Hindawi Publishing Corporation
Advances in High Energy Physics
Volume 2012, Article ID 309304, 6 pages
doi:10.1155/2012/309304

Research Article

On Possible Reduction of Equilibrium Radius of a Neutron Star Influenced by Superstrong Magnetic Field

Vladimir V. Skobelev

Moscow State Industrial University, Faculty of Applied Mathematics and the Technical Physics, 115280, Russia

Correspondence should be addressed to Vladimir V. Skobelev, v.skobelev@inbox.ru

Received 16 September 2011; Revised 19 December 2011; Accepted 19 December 2011

Academic Editor: C. Q. Geng

Necessary condition for β-decay suppression of a neutron in degenerate magnetized electron gas is formulated. Based on this, it is shown that, in superstrong magnetic field, equilibrium radius of a neutron star is approximately several times smaller than without the field influence. Therefore, we can make a prediction that in short-period pulsars, such fields can be observed. In fact, possible existence of new class of stellar objects is noted, the objects with superstrong magnetic field and supersmall radius about 1 km which we named *minimagnetars*. They can be detected by gravitational red shift of their radiation.

1. Introduction

It is well known that the main contribution in total pressure in neutron star is made by degenerate neutron gas [1], and equilibrium radius about 10 km is formed only under this condition available. On the other hand, existence of superstrong magnetic field is possible there, B up to 10^{17} G [2, 3]. Therefore, the interest for exploration of such field influence onto equilibrium radius by interaction of neutron star matter, in particular, with degenerate electron gas, is arising. The latter might be connected with star dynamics in implicit way, by posing influence on neutron β-decay $n \rightarrow p + e^- + \overline{\nu}_e$ and on reverse reaction $e^- + p \rightarrow n + \nu_e$ (the so called *direct URCA processes*, that dominate in case of rather high concentration of electrons and protons [4]), that is, on the process of *neutronization* and equilibrium radius (in this research we do not cover the role of modified URCA processes [5] with one extra nucleon in such reactions).

We underline the fact that in the nucleus of the neutron star direct process dominates over modified process, being more specific; in the nucleus, there is superstrong magnetic field

[3]—with higher probability. As for the influence of magnetic field on the neutron star radius, we are familiar only with one dedicated paper—our own paper [6, 7], where we consider interaction of the magnetic field with anomalous magnetic moment of the neutron, leading to increasing of neutron pressure and, consequently, leading to radius increase. But the effect is too insignificant for observations to be performed. In other papers, this problem under the aspect considered has not been discussed.

We start from the fact that the wave function of the electron in the homogeneous magnetic field (oriented to the third axis) on the ground Landau level is the following [8] (see [9] also):

$$\Psi = \frac{(\gamma/\pi)^{1/4}}{\sqrt{2EL_2L_3}} \exp\left[i(p_3 z - p_2 y) - \frac{1}{2}\left(x\sqrt{\gamma} - \frac{p_2}{\sqrt{\gamma}}\right)^2\right] u, \tag{1.1}$$

where u is the two-component spinor in the subspace $(0,3), \gamma = |eB|$, p_3 is the momentum along the field, p_2 is the quasimomentum that determines the position of the center of the wave package on the x-axis:

$$X = \frac{p_2}{\gamma}. \tag{1.2}$$

As a result, the integration over p_2 gives

$$\int_{-\infty}^{\infty} dp_2 \longrightarrow \int_0^{L_1} \gamma dX = \gamma L_1. \tag{1.3}$$

Further, we will formulate the condition according to which the neutron decay is impossible in the degenerate magnetized electron gas.

Here, it is necessary to note that the total number of electrons can be represented in the form of a sum over all quantum states [10] (spin projection on ground Landau level is fixed):

$$N = \sum_{\text{states}} f_e \longrightarrow \frac{L_2}{2\pi}\frac{L_3}{2\pi}\int_{-\infty}^{\infty} dp_2 \int_{-\infty}^{\infty} f_e dp_3, \tag{1.4}$$

where

$$f_e = \left[\exp\left(\frac{E-\mu}{T}\right) + 1\right]^{-1} \tag{1.5}$$

is the distribution function and

$$E = \sqrt{m^2 + p_3^2} \tag{1.6}$$

the relativistic electron energy on the ground Landau level.

By using the ratio (1.3), we receive the following for concentration:

$$n \equiv \frac{N}{V} = \frac{\gamma}{2\pi^2}\int_0^\infty f_e dp_3. \tag{1.7}$$

For completely degenerate electron gas ($T = 0, f_e \to 1, E < E_F \equiv \mu|_{T=0}$), we find

$$n = \frac{\gamma p_F}{2\pi^2}, \tag{1.8a}$$

where

$$p_F = \frac{2\pi^2 n}{\gamma} \tag{1.8b}$$

is the Fermi momentum.

It follows from the general formula for the electron energy in the magnetic field

$$E = \sqrt{m^2 + p_3^2 + 2\gamma\tilde{n}}, \quad \tilde{n} = 0, 1, 2, \ldots, \tag{1.9}$$

that the states $\tilde{n} = 1, 2, \ldots$ do not energize if $B > B_{\rm cr}$, at the same time, the "critical" field can be found from the following equation

$$p_F^2 = 2\gamma \tag{1.10}$$

or

$$4\pi^2 n = (2\gamma)^{3/2}. \tag{1.10a}$$

On the basis of this equation, we can find the following:

$$B_{\rm cr} = \frac{B_0}{2}\left(4\pi^2 n\lambda_C^3\right)^{2/3}, \qquad B_0 = \frac{m^2}{e} = 4.41 * 10^{13}\,\mathrm{G} \tag{1.11}$$

(λ_C is the Compton electron wavelength), and for the standard electron concentration $n \sim 10^{35}\,\mathrm{cm}^{-3}$, which is 10^{-3} [11] from the neutron concentration in the center of star under nuclear density, we obtain $B_{\rm cr} \approx 8 * 10^{16}$ G. That is why under out assumptions and under $B \approx 10^{17}$ G, we can consider that the majority of electrons is concentrated on the ground Landau level.

Furthermore, we can add to the above-mentioned "field" condition with "temperature" condition

$$\sqrt{m^2 + 2\gamma} - m \gg T, \tag{1.12}$$

which is valid for "old" neutron stars with temperature $T \sim 1$ MeV [12].

The neutron decay is impossible when $n > n_{\min}$. At the same time, the minimum concentration can be found on the basis of the following equation:

$$\Delta = \sqrt{m^2 + p_F^2}, \tag{1.13}$$

and the formula (1.8b). Here, $\Delta = m_n - m_p \approx 1.3\,\text{MeV}$ stands for the energy output in the neutron β-decay. With the use of (1.8b) and (1.13) equations and maximum field value $B \sim 10^{17}\,\text{G}$, we have

$$n_{\min}{}^{(17)} \approx 4.7 * 10^{33}\,\text{cm}^{-3}. \tag{1.14}$$

On the other hand, in the absence of the magnetic field, the following assessment can be received from (1.13) and standard expression $p_F = (3\pi^2 n)^{1/3}$ [10]:

$$n_{\min}{}^{(0)} \approx 7.6 * 10^{30}\,\text{cm}^{-3}. \tag{1.15}$$

Thus, in the process of gravitational compression of the star, its substance is "neutronized" in the absence of the magnetic field with larger value of the star radius (R_0) as compared to the condition when the magnetic field is present (R_{17}). In particular, we have the following result for star radii at the beginning of "neutronization" (under other equal conditions):

$$\frac{R_0}{R_{17}} \approx \left(\frac{n_{\min}{}^{(17)}}{n_{\min}{}^{(0)}}\right)^{1/3} \cdot \approx 8.5. \tag{1.16}$$

We can also note that the interaction of the anomalous magnetic moment of the neutron with the magnetic field is characterized by the dimensionless parameter $y \pm x$ (see the appendix to the paper [13]) which is equal to 1 in the absence of the magnetic field and has the same order up to the field value $10^{17}\,\text{G}$. That is why we can abandon the effect which the magnetic field exercises over the pressure of neutrons and, in this sense, over the value of the equilibrium radius $R^{(\text{eq})}$ taken as the final result of the gravitational compression. If we also assume that the equilibrium radius is proportional to the neutronization radius and the superstrong magnetic field is generated prior to the neutronization, this and (1.16) take us to the final result:

$$\frac{R_0{}^{(\text{eq})}}{R_{17}{}^{(\text{eq})}} \sim 10. \tag{1.17}$$

This assessment is more likely to be overstated since the gravitational compression continues after the neutronization as well, but still it seems that the significant difference in the sizes is still present and we can observe it empirically (see below). In particular pulsars with short period and, therefore, according to the angular momentum rule ($R^2\omega = \text{const}$), with small radius can have superstrong magnetic field. On the other hand, pulsars with large period must have large radius without superstrong magnetic field.

Furthermore, on the basis of equations (1.8b) and (1.13) and the condition of suppression of energized states of the final electron ($\Delta < \sqrt{m^2 + 2\gamma}$, see (1.9)), we can generalize (1.14) and (1.16) in the following way:

$$n_{\min}{}^{(B)} \approx 0.47 * 10^{30} B_{13}\,\mathrm{cm}^{-3}, \qquad B_{13} = \frac{B}{10^{13}\,\mathrm{G}}, \tag{1.18}$$

$$\frac{R_0}{R_B} \approx 0.4 B_{13}{}^{1/3}, \qquad B > 1.2 * 10^{14}\,\mathrm{G}. \tag{1.19}$$

In the particular case $B \approx 10^{17}$ G, we obtain the maximum results (1.14) and (1.16) again.

It will be also observed that our approximation of completely degenerate electron gas features an additional limitation (in addition to (1.12)) for the temperature value on the ground Landau level:

$$K_F \equiv E_F - m \gg T. \tag{1.20}$$

Since the minimum value K_F is equal to $K_F{}^{(\min)} = \Delta - m \approx 0.8$ MeV, the condition (1.20) amounts to the limitation

$$\left(\frac{T}{m}\right) \ll 1.57, \tag{1.21}$$

which is broadly fulfilled for values $T \leq m$ but not for $T \sim 1$ MeV. However, even in the last case, formulas (1.16) and (1.17) are approximately correct because $n_{\min}{}^{(17)}, n_{\min}{}^{(0)}$ are increased and $R_{17}{}^{(\mathrm{eq})}, R_0{}^{(\mathrm{eq})}$ are decreased at $T \neq 0$ to the same extent, so our main conclusion remains unchanged.

As regards suppression of neutron decay with the influence of nonrelativistic proton gas, this phenomenon occurs (if it does at all) at $n_{\min}{}^{(0,17)} \sim 10^{35}\,\mathrm{cm}^{-3}$. This results from formulas for calculation of the minimum concentration in the absence of the magnetic field

$$\frac{\left(3\pi^2 n_{\min}{}^{(0)}\right)^{2/3}}{2M} = \Delta - m \tag{1.22a}$$

and in the presence thereof (for protons at the ground Landau level)

$$\frac{1}{2M}\left(\frac{2\pi^2 n_{\min}{}^{(B)}}{\gamma}\right)^2 = \Delta - m, \tag{1.22b}$$

where M is proton mass.

Therefore, the electron component of a neutron star is critical under the aspect considered since the relevant suppression occurs “earlier” with larger radius of a neutron star and our conclusions remain unchanged. Thus, there is a double suppression of neutron decay at $n_e \approx n_p \sim 10^{35}\,\text{cm}^{-3}$ both by electron and by proton gas, so we may conclude based on our computations that neutron stars with superstrong magnetic field may shrink to such concentration in the process of a gravitational collapse.

The stellar objects considered herein may be called minimagnetars whose radius (about 1 km instead of 10 km in the absence of the magnetic field, e.g., Crab nebula pulsar), according to equations (1.16), (1.17), and (1.19), is close to the gravitational one that is why these objects may be identified by gravitational red shift in their radiation.

References

[1] S. L. Shapiro and S. A. Teukolsky, *Black Holes, White Dwarfs and Neutron Stars*, Wiley-Interscience, New York, NY, USA, 1982.

[2] R. C. Duncan and C. Thompson, "Formation of very strongly magnetized neutron stars: implications for gamma-ray bursts," *Astrophysical Journal Letters*, vol. 392, no. 1, part 2, pp. L9–L13, 1992.

[3] S. Dall'Osso, S. N. Shore, and L. Stella, "Early evolution of newly born magnetars with a strong toroidal field," *Monthly Notices of the Royal Astronomical Society*, vol. 398, no. 4, pp. 1869–1885, 2009.

[4] J. M. Lattimer, C. J. Pethick, M. Prakash, and P. Haensel, "Direct URCA process in neutron stars," *Physical Review Letters*, vol. 66, no. 21, pp. 2701–2704, 1991.

[5] H. Y. Chiu and E. E. Salpeter, "Surface X-Ray emission from neutron stars," *Physical Review Letters*, vol. 12, no. 15, pp. 413–415, 1964.

[6] V. V. Skobelev, "Thermodynamic functionsof a nonrelativistic degenerate neutron gas in a magnetic field," *Journal of Experimental and Theoretical Physics*, vol. 111, no. 6, pp. 962–966, 2010.

[7] V. V. Skobelev, "Temperature corrections to the thermodynamic functions of a degenerate neutron gas in a magnetic field," *Journal of Experimental and Theoretical Physics*, vol. 113, no. 2, pp. 227–231, 2011.

[8] V. V. Skobelev, "Photogeneration of neutrinos and axions under the stimulating effect of a strong magnetic field," *Journal of Experimental and Theoretical Physics*, vol. 93, no. 4, pp. 685–694, 2001.

[9] L. D. Landau and E. M. Lifshits, *Course of Theoretical Physics*, vol. 3 of *Quantum Mechanics: Non-Relativistic Theory*, Butterworth-Heinemann, Oxford, UK, 1977.

[10] L. D. Landau and E. M. Lifshits, *Statistical Physics*, vol. 5, part 1 of *Course of Theoretical Physics*, Butterworth-Heinemann, Oxford, UK, 3rd edition, 2000.

[11] S. Weinberg, *Gravitation and Cosmology*, John Wiley & Sons, New York, NY, USA, 1972.

[12] D. G. Yakovlev, O. Y. Gnedin, A. D. Kaminker, and A. Y. Potekhin, "Theory of cooling neutron stars versus observations," in *Proceedings of the AIP Conference*, vol. 983, pp. 379–387, 2008.

[13] V. V. Skobelev, "Emission and absorption of neutrinos by nonrelativistic neutrons in a magnetic field," *Journal of Experimental and Theoretical Physics*, vol. 111, no. 1, pp. 64–69, 2010.