3^\lambda \left[2p_{1\nu} g_{\alpha\beta} - (p_1 + p_2)_\alpha g_{\nu\beta} + 2p_{2\beta} g_{\alpha\nu}\right], \\
\langle D(p_3)\psi(\epsilon_2, p_2)|i\pounds|D(p_1)\rangle &= -ig_{\psi DD}\epsilon_2 \cdot (p_1 + p_3), \\
\langle D^*(\epsilon_3, p_3)\psi(\epsilon_2, p_2)|i\pounds|D(p_1)\rangle &= -ig_{\psi D^* D}\epsilon_{\mu\nu\alpha\beta}\epsilon_2^\mu \epsilon_3^{*\nu} p_1^\alpha p_2^\beta, \\
\langle D^*(\epsilon_3, p_3)\psi(\epsilon_2, p_2)|i\pounds|D^*(\epsilon_1, p_1)\rangle &= -i\epsilon_1^\beta \epsilon_2^\nu \epsilon_3^\lambda \left[2p_{1\nu} g_{\alpha\beta} - (p_1 + p_2)_\alpha g_{\nu\beta} + 2p_{2\beta} g_{\alpha\nu}\right].
\end{aligned} \tag{4.11}$$

The dispersive part of the rescattering amplitude can be obtained from the absorptive part via the dispersion relation [6, 20]:

$$\mathrm{Dis}M\left(m_B^2\right) = \frac{1}{\pi}\int_s^\infty \frac{\mathrm{Abs}M(s')}{s' - m_B^2} ds', \tag{4.12}$$

where s' is the square of the momentum carried by the exchanged particle and s is the threshold of intermediate states, in this case $s \sim m_B^2$. Unlike the absorptive part, the dispersive contribution suffers from the large uncertainties arising from the complicated integrations. So the amplitudes of the mode $B^+ \rightarrow D^+(p_1)\overline{D}^0(p_2) \rightarrow J/\psi(p_3, \epsilon_3)\rho^+(p_4, \epsilon_4)$, where D^- and D^{-*} mesons are exchanged, respectively, are given by

$$\begin{aligned}
\mathrm{Abs}(4\mathrm{a}) &= \int_{-1}^{1} \frac{|\vec{p_1}| d(\cos\theta)}{16\pi m_B} A\left(B^+ \longrightarrow D^+\overline{D}^0\right)(-i)g_{\psi DD}(2\epsilon_3 \cdot p_1) \\
&\quad \times (-i)g_{DD\rho}(2\epsilon_4 \cdot p_2)\frac{F^2(q^2, m_D^2)}{q^2 - m_D^2} \\
&= -\int_{-1}^{1} \frac{|\vec{p_1}| d(\cos\theta)}{4\pi m_B} g_{\psi DD} g_{DD\rho} A\left(B^+ \longrightarrow D^+\overline{D}^0\right)\frac{F^2(q^2, m_D^2)}{q^2 - m_D^2} H_1,
\end{aligned} \tag{4.13}$$

where θ is the angel between $\vec{p}_1$ and $\vec{p}_3$, q is the momentum of the exchange D meson, and

$$H_1 = \epsilon_{3\mu} p_1^{\mu} \epsilon_{4\nu} p_2^{\nu} = \left(p_1^0 - |\vec{p}_1|\right)\left(p_2^0 - |\vec{p}_2|\right),$$
$$q^2 = m_1^2 + m_3^2 - 2E_1E_3|\vec{p}_1||\vec{p}_3|\cos\theta, \tag{4.14}$$

and $F(q^2, m_D^2)$ is the form factor defined to take care of the off-shell of the exchange particles, which introduced as [6, 21]

$$F\left(q^2, m_D^2\right) = \left(\frac{\Lambda^2 - m_D^2}{\Lambda^2 - q^2}\right)^n. \tag{4.15}$$

The form factor (i.e., $n = 1$) normalized to unity at $q^2 = m_D^2$. m_D and q are the physical parameters of the exchange particle and Λ is a phenomenological parameter. It is obvious that for $q^2 \to 0$, $F(q^2, m_D^2)$ becomes a number. If $\Lambda \gg m_i$ then $F(q^2, m_D^2)$ turns to be unity, whereas, as $q^2 \to \infty$ the form factor approaches to zero and the distance becomes small and the hadron interaction is no longer valid. Since Λ should not be far from the m_D and q, we choose

$$\Lambda = m_D + \eta\Lambda_{\mathrm{QCD}}, \tag{4.16}$$

where η is the phenomenological parameter that its value in the form factor is expected to be of the order of unity and can be determined from the measured rates, and

$$\begin{aligned}
\mathrm{Abs(4b)} &= \int_{-1}^{1} \frac{|\vec{p}_1| d(\cos\theta)}{16\pi m_B} A\left(B^+ \longrightarrow D^+\overline{D}^0\right)(-i)\sqrt{2} g_{D^*D\rho}\epsilon_{\mu\nu\alpha\beta}\epsilon_4^{\mu} p_2^{\alpha} p_4^{\beta} \\
&\quad \times (-i) g_{\psi D^* D}\epsilon_{\rho\sigma\lambda\eta}\epsilon_3^{\rho} p_1^{\lambda} p_3^{\eta}\left(g_{\sigma\nu} - \frac{q_\sigma q_\nu}{m_{D^*}^2}\right)\frac{F^2(q^2, m_{D^*}^2)}{q^2 - m_{D^*}^2} \\
&= -\int_{-1}^{1} \frac{|\vec{p}_1| d(\cos\theta)}{16\pi m_B} g_{\psi D^* D} g_{D^* D\rho} A\left(B^+ \longrightarrow D^+\overline{D}^0\right)\frac{F^2(q^2, m_{D^*}^2)}{q^2 - m_{D^*}^2} H_2,
\end{aligned} \tag{4.17}$$

where

$$\begin{aligned}
H_2 = &-2\left[(p_1\cdot p_4)(p_2\cdot p_3) - (p_1\cdot p_2)(p_3\cdot p_4)\right] + p_3^0\left[p_4^0(p_1\cdot p_2) - \left(p_2^0 - |\vec{p}_2|\right)(p_1\cdot p_4)\right] \\
&+ \left(p_1^0 - |\vec{p}_1|\right)\left[\left(p_2^0 - |\vec{p}_2|\right)(p_3\cdot p_4) - p_4^0(p_2\cdot p_3)\right].
\end{aligned} \tag{4.18}$$

For diagrams (4c) and (4d) in Figure 4 the absorptive part of the amplitude corresponding to the process of $B^+ \to D^{+*}(p_1,\epsilon_1)\overline{D}^{0*}(p_2,\epsilon_2) \to J/\psi(p_3,\epsilon_3)\rho^+(p_4,\epsilon_4)$, where D^- and D^{-*} mesons are exchanged, respectively, is read as

$$\begin{aligned}
\mathrm{Abs}(4c) = & -i\frac{G_F}{2\sqrt{2}}\int \frac{d^3\vec{p_1}}{2E_1(2\pi)^3}\frac{d^3\vec{p_2}}{2E_2(2\pi)^3}(2\pi)^4\delta^4(p_B-p_1-p_2) \\
& \times(-i)g_{\psi D^* D}\epsilon_{\mu\nu\alpha\beta}\epsilon_3^{\mu}p_3^{\nu}\epsilon_1^{\alpha}p_1^{\beta}\left(-i\sqrt{2}\right)g_{D^*D\rho}\epsilon_{\rho\sigma\lambda\eta}\epsilon_4^{\rho}p_4^{\sigma}\epsilon_2^{\lambda}p_2^{\eta}\frac{F^2(q^2,m_D^2)}{q^2-m_D^2} \\
& \times f_{D^*}m_{D^*}V_{cb}V_{cd}^*\left\{(\epsilon_1^*\cdot\epsilon_2^*)(m_B+m_{D^*})A_1^{BD^*}\left(m_{D^*}^2\right)-(\epsilon_1\cdot p_B)(\epsilon_2\cdot p_B)\frac{2A_2^{BD^*}(m_{D^*}^2)}{m_B+m_{D^*}}\right\} \\
= & \, i\frac{G_F}{16\pi m_B}f_{D^*}m_{D^*}g_{\psi D^*D}g_{D^*D\rho}V_{cb}V_{cd}^* \\
& \times\int_{-1}^{1}|\vec{p_1}|d(\cos\theta)\frac{F^2(q^2,m_D^2)}{q^2-m_D^2}\times\left\{(m_B+m_{D^*})A_1^{BD^*}\left(m_{D^*}^2\right)H_3-\frac{2A_2^{BD^*}(m_{D^*}^2)}{m_B+m_{D^*}}H_3'\right\},
\end{aligned} \tag{4.19}$$

where

$$\begin{aligned}
H_3 = & \, \epsilon_{\mu\nu\alpha\beta}\epsilon_{\rho\sigma\lambda\eta}\epsilon_3^{\mu}p_3^{\nu}p_1^{\beta}\epsilon_4^{\rho}p_4^{\sigma}p_2^{\eta}\left(-g_{\chi}^{\alpha}+\frac{p_1^{\alpha}p_{1\chi}}{m_{D^*}^2}\right)\left(-g^{\lambda\chi}+\frac{p_2^{\lambda}p_2^{\chi}}{m_{D^*}^2}\right) \\
= & -2\left[(p_1\cdot p_2)(p_3\cdot p_4)-(p_1\cdot p_4)(p_2\cdot p_3)\right]+p_4^0\left[\left(p_1^0-|\vec{p_1}|\right)(p_2\cdot p_3)-p_3^0(p_1\cdot p_2)\right] \\
& +\left(p_2^0-|\vec{p_2}|\right)\left[p_3^0(p_1\cdot p_4)-\left(p_1^0-|\vec{p_1}|\right)(p_3\cdot p_4)\right], \\
H_3' = & \, \epsilon_{\mu\nu\alpha\beta}\epsilon_{\rho\sigma\lambda\eta}\epsilon_3^{\mu}p_3^{\nu}p_1^{\beta}\epsilon_4^{\rho}p_4^{\sigma}p_2^{\eta}\left(-p_B^{\alpha}+\frac{(p_1\cdot p_B)p_1^{\alpha}}{m_{D^*}^2}\right)\left(-p_B^{\lambda}+\frac{(p_2\cdot p_B)p_2^{\lambda}}{m_{D^*}^2}\right) \\
= & \, m_B^2\Big\{\left(p_1^0-|\vec{p_1}|\right)\left[p_4^0(p_2\cdot p_3)-\left(p_2^0-|\vec{p_2}|\right)(p_3\cdot p_4)\right]-2\left[(p_1\cdot p_2)(p_3\cdot p_4)-(p_1\cdot p_4)(p_2\cdot p_3)\right] \\
& +p_3^0\left[\left(p_2^0-|\vec{p_2}|\right)(p_1\cdot p_4)-p_4^0(p_1\cdot p_2)\right] \\
& +\left[p_3\cdot p_4-p_3^0p_4^0, p_1^0\left(p_2^0-|\vec{p_2}|\right)+p_2^0\left(p_1^0-|\vec{p_1}|\right)-p_1\cdot p_2\right] \\
& +3p_4^0\left[p_3^0(p_1\cdot p_2)-p_1^0(p_2\cdot p_3)\right]+\left[p_1\cdot p_4-p_4^0\left(p_1^0-|\vec{p_1}|\right)\right]\left[p_2\cdot p_3-p_3^0\left(p_2^0-|\vec{p_2}|\right)\right] \\
& -2p_2^0\left[p_3^0(p_1\cdot p_4)-p_1^0(p_3\cdot p_4)\right]+p_2^0p_3^0\left[p_1^0p_4^0-p_1\cdot p_4\right]\Big\}, \\
\mathrm{Abs}(4d) = & -i\frac{G_F}{2\sqrt{2}}\int\frac{d^3\vec{p_1}}{2E_1(2\pi)^3}\frac{d^3\vec{p_2}}{2E_2(2\pi)^3}(2\pi)^4\delta^4(p_B-p_1-p_2) \\
& \times(-i)g_{\psi D^*D^*}\epsilon_1^{\mu}\epsilon_3^{\rho}\left[2p_{1\rho}g_{\mu\beta}-(p_1+p_3)_{\beta}g_{\mu\rho}+2p_{3\mu}g_{\beta\rho}\right]\left(-g^{\alpha\beta}+\frac{q^{\alpha}q^{\beta}}{m_{D^*}^2}\right)
\end{aligned}$$

$$\times (-i) g_{D^*D^*\rho} \epsilon_2^\nu \epsilon_4^\lambda \left[2p_{2\lambda} g_{\alpha\nu} - (p_2 + p_4)_\alpha g_{\nu\lambda} + 2p_{4\nu} g_{\alpha\lambda}\right] \frac{F^2(q^2, m_{D^*}^2)}{q^2 - m_{D^*}^2}$$

$$\times f_{D^*} m_{D^*} V_{cb} V_{cd}^* \left\{ (\epsilon_1^* \cdot \epsilon_2^*)(m_B + m_{D^*}) A_1^{BD^*}\left(m_{D^*}^2\right) - (\epsilon_1 \cdot p_B)(\epsilon_2 \cdot p_B) \frac{2A_2^{BD^*}(m_{D^*}^2)}{m_B + m_{D^*}} \right\}$$

$$= i \frac{G_F}{16\sqrt{2}\pi m_B} f_{D^*} m_{D^*} V_{cb} V_{cd}^* \int_{-1}^{1} |\vec{p_1}| d(\cos\theta) \frac{F^2(q^2, m_{D^*}^2)}{q^2 - m_{D^*}^2}$$

$$\times \left\{ (m_B + m_{D^*}) A_1^{BD^*}\left(m_{D^*}^2\right) H_4 - \frac{2A_2^{BD^*}(m_{D^*}^2)}{m_B + m_{D^*}} H_4' \right\}, \tag{4.20}$$

where

$$H_4 = \left(-g^{\alpha\beta} + \frac{q^\alpha q^\beta}{m_{D^*}^2}\right)\left(-g^{\mu\chi} + \frac{p_1^\mu p_1^\chi}{m_{D^*}^2}\right)\left(-g_\chi^\nu + \frac{p_2^\nu p_{2\chi}}{m_{D^*}^2}\right)\epsilon_3^\rho \epsilon_4^\lambda$$

$$\times \left(2p_{1\rho} g_{\mu\beta} - (p_1 + p_3)_\beta g_{\mu\rho} + 2p_{3\mu} g_{\beta\rho}\right)\left(2p_{2\lambda} g_{\alpha\nu} - (p_2 + p_4)_\alpha g_{\nu\lambda} + 2p_{4\nu} g_{\alpha\lambda}\right),$$

$$H_4' = \left(-g^{\alpha\beta} + \frac{q^\alpha q^\beta}{m_{D^*}^2}\right)\left(-p_B^\mu + \frac{(p_1 \cdot p_B) p_1^\mu}{m_{D^*}^2}\right)\left(-p_B^\nu + \frac{(p_2 \cdot p_B) p_2^\nu}{m_{D^*}^2}\right)\epsilon_3^\rho \epsilon_4^\lambda \tag{4.21}$$

$$\times \left(2p_{1\rho} g_{\mu\beta} - (p_1 + p_3)_\beta g_{\mu\rho} + 2p_{3\mu} g_{\beta\rho}\right)\left(2p_{2\lambda} g_{\alpha\nu} - (p_2 + p_4)_\alpha g_{\nu\lambda} + 2p_{4\nu} g_{\alpha\lambda}\right).$$

For diagrams (4e) and (4f) in Figure 4 the absorptive part of the amplitude corresponding to the process of $B^+ \to D^{+*}(p_1, \epsilon_1)\overline{D}^0(p_2) \to J/\psi(p_3, \epsilon_3)\rho^+(p_4, \epsilon_4)$, where D^- and D^{-*} mesons are exchanged, respectively, becomes

$$\mathrm{Abs}(4e) = \frac{G_F}{\sqrt{2}} \int \frac{d^3\vec{p_1}}{2E_1(2\pi)^3} \frac{d^3\vec{p_2}}{2E_2(2\pi)^3} (2\pi)^4 \delta^4(p_B - p_1 - p_2)$$

$$\times V_{cb} V_{cd}^* m_{D^*} (-i) g_{\psi D^* D} \epsilon_{\mu\nu\alpha\beta} \epsilon_3^\mu \epsilon_1^\nu p_1^\alpha p_3^\beta (-i) g_{DD\rho} (\epsilon_4 \cdot q)(\epsilon_1 \cdot p_2)$$

$$\times \left[f_{D^*} F_1^{BD}\left(m_{D^*}^2\right) + f_D A_0^{BD^*}\left(m_D^2\right)\right] \frac{F^2(q^2, m_D^2)}{q^2 - m_D^2} \tag{4.22}$$

$$= -\frac{G_F}{4\sqrt{2}\pi m_B} \left(f_{D^*} F_1^{BD}\left(m_{D^*}^2\right) + f_D A_0^{BD^*}\left(m_D^2\right)\right) V_{cb} V_{cd}^* m_{D^*} g_{\psi D^* D} g_{DD\rho}$$

$$\times \int_{-1}^{1} |\vec{p_1}| d(\cos\theta) \frac{F^2(q^2, m_D^2)}{q^2 - m_D^2} H_5,$$

where

$$H_5 = \epsilon_{\mu\nu\alpha\beta}\epsilon_3^{\mu}p_1^{\alpha}p_3^{\beta}(\epsilon_4\cdot p_2)\left(-p_2^{\nu}+\frac{(p_1\cdot p_2)p_1^{\nu}}{m_{D^*}^2}\right),$$

$$\begin{aligned}
\mathrm{Abs(4f)} = {} & \frac{G_F}{\sqrt{2}}\int\frac{d^3\vec{p_1}}{2E_1(2\pi)^3}\frac{d^3\vec{p_2}}{2E_2(2\pi)^3}(2\pi)^4\delta^4(p_B-p_1-p_2)\\
& \times V_{cb}V_{cd}^{*}m_{D^*}(-i)g_{\psi D^*D^*}\epsilon_1^{\mu}\epsilon_3^{\nu}\epsilon^{\alpha}\left[2p_{1\nu}g_{\mu\beta}-(p_1+p_3)_{\beta}g_{\mu\nu}+2p_{3\mu}g_{\beta\nu}\right]\\
& \times(i)\sqrt{2}g_{D^*D\rho}\epsilon_{\rho\sigma\lambda\eta}\epsilon_4^{\rho}\epsilon^{\sigma}p_2^{\lambda}p_4^{\eta}\left[f_{D^*}F_1^{BD}(m_{D^*}^2)+f_DA_0^{BD^*}(m_D^2)\right]\frac{F^2(q^2,m_{D^*}^2)}{q^2-m_{D^*}^2}\\
= {} & \frac{G_F}{8\pi m_B}\left(f_{D^*}F_1^{BD}(m_{D^*}^2)+f_DA_0^{BD^*}(m_D^2)\right)V_{cb}V_{cd}^{*}m_{D^*}g_{\psi D^*D^*}g_{D^*D\rho}\\
& \times\int_{-1}^{1}|\vec{p_1}|d(\cos\theta)\frac{F^2(q^2,m_{D^*}^2)}{q^2-m_{D^*}^2}H_6,
\end{aligned} \tag{4.23}$$

where

$$\begin{aligned}
H_6 = {} & \epsilon_{\rho\sigma\lambda\eta}\epsilon_4^{\rho}\epsilon_3^{\nu}p_2^{\lambda}p_4^{\eta}\left[2p_{1\nu}g_{\mu\beta}-(p_1+p_3)_{\beta}g_{\mu\nu}+2p_{3\mu}g_{\beta\nu}\right]\\
& \times\left(-g^{\alpha\sigma}+\frac{q^{\alpha}q^{\sigma}}{m_{D^*}^2}\right)\left(-p_2^{\mu}+\frac{(p_1\cdot p_2)p_1^{\mu}}{m_{D^*}^2}\right).
\end{aligned} \tag{4.24}$$

For diagrams (4g) and (4h) in Figure 4 the absorptive part of the amplitude corresponding to the process of $B^+ \to D^+(p_1)\overline{D}^{0*}(p_2,\epsilon_2) \to J/\psi(p_3,\epsilon_3)\rho^+(p_4,\epsilon_4)$, where D^- and D^{-*} mesons are exchanged, respectively, is given

$$\begin{aligned}
\mathrm{Abs(4g)} = {} & \frac{G_F}{\sqrt{2}}\int\frac{d^3\vec{p_1}}{2E_1(2\pi)^3}\frac{d^3\vec{p_2}}{2E_2(2\pi)^3}(2\pi)^4\delta^4(p_B-p_1-p_2)\\
& \times V_{cb}V_{cd}^{*}m_{D^*}\left(i\sqrt{2}\right)g_{D^*D\rho}\epsilon_{\mu\nu\alpha\beta}\epsilon_4^{\mu}\epsilon_2^{\nu}p_2^{\alpha}p_4^{\beta}(-i)g_{\psi DD}(\epsilon_3\cdot q)(\epsilon_2\cdot p_1)\\
& \times\left[f_{D^*}F_1^{BD}\left(m_{D^*}^2\right)+f_DA_0^{BD^*}\left(m_D^2\right)\right]\frac{F^2(q^2,m_D^2)}{q^2-m_D^2}\\
= {} & \frac{G_F}{4\pi m_B}\left(f_{D^*}F_1^{BD}\left(m_{D^*}^2\right)+f_DA_0^{BD^*}\left(m_D^2\right)\right)V_{cb}V_{cd}^{*}m_{D^*}g_{\psi DD}g_{D^*D\rho}\\
& \times\int_{-1}^{1}|\vec{p_1}|d(\cos\theta)\frac{F^2(q^2,m_D^2)}{q^2-m_D^2}H_7,
\end{aligned} \tag{4.25}$$

where

$$H_7 = \epsilon_{\mu\nu\alpha\beta}\epsilon_4^{\mu}p_2^{\alpha}p_4^{\beta}(\epsilon_3\cdot p_1)\left(-p_1^{\nu}+\frac{(p_1\cdot p_2)p_2^{\nu}}{m_{D^*}^2}\right),$$

$$\begin{aligned}
\mathrm{Abs(4h)} = {} & \frac{G_F}{\sqrt{2}}\int\frac{d^3\vec{p_1}}{2E_1(2\pi)^3}\frac{d^3\vec{p_2}}{2E_2(2\pi)^3}(2\pi)^4\delta^4(p_B-p_1-p_2) \\
& \times V_{cb}V_{cd}^{*}m_{D^*}(-i)g_{D^*D^*\rho}\epsilon_2^{\mu}\epsilon_4^{\nu}\epsilon^{\beta}\left[2p_{2\mu}g_{\alpha\nu}-(p_2+p_4)_{\alpha}g_{\mu\nu}+2p_{4\nu}g_{\alpha\mu}\right] \\
& \times(-i)g_{\psi D^*D}\epsilon_{\rho\sigma\lambda\eta}\epsilon_3^{\rho}\epsilon^{\sigma}p_1^{\lambda}p_3^{\eta}\left[f_{D^*}F_1^{BD}(m_{D^*}^2)+f_DA_0^{BD^*}(m_D^2)\right]\frac{F^2(q^2,m_{D^*}^2)}{q^2-m_{D^*}^2} \\
= {} & \frac{G_F}{8\sqrt{2}\pi m_B}\left(f_{D^*}F_1^{BD}(m_{D^*}^2)+f_DA_0^{BD^*}(m_D^2)\right)V_{cb}V_{cd}^{*}m_{D^*}g_{\psi D^*D}g_{D^*D^*\rho} \\
& \times\int_{-1}^{1}|\vec{p_1}|d(\cos\theta)\frac{F^2(q^2,m_{D^*}^2)}{q^2-m_{D^*}^2}H_8,
\end{aligned} \tag{4.26}$$

where

$$\begin{aligned}
H_8 = {} & \epsilon_{\rho\sigma\lambda\eta}\epsilon_3^{\rho}\epsilon_4^{\mu}p_1^{\lambda}p_3^{\eta}\left[2p_{2\mu}g_{\alpha\nu}-(p_2+p_4)_{\alpha}g_{\mu\nu}+2p_{4\nu}g_{\alpha\mu}\right] \\
& \times\left(-g^{\beta\sigma}+\frac{q^{\beta}q^{\sigma}}{m_{D^*}^2}\right)\left(-p_1^{\nu}+\frac{(p_1\cdot p_2)p_2^{\nu}}{m_{D^*}^2}\right).
\end{aligned} \tag{4.27}$$

As the bridge between the dispersive part of FSI amplitude and the absorptive part, the dispersion relation is

$$\mathrm{Dis4}\left(m_B^2\right) = \frac{1}{\pi}\int_s^{\infty}\frac{\mathrm{Abs}_{(4a)}(s')+\mathrm{Abs}_{(4b)}(s')+\mathrm{Abs}_{(4c)}(s')\cdots\mathrm{Abs}_{(4h)}(s')}{s'-m_B^2}ds'. \tag{4.28}$$

According to the FSI, the amplitude of $B^+ \to J/\psi\rho^+$ decay is given by

$$\begin{aligned}
A_{\mathrm{FSI}}\left(B^+\longrightarrow D^{+(*)}\overline{D}^{0(*)}\longrightarrow\frac{J}{\psi\rho^+}\right) = {} & \mathrm{Abs(4a)}+\mathrm{Abs(4b)}+\mathrm{Abs(4c)}+\mathrm{Abs(4d)}+\mathrm{Abs(4e)} \\
& +\mathrm{Abs(4f)}+\mathrm{Abs(4g)}+\mathrm{Abs(4h)}+\mathrm{Dis4}\left(m_B^2\right).
\end{aligned} \tag{4.29}$$

Concerning the QCDF amplitude and FSI correction, then the total decay amplitude turns to

$$A\left(B^+\longrightarrow\frac{J}{\psi\rho^+}\right) = A_{\mathrm{QCDF}}+iA_{\mathrm{FSI}}. \tag{4.30}$$

5. Numerical Results

Numerical values of effective coefficients a_i for $\overline{b} \to \overline{d}$ transition at $N_c = 3$ are given by [22]

$$\begin{aligned}
&a_1 = 1.05, \qquad a_2 = 0.053,\\
&a_3 = 0.0048, \qquad a_4 = -0.0461 - 0.0124i,\\
&a_5 = -0.0045, \qquad a_6 = -0.0597 - 0.0124i,\\
&a_7 = 0.00003 - 0.00018i, \qquad a_8 = 0.00045 - 0.00006i,\\
&a_9 = -0.0095 - 0.00018i, \qquad a_{10} = -0.0014 - 0.00006i.
\end{aligned} \tag{5.1}$$

The Cabibbo-Kobayashi-Maskawa (CKM) matrix is a 3×3 unitary matrix as [9]

$$\mathbf{V} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \tag{5.2}$$

The elements of the CKM matrix can be parameterized by three mixing angles A, λ, ρ and a CP-violating phase η

$$\begin{aligned}
&V_{ud} = 1 - \frac{\lambda^2}{2}, \qquad V_{us} = \lambda, \qquad V_{ub} = A\lambda^3(\rho - i\eta),\\
&V_{cd} = -\lambda, \qquad V_{cs} = 1 - \frac{\lambda^2}{2}, \qquad V_{cb} = A\lambda^2,\\
&V_{td} = A\lambda^3(1 - \rho - i\eta), \qquad V_{ts} = -A\lambda^2, \qquad V_{tb} = 1.
\end{aligned} \tag{5.3}$$

The results for the Wolfenstein parameters are

$$\lambda = 0.2257 \pm 0.001, \qquad A = 0.814 \pm 0.02, \qquad \overline{\rho} = 0.135 \pm 0.023, \qquad \overline{\eta} = 0.349 \pm 0.016. \tag{5.4}$$

We use the central values of the Wolfenstein parameters and obtain

$$\begin{aligned}
&V_{ud} = 0.9745, \qquad V_{us} = 0.2257, \qquad V_{ub} = 0.0035,\\
&V_{cd} = 0.2257, \qquad V_{cs} = 0.9745, \qquad V_{cb} = 0.0415,\\
&V_{td} = 0.0087, \qquad V_{ts} = 0.0415, \qquad V_{tb} = 1.
\end{aligned} \tag{5.5}$$

The meson masses and decay constants needed in our calculations are taken as (in units of Mev)

$$\begin{gathered} m_{B^+} = 5278 \pm 0.3, \qquad m_{D^\pm} = 1869 \pm 0.2, \\ m_{\overline{D}^0} = 1864.84 \pm 0.17, \qquad m_{D^{\pm *}} = 2010.27 \pm 0.17, \\ m_{\overline{D}^{0*}} = 2006.97 \pm 0.19, \qquad m_{J/\psi} = 3096.916 \pm 0.011, \\ m_b = 4.20 \pm 0.12, \qquad m_\rho = 775.49 \pm 0.34, \\ f_{B^+} = 176 \pm 42, \quad f_{J/\psi} = 0.416, \\ f^{\perp}_{J/\psi} = 0.405, \qquad f_{D^\pm} = f_{\overline{D}^0} = 222.6 \pm 19.5, \\ f_{D^{\pm *}} = f_{\overline{D}^{0*}} = 230 \pm 20, \end{gathered} \tag{5.6}$$

The Borel mass squares M_1^2 and M_2^2 and continuum thresholds s_0 and s_0' are auxiliary parameters, hence the physical quantities should be independent of them. The parameters s_0 and s_0' are determined from the conditions that guarantee the sum rules for form factors to have the best stability in the allowed M_1^2 and M_2^2 region. The working regions for M_1^2 and M_2^2 as well as the values for continuum thresholds are determined in [13]. They choose the values $s_0 = 35 \pm 5\,\mathrm{GeV}^2$, $s_0' = 7 \pm 1\,\mathrm{GeV}^2$, $M_1^2 = 17.0 \pm 2.5\,\mathrm{GeV}^2$, $M_2^2 = 7 \pm 1\,\mathrm{GeV}^2$ from those working values for auxiliary parameters. The values of the form factors are given by

$$\begin{gathered} F_0^{BD}\left(m_D^2\right) = 0.86 \pm 0.22, \qquad F_1^{BD}\left(m_{D^*}^2\right) = 0.95 \pm 0.23, \\ A_0^{BD^*}\left(m_D^2\right) = 2.50 \pm 0.60, \qquad A_1^{BD^*}\left(m_{D^*}^2\right) = 1.09 \pm 0.26, \\ A_2^{BD^*}\left(m_{D^*}^2\right) = 1.82 \pm 0.46[13], \qquad A_1^{B\rho} = A_2^{B\rho} = 0.26, \\ V^{B\rho} = 0.33[10]. \end{gathered} \tag{5.7}$$

Nonuniversal annihilation phases are [23]

$$\phi = -55(PP), \qquad \phi = -20^\circ(PV), \qquad \phi = -70^\circ(VP). \tag{5.8}$$

The signs of these phases are not predicted. As a result in [23], their predictions for the signs of CP asymmetries must be taken with caution. We use other input parameters as follows:

$$\begin{gathered} \Lambda_{\mathrm{QCD}} = 0.225\,\mathrm{Gev}, \qquad C_F = \frac{4}{3}, \qquad N = 3, \\ G_F = 1.166 \times 10^{-5}, \qquad \alpha_s = 0.2244, \qquad \rho = 0.5, \qquad r_\chi^{D^\pm} = 1, \\ g_{DD\rho} = g_{D^*D^*\rho} = 2.52, \qquad g_{\psi DD} = g_{\psi D^*D^*} = 7.71, \\ g_{D^*D\rho} = 2.82, \qquad g_{\psi D^*D} = 8.64[5]. \end{gathered} \tag{5.9}$$

Table 1: The branching ratio of $B^+ \to J/\psi\rho^+$ decay with $\eta = 1 \sim 2$ and experimental data (in units of 10^{-5}).

η	1	1.2	1.4	1.6	1.8	2	EXP [9]
BR	$1.88^{+0.37}_{-0.35}$	$2.33^{+0.37}_{-0.35}$	$3.05^{+0.36}_{-0.35}$	$4.00^{+0.36}_{-0.35}$	$5.39^{+0.37}_{-0.35}$	$7.32^{+0.36}_{-0.35}$	$5^{+0.80}_{-0.80}$

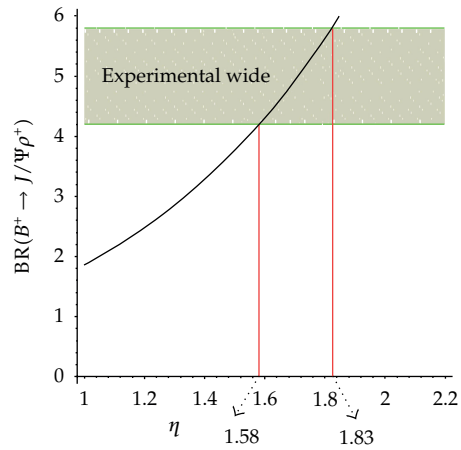

Figure 5: The variation of the branching ratio of $B^+ \to J/\psi\rho^+$ with $\eta = 1 \sim 2.2$.

By using the input parameters and according to the QCDF method of $B^+ \to J/\psi\rho^+$ decay in Section 2, we get

$$\mathrm{BR}\left(B^+ \longrightarrow \frac{J}{\psi\rho^+}\right) = (1.42 \pm 0.36) \times 10^{-5}. \tag{5.10}$$

We note that our estimate of branching ratio of $B^+ \to J/\psi\rho^+$ decay according to QCDF method seems less than the experimental result. Before calculating the $B^+ \to J/\psi\rho^+$ decay amplitude via FSI, we have to compute the intermediate state amplitude, for the $B^+ \to D^+\overline{D}^0$ decay amplitude we get

$$A\left(B^+ \longrightarrow D^+\overline{D}^0\right) = (2.31 \pm 0.27) \times 10^{-7}. \tag{5.11}$$

Now, using the above amplitude and according to the FSI, we are able to calculate the branching ratio of $B^+ \to J/\psi\rho^+$ decay with different values of η which are shown in Table 1 and Figure 5.

6. Conclusion

In this work, we have calculated the contribution of the t-channel FSI, that is, inelastic rescattering processes to the branching ratio of $B^+ \to J/\psi\rho^+$ decay. For evaluating the FSI effects, we have only considered the absorptive part of the HLL because both hadrons which produced via the weak interaction are on their mass shells.

We have calculated the branching ratio of $B^+ \to J/\psi\rho^+$ decay by using FSI. The experimental result of this decay is $\mathrm{BR}(B^+ \to J/\psi\rho^+) = (5 \pm 0.8) \times 10^{-5}$ and according to QCDF method and assuming the FSI corrections into account, our results of the branching ratio are $(1.42 \pm 0.36) \times 10^{-5}$ and $(4.2 \sim 5.8) \times 10^{-5}$, respectively.

There exist some phenomenological parameters in our calculations on FSI, such as, η in (4.16). We have introduced the phenomenological parameter η that its value in the form factor is expected to be of order unity and can be determined from the measured rates. For a given exchanged particle, we have used $\eta = 1 \sim 2$, and the branching ratios are $1.88 \sim 7.32$. We have considered that the factor of η has important role to obtain of branching ratio. Note that [6] for the exchange particles D^* and D they have fixed $\eta = 2.2$ and according to Figure 5, we have found that if $\eta = 1.58 \sim 1.83$ is selected (which is in good agreement with the reference [6] selection), the numbers of the branching ratio of the $B^+ \to J/\psi\rho^+$ decay are placed in the experimental range.

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