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Research Article

Matter Instability in Theories with Modified Induced Gravity

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Matter stability is a necessary condition to have a cosmologically viable model. Modified gravity in the spirit of $f(R)$ theories suffers from matter instability in some subdomains of the model parameter space. It has been shown recently that the late-time cosmic speedup can be explained through an $f(R)$-modified induced gravity program. In this paper, we study the issue of matter instability in a braneworld setup with modified induced gravity.

1. Introduction

There are many lines of astronomical evidence supporting the idea that our universe is currently undergoing a speedup expansion [1–4]. Several approaches are proposed in order to explain the origin of this novel phenomenon. These approaches can be classified in two main categories: models based on the notion of dark energy which modify the matter sector of the gravitational field equations and those models that modify the geometric part of the field equations are generally dubbed as dark geometry in the literature [5–13]. From a relatively different viewpoint (but in the spirit of dark geometry proposal), the braneworld model proposed by Dvali, Gabadadze, and Porrati (DGP) [14] explains the late-time cosmic speedup phase in its self-accelerating branch without recourse to dark energy [15, 16]. However, existence of ghost instabilities in this branch of the solutions makes its unfavorable in some senses [17, 18]. Fortunately, it has been revealed recently that the normal, ghost-free DGP branch has the potential to explain late-time cosmic speedup if we incorporate possible modification of the induced gravity on the brane in the spirit of $f(R)$ theories [19–24]. This extension can be considered as a manifestation of the scalar-tensor gravity on the brane. Some features of this extension are studied recently [25–29].

Modified gravity in the spirit of $f(R)$ theories have the capability to provide a unified gravitational alternative to dark energy and inflation [30–37]. A number of viable modified $f(R)$ gravities are proposed in recent years (see [38, 39] and references therein). The cosmological viability of these theories is a necessary condition, and in this respect, there are important criteria for viability such as the fulfillment of the solar system tests. Among these requirements, one of the most important ones is related to the so-called matter instability [40–57] in $f(R)$ gravity. Matter instability is related to the fact that spherical body solution in general relativity may not be the solution in modified theory in general. This instability may appear when the energy density or the curvature is large compared with the average one in the universe, as is the case inside of a star [58]. In a simple term, matter instability means that the curvature inside a matter sphere becomes very large, leading to a very strong gravitational field. It was indicated that such matter instability may be dangerous in the relativistic star formation processes [59–61] due to the appearance of the corresponding singularity. In this respect, for a model to be cosmologically viable, it is necessary to have matter stability in the model. For a detailed study of the issue of matter instability in $f(R)$ theories, see [40–58].

Since the $f(R)$-modified induced gravity (brane $f(R)$ gravity) has the capability to bring the normal DGP solutions to be self-accelerating, it is desirable to see whether this model is cosmologically viable from matter instability viewpoint. So, this letter is devoted to the issue of matter instability in a brane $f(R)$ gravity.

2. DGP-Inspired $f(R)$ Gravity

2.1. The Basic Equations

Modified gravity in the form of $f(R)$ theories is derived by generalization of the Einstein-Hilbert action so that R (the Ricci scalar) is replaced by a generic function $f(R)$ in the action

$$S = \int d^4x\sqrt{-g}\left(\frac{f(R)}{2\kappa^2} + \mathcal{L}_m\right), \tag{2.1}$$

where $\mathcal{L}_m$ is the matter Lagrangian and $\kappa^2 = 8\pi G$. Varying this action with respect to the metric gives

$$G_{\mu\nu} = \kappa^2 T^{(\mathrm{tot})}_{\mu\nu} = \kappa^2\left(T^{(m)}_{\mu\nu} + T^{(f)}_{\mu\nu}\right) = \kappa^2\frac{\widetilde{T}^{(m)}_{\mu\nu} + \widetilde{T}^{(f)}_{\mu\nu}}{f'}, \tag{2.2}$$

where $\widetilde{T}^{(m)}_{\mu\nu} = \mathrm{diag}(\rho, -p, -p, -p)$ is the stress-energy tensor for standard matter, which is assumed to be a perfect fluid and by definition $f' \equiv df/dR$. Also, $\widetilde{T}^{(f)}_{\mu\nu}$ is the stress-energy tensor attributed to the curvature defined as follows:

$$\widetilde{T}^{(f)}_{\mu\nu} = \frac{1}{2}g_{\mu\nu}\left[f(R) - Rf'\right] + f'^{;\alpha\beta}\left(g_{\alpha\mu}g_{\beta\nu} - g_{\alpha\beta}g_{\mu\nu}\right). \tag{2.3}$$

By substituting a flat FRW metric into the field equations, one achieves the analogue of the Friedmann equations as follows [30–37]:

$$\begin{aligned} 3f'H^2 &= \kappa^2\rho_m + \left[\frac{1}{2}\left(f(R) - Rf'\right) - 3H\dot{f'}\right] \\ -2f'\dot{H} &= \kappa^2\rho_m + \dot{R}^2 f''' + \left(\ddot{R} - H\dot{R}\right)f'', \end{aligned} \tag{2.4}$$

where a dot marks the differentiation with respect to the cosmic time. In the next step, following [28, 29] we suppose that the induced gravity on the DGP brane is modified in the spirit of $f(R)$ gravity. The action of this DGP-inspired $f(R)$ gravity is given by

$$S = \frac{1}{2\kappa_5^3}\int d^5x\sqrt{-g}\mathcal{R} + \int d^4x\sqrt{-q}\left(\frac{f(R)}{2\kappa^2} + \mathcal{L}_m\right), \tag{2.5}$$

where g_{AB} is a five-dimensional bulk metric with Ricci scalar $\mathcal{R}$, while q_{ab} is an induced metric on the brane with induced Ricci scalar R. The Friedmann equation in the *normal branch* of this scenario is written as [28, 29]

$$3f'H^2 = \kappa^2\left(\rho_m + \rho^{(f)}\right) - \frac{3H}{r_c}, \tag{2.6}$$

where $r_c = G^{(5)}/G^{(4)} = \kappa_5^2/2\kappa^2$ is the DGP crossover scale which has the dimension of $[length]$ and marks the IR (infrared) behavior of the DGP model. The Raychaudhuri equation is written as follows:

$$\dot{H}\left(1 + \frac{1}{2Hr_c f'}\right) = -\frac{\kappa^2\rho_m}{2f'} - \frac{\dot{R}^2 f''' + \left(\ddot{R} - H\dot{R}\right)f''}{2f'}. \tag{2.7}$$

To achieve this equation, we have used the continuity equation for $\rho^{(f)}$ as

$$\dot{\rho}^{(f)} + 3H\left(\rho^{(f)} + p^{(f)} + \frac{\dot{R}f''}{r_c\left(f'\right)^2}\right) = \frac{\kappa^2\rho_m\dot{R}f''}{\left(f'\right)^2}, \tag{2.8}$$

where the energy density and pressure of the curvature *fluid* are defined as follows:

$$\begin{aligned} \rho^{(f)} &= \frac{1}{\kappa^2}\left(\frac{1}{2}\left[f(R) - Rf'\right] - 3H\dot{f'}\right), \\ p^{(f)} &= \frac{1}{\kappa^2}\left(2H\dot{f'} + \ddot{f'} - \frac{1}{2}\left[f(R) - Rf'\right]\right). \end{aligned} \tag{2.9}$$

After presentation of the necessary field equations, in what follows we study the issue of matter instability and cosmological viability in this setup.

3. The Issue of Matter Instability

In 4 dimensions, the conditions $f'(R) > 0$ and $f''(R) > 0$ are necessary conditions for $f(R)$ theories to be free from ghosts and other instabilities [40–44, 62]. In our brane $f(R)$ scenario, in addition to $f(R)$, there is another piece of information in the action (2.5) (the DGP character of the model) which should be taken into account when discussing the issue of instabilities. To study possible instabilities in this setup, we proceed as follows: variation of the action (2.5) with respect to the metric yields the induced modified Einstein equations on the brane

$$G_{\alpha\beta} = \frac{1}{M_5^6}\mathcal{S}_{\alpha\beta} - \mathcal{E}_{\alpha\beta}, \tag{3.1}$$

where $\mathcal{E}_{\alpha\beta}$ (which we neglect it in our forthcoming arguments) is the projection of the bulk Weyl tensor on the brane

$$\mathcal{E}_{\alpha\beta} =^{(5)} C^{M}_{RNS} n_M n^R g^N_\alpha g^S_\beta, \tag{3.2}$$

where n^M is a unit vector normal to the brane, and $\mathcal{S}_{\alpha\beta}$ as the quadratic energy-momentum correction into the Einstein field equations is defined as follows:

$$\mathcal{S}_{\alpha\beta} = -\frac{1}{4}\tau_{\alpha\mu}\tau^{\mu}_{\beta} + \frac{1}{12}\tau\tau_{\alpha\beta} + \frac{1}{8}g_{\alpha\beta}\tau_{\mu\nu}\tau^{\mu\nu} - \frac{1}{24}g_{\alpha\beta}\tau^2. \tag{3.3}$$

$\tau_{\alpha\beta}$ as the effective energy-momentum tensor localized on the brane is defined as [25–27]

$$\tau_{\alpha\beta} = -m_p^2 f'(R)G_{\alpha\beta} + \frac{m_p^2}{2}\left[f(R) - Rf'(R)\right]g_{\alpha\beta} + T_{\alpha\beta} + m_p^2\left[\nabla_\alpha\nabla_\beta f'(R) - g_{\alpha\beta}f'(R)\right]. \tag{3.4}$$

Following [40–44], the trace of (3.1), which can be interpreted as the equation of motion for $f'(R)$, is obtained as

$$R = \frac{5}{6}r_c^2\left(\left[2f - Rf'\right]^2 + 9(\Box f')^2 + 6Rf'\Box f' - 12f\Box f'\right) + \frac{5}{3}\frac{r_c^2}{m_p^2}(Rf' - 2f + 3\Box f')T + \frac{5}{24M_5^6}T^2. \tag{3.5}$$

We parameterize the deviation from Einstein gravity as

$$f(R) = R + \epsilon\varphi(R), \tag{3.6}$$

where ϵ is a small parameter with the dimension of an inverse-squared length, and φ is arranged to be dimensionless. Typically, $\epsilon \approx H_0^2 \simeq 10^{-66}(\text{eV})^2$ (see, for instance, [42]). By evaluating $\Box f'$ as

$$\Box f' = \epsilon\Box\varphi'(R) = \epsilon\left(\varphi'''\nabla^{\alpha}R\,\nabla_\alpha R + \varphi''\Box R\right), \tag{3.7}$$

(3.5) can be rewritten as follows:

$$R = \frac{5}{6} r_c^2 \Big(\left[R + \epsilon(2\varphi - R\varphi') \right]^2 + 9\epsilon^2 \left[\varphi''' \nabla^\alpha R \nabla_a R + \varphi'' \Box R \right]^2 + 6\epsilon\left(\epsilon(R\varphi' - 2\varphi) - R\right)\left[\varphi''' \nabla^\alpha R \nabla_a R + \varphi'' \Box R\right] \Big) + \frac{5}{24M_5^6} T^2 + \frac{5}{3} \frac{r_c^2}{m_p^2} \left(R(\epsilon\varphi' - 1) - 2\epsilon\varphi + 3\epsilon\left[\varphi''' \nabla^\alpha R \nabla_a R + \varphi'' \Box R\right] \right) T. \tag{3.8}$$

This equation to first order of ϵ gives

$$R = \frac{5}{24M_5^6} T^2 + \frac{5}{6} r_c^2 \Big(\left[R^2 + 2\epsilon R(2\varphi - R\varphi') \right] - 6\epsilon R\left[\varphi''' \nabla^\alpha R \nabla_a R + \varphi'' \Box R\right] \Big) + \frac{5}{3} \frac{r_c^2}{m_p^2} \left(R(\epsilon\varphi' - 1) - 2\epsilon\varphi + 3\epsilon\left[\varphi''' \nabla^\alpha R \nabla_a R + \varphi'' \Box R\right] \right) T. \tag{3.9}$$

Now we consider a small region of spacetime in the weak-field regime in which curvature and the metric can locally be approximated by

$$R = -\kappa^2 T + R_1, \qquad g_{ab} = \eta_{ab} + h_{ab}, \tag{3.10}$$

respectively, where R_1 is curvature perturbation, and η_{ab} is the Minkowski metric. In this case, the metric can be approximately taken as a flat one, so $\Box = \partial_t^2 - \nabla^2$ and $(\nabla^\alpha R)(\nabla_\alpha R) = \dot{R}^2 - (\nabla R)^2$. Now, (3.9) up to first order of R_1 gives (we set for simplicity $\kappa^2 = 1$)

$$\begin{aligned} R_1 - T = {} & \frac{5}{24M_5^6} T^2 + \frac{5}{6} r_c^2 \left[T^2 - 2TR_1 + 4\epsilon\varphi R_1 - 4\epsilon\varphi T - 2\epsilon\varphi' T^2 + 4\epsilon T\varphi' R_1 \right] \\ & + 5\epsilon r_c^2 T \Big[\varphi'''(R)\Big(\dot{T}^2 - 2\dot{T}\dot{R}_1 - (\nabla T)^2 + 2\nabla R_1\ \nabla T\Big) + \varphi''(R)\Big(\ddot{R}_1 - \ddot{T} - \nabla^2 R_1 + \nabla^2 T\Big)\Big] \\ & - 5\epsilon r_c^2 R_1 \times \Big[\varphi'''(R)\Big(\dot{T}^2 - (\nabla T)^2\Big) + \varphi''(R)\Big(\nabla^2 T - \ddot{T}\Big)\Big] + \frac{5r_c^2}{3m_p^2} \\ & \times \Big(3\epsilon\Big[\varphi'''(R)\Big(\dot{T}^2 - 2\dot{T}\dot{R}_1 - (\nabla T)^2 + 2\nabla R_1\ \nabla T\Big) + (R_1 - T)(\epsilon\varphi' - 1) \\ & \quad - 2\epsilon\varphi + \varphi''(R)\Big(\ddot{R}_1 - \ddot{T} - \nabla^2 R_1 + \nabla^2 T\Big)\Big]\Big) T. \end{aligned} \tag{3.11}$$

This relation can be recast in the following suitable form:

$$\ddot{R}_1-\nabla^2 R_1+\frac{\varphi'''}{\varphi''}\left(\dot{T}^2-(\nabla T)^2+2\nabla R_1\ \nabla T-2\dot{T}\dot{R}_1\right)-\left[\frac{1}{10\epsilon\ r_c^2\ T\varphi''}+\frac{1}{\epsilon\varphi''}-\frac{1}{3T\varphi''}\left(\varphi+\frac{9}{2}T\varphi'\right)\right]R_1$$
$$-\frac{1}{2T\varphi''}\left[\varphi'''\left(\dot{T}^2-(\nabla T)^2\right)+\varphi''\left(\nabla^2 T-\ddot{T}\right)\right]R_1$$
$$=\ddot{T}-\nabla^2 T-\left(\ddot{R}_1-\nabla^2 R_1+\left(\nabla^2 T-\ddot{T}\right)\right)+\frac{1}{3\varphi''}\left(T\varphi'+2\varphi\right)-\frac{1}{\epsilon\varphi''}\left(\frac{1}{10r_c^2}+\frac{5T}{3}\right). \tag{3.12}$$

The coefficient of R_1 in the fourth term on the left-hand side is the square of an effective mass defined as

$$m_{\text{eff}}^2=-\frac{1}{10\ \epsilon\ r_c^2\ T\varphi''}-\frac{1}{\epsilon\varphi''}+\frac{1}{3T\varphi''}\left(\varphi+\frac{9}{2}T\varphi'\right)-\frac{1}{2T\varphi''}\left[\varphi'''\left(\dot{T}^2-(\nabla T)^2\right)+\varphi''\left(\nabla^2 T-\ddot{T}\right)\right]. \tag{3.13}$$

This quantity is dominated by the term $1/\epsilon\varphi''$ due to the extremely small value of ϵ needed for these theories. It is therefore obvious that the theory will be stable (i.e., $m_{\text{eff}}^2 > 0$) if $\varphi'' = f'' < 0$, while an instability arises if this effective mass squared is negative, that is, if $\varphi'' = f'' > 0$. Based on this fact and as an example, the $f(R) = R + \gamma R^{-n}$ function (with γ as a positive quantity) in the spirit of normal DGP braneworld scenario suffers from a matter instability for $n > 0$ and $n < -1$. For this kind of $f(R)$ function, γ plays the same role as ϵ in (3.6) and is supposed to be positive (note that ϵ is a small parameter with dimension of an inverse length squared), so $\varphi(R) = R^{-n}$. The condition for matter stability $\varphi'' = f'' < 0$ leads to $-1 < n < 0$.

As another important point, we focus on the stability of the de Sitter accelerating solution under small homogeneous perturbations in the normal branch (see [28, 62, 63] for a similar argument). It is useful to rewrite the Friedmann equation corresponding to a de Sitter brane with Hubble rate H_0 in a form that exhibits the effect of an extra dimension on a 4D $f(R)$ model as follows [28, 62, 63]:

$$H_0^2 = H_{(4)}^2 + \frac{1-\sqrt{1+(2/3)f_0' f_0}}{2{f_0'}^2}. \tag{3.14}$$

Subscript 0 stands for quantities evaluated in the de Sitter space time. We also note that the de Sitter brane is described by the scalar factor $a(t) = a_0 \exp(\xi t)$ which leads to $R_0 = 12\xi^2$. $H_{(4)}^2$ is defined as

$$H_{(4)}^2 = \frac{f_0'}{6f_0}. \tag{3.15}$$

Therefore, the presence of the extra dimension implies a shift on the Hubble rate. One can perturb the Hubble parameter up to the first order as $\delta H = H(t) - H_0$. Also by a perturbed Friedmann equation based on (2.6), one can achieve an evolution equation for δH as [64]

$$\delta\ddot{H} + 3H_0\delta\dot{H} + (M_{\text{eff}})^2\delta H = 0. \tag{3.16}$$

The stability condition for the de Sitter solution in the DGP normal branch is positivity of the effective mass squared, $(M_{\text{eff}})^2 > 0$. Now $(M_{\text{eff}})^2$ can be written as the sum of three terms $(M_{\text{eff}})^2 = m^2_{(4)} + m^2_{\text{back}} + m^2_{\text{pert}}$. In the 4D version of the $f(R)$ gravity, this summation reduces to [64]

$$m^2_{\text{eff}} = m^2_{(4)} = \frac{f_0'^2 - 2f_0 f''}{3f_0' f''}. \tag{3.17}$$

In the braneworld version, we except the crossover distance to affect the effective mass. In this respect, m^2_{back} is a purely *background* effect due to the shift on the Hubble parameter with respect to the standard 4D case, while m^2_{pert} is a purely *perturbative* extradimensional effect [28, 63]. In our setup, these quantities are defined as follows:

$$\begin{aligned} m^2_{\text{back}} &= -\frac{2}{r_c^2 f'^2_0}\left[1 - \sqrt{1 + \frac{2}{3}r_c^2 f_0'^2 f_0}\right], \\ m^2_{\text{pert}} &= -\frac{f'_0}{3f''}\left[1 - \sqrt{1 + \frac{2}{3}r_c^2 f_0'^2 f_0}\right]^{-1}. \end{aligned} \tag{3.18}$$

The de Sitter brane is close to the standard 4D regime as long as $H_0^2 \sim H^2_{(4)}$, which leads to

$$\left|\frac{3\left(1 - \sqrt{1 + (2/3)f_0' f_0}\right)}{f_0 f_0'}\right| \ll 1. \tag{3.19}$$

The assumption that we are slightly perturbing the Hilbert-Einstein action of the brane, that is, $f_0 \sim R_0 > 0$, and also the positivity of the effective gravitational constant on the brane at late time, that is, $f_0' > 0$, implies that $f_0 f_0' \gg 1$ for the last inequality. Now the stability of the de Sitter accelerating solution under small homogeneous perturbations in the normal DGP branch of the model (since $m^2_{\text{back}} > 0$ and $m^2_{\text{pert}} < 0$) is guaranteed if $m^2_{\text{eff}} > m^2_{(4)}$ which leads to the following condition (for more details, see [28, 63, 64]):

$$f_0'^2 < 4f_0 f_0''. \tag{3.20}$$

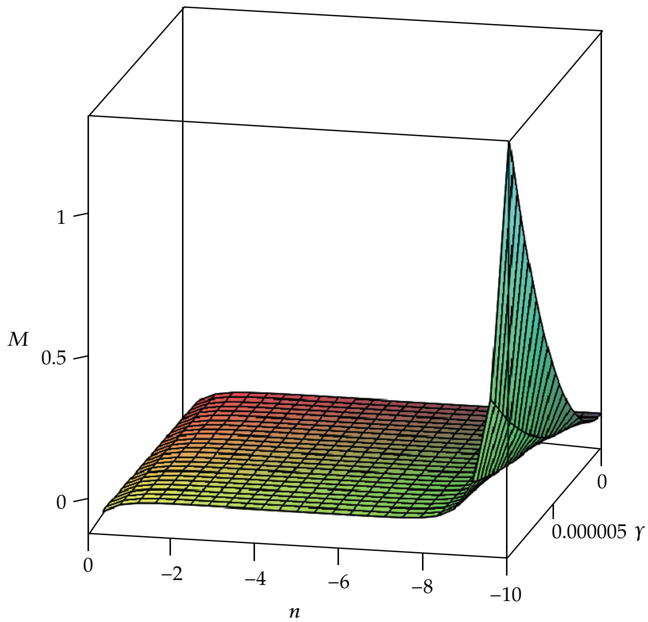

Figure 1: The behavior of $M \equiv m_{\text{eff}}^2 - m_{(4)}^2$ versus n and γ.

Based on this condition, $f(R) = R + \gamma R^{-n}$ function in the spirit of the normal DGP braneworld exhibits a de Sitter stability if the following condition is satisfied:

$$R_0^{n+1} - n(3n+4)\gamma^2 R_0^{-(n+1)} < n\gamma(6+4n). \tag{3.21}$$

Figure 1 shows the behavior of $M \equiv m_{\text{eff}}^2 - m_{(4)}^2$ versus n and γ. The de Sitter phase is stable in this setup if $n \lesssim -4.12$. This is shown with more resolution in Figure 2. Note that the matter stability in this induced gravity braneworld scenario occurs in those values of n that the corresponding de Sitter phase is not stable.

4. Summary and Conclusion

Matter stability is a necessary condition for cosmological viability of a gravitational theory. Recently, it has been shown that the normal, non-self-accelerating branch of the DGP cosmological solutions self-accelerates if the induced gravity on the brane is modified in the spirit of $f(R)$ gravity. In this letter, we have studied the issue of matter stability in an induced gravity, brane-$f(R)$ scenario. We obtained the condition for matter stability in this setup via a perturbative scheme, and we applied our condition for an specific model of the type $f(R) = R + \gamma R^{-n}$. For this type of modified induced gravity, the matter is stabilized on the brane for $-1 < n < 0$. We have also studied the stability of the de Sitter phase for this type of modified induced gravity. For this type of the modified induced gravity, the de Sitter phase is stable for $n \lesssim -4.12$. Albeit for these values of n, matter is not stable. So, these types of modified induced gravity are not suitable candidates for late-time cosmological evolution. We note however that other types of modified induced gravity such as $f(R) = R^n \exp(\eta/R)$ with

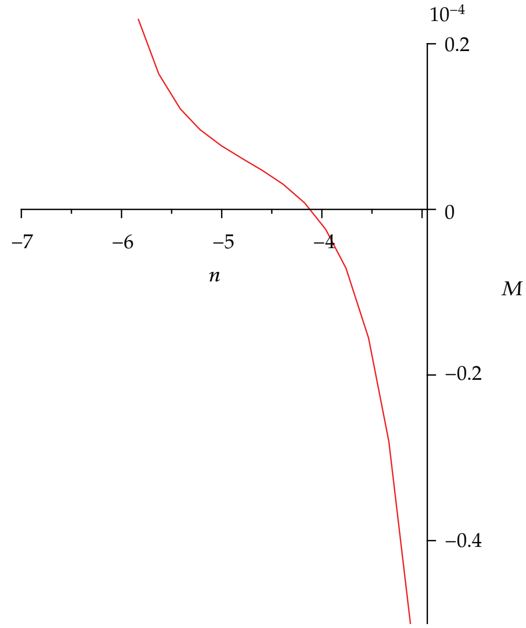

Figure 2: Stability of the de Sitter phase. The de Sitter phase is stable for $n \lesssim -4.12$.

η a constant have simultaneous matter stability and stable de Sitter phase in some subspaces of the model parameter space (see [65]).

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