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Research Article

On Radiation by a Heavy Quark in $\mathcal{N} = 4$ SYM

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A short note on radiation by a moving classical particle in $\mathcal{N} = 4$ supersymmetric Yang-Mills theory is discussed in this paper.

1. Introduction

In papers [1–3], radiation by a point-like quark in $\mathcal{N} = 4$ supersymmetric Yang-Mills theory at strong coupling is investigated using the AdS/CFT correspondence in the supergravity approximation [4–6]. In this paper, modifications of the published radiation pattern are suggested, which are consistent with the results in [7]. This analysis is motivated by the description of electrodynamic radiation in classical electrodynamics [8, 9].

The important result in the context of radiation by an accelerated charge e is given by the Abraham-Lorentz four-vector force [8, 10, 11] in classical relativistic electrodynamics [12] as

$$f^{\mu} = \frac{2e^2}{3}\left(a^{\nu} a_{\nu} v^{\mu} + \dot{a}^{\mu}\right), \tag{1.1}$$

where the particle velocity is $\dot{x}^{\mu} = v^{\mu} \equiv dx^{\mu}/d\tau$ and the acceleration $\dot{v}^{\mu} = a^{\mu} \equiv dv^{\mu}/d\tau$ with the proper time τ for the particle [13] (the signature used here is $(+---)$). It is important to note the orthogonality of the force to the velocity as

$$v_{\mu} f^{\mu} = 0. \tag{1.2}$$

This force vanishes for uniformly accelerated motion, $f^{\mu} = 0$ [8].

2. Classical Radiation of Accelerated Electrons

In order to set the framework, it is helpful to discuss radiation in classical electrodynamics. A usefull approach is found in the 1949 paper by Schwinger [9].

Assume sources, restricted to a finite domain, which emit radiation. The four-momentum of the classical electromagnetic field is given in terms of the energy-momentum tensor by integrating on a hyper-surface as follows:

$$P^{\nu} = \oint T^{\mu\nu}\, d\sigma_{\mu}. \tag{2.1}$$

Gauß and Maxwell allow us to rewrite it as

$$P^{\nu} = \int \partial_{\mu} T^{\mu\nu}\, d^4x = \int j_{\mu} F^{\mu\nu}\, d^4x, \tag{2.2}$$

in terms of the current $j_{\mu} = (\rho, \vec{j})$ and the field-strength $F^{\mu\nu}$.

In the following it is important in order to determine the radiation force that the radiation field tensor and the vector potential are introduced in terms of the retarded and advanced fields [10, 11] as follows:

$$F^{\mu\nu}_{\text{rad}} = \frac{1}{2}\left(F^{\mu\nu}_{\text{ret}} - F^{\mu\nu}_{\text{adv}}\right). \tag{2.3}$$

Replacing $F^{\mu\nu}$ by $F^{\mu\nu}_{\text{rad}}$ in (2.2) gives P^{ν}_{rad}, which for point-like charges satisfies (compare to (1.2))

$$v_{\nu} \frac{dP^{\nu}_{\text{rad}}}{d\tau} \propto v_{\mu} F^{\mu\nu}_{\text{rad}} v_{\nu} = 0. \tag{2.4}$$

Using current conservation $\partial_{\mu} j^{\mu} = 0$ and introducing the vector potential

$$A^{\mu}_{\text{rad}} = \frac{1}{2}\left(A^{\mu}_{\text{ret}} - A^{\mu}_{\text{adv}}\right) \equiv \left(\phi, \vec{A}\right), \tag{2.5}$$

one obtains the power

$$\frac{dP^{0}_{\text{rad}}}{dt} = \int \left[\vec{j} \cdot \frac{\partial \vec{A}}{\partial t} - \rho \frac{\partial \phi}{\partial t}\right] d^3x + \frac{d}{dt} \int \rho \phi\, d^3x. \tag{2.6}$$

In [9, equation (I.17)], Schwinger discards the second term of this formula, which has the form of a total time derivative.

The radiation vector potential [9] is expressed by

$$A^{\mu}_{\text{rad}}(t, \mathbf{x}) = \frac{i}{2\pi} \int \exp\left[i\omega\left(\mathbf{n} \cdot \left(\mathbf{x} - \mathbf{x}'\right) - \left(t - t'\right)\right)\right] j^{\mu}\left(t', \mathbf{x}'\right) d^3x'\, dt'\, \omega\, d\omega \frac{d\Omega}{4\pi}. \tag{2.7}$$

A point-particle current is assumed as

$$j^{\mu} = e\left(1, \mathbf{v}(t) = \frac{d\mathbf{x}}{dt}\right)\delta(\mathbf{x} - \mathbf{R}(t)). \tag{2.8}$$

The integrals in (2.6) are as follows:

$$\int \rho\phi\, d^3x = e^2 \int \delta'\left(t' - t + \mathbf{n}\cdot\left(\mathbf{R}(t) - \mathbf{R}(t')\right)\right)dt'd\Omega = -e^2 \int \frac{d\Omega}{4\pi}\frac{\mathbf{n}\cdot\mathbf{a}}{\xi^3}, \tag{2.9}$$

with $\xi = (1 - \mathbf{n}\cdot\mathbf{v})$, $\mathbf{a} = d\mathbf{v}/dt$, and from equation (I.41) of [9] one obtains

$$\begin{aligned}\int\left[\vec{j}\cdot\frac{\partial\vec{A}}{\partial t} - \rho\frac{\partial\phi}{\partial t}\right]d^3x &= e^2\int\frac{d\Omega}{4\pi}\left[\frac{\mathbf{a}^2}{\xi^3} + 2\frac{\mathbf{n}\cdot\mathbf{a}\mathbf{v}\cdot\mathbf{a}}{\xi^4} - \frac{(\mathbf{n}\cdot\mathbf{a})^2}{\gamma^2\xi^5}\right] \\ &\quad + \frac{d}{dt}e^2\int\frac{d\Omega}{4\pi}\left[-\frac{\mathbf{v}\cdot\mathbf{a}}{\xi^3} + \frac{\mathbf{n}\cdot\mathbf{a}}{\gamma^2\xi^4}\right],\end{aligned} \tag{2.10}$$

with $\gamma = 1/\sqrt{1-\mathbf{v}^2}$ and $v^{\mu} = (\gamma, \gamma\mathbf{v})$.

Finally, using the notion of emitted power and a Schott-type term (e.g., in the notation of [14]), the result of the angular radiation pattern is

$$\frac{dP^0_{\text{rad}}}{dt\, d\Omega} \equiv P_{\text{rad}}(\mathbf{n}, t) = P_{\text{emitt}}(\mathbf{n}, t) + P_{\text{Schott}}(\mathbf{n}, t), \tag{2.11}$$

where

$$\begin{aligned}P_{\text{emitt}}(\mathbf{n}, t) &= \frac{e^2}{4\pi}\left[\frac{\mathbf{a}^2}{\xi^3} + 2\frac{\mathbf{n}\cdot\mathbf{a}\mathbf{v}\cdot\mathbf{a}}{\xi^4} - \frac{(\mathbf{n}\cdot\mathbf{a})^2}{\gamma^2\xi^5}\right], \\ P_{\text{Schott}}(\mathbf{n}, t) &= \frac{e^2}{4\pi}\frac{d}{dt}\left[-\frac{\mathbf{v}\cdot\mathbf{a} + \mathbf{n}\cdot\mathbf{a}}{\xi^3} + \frac{\mathbf{n}\cdot\mathbf{a}}{\gamma^2\xi^4}\right].\end{aligned} \tag{2.12}$$

Indeed, there are two terms contributing to the radiation power. Schwinger [9] claims that only the first one $P_{\text{emitt}}(\mathbf{n}, t)$, the one denoting the emission, should be retained. It has the characteristics of an irreversible energy transfer. The second one in the form of a total time derivative is reversible in nature.

Following Jackson [15], to obtain the radiated energy density of a charged particle, one starts from the large distance $-1/R$ contribution of the Liénard-Wiechert electric field as

$$\begin{aligned}\mathbf{E}_{\text{rad}} &= \frac{e}{R}\frac{\mathbf{n}\wedge[(\mathbf{n}-\mathbf{v})\wedge\mathbf{a}]}{(1-\mathbf{n}\cdot\mathbf{v})^3} \\ &= \frac{e}{R}\left[-\frac{\mathbf{a}}{(1-\mathbf{n}\cdot\mathbf{v})^2} + \frac{(\mathbf{n}\cdot\mathbf{a})(\mathbf{n}-\mathbf{v})}{(1-\mathbf{n}\cdot\mathbf{v})^3}\right],\end{aligned} \tag{2.13}$$

to obtain from

$$\mathcal{E}_{\text{vector}} = \frac{1}{8\pi}\left(\mathbf{E}^2 + \mathbf{B}^2\right), \tag{2.14}$$

with $|\mathbf{B}_{\text{rad}}| = |\mathbf{E}_{\text{rad}}|$

$$\mathcal{E}_{\text{vector}} = \frac{e^2}{4\pi R^2}\left[\frac{\mathbf{a}^2}{(1-\mathbf{n}\cdot\mathbf{v})^4} + 2\,\frac{(\mathbf{v}\cdot\mathbf{a})(\mathbf{n}\cdot\mathbf{a})}{(1-\mathbf{n}\cdot\mathbf{v})^5} - \frac{(\mathbf{n}\cdot\mathbf{a})^2}{\gamma^2(1-\mathbf{n}\cdot\mathbf{v})^6}\right]. \tag{2.15}$$

There is agreement between

$$R^2(1-\mathbf{n}\cdot\mathbf{v})\mathcal{E}_{\text{vector}} = P_{\text{emitt}}(\mathbf{n}, t). \tag{2.16}$$

Integrating the angular dependence (see, e.g., the useful integrals in Appendix A in [2]), one obtains

$$\gamma\frac{dP^0_{\text{rad}}}{dt} = \frac{dP^0_{\text{rad}}}{d\tau} = \frac{2e^2}{3}\gamma^4\left[\mathbf{a}^2 + \gamma^2(\mathbf{v}\cdot\mathbf{a})^2\right]\gamma - \frac{2e^2}{3}\frac{d}{d\tau}\left[\gamma^4(\mathbf{v}\cdot\mathbf{a})\right], \tag{2.17}$$

which, with $a^\mu = (\gamma^4\mathbf{v}\cdot\mathbf{a}, \gamma^2\mathbf{a} + \gamma^4(\mathbf{v}\cdot\mathbf{a})\mathbf{v})$, can be written as

$$\frac{dP^0_{\text{rad}}}{d\tau} = -f^0 = -\frac{2e^2}{3}\left[a^\mu a_\mu v^0 + \dot{a}^0\right], \tag{2.18}$$

that is, the zero component of the (negative) relativistic Abraham-Lorentz vector force (1.1) [8, 10, 12, 14], as

$$f^\mu = f^\mu_{\text{emitt}} + f^\mu_{\text{Schott}} = \frac{2e^2}{3}\left[\left(\frac{d^2x}{d\tau^2}\right)^2\frac{dx^\mu}{d\tau} + \frac{d^3x^\mu}{d\tau^3}\right]. \tag{2.19}$$

The first term represents an irretrievable loss of energy, and the second, the Schott contribution, is a total time differential, which contributes nothing to an integral by $d\tau$, when the initial value of a^μ is returned at the end [12].

In summary, as Schwinger stated already in 1949, only the spectrum for the irreversible transfer $P_{\text{emitt}}(\mathbf{n}, t)$ is the relevant one for discussing the emitted radiation and, therefore, should be retained. It is consistent with the derivation via the radiative electric field (2.13) as given in [15].

3. Classical $\mathcal{N} = 4$ SYM Radiation

In order to calculate the radiation power, one has to add to the vector part a contribution due to a massless scalar field χ [1, 2] and the replacement $e^2 \to e^2_{\text{eff}} = \lambda/8\pi$. This leads to

$$\partial_\mu T^{\mu\nu}_{\text{scalar}} = j_\chi\partial^\nu\chi, \tag{3.1}$$

with the current

$$j_X = \rho_X = e_{\text{eff}}\sqrt{1-v^2}\delta(\mathbf{x}-\mathbf{R}(t)). \tag{3.2}$$

This scalar contribution leads to

$$\begin{aligned} P_{\text{scalar}}(\mathbf{n},t) &= \frac{e_{\text{eff}}^2}{4\pi}\left[\frac{\gamma^2(\mathbf{v}\cdot\mathbf{a})^2}{\xi^3} - 2\frac{(\mathbf{v}\cdot\mathbf{a})(\mathbf{n}\cdot\mathbf{a})}{\xi^4} + \frac{(\mathbf{n}\cdot\mathbf{a})^2}{\gamma^2\xi^5}\right] \\ &\quad + \frac{e_{\text{eff}}^2}{4\pi}\frac{d}{dt}\left[\frac{\mathbf{v}\cdot\mathbf{a}}{\xi^3} - \frac{\mathbf{n}\cdot\mathbf{a}}{\gamma^2\xi^4}\right], \end{aligned} \tag{3.3}$$

(see also [1, 2]). Adding the vector parts (2.12) from the previous section, the weak coupling angular spectrum is given by

$$P_{\text{rad}}(\mathbf{n},t) = \frac{\lambda}{32\pi^2}\frac{\mathbf{a}^2 + \gamma^2(\mathbf{v}\cdot\mathbf{a})^2}{\xi^3} - \frac{\lambda}{32\pi^2}\frac{d}{dt}\left[\frac{\mathbf{n}\cdot\mathbf{a}}{\xi^3}\right]. \tag{3.4}$$

The term for $P_{\text{emitt}}(\mathbf{n},t)$ may also be expressed as

$$P_{\text{emitt}}(\mathbf{n},t) = \frac{\lambda}{32\pi^2}\frac{\gamma^2\left[\mathbf{a}^2 - (\mathbf{v}\wedge\mathbf{a})^2\right]}{(1-\mathbf{n}\cdot\mathbf{v})^3}. \tag{3.5}$$

Performing the angular integration gives

$$\int \gamma P_{\text{rad}}(\mathbf{n},t)\, d\Omega = \frac{\lambda}{8\pi}\gamma^4\left[\mathbf{a}^2 + \gamma^2(\mathbf{v}\cdot\mathbf{a})\right]\gamma - \frac{\lambda}{8\pi}\frac{d}{d\tau}\left[\gamma^4(\mathbf{v}\cdot\mathbf{a})\right]. \tag{3.6}$$

Up to the coupling, this expression agrees with the one from classical electrodynamics, given by (2.17). The $\mathcal{N} = 4$ SYM Abraham-Lorentz force [16, 17] in the weak coupling limit reads

$$f^{\mu}_{\text{SYM, weak}} = \frac{\lambda}{8\pi}\left[a^{\nu}a_{\nu}v^{\mu} + \dot{a}^{\mu}\right], \tag{3.7}$$

allowing the same interpretation as in the classical electrodynamics given above. $f^{\mu}_{\text{SYM,weak}}$ satisfies the constraint (1.2).

4. Radiation in $\mathcal{N} = 4$ SYM at Strong Coupling

Based on the work by [1], Hatta et al. [2] performed a detailed and transparent calculation of the radiation pattern by a heavy quark in $\mathcal{N} = 4$ SYM at strong coupling, to be followed rather closely. The result consists of two parts for the energy density, to be identified as

$$P_{\text{emitt}}(\mathbf{n}, t) = \frac{\sqrt{\lambda}}{8\pi^2} \frac{\gamma^2 \left[\mathbf{a}^2 - (\mathbf{v} \wedge \mathbf{a})^2\right]}{(1 - \mathbf{n} \cdot \mathbf{v})^3}, \tag{4.1}$$

and a term in the form of a total time derivative

$$P_{tt}(\mathbf{n}, t) = \frac{\sqrt{\lambda}}{24\pi^2} \frac{d}{dt} \left[\frac{\mathbf{v} \cdot \mathbf{a}}{\xi^3} - \frac{\mathbf{n} \cdot \mathbf{a}}{\gamma^2 \xi^4}\right], \tag{4.2}$$

which is, up to the couplings, the same given by (2.10) in the classical electrodynamics [9].

In the notation of [2], $P_{\text{emitt}}(\mathbf{n}, t) = R^2 \xi \mathcal{E}^{(1)}_{\text{rad}}(t, \mathbf{r})$ and $P_{tt}(\mathbf{n}, t) = R^2 \xi \mathcal{E}^{(2)}_{\text{rad}}(t, \mathbf{r})$. It is noted in [2] that integrating the sum of these two terms with respect to $d\Omega$ does not give a proper Abraham-Lorentz force [16, 17], and the constraint (1.2) is not satisfied, as it is the case in the weak coupling limit, when compared with (3.7).

A possible source of this deficiency may be found that only retarded contributions for the radiation are taken into account, instead of following the prescription given, for example, by (2.3) and (2.5) in the previous sections.

There is no need to repeat the derivations given in [2], but instead relying on the expressions of the energy density in the gauge theory, that is, on the Minkowski boundary given therein.

First consider the following quantity:

$$\mathcal{E}_{\text{A}} = \frac{\sqrt{\lambda}}{4\pi^2} \int \mathrm{d}t_q \, \delta(\mathcal{W}_q) \left(\frac{A_1}{\gamma^2 \Xi^2} + \frac{\partial}{\partial t_q} \frac{A_0}{\gamma \Xi^2} \right), \tag{4.3}$$

with the following definitions:

$$\mathcal{W}_q \equiv -(t - t_q)^2 + \left|\mathbf{r} - \mathbf{r}_q\right|^2, \qquad \Xi \equiv (t - t_q) - \boldsymbol{v}_q \cdot (\mathbf{r} - \mathbf{r}_q) = \frac{1}{2} \frac{\mathrm{d}\mathcal{W}_q}{\mathrm{d}t_q}. \tag{4.4}$$

In [2], the integral is evaluated by the retarded condition: $t_r = t_r(t, \mathbf{r})$ which denotes the value of t_q for which $\mathcal{W}_q(t_q) = 0$, with

$$t - t_r = \left|\mathbf{r} - \mathbf{r}_q(t_r)\right| = R. \tag{4.5}$$

Writing $\delta(\mathcal{W}_q) = \delta(t_q - t_r)/2|\Xi|$, the result in the large R-limit taken from [2] is

$$\mathcal{E}^{\text{ret}}_{\text{A}} = \frac{\sqrt{\lambda}}{8\pi^2 R^2 |\xi|} \left(\frac{\gamma^4 \left[\mathbf{a}^2 - (\mathbf{v} \wedge \mathbf{a})^2\right](2 - \xi)}{\xi^2} \right) + \frac{\sqrt{\lambda}}{8\pi^2 R^2 |\xi|} \frac{\partial}{\partial t_r} \left(\frac{\mathbf{n} \cdot \mathbf{a} + \gamma^2 (\mathbf{v} \cdot \mathbf{a})(2 - \xi)}{\xi^2} \right). \tag{4.6}$$

As a conjecture, let us consider

$$\mathcal{E}_{\mathrm{A}}^{\mathrm{rad}} = \frac{1}{2}\left(\mathcal{E}_{\mathrm{A}}^{\mathrm{ret}} - \mathcal{E}_{\mathrm{A}}^{\mathrm{adv}}\right), \tag{4.7}$$

by performing the integral for $\mathcal{E}_{\mathrm{A}}^{\mathrm{rad}}$ starting from (4.3), but using the advanced condition with

$$t - t_r = -\left|\mathbf{r} - \mathbf{r}_q(t_r)\right| = -R. \tag{4.8}$$

This amounts to the substitutions, when on top $\mathbf{n} \to -\mathbf{n}$, which do not affect the force as

$$\Xi \longrightarrow -R(1 - \mathbf{n} \cdot \mathbf{v}), \tag{4.9}$$

that is,

$$\xi \longrightarrow -\xi, \tag{4.10}$$

whereas $\partial / |\xi| \partial t_r$ remains unchanged.

From (4.6), one obtains

$$\begin{aligned} \mathcal{E}_{\mathrm{A}}^{\mathrm{adv}} &= \frac{\sqrt{\lambda}}{8\pi^2 R^2 |\xi|}\left(\frac{\gamma^4\left[\mathbf{a}^2 - (\mathbf{v} \wedge \mathbf{a})^2\right](2+\xi)}{\xi^2}\right) + \frac{\sqrt{\lambda}}{8\pi^2 R^2 |\xi|}\frac{\partial}{\partial t_r}\left(\frac{-\mathbf{n}\cdot\mathbf{a} + \gamma^2(\mathbf{v}\cdot\mathbf{a})(2+\xi)}{\xi^2}\right), \\ \mathcal{E}_{\mathrm{A}}^{\mathrm{rad}} &= -\frac{\sqrt{\lambda}}{8\pi^2}\frac{\gamma^4\left[\mathbf{a}^2 - (\mathbf{v} \wedge \mathbf{a})^2\right]}{R^2 \xi^2} + \frac{\sqrt{\lambda}}{8\pi^2 R^2 \xi}\frac{\partial}{\partial t_r}\left[\frac{\mathbf{n}\cdot\mathbf{a}}{\xi^2} - \frac{\gamma^2 \mathbf{v}\cdot\mathbf{a}}{\xi}\right]. \end{aligned} \tag{4.11}$$

In an analogous way, the contribution $\mathcal{E}_{\mathrm{B}}^{\mathrm{rad}}$ is evaluated, starting from

$$\begin{aligned} \mathcal{E}_{\mathrm{B}}^{\mathrm{ret}} = &-\frac{\sqrt{\lambda}}{8\pi^2 R^2 |\xi|}\left(\frac{\gamma^4\left[\mathbf{a}^2 - (\mathbf{v} \wedge \mathbf{a})^2\right]\left(-(1/\gamma^2) + 2\xi - \xi^2\right)}{\xi^3}\right) \\ &- \frac{\sqrt{\lambda}}{8\pi^2 R^2 |\xi|}\frac{\partial}{\partial t_r}\left(\frac{\mathbf{n}\cdot\mathbf{a}(\xi - 1)}{\xi^3} + \frac{\gamma^2 \mathbf{v}\cdot\mathbf{a}(2-\xi)}{\xi^2} + \frac{1}{|\xi|}\frac{\partial}{\partial t_r}\left[\frac{1}{6\gamma^2\xi^2} + \frac{1}{\xi}\right]\right). \end{aligned} \tag{4.12}$$

After the substitution (4.10), $\mathcal{E}_{\mathrm{B}}^{\mathrm{adv}}$ and then $\mathcal{E}_{\mathrm{B}}^{\mathrm{rad}}$ are obtained as

$$\begin{aligned}\mathcal{E}_{\mathrm{B}}^{\mathrm{rad}} = &-\frac{\sqrt{\lambda}}{8\pi^2 R^2}\left(\frac{\gamma^4\left[\mathbf{a}^2-(\mathbf{v}\wedge\mathbf{a})^2\right](-(1/\gamma^2)-\xi^2)}{\xi^4}\right)\\ &-\frac{\sqrt{\lambda}}{8\pi^2 R^2\xi}\frac{\partial}{\partial t_r}\left(\frac{\mathbf{n}\cdot\mathbf{a}}{\xi^2}-\frac{\gamma^2\mathbf{v}\cdot\mathbf{a}}{\xi}+\frac{1}{|\xi|}\frac{\partial}{\partial t_r}\frac{1}{\xi}\right).\end{aligned} \tag{4.13}$$

Finally, adding $\mathcal{E}_{\mathrm{A}}^{\mathrm{rad}}$ and $\mathcal{E}_{\mathrm{B}}^{\mathrm{rad}}$, the angular radiation power in the strong coupling limit is obtained as follows:

$$\begin{aligned}P_{\mathrm{strong}}^{\mathrm{rad}}(\mathbf{n},t) &= P_{\mathrm{emitt}}(\mathbf{n},t)+P_{\mathrm{Schott}}(\mathbf{n},t)\\ &= \frac{\sqrt{\lambda}}{8\pi^2}\frac{\gamma^2\left[\mathbf{a}^2-(\mathbf{v}\wedge\mathbf{a})^2\right]}{\xi^3}-\frac{\sqrt{\lambda}}{8\pi^2}\frac{d}{dt}\left[\frac{\mathbf{n}\cdot\mathbf{a}}{\xi^3}\right].\end{aligned} \tag{4.14}$$

The total time derivative term $P_{\mathrm{Schott}}(\mathbf{n},t)$ differs from $P_{tt}(\mathbf{n},t)$ in (4.2).

Up to the dependence on the coupling λ, the same angular radiative spectrum is found in the weak as well as in the strong coupling limit of the $\mathcal{N}=4$ supersymmetric Yang-Mills theory, that is, $\lambda/4 \to \sqrt{\lambda}$. As in electrodynamics [9], it is suggestive that for strong coupling as well only the emission spectrum $P_{\mathrm{emitt}}(\mathbf{n},t)=(\sqrt{\lambda}/8\pi^2)(\gamma^2[\mathbf{a}^2-(\mathbf{v}\wedge\mathbf{a})^2]/\xi^3)$ is the relevant one for radiation, that is, for the irreversible energy transfer [7].

Furthermore the force [16, 17] is

$$f_{\mathrm{SYM,\,strong}}^{\mu} = \frac{\sqrt{\lambda}}{2\pi}\left[a^{\nu}a_{\nu}v^{\mu}+\dot{a}^{\mu}\right]. \tag{4.15}$$

Up to the coupling dependence, the Abraham-Lorentz forces in classical electrodynamics as well as in the Yang-Mills theory have the same dependence on the acceleration a^{μ} and the velocity v^{μ}, when comparing (1.1), (3.7), and (4.15). All do satisfy the constraint (1.2).

As in electrodynamics [8, 18] the forces $f_{\mathrm{SYM,\,weak}}^{\mu}$ and $f_{\mathrm{SYM,\,strong}}^{\mu}$ vanish in weak and strong coupling $\mathcal{N}=4$ SYM, respectively, for uniformly accelerated motion [2, 19], for example, along the x direction, $x^{\mu}=((1/g)\sinh(g\tau),(1/g)\cosh(g\tau)=\sqrt{t^2+(1/g^2)},0,0)$, that is, $a_{\mu}a^{\mu}=-v_{\mu}\dot{a}^{\mu}=-g^2$, although radiation is emitted.

In the spirit of [20], a phenomenological derivation of $P_{\mathrm{strong}}^{\mathrm{rad}}$ (4.14) could be done as follows: retain only $P_{\mathrm{emitt}}(\mathbf{n},t)$ of (4.1) as derived in [2], calculate after the angular integration the force $f_{\mathrm{emitt}}^{\mu}=(\sqrt{\lambda}/2\pi)a_{\nu}a^{\nu}v^{\mu}$. Enforce the orthogonality (1.2) to v^{μ}, together with $f_{\mathrm{SYM,\,strong}}^{\mu}=0$ for uniformly accelerated motion, by adding the Schott-type term $(\sqrt{\lambda}/2\pi)\dot{a}^{\mu}$. This one is consistently obtained after integrating $P_{\mathrm{Schott}}(\mathbf{n},t)$ which is assumed to be of the same form in strong as well as in weak coupling (compare with (3.4)).

In [3], the time averaged energy density of an oscillating quark with small linear oscillations is derived, $v_q(t) = \epsilon\Omega \cos\Omega T,\ a_q = -\epsilon\Omega^2 \sin\Omega t$ and $\epsilon \ll 1$. It is asymptotically isotropic and, after correcting the numerical coefficient by a factor 6, given by

$$\int_{-\infty}^{+\infty} dt \, \langle T_{00}(t,\vec{x})\rangle = \frac{\epsilon^2\Omega^4\sqrt{\lambda}}{16\pi^2 R^2}\int_{-\infty}^{+\infty} dt, \tag{4.16}$$

which is consistent with the result by Mikhailov [7], namely, for

$$P_{\text{emitt}} = \frac{\sqrt{\lambda}}{2\pi} a_q^2(t). \tag{4.17}$$

5. Conclusion

The essence of this paper is based on the structure of the Abraham-Lorentz force f^{μ} (1.1), which holds even for strong coupling, with the properties of the orthogonality to the velocity and its vanishing for uniformly accelerated motion. It is surprising that the force, up to its strength, is the same in relativistic electrodynamics as well as in weak and strong coupling $\mathcal{N} = 4$ SYM, although the underlying angular distributions $P(\mathbf{n}, t)$ are different.

For the $\mathcal{N} = 4$ SYM model in the strong coupling limit, the special case of synchrotron radiation with frequency ω_0 is considered in [1]. An independent derivation of the synchrotron radiation in this model is given in [21].

In this case, with $\mathbf{v} \cdot \mathbf{a} = 0$, $\mathbf{a}^2 = v^2\omega_0^2$ the energy density (4.14) reads as

$$P_{\text{strong}}^{\text{rad}}(\mathbf{n}, t) = \frac{\sqrt{\lambda}\omega_0^2}{8\pi^2\xi^4}\left[3 - \left(4 + \gamma^{-2}\right)\xi - 3v^2 \sin^2\Theta + 2\xi^2\right], \tag{5.1}$$

which differs from the one using P_{tt} of (4.2) (compare with equation (3.71) in [1] and with equation (6.5) in [2]).

A quantity of interest considered in [1] is the time-averaged angular distribution of power, given by

$$\frac{dP_{\text{emitt}}(\mathbf{n})}{d\Omega} = \frac{\omega_0}{2\pi}\int_0^{2\pi/\omega_0} dt \frac{\sqrt{\lambda}}{8\pi^2}\frac{\mathbf{a}^2}{(1 - \mathbf{n} \cdot \mathbf{v})^3}, \tag{5.2}$$

with $1 - \mathbf{n} \cdot \mathbf{v} = 1 - v \sin\Theta \sin(\phi - \omega_0 t)$.

For this periodic motion for synchrotron radiation, the contributions of the total time derivatives P_{Schott} as well as P_{tt} vanish in the time-averaged distribution. In any case, when emitted radiation is considered, total time derivative terms should not be retained. They do not represent irreversible loss of energy in contrast to P_{emitt} [9, 12].

Integration in (5.2) leads to

$$\frac{dP_{\text{emitt}}(\mathbf{n})}{d\Omega} = \frac{\sqrt{\lambda}}{8\pi^2}\mathbf{a}^2\gamma^5 \frac{1 + (v^2/2)\sin^2\Theta}{\left(\gamma^2\cos^2\Theta + \sin^2\Theta\right)^{5/2}}, \tag{5.3}$$

which agrees with the resulting equation (3.72) derived in [1]. The total power emitted,

$$P_{\text{emitt}} = \frac{\sqrt{\lambda}}{2\pi}\left[\gamma^2 v\omega_0\right]^2, \tag{5.4}$$

is the same as the result obtained in [1, 2, 7].

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