

Research Article

Nonstandard Neutrinos Interactions in a 331 Model with Minimum Higgs Sector

M. Medina and P. C. de Holanda

Instituto de Física Gleb Wataghin-UNICAMP, 13083-859 Campinas, SP, Brazil

Correspondence should be addressed to M. Medina, mmedina@ifi.unicamp.br

Received 26 August 2011; Revised 2 December 2011; Accepted 9 December 2011

Academic Editor: Hoang Ngoc Long

Copyright © 2012 M. Medina and P. C. de Holanda. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

We present a detailed analysis of a class of extensions to the SM Gauge chiral symmetry $SU(3)_C \times SU(3)_L \times U(1)_X$ (331 model), where the neutrino electroweak interaction with matter via charged and neutral current is modified through new gauge bosons of the model. We found the connections between the nonstandard contributions on 331 model with nonstandard interactions. Through limits of such interactions in cross-section experiments, we constrained the parameters of the model, obtaining that the new energy scale of this theory should obey $V > 1.3$ TeV and the new bosons of the model must have masses greater than 610 GeV.

1. Introduction

Although the standard model (SM) is a good phenomenological theory, describing very well all experimental results, it leaves several unanswered questions that suggest that the SM might be an effective model at low energies, originating from a more fundamental theory. Some of the unexplained aspects in the SM are the existence of three families and lepton flavour violation observed in solar [1–5], atmospheric [6–11], and reactor [12–17] neutrino experiments. These results demonstrate that new physics is required, being interpreted as a sign of physics beyond the SM.

In principle neutrinos new interactions not described by Standard Model can arise in extensions of the SM. We assume that the new physics which induces the nonstandard neutrino interactions (NSIs) [18–29] arises in some models enlarging the symmetry group where the SM is embedded. Models with larger symmetries that may allow us to understand the origin of the families have been proposed [30–34]. In some models, it is also possible to understand the number of families from the cancellation of chiral anomalies, necessary to preserve the renormalizability of the theory [35–37]. This is the case of the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$

or 331 models, which are an immediate extension of the SM [38–46]. There are a great variety of such models, which have generated new expectations and possibilities of solving several problems of the SM.

Our goal is to investigate how NSI with matter can be induced by new physics generated by 331 models. Through the constraints from neutrino elastic scattering experiments on this NSI parameters, we can constrain some values expected for 331 model parameters. We find that the constraints on vacuum expectation values of the model, as well as for the mass of the new bosons, are in full agreement with the limits found in the literature, which makes this class of models a viable theory for a higher energy level.

The paper is organized as follows. In Section 2 we briefly review NSI and present how new interactions can contribute to new matter effects, in addition to the SM electroweak ones. In Section 3 we introduce a specific 331 model and we give the fermion gauge-boson couplings. In Section 4 we calculate the interactions involving neutrinos and how these interactions can be interpreted as new terms beyond SM. Finally, in Section 5 we summarize our main results.

2. Nonstandard Neutrino Interactions

One convenient way to describe neutrino new interactions with matter in the electro-weak (EW) broken phase are the so-called nonstandard neutrino interactions (NSIs), which is a very widespread and convenient way of parameterizing the effects of new physics in neutrino oscillations [18–29]. NSIs with first generation of leptons and quarks for four-fermion operators are contained in the following Lagrangian density [18–22, 24, 25, 28]:

$$\mathcal{L}_{\text{eff}}^{\text{NSI}} = -2\sqrt{2}G_F \sum_{f,P} \varepsilon_{\alpha\beta}^{fP} [\bar{f}\gamma^\mu P f] [\bar{\nu}_\alpha \gamma^\mu L \nu_\beta], \quad (2.1)$$

where G_F is the Fermi constant, $f = u, d, e$, and $P = L, R$ with $2L = (1 - \gamma^5)$, $2R = (1 + \gamma^5)$, and the coefficients $\varepsilon_{\alpha\beta}^{fP}$ encode the deviation from standard interactions between neutrinos of flavor α with component P -handed of fermions f , resulting in a neutrinos of flavor β . Then, the neutrino oscillations in the presence of nonstandard matter effects can be described by an effective Hamiltonian, parameterized as

$$\widetilde{H} = \frac{1}{2E} \left[U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix} U^\dagger + a \begin{pmatrix} 1 + \varepsilon_{ee} & \varepsilon_{e\mu} & \varepsilon_{e\tau} \\ \varepsilon_{e\mu}^* & \varepsilon_{\mu\mu} & \varepsilon_{\mu\tau} \\ \varepsilon_{e\tau}^* & \varepsilon_{\mu\tau}^* & \varepsilon_{\tau\tau} \end{pmatrix} \right], \quad (2.2)$$

where $a = \sqrt{2}G_F n_f$, E is the neutrino energy and $\varepsilon_{\alpha\beta} = \sum_{f,P} \varepsilon_{\alpha\beta}^{fP} n_f / n_e$ with n_e and n_f the electrons and fermions f density in the medium, respectively. These parameters $\varepsilon_{\alpha\beta}$ can be found in solar [22, 47], atmospheric [20, 48], accelerator [18, 19, 22, 49], and cross-section [18, 19, 21, 50, 51] neutrino data experiment.

We focus on cross-section neutrino experiment, where at low energies the standard differential cross-section for $\nu_\alpha e \rightarrow \nu_\alpha e$ scattering processes has the well-know form:

$$\frac{d\sigma_\alpha}{dT} = \frac{2G_F m_e}{\pi} \left[(g_1^\alpha)^2 + (g_2^\alpha)^2 \left(1 - \frac{T}{E_\nu}\right)^2 - g_1^\alpha g_2^\alpha \frac{m_e T}{E_\nu^2} \right], \quad (2.3)$$

where m_e is the electron mass, E_ν is the incident neutrino energy, and T_e is the electron recoil energy. The quantities g_1^α and g_2^α are related to the SM neutral current couplings of the electron $g_L^e = -1/2 + \sin^2\theta_W$ and $g_R^e = \sin^2\theta_W$, with $\sin^2\theta_W = 0,23119$. For $\nu_{\mu,\tau}$ neutrinos, which take part only in neutral current interactions, we have $g_1^{\mu,\tau} = g_L^e$ and $g_2^{\mu,\tau} = g_R^e$ while for electron neutrinos, which take part in both charge current (CC) and neutral current (NC) interactions, $g_1^e = 1 + g_L^e$, $g_2^e = g_R^e$. In the presence of nonuniversal standard interaction, the cross-section can be written in the same form of (2.3) but with $g_{1,2}^\alpha$ replaced by the effective nonstandard couplings $\tilde{g}_1^\alpha = g_1^\alpha + \varepsilon_{\alpha\alpha}^{eL}$ and $\tilde{g}_2^\alpha = g_2^\alpha + \varepsilon_{\alpha\alpha}^{eR}$, leading to the following differential scattering cross-section [19, 21, 50, 51]

$$\frac{d\sigma_\alpha}{dT} = \frac{2G_F m_e}{\pi} \left\{ \left(g_1^\alpha + \varepsilon_{\alpha\alpha}^{eL} \right)^2 + \left(g_2^\alpha + \varepsilon_{\alpha\alpha}^{eR} \right)^2 \left(1 - \frac{T_e}{E_\nu} \right)^2 - \left(g_1^\alpha + \varepsilon_{\alpha\alpha}^{eL} \right) \left(g_2^\alpha + \varepsilon_{\alpha\alpha}^{eR} \right) \frac{m_e T_e}{E_\nu} \right\}. \quad (2.4)$$

3. 331 Model

The success of the standard model (SM) implies that any new theory should contain the symmetry $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ (G_{321}) in a low energy limit. Then, it is natural that one possible modification of SM involves extensions of the representation content in matter and Higgs sector, leading to extension of symmetry group G_{321} to groups $SU(N_C)_C \otimes SU(m)_L \otimes U(1)_X$ with $SU(N_C)_C \otimes SU(m)_L \otimes U(1)_X \supset G_{321}$.

In early 90's, Pisano and Pleitez [38, 39] and Frampton [40] suggested an extension of the symmetry group $SU(2)_L \otimes U(1)_Y$ of electroweak sector to a group $SU(3)_L \otimes U(1)_X$, that is, with $N_C = m = 3$. The 331 models present some interesting features; for instance, they associate the number of families to internal consistence of the theory, preserving asymptotic freedom.

In these models, the SM doublets are part of triplets. In quark sector three new quarks are included to build the triplets, while in lepton sector we can use the right-handed neutrino to such role [38, 40]. Another option is to invoke three new heavy leptons, charged or not, depending on the choice of charge operator [41, 42]. In SM the electric charge operator is constructed as a combination of diagonal generators of $SU(2) \otimes U(1)_Y$. Then, it is natural to assume that this operator in $SU(3)_L \otimes U(1)_X$ is defined in the same way. The most general charge operator in $SU(3)_L \otimes U(1)_X$ is a linear superposition of diagonal generators of symmetry groups, given by

$$Q \equiv aT_{3L} + \frac{2}{\sqrt{3}}bT_{8L} + XI_3, \quad (3.1)$$

where the group generator is defined as $T_{iL} \equiv \lambda_{iL}/2$ with λ_{iL} , $i = 1, \dots, 8$, being the Gell-Mann matrices for $SU(3)_L$, where the normalization chosen is $\text{Tr}(\lambda_{iL}\lambda_{jL}) = 2\delta_{ij}$ and $I_3 = \text{diag}(1, 1, 1)$ is the identity matrix, and a and b are two parameters to be determined. Then the charge operator in (3.1) acts on the representations 3 and 3^* of $SU(3)_L$ having the following form:

$$Q[3] = \text{diag} \left[\frac{a}{2} + \frac{b}{3} + X, -\frac{a}{2} + \frac{b}{3} + X, -\frac{2b}{3} + X \right], \quad (3.2)$$

$$Q[3^*] = \text{diag} \left[-\frac{a}{2} - \frac{b}{3} + X, +\frac{a}{2} - \frac{b}{3} + X, +\frac{2b}{3} + X \right], \quad (3.3)$$

where we have two free parameters to obtain the charge of fermions, a and b (X can be determined by anomalies cancellation). However, $a = 1$ is necessary to obtain doublets of isospins $SU(2) \otimes U(1)_Y$ correctly incorporated in the model $SU(3)_L \otimes U(1)_X$ [41, 42, 45]. Then we can vary b to create different models in 331 context, being a signature that differentiates such models. For $b = -3/2$, we have the original 331 model [38, 39].

To have local gauge invariance, we have the following covariant derivative: $D_\mu = \partial_\mu - i(g/2)\lambda_a W_\mu^\alpha - ig_x X B_\mu$ and a total of 17 mediator bosons: one field B_μ associated with $U(1)_X$, eight fields associated with $SU(3)_C$, and another eight fields associated with $SU(3)_L$, written in the following form:

$$W_\mu \equiv W_\mu^\alpha \lambda_\alpha = \begin{pmatrix} W_\mu^3 + \frac{1}{\sqrt{3}}W_\mu^8 & \sqrt{2}W_\mu^+ & \sqrt{2}K_\mu^{Q_1} \\ \sqrt{2}W_\mu^- & -W_\mu^3 + \frac{1}{\sqrt{3}}W_\mu^8 & \sqrt{2}K_\mu^{Q_2} \\ \sqrt{2}K_\mu^{-Q_1} & \sqrt{2}K_\mu^{-Q_2} & -\frac{2}{\sqrt{3}}W_\mu^8 \end{pmatrix}, \quad (3.4)$$

where

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_{1\mu} \mp iW_{2\mu}), \quad K_\mu^{\pm Q_1} = \frac{1}{\sqrt{2}}(W_{4\mu} \mp iW_{5\mu}), \quad K_\mu^{\pm Q_2} = \frac{1}{\sqrt{2}}(W_{6\mu} \mp iW_{7\mu}). \quad (3.5)$$

Therefore, charge operator in (3.2) applied over (3.4) leads to $Q_1 = 1/2 + b$ and $Q_2 = (-1/2) + b$. Then the mediator bosons will have integer electric charge only if $b = \pm 1/2, \pm 3/2, \pm 5/2, \dots, \pm (2n+1)/2, n = 0, 1, 2, 3, \dots$. A detailed analysis shows that if a and b are associated with the fundamental representation 3, then $-a$ and $-b$ will be associated with antisymmetric representation 3^* .

3.1. The Representation Content

There are many representations for the matter content [46], for instance, $b = 3/2$ [38]. But we note that if we accommodate the doublets of $SU(2)_L$ in the superior components of triplets and antitriplets of $SU(3)_L$, and if we forbid exotic charges for the new fermions, we obtain from (3.2) the constrain $b = \pm 1/2$ (assuming $a = 1$). Since a negative value of b can be associated to the antitriplet, we obtain that $b = 1/2$ is a necessary and sufficient condition to exclude exotic electric charges in fermion and boson sector [41].

The fields left- and right-handed components transform under $SU(3)_L$ as triplets and singlets, respectively. Therefore the theory is chiral and can present anomalies of Adler-Bell-Jackiw [52, 53]. In a non-abelian theory, in the fermionic representation \mathcal{R} , the divergent anomaly is given by

$$\mathcal{A}^{abc} \propto \sum_{\mathcal{R}} \text{Tr} \left[\left\{ T_L^a(\mathcal{R}), T_L^b(\mathcal{R}) \right\} T_L^c(\mathcal{R}) - \left\{ T_R^a(\mathcal{R}), T_R^b(\mathcal{R}) \right\} T_R^c(\mathcal{R}) \right], \quad (3.6)$$

where $T^a(\mathcal{R})$ are the matrix representations for each group generator acting on the basis \mathcal{R} with helicity left or right. Therefore, to eliminate the pure anomaly $[SU(3)_L]^3$, we should have that $\mathcal{A}^{abc} \propto \sum_{\mathcal{R}'} \text{Tr}[\{T_L^a(\mathcal{R}'), T_L^b(\mathcal{R}')\} T_L^c(\mathcal{R}')] = 0$. We use the fact that $SU(3)_L$ has two fundamental representations, 3 and 3^* , then its generators should be associated to T^a and T^{a*} , respectively, that is,

$$\begin{aligned} \sum_{\mathcal{R}'} \text{Tr} \left[\left\{ T_L^a(\mathcal{R}'), T_L^b(\mathcal{R}') \right\} T_L^c(\mathcal{R}') \right] &= \sum_{\mathcal{R}} \text{Tr} \left[\left\{ T_L^a(\mathcal{R}), T_L^b(\mathcal{R}) \right\} T_L^c(\mathcal{R}) \right] \\ &- \sum_{\mathcal{R}^*} \text{Tr} \left[\left\{ T_L^{a*}(\mathcal{R}^*), T_L^{b*}(\mathcal{R}^*) \right\} T_L^{c*}(\mathcal{R}^*) \right], \end{aligned} \quad (3.7)$$

but we know that the matrix representations for each group generator satisfies that $T_L^{a*}(\mathcal{R}^*) = -T_L^a(\mathcal{R})$ [54]. So, we can see that for the anomalies to be canceled, the number of fields that transform as triplets (first term in equation above) and antitriplets under $SU(3)_L$ has to be the same; that is, two triplets quark families $\times 3$ (color) = one antitriplet quark family $\times 3$ (color) + 3 antitriplet lepton families. This implies that two families of quarks should transform differently than the third family, as will be discussed in next paragraph.

Usually the third quark family is chosen to transform in a different way than the first two families. But we will assume that the first family transform differently, to address the fact that $m_u < m_d$, $m_{\nu_e} < m_\ell$ while $m_c \gg m_s$ and $m_t \gg m_b$. To state this in a clearer way, we recall that in SM the $SU(2)_L$ doublets are $(\nu_\ell, \ell)^T$, $(u, d)^T$, $(c, s)^T$, $(t, b)^T$, with $\ell = e, \mu, \tau$. We can see that the first component of leptons doublets and first quark family is lighter than the second component. But for the second and third quark families, the opposite occurs. Then we use this idea to justify that first quark family transform as leptons.

3.2. Minimal 331 Model on Scalar Sector

Among the different possibilities of 331 models, we will present a detailed study on a minimal model on scalar sector without exotic electric charges for quarks and with three new leptons

without charged [41] ($b = 1/2$), where the fermions present the following transformation structure under $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$:

$$\begin{aligned}
\psi_{\ell L} &= \left(\ell^-, \nu_\ell, N_\ell^0 \right)_L^T \sim \left(1, 3^*, -\frac{1}{3} \right), \\
\nu_{\ell R} &\sim (1, 1, 0), \\
\ell_R^- &\sim (1, 1, -1), \\
N_{\ell R}^0 &\sim (1, 1, 0), \\
Q_{1L} &= (d, u, U_1)_L^T \sim \left(3, 3^*, \frac{1}{3} \right), \\
u_{iR} &\sim \left(3, 1, \frac{2}{3} \right), \\
d_{iR} &\sim \left(3, 1, -\frac{1}{3} \right), \\
U_{1R} &\sim \left(3, 1, \frac{2}{3} \right), \\
Q_{aL} &= (u_a, d_a, D_a)_L^T \sim (3, 3, 0), \\
D_{aR} &\sim \left(3, 1, -\frac{1}{3} \right),
\end{aligned} \tag{3.8}$$

where $i = 1, 2, 3$, $\ell = e, \mu, \tau$, $a = 2, 3$. We note that the leptons multiplets $\psi_{\ell L}$ consist of three fields $\ell = \{e, \mu, \tau\}$, the corresponding neutrinos $\nu_\ell = \{\nu_e, \nu_\mu, \nu_\tau\}$, and new neutral leptons $N_\ell^0 = \{N_e^0, N_\mu^0, N_\tau^0\}$. We can also see that the multiplet associated with the first quark family Q_{1L} consists of down and up quarks and a new quark with the same electric charge of quark up (named U_1), while the multiplet associated with second (third) family Q_{aL} consists of SM quarks of second (third) family and a new quark with the same electric charge of down quark (named D_2 (D_3)). The numbers on parenthesis refer to the transformation properties under $SU(3)_C$, $SU(3)_L$, and $U(1)_X$, respectively. With this choice, the anomalies are cancelled in a nontrivial way [55], and asymptotic freedom is guaranteed [56–59].

3.2.1. Scalar Sector and the Yukawa Couplings

The scalar fields have to be coupled to fermions by the Yukawa terms, invariants under $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$. In lepton sector, these couplings can be written as

$$\begin{aligned}
\bar{\psi}_{\ell L} \ell_R &\sim \left(1, 3, \frac{1}{3} \right) \otimes (1, 1, -1) = \underbrace{\left(1, 3, -\frac{2}{3} \right)}_{\rho^*}, \\
\bar{\psi}_{\ell L} \nu_{\ell R} &\sim \left(1, 3, \frac{1}{3} \right) \otimes (1, 1, 0) = \underbrace{\left(1, 3, \frac{1}{3} \right)}_{\eta}, \\
\bar{\psi}_{\ell L} N_{\ell R}^0 &\sim \left(1, 3, \frac{1}{3} \right) \otimes (1, 1, 0) = \underbrace{\left(1, 3, \frac{1}{3} \right)}_x,
\end{aligned} \tag{3.9}$$

and writing only three terms in quarks sector, for example,

$$\begin{aligned}
\bar{Q}_{1L}u_{iR} &= \left(3^*, 3, -\frac{1}{3}\right) \otimes \left(3, 1, \frac{2}{3}\right) = \underbrace{\left(1, 3, \frac{1}{3}\right)}_{\chi} \oplus \underbrace{\left(8, 3, \frac{1}{3}\right)}_{\text{Color Higgs}}, \\
\bar{Q}_{1L}d_{iR} &= \left(3^*, 3, -\frac{1}{3}\right) \otimes \left(3, 1, -\frac{1}{3}\right) = \underbrace{\left(1, 3, -\frac{2}{3}\right)}_{\rho^*} \oplus \dots, \\
\bar{Q}_{aL}d_{iR} &= \left(3^*, 3^*, 0\right) \otimes \left(3, 1, -\frac{1}{3}\right) = \underbrace{\left(1, 3^*, -\frac{1}{3}\right)}_{\eta^*} \oplus \dots, \dots
\end{aligned} \tag{3.10}$$

As usual in these class of models, we impose colorless Higgs (i.e., selecting only the multiplets that transform as singlets under $SU(3)_C$). We note that we need only three Higgs multiplets, ρ , χ , and η , to couple the different fermionic fields and generate mass through spontaneous symmetry breaking. In (3.9) and (3.10) we note that quantum numbers of triplets χ and η are the same, which leads us to consider models with two or three Higgs triplets. We will adopt the first option, two Higgs triplets, due to the simpler scalar sector in comparison with the scenario with three triplets [41–44].

3.3. Model with Two Higgs Triplets

For the models with two Higgs triplets, we obtain (note that in this model we assumed $\Phi_1 = \chi, \eta$ e $\Phi_2 = \rho$)

$$\begin{aligned}
\Phi_1 &= \left(\phi_1^-, \phi_1^0, \phi_1^0\right)^T \sim \left(1, 3^*, -\frac{1}{3}\right), \\
\Phi_2 &= \left(\phi_2^0, \phi_2^+, \phi_2^+\right)^T \sim \left(1, 3^*, \frac{2}{3}\right).
\end{aligned} \tag{3.11}$$

Assuming the following choice to the Higgs triplets vacuum expectation value (VEV) [41] $\langle \Phi_1 \rangle_0 = (0, \vartheta_1, V)^T$ and $\langle \Phi_2 \rangle_0 = (\vartheta_2, 0, 0)^T$, we associate V with the mass of the new fermions, which lead us to assume $V \gg \vartheta_1, \vartheta_2$. We expand the scalar VEVs in the following way:

$$\phi_1^0 = V + \frac{H_{\phi_1}^0 + iA_{\phi_1}^0}{\sqrt{2}}, \quad \phi_1^{\prime 0} = \vartheta_1 + \frac{H_{\phi_1}^{\prime 0} + iA_{\phi_1}^{\prime 0}}{\sqrt{2}}, \quad \phi_2^0 = \vartheta_2 + \frac{H_{\phi_2}^0 + iA_{\phi_2}^0}{\sqrt{2}}. \tag{3.12}$$

The real (imaginary) part H_{ϕ_i} (A_{ϕ_i}) is usually called CP-even (CP-odd) scalar field. The most general potential can be written as

$$\begin{aligned}
V(\Phi_1, \Phi_2) &= \mu_1^2 \Phi_1^\dagger \Phi_1 + \mu_2^2 \Phi_2^\dagger \Phi_2 + \lambda_1 \left(\Phi_1^\dagger \Phi_1\right)^2 + \lambda_2 \left(\Phi_2^\dagger \Phi_2\right)^2 \\
&+ \lambda_3 \left(\Phi_1^\dagger \Phi_1\right) \left(\Phi_2^\dagger \Phi_2\right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2\right) \left(\Phi_2^\dagger \Phi_1\right).
\end{aligned} \tag{3.13}$$

Demanding that in the displaced potential $V(\Phi_1, \Phi_2)$ the linear terms on the field should be absent, we have, in tree-level approximation, the following constraints:

$$\begin{aligned}\mu_1^2 + 2\lambda_1(\vartheta_1^2 + V^2) + \lambda_3\vartheta_2^2 &= 0, \\ \mu_2^2 + \lambda_3(\vartheta_1^2 + V^2) + 2\lambda_2\vartheta_2^2 &= 0.\end{aligned}\tag{3.14}$$

The analysis of such equations shows that they are related to a minimum in scalar potential with the value $V_{\min} = -\vartheta_2^4\lambda_2 - (\vartheta_1^2 + V^2)[(\vartheta_1^2 + V^2)\lambda_1 + \vartheta_2^2\lambda_3]$. Then, replacing (3.12) and (3.14) in (3.13), we can calculate the mass matrix in $(H_{\phi_1}^0, H_{\phi_2}^0, H_{\phi_1}^0)$ basis through the relation $M_{ij}^2 = 2(\partial^2 V(\Phi_1, \Phi_2)/\partial H_{\phi_i}^0 \partial H_{\phi_j}^0)$, obtaining

$$M_H^2 = 2 \begin{pmatrix} 2\lambda_1 V^2 & \lambda_3 \vartheta_2 V & 2\lambda_1 \vartheta_1 V \\ \lambda_3 \vartheta_2 V & 2\lambda_2 \vartheta_2^2 & \lambda_3 \vartheta_1 \vartheta_2 \\ 2\lambda_1 \vartheta_1 V & \lambda_3 \vartheta_1 \vartheta_2 & 2\lambda_1 \vartheta_1^2 \end{pmatrix}.\tag{3.15}$$

Since (3.15) has vanishing determinant, we have one Goldstone boson G_1 and two massive neutral scalar fields H_1 and H_2 with masses (note that if $\lambda_3^2 = 4\lambda_1\lambda_2$, we obtain two Goldstone bosons, G_1 and H_2 , and a massive scalar field H_1 with mass $M_{H_1}^2 = 4[\lambda_1(\vartheta_1^2 + V^2) + \lambda_2\vartheta_2^2]$, where $\lambda_1\lambda_2 > 0$; then imposing $M_{H_1}^2 > 0$ leads to $\lambda_1 > 0$ and $\lambda_2 > 0$)

$$\begin{aligned}M_{H_1, H_2}^2 &= 2\lambda_1(\vartheta_1^2 + V^2) + 2\lambda_2\vartheta_2^2 \\ &\pm 2\sqrt{[\lambda_1(\vartheta_1^2 + V^2) + \lambda_2\vartheta_2^2]^2 + \vartheta_2^2(\vartheta_1^2 + V^2)(\lambda_3^2 - 4\lambda_1\lambda_2)},\end{aligned}\tag{3.16}$$

where real values for λ 's produce positive mass to neutral scalar fields only if $\lambda_1 > 0$ and $4\lambda_1\lambda_2 > \lambda_3^2$, which implies that $\lambda_2 > 0$. A detailed analysis shows that when $V(\Phi_1, \Phi_2)$ in (3.13) is expanded around the most general vacuum, given by (3.12) and using constrains in (3.14), we do not obtain pseudoscalar fields $A_{\phi_i}^0$. This allows us to identify three more Goldstone bosons, $G_2 = A_{\phi_1}^0$, $G_3 = A_{\phi_2}^0$, and $G_4 = A_{\phi_1}^0$. For the mass spectrum in charged scalar sector on $(\phi_1^-, \phi_2^+, \phi_2'^+)$ basis, the mass matrix will be given by

$$M_+^2 = 2\lambda_4 \begin{pmatrix} \vartheta_2^2 & \vartheta_1 \vartheta_2 & \vartheta_2 V \\ \vartheta_1 \vartheta_2 & \vartheta_1^2 & \vartheta_1 V \\ \vartheta_2 V & \vartheta_1 V & V^2 \end{pmatrix},\tag{3.17}$$

with two eigenvalues equal to zero, equivalent to four Goldstone bosons G_5^\pm , G_6^\pm and two physical charged scalar fields with large masses given by $\lambda_4(\vartheta_1^2 + \vartheta_2^2 + V^2)$, which leads to the constrain $\lambda_4 > 0$.

This analysis shows that, after symmetry breaking, the original twelve degrees of freedom in scalar sector leads to eight Goldstone bosons (four electrically neutral and four electrically charged), four physical scalar fields, two neutral (one of which being the SM Higgs scalar), and two charged. Eight Goldstone bosons should be absorbed by eight gauge fields as we will see in next section.

3.3.1. Gauge Sector with Two Higgs Triplets

The gauge bosons interaction with matter in electroweak sector appears with the covariant derivative for a matter field φ as

$$D_\mu^\varphi = \partial_\mu - \frac{i}{2}gW_\mu^a\lambda_{aL} - ig_X X_\varphi B_\mu = \partial_\mu - \frac{i}{2}g\mathcal{M}_\mu^\varphi, \quad (3.18)$$

where λ_{aL} , $a = 1, \dots, 8$ are Gell-Mann matrices of $SU(3)_L$ algebra and X_φ is the charge of abelian factor $U(1)_X$ of the multiplet φ in which D_μ acts. The matrix \mathcal{M}_μ^φ contains the gauge bosons with electric charges q , defined by the generic charge operator in (3.1). For $b = 1/2$ the matrix \mathcal{M}_μ^φ will have the following form:

$$\mathcal{M}_\mu^\varphi = \begin{pmatrix} W_{3\mu} + \frac{W_{8\mu}}{\sqrt{3}} + 2tX_\varphi B_\mu & \sqrt{2}W_\mu^+ & \sqrt{2}K_\mu^+ \\ \sqrt{2}W_\mu^- & -W_{3\mu} + \frac{W_{8\mu}}{\sqrt{3}} + 2tX_\varphi B_\mu & \sqrt{2}K_\mu^0 \\ \sqrt{2}K_\mu^- & \sqrt{2}\bar{K}_\mu^0 & \frac{-2W_{8\mu}}{\sqrt{3}} + 2tX_\varphi B_\mu \end{pmatrix}, \quad (3.19)$$

where $t = g_X/g$ and nonphysical gauge bosons on nondiagonal entries, W_μ^\pm and K_μ^\pm , are defined in (3.5) with $Q_1 = 1$, and

$$K_\mu^0 = \frac{1}{\sqrt{2}}(W_{6\mu} - iW_{7\mu}), \quad \bar{K}_\mu^0 = \frac{1}{\sqrt{2}}(W_{6\mu} + iW_{7\mu}). \quad (3.20)$$

Then for the 331 model we are considering ($b = 1/2$), we have two neutral gauge bosons, K_μ^0 and \bar{K}_μ^0 , and four charged gauge bosons, W_μ^\pm and K_μ^\pm . The three physical neutral eigenstates will be a linear combination of $W_{3\mu}$, $W_{8\mu}$, and B_μ . After breaking the symmetry with $\langle \Phi_i \rangle$, $i = 1, 2$, and using covariant derivative $D_\mu = \partial_\mu - (i/2)g\mathcal{M}_\mu^\varphi$ for the triplets Φ_i , we obtain the following masses for the charged physical fields:

$$M_{W'}^2 = \frac{1}{2}g^2\vartheta_2^2, \quad M_{K'}^2 = \frac{1}{2}g^2(\vartheta_1^2 + \vartheta_2^2 + V^2), \quad (3.21)$$

and the following physical eigenstates:

$$W_\mu'^{\pm} = \frac{1}{\sqrt{\vartheta_1^2 + V^2}}(-\vartheta_1 K_\mu^\pm + V W_\mu^\pm), \quad K_\mu'^{\pm} = \frac{1}{\sqrt{\vartheta_1^2 + V^2}}(V K_\mu^\pm + \vartheta_1 W_\mu^\pm). \quad (3.22)$$

The neutral sector in approximation $(\vartheta_i/V)^n \approx 0$ for $n > 2$ leads to the following masses for the neutral physical fields:

$$\begin{aligned}
M_{\text{photon}}^2 &= 0, \\
M_{K_R^0}^2 &= \frac{1}{2}g^2(V^2 + \vartheta_1^2), \\
M_Z^2 &\approx \frac{1}{2}g^2\vartheta_2^2 \left(\frac{3g^2 + 4g_x^2}{3g^2 + g_x^2} \right), \\
M_{Z'}^2 &\approx \frac{2}{9}(V^2 + \vartheta_1^2)(3g^2 + g_x^2) + \frac{\vartheta_2^2(3g^2 + 4g_x^2)^2}{18(3g^2 + g_x^2)}, \\
M_{K_I^0}^2 &= \frac{1}{2}g^2(V^2 + \vartheta_1^2).
\end{aligned} \tag{3.23}$$

We can see from (3.21) and (3.23) that we have one nonmassive boson, which we associate with the photon, and four massive neutral fields, where the mass of one of them is proportional to ϑ_2 while the other three have masses proportional to V (new energy scale). Therefore we can associate the field Z with SM Z_μ and the fields Z' , K_I^0 , and K_R^0 with three new neutral bosons. We note that (3.23) contains two same of the eigenvalues; thus, the K_I^0 and K_R^0 components have the same mass, and this conclusion contradicts the previous analysis in [41], but this is in agreement with [43, 44]. We also have four massive charged fields, where two of them have masses proportional to ϑ_2 . Thus we can associate the fields W_μ^\pm to the SM fields W_μ^\pm , while the fields K_μ^\pm are new bosons. The eigenstates B_μ , $W_{3\mu}$, $W_{8\mu}$, and $K_{R\mu}^0$ can be related to the physical eigenstates A_μ , $K_{R\mu}^0$, Z_μ^0 , and Z_μ^0 by

$$\begin{pmatrix} B_\mu \\ W_{3\mu} \\ W_{8\mu} \\ K_{R\mu}^0 \end{pmatrix} = \mathbf{M}^{-1} \begin{pmatrix} A_\mu \\ K_{R\mu}^0 \\ Z_\mu^0 \\ Z_\mu^0 \end{pmatrix}. \tag{3.24}$$

Assuming $(\vartheta_i/V)^n \sim 0$ for $n > 2$, we obtain

$$\mathbf{M}^{-1} = \begin{pmatrix} -\frac{1}{t}S_W & 0 & \frac{1}{t}T_W^2C_W + \beta_1 & -\frac{1}{\sqrt{3}}T_W + \beta_2 \\ S_W & \frac{-\vartheta_1}{V} & C_W + \beta_3 & \beta_4 \\ \frac{1}{\sqrt{3}}S_W & \frac{\sqrt{3}\vartheta_1}{V} & -\frac{1}{\sqrt{3}}T_WS_W + \beta_5 & -\frac{1}{t}T_W + \beta_6 \\ 0 & 1 - \beta_7 & \frac{\vartheta_1}{V}C_W^{-1} & \frac{\sqrt{3}\vartheta_1}{tV}T_W \end{pmatrix}, \tag{3.25}$$

where, again, $t = g_x/g$ and

$$\begin{aligned}
S_W &= \frac{\sqrt{3}g_x}{\sqrt{3g^2 + 4g_x^2}}, & C_W &= \sqrt{1 - S_W^2}, & T_W &= \frac{S_W}{C_W}, \\
\beta_1 &= -\frac{\vartheta_2^2}{4tV^2}T_W^2C_W^{-3}, & \beta_2 &= -\frac{\sqrt{3}\vartheta_2^2}{4t^2V^2}T_W^3C_W^{-2}, \\
\beta_3 &= -\frac{\vartheta_1^2}{2V^2}C_W^{-1}, & \beta_4 &= -\frac{\sqrt{3}(2C_W^2\vartheta_1^2 + \vartheta_2^2)}{4tV^2}T_WC_W^{-2}, \\
\beta_5 &= \frac{6C_W^4\vartheta_1^2 - (3 - 4S_W^2)\vartheta_2^2}{4\sqrt{3}V^2C_W^5}, & \beta_6 &= \frac{(6C_W^4\vartheta_1^2 + S_W^2\vartheta_2^2)}{4tV^2C_W^4}T_W, \\
\beta_7 &= -\frac{2\vartheta_2^2}{V^2}.
\end{aligned} \tag{3.26}$$

We note that all β_i are of order $\mathcal{O}((\vartheta_i/V)^2)$. So, assuming $\vartheta_i \sim \mathcal{O}(10^{-1})$ TeV, for a new energy scale of order $V \sim 10$ TeV, all the β_i 's are negligible.

3.3.2. Charged and Neutral Currents

The interaction between gauge bosons and fermions in flavor basis is given by the following Lagrangian density:

$$\mathcal{L}_f = \bar{R}i\gamma^\mu(\partial_\mu + ig_x B_\mu X_R)R + \bar{L}i\gamma^\mu\left(\partial_\mu + ig_x B_\mu X_L + \frac{ig}{2}\lambda_a W_\mu^a\right)L, \tag{3.27}$$

where R represents any right-handed singlet and L any left-handed triplet. We can write $\mathcal{L}_f = \mathcal{L}_{\text{lep}} + \mathcal{L}_{Q_1} + \mathcal{L}_{Q_a}$, and in lepton sector, we obtain

$$\mathcal{L}_{\text{lep}} = \mathcal{L}_{\text{lep}}^{\text{kin}} + \mathcal{L}_{\text{lep}}^{\text{CC}} + \mathcal{L}_{\text{lep}}^{\text{NC}}, \tag{3.28}$$

where

$$\mathcal{L}_{\text{lep}}^{\text{kin}} = \bar{R}i\gamma^\mu\partial_\mu R + \bar{L}i\gamma^\mu\partial_\mu L, \tag{3.29}$$

$$\mathcal{L}_{\text{lep}}^{\text{CC}} = -\frac{g}{\sqrt{2}}\bar{\ell}_L\gamma^\mu\nu_{\ell L}W_\mu^+ - \frac{g}{\sqrt{2}}\bar{\ell}_L\gamma^\mu N_{\ell L}^0 K_\mu^+ + \text{h.c.}, \tag{3.30}$$

$$\begin{aligned}
\mathcal{L}_{\text{lep}}^{\text{NC}} &= \frac{g_x}{3}\left[\bar{\ell}_L\gamma^\mu\ell + \bar{\nu}_{\ell L}\gamma^\mu\nu_{\ell L} + \bar{N}_{\ell L}^0\gamma^\mu N_{\ell L}^0\right]B_\mu + g_x\bar{\ell}_R\gamma^\mu\ell_R B_\mu \\
&\quad - \frac{g}{2\sqrt{3}}\left[\bar{\ell}_L\gamma^\mu\ell_L + \bar{\nu}_{\ell L}\gamma^\mu\nu_{\ell L} - 2t\bar{N}_{\ell L}^0\gamma^\mu N_{\ell L}^0\right]W_{8\mu} - \frac{g}{\sqrt{2}}\bar{\nu}_{\ell L}\gamma^\mu N_{\ell L}^0 K^{0\mu} \\
&\quad - \frac{g}{2}\left[\bar{\ell}_L\gamma^\mu\ell_L - \bar{\nu}_{\ell L}\gamma^\mu\nu_{\ell L}\right]W_{3\mu} - \frac{g}{\sqrt{2}}\bar{N}_{\ell L}^0\gamma^\mu\nu_{\ell L}\bar{K}_\mu^0.
\end{aligned} \tag{3.31}$$

In quark sector we have that for the first family triplet $X = 1/3$, and for the singlets d , u , and U_1 , we have $X = -1/3$, $2/3$ and $2/3$, respectively. Then we have

$$\begin{aligned}\mathcal{L}_{Q_1}^{\text{kin}} &= \bar{Q}_{1R} i\gamma^\mu \partial_\mu Q_{1R} + \bar{Q}_{1L} i\gamma^\mu \partial_\mu Q_{1L}, \\ \mathcal{L}_{Q_1}^{\text{CC}} &= -\frac{g}{\sqrt{2}} \bar{d}_L \gamma^\mu u_L W_\mu^+ - \frac{g}{\sqrt{2}} \bar{d}_L \gamma^\mu U_{1L} K_\mu^+ + \text{h.c.},\end{aligned}\quad (3.32)$$

$$\begin{aligned}\mathcal{L}_{Q_1}^{\text{NC}} &= \frac{g^x}{3} \left(\bar{d}_R \gamma^\mu d_R - 2\bar{u}_R \gamma^\mu u_R - 2\bar{U}_{1R} \gamma^\mu U_{1R} \right) B_\mu + \frac{g}{2} \bar{u}_L \gamma^\mu u_L W_{3\mu} \\ &\quad - \frac{g^x}{3} \left(\bar{d}_L \gamma^\mu d_L + \bar{u}_L \gamma^\mu u_L + \bar{U}_{1L} \gamma^\mu U_{1L} \right) B_\mu - \frac{g}{2} \bar{d}_L \gamma^\mu d_L W_{3\mu} - \frac{g}{\sqrt{2}} \bar{U}_{1L} \gamma^\mu u_L \bar{K}_\mu^0 \\ &\quad - \frac{g}{2\sqrt{3}} \left(\bar{d}_L \gamma^\mu d_L + \bar{u}_L \gamma^\mu u_L - 2\bar{U}_{1L} \gamma^\mu U_{1L} \right) W_{8\mu} - \frac{g}{\sqrt{3}} \bar{u}_L \gamma^\mu U_{1L} K_\mu^0.\end{aligned}\quad (3.33)$$

For second and third families we know that $X = 0$ for the triplets and $X = 2/3$, $-1/3$ and $-1/3$, for the singlets $u_{2,3}$, $d_{2,3}$, $D_{2,3}$, respectively, where $u_2 = c$, $u_3 = t$, $d_2 = s$, $d_3 = b$. Then we obtain for $a = 2, 3$

$$\begin{aligned}\mathcal{L}_{Q_a}^{\text{kin}} &= \bar{Q}_{aR} i\gamma^\mu \partial_\mu Q_{aR} + \bar{Q}_{aL} i\gamma^\mu \partial_\mu Q_{aL}, \\ \mathcal{L}_{Q_a}^{\text{CC}} &= -\frac{g}{\sqrt{2}} \bar{u}_{aL} \gamma^\mu d_{aL} W_\mu^+ - \frac{g}{\sqrt{2}} \bar{u}_{aL} \gamma^\mu D_{aL} K_\mu^+ + \text{h.c.}, \\ \mathcal{L}_{Q_a}^{\text{NC}} &= \frac{g^x}{3} \left[-2\bar{u}_{aR} \gamma^\mu u_{aR} + \bar{d}_{aR} \gamma^\mu d_{aR} + \bar{D}_{aR} \gamma^\mu D_{aR} \right] B_\mu \\ &\quad - \frac{g}{2\sqrt{3}} \left[\bar{u}_{aL} \gamma^\mu u_{aL} + \bar{d}_{aL} \gamma^\mu d_{aL} - 4\bar{D}_{aL} \gamma^\mu D_{aL} \right] W_{8\mu} - \frac{g}{\sqrt{2}} \bar{d}_{aL} \gamma^\mu D_{aL} K_\mu^0 \\ &\quad - \frac{g}{2} \left[\bar{u}_{aL} \gamma^\mu u_{aL} - \bar{d}_{aL} \gamma^\mu d_{aL} \right] W_{3\mu} - \frac{g}{\sqrt{2}} \bar{D}_{aL} \gamma^\mu d_{aL} \bar{K}_\mu^0.\end{aligned}\quad (3.34)$$

4. Neutrinos Interactions with Matter in 331 Model

It is well known that neutrino oscillation phenomenon in a material medium, as the sun, earth, or in a supernova, can be quite different from the oscillation that occurs in vacuum, since the interactions in the medium modify the dispersion relations of the particles traveling through it [60]. From the macroscopic point of view, the modifications of neutrino dispersion relations can be represented in terms of a refractive index or an effective potential. And according to [60, 61], the effective potential can be calculated from the amplitudes of coherent elastic scattering in relativistic limit.

In the present 331 model, the coherent scattering will be induced by neutral currents, NC, mediated by bosons Z_μ^0 , Z_μ^0 , and $K_{R\mu}^0$ and by charged currents, CC, mediated by bosons W_μ^\pm and K_μ^\pm . Following [61], we calculate in next sections the neutrino effective potentials in coherent scattering.

4.1. Charged Currents

The first term of (3.30) shows that the interaction of charged leptons with neutrinos occurs only through the gauge bosons W_μ^\pm ; then, by (3.22) we obtain that the interaction through charged bosons is given by

$$-\frac{g}{\sqrt{2}}\bar{\ell}_L\gamma^\mu\nu_{eL}W_\mu^+ = -\frac{Vg}{\sqrt{2}\sqrt{\vartheta_1^2+V^2}}\bar{\ell}_L\gamma^\mu\nu_{eL}W_\mu^\pm - \frac{g\vartheta_1}{\sqrt{2}\sqrt{\vartheta_1^2+V^2}}\bar{\ell}_L\gamma^\mu\nu_{eL}K_\mu^\pm. \quad (4.1)$$

The amplitude for the neutrino elastic scattering with charged leptons in tree level through CC is given by (note from (4.1) that only left-handed leptons interact with neutrinos, as in SM)

$$\begin{aligned} \mathcal{L}_{\text{int}}^{\text{cc}} = & -\left(-\frac{Vg}{\sqrt{2}\sqrt{\vartheta_1^2+V^2}}\right)^2\bar{\ell}_L(p_1)\gamma^\mu\nu_{eL}(p_2)\frac{-ig_{\mu\lambda}}{(p_2-p_1)^2-M_W^2}\bar{\nu}_{eL}(p_3)\gamma^\lambda\ell_L(p_4) \\ & -\left(-\frac{g\vartheta_1}{\sqrt{2}\sqrt{\vartheta_1^2+V^2}}\right)^2\bar{\ell}_L(p_1)\gamma^\mu\nu_{eL}(p_2)\frac{-ig_{\mu\lambda}}{(p_2-p_1)^2-M_{K'}^2}\bar{\nu}_{eL}(p_3)\gamma^\lambda\ell_L(p_4). \end{aligned} \quad (4.2)$$

For low energies $M_{W'}^2, M_{K'}^2 \gg (p_2-p_1)^2$, the effective Lagrangian is given by

$$\mathcal{L}_{\text{eff}}^{\text{cc}} \approx -\frac{g^2}{2(\vartheta_1^2+V^2)}\left(\frac{V^2}{M_{W'}^2} + \frac{\vartheta_1^2}{M_{K'}^2}\right)\left[\bar{\ell}_L(p_1)\gamma^\mu\ell_L(p_4)\right]\left[\bar{\nu}_{eL}(p_3)\gamma_\mu\nu_{eL}(p_2)\right], \quad (4.3)$$

where we used the Fierz transformation [62] to go from (4.2) to (4.3). Replacing (3.21) in (4.3), we obtain

$$-\mathcal{L}_{\text{eff}}^{\text{cc}} \approx \left[\frac{1}{\vartheta_2^2} - \frac{\vartheta_1^2}{V^2\vartheta_2^2} + \left(\frac{\vartheta_1^2}{V^4}\right)_{K'} + \mathcal{O}\left(\frac{1}{V^4}\right)\right]\left\langle\bar{\ell}\gamma^\mu\frac{(1-\gamma_5)}{2}\ell\right\rangle\{\bar{\nu}_{eL}(p)\gamma^\mu\nu_{eL}(p)\}, \quad (4.4)$$

where we used $(\)_{K'}$ to denote the term that appears from the new charged boson. We can see that for a new energy scale $V \gg \vartheta_1$ the term that comes from the new boson does not contribute to the process, as expected, since the new charged boson K_μ^\pm has a mass of the order of the new energy scale of the theory (see (3.21)).

Now, since usual matter has only leptons from first family, we will restrain our calculations to the neutrino interactions with first family standard model particles. The term $\langle \ \rangle$ in (4.4) can be calculated following [61], where we have the correspondence $\langle\bar{e}\gamma^\mu\gamma_5e\rangle \sim$ spin, $\langle\bar{e}\gamma^i e\rangle \sim$ velocity, and $\langle\bar{e}\gamma^0 e\rangle \sim n_e$, where n_e is the electronic density. Assuming nonpolarized medium and vanishing average velocity, we obtain that (4.4) can be written as

$$\mathcal{L}_{\text{eff}}^{\text{cc}} \approx -\left[\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2\vartheta_2^2} + \left(\frac{\vartheta_1^2}{2V^4}\right)_{K'} + \mathcal{O}(V^{-4})\right]n_e\bar{\nu}_{eL}\gamma^0\nu_{eL}. \quad (4.5)$$

The modifications on electronic neutrino dispersion relations can be represented by the following effective potential:

$$V_{\text{CC}}^e \approx \frac{1}{2\vartheta_2^2} n_e - \frac{\vartheta_1^2}{2V^2\vartheta_2^2} n_e + \left(\frac{\vartheta_1^2}{2V^4} \right)_{K'} n_e + \mathcal{O}(V^{-4}). \quad (4.6)$$

Disregarding the term $()_{K'}$, since we are assuming $V \gg \vartheta_i$, and remembering that in Section 3.3.1 we associated boson W' with SM boson W , we can easily associate

$$\sqrt{2}G_F \approx \frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2\vartheta_2^2}. \quad (4.7)$$

We note that (4.7) gives limits for the VEV of one of the Higgs triplets. Under assumption $\vartheta_1, \vartheta_2 \ll V$, we can write $G_F \approx (1/2\sqrt{2}\vartheta_2^2)(1 - \vartheta_1^2/V^2)$, from which we can see that the maximum value of ϑ_2^2 is achieved when we consider $(\vartheta_1^2/V^2) \rightarrow 0$, in which replacing $G_F = 1.16637(1) \times 10^{-5} \text{ GeV}^{-2}$ leads to

$$\vartheta_2 \lesssim 174.105 \text{ GeV}. \quad (4.8)$$

4.2. Neutral Current

The Lagrangian for neutrino elastic scattering with fermions $f = e, u, d$ through NC is given by

$$\begin{aligned} -\mathcal{L}_{\text{int}}^{\text{NC}} = & \bar{f}(p_1)\gamma^\mu (g_{z'L}^f + g_{z'R}^f) f(p_2) \frac{-ig_{\mu\lambda}}{(p_2 - p_1)^2 - M_{z'}^2} \bar{\nu}_{\ell L}(p_3)\gamma^\lambda g_{\nu z'} \nu_{\ell L}(p_4) \\ & + \bar{f}(p_1)\gamma^\mu (g_{z'L}^f + g_{z'R}^f) f(p_2) \frac{-ig_{\mu\lambda}}{(p_2 - p_1)^2 - M_z^2} \bar{\nu}_{\ell L}(p_3)\gamma^\lambda g_{\nu z} \nu_{\ell L}(p_4) \\ & + \bar{f}(p_1)\gamma^\mu (g_{k'L}^f + g_{k'R}^f) f(p_2) \frac{-ig_{\mu\lambda}}{(p_2 - p_1)^2 - M_{k'}^2} \bar{\nu}_{\ell L}(p_3)\gamma^\lambda g_{\nu k'} \nu_{\ell L}(p_4). \end{aligned} \quad (4.9)$$

For low energies, we have that $M_{k'}^2, M_z^2, M_{z'}^2 \gg (p_2 - p_1)^2$ with $p_3 = p_4 = p$ and (4.9), and following the same procedure of Section 4.1, we obtain

$$\mathcal{L}_{\text{eff}}^{\text{NC}} \approx - \sum_{P=L,R} \left(g_{z'P}^f \frac{G_{\nu z'}}{M_{z'}^2} + g_{zP}^f \frac{G_{\nu z}}{M_z^2} + g_{k'P}^f \frac{G_{\nu k'}}{M_{k'}^2} \right) \frac{1}{2} n_f \bar{\nu}_{\ell L} \gamma_0 \nu_{\ell L}. \quad (4.10)$$

4.2.1. Leptons Sector

From (3.31) and (3.24), we obtain that for the known neutral leptons

$$\begin{aligned} \frac{g_x}{3} \bar{\nu}_{eL} \gamma^\mu \nu_{eL} B_\mu = \bar{\nu}_{eL} \gamma^\mu \nu_{eL} \left[-\frac{g}{3} S_W A_\mu + \left(\frac{g}{3} T_W^2 C_W + \frac{g_x}{3} \beta_1 \right) Z_\mu^0 \right. \\ \left. - \frac{g_x}{3} \left(\frac{1}{\sqrt{3}} T_W - \beta_2 \right) Z_\mu^{\prime 0} \right], \end{aligned} \quad (4.11)$$

$$\frac{g}{2} \bar{\nu}_{eL} \gamma^\mu \nu_{eL} W_3^\mu = \bar{\nu}_{eL} \gamma^\mu \nu_{eL} \left[\frac{g}{2} S_W A_\mu - \frac{g \vartheta_1}{2V} K_{R\mu}^{\prime 0} + \frac{g(C_W + \beta_3)}{2} Z_\mu^0 + \frac{g \beta_4}{2} Z_\mu^{\prime 0} \right], \quad (4.12)$$

$$\begin{aligned} \frac{-g}{2\sqrt{3}} \bar{\nu}_{eL} \gamma^\mu \nu_{eL} W_8^\mu = \bar{\nu}_{eL} \gamma^\mu \nu_{eL} \left[-\frac{g}{6} S_W A_\mu - \frac{g \vartheta_1}{2V} K_{R\mu}^{\prime 0} + \left(\frac{g}{6} \frac{S_W^2}{C_W} - \frac{g \beta_5}{2\sqrt{3}} \right) Z_\mu^0 \right. \\ \left. + \frac{g}{2\sqrt{3}} \left(\frac{1}{t} T_W - \beta_6 \right) Z_\mu^{\prime 0} \right]. \end{aligned} \quad (4.13)$$

By (4.11), (4.12), and (4.13), we obtain that vertex interactions with neutrinos can be written as

$$\bar{\nu}_{eL} \gamma^\mu \nu_{eL} A_\mu \propto 0, \quad (4.14)$$

$$\bar{\nu}_{eL} \gamma^\mu \nu_{eL} K_{R\mu}^{\prime 0} \propto -\frac{g \vartheta_1}{V} \equiv G_{\nu K'}, \quad (4.15)$$

$$\bar{\nu}_{eL} \gamma^\mu \nu_{eL} Z_\mu^0 \propto \frac{1}{2} g C_W^{-1} + \eta_1 \equiv G_{\nu Z}, \quad (4.16)$$

$$\bar{\nu}_{eL} \gamma^\mu \nu_{eL} Z_\mu^{\prime 0} \propto \left(\frac{3g - 2g_x t}{6 \sqrt{3} t} \right) T_W + \eta_2 \equiv G_{\nu Z'}, \quad (4.17)$$

where

$$\begin{aligned} \eta_1 = \frac{-4gtC_W^2 \vartheta_1^2 + g_x(1 - 2S_W^2) \vartheta_2^2}{8tV^2 C_W^5}, \\ \eta_2 = \frac{gt(1 - 4C_W^2) \vartheta_1^2}{2\sqrt{3}V^2 C_W S_W} - \frac{(-gt^3 + 2gt^3 C_W^2 + 8gt^3 C_W^4 + 6g_x S_W^4) \vartheta_2^2}{24\sqrt{3}t^2 V^2 C_W^5 S_W}. \end{aligned} \quad (4.18)$$

We note from (4.14) that neutrinos do not interact electrically, as expected. For charged leptons, from (3.31) and (3.24), we obtain

$$\begin{aligned}
\frac{g_x}{3} \bar{\ell}_L \gamma^\mu \ell_L B_\mu &= \bar{\ell}_L \gamma^\mu \ell_L \left[\frac{-g}{3} S_W A_\mu + \left(\frac{g}{3} T_W^2 C_W + \frac{g_x}{3} \beta_1 \right) Z_\mu^0 \right. \\
&\quad \left. - \frac{g_x}{3} \left(\frac{1}{\sqrt{3}} T_W - \beta_2 \right) Z_\mu^{\prime 0} \right], \\
-\frac{g}{2} \bar{\ell}_L \gamma^\mu \ell_L W_3^\mu &= \bar{\ell}_L \gamma^\mu \ell_L \left[\frac{-g}{2} S_W A_\mu + \frac{g \vartheta_1}{2V} K_{R\mu}^{\prime 0} - \frac{g(C_W + \beta_3)}{2} Z_\mu^0 - \frac{g\beta_4}{2} Z_\mu^{\prime 0} \right], \\
\frac{-g}{2\sqrt{3}} \bar{\ell}_L \gamma^\mu \ell_L W_8^\mu &= \bar{\ell}_L \gamma^\mu \ell_L \left[\frac{-g}{6} S_W A_\mu - \frac{g \vartheta_1}{2V} K_{R\mu}^{\prime 0} + \left(\frac{g}{6} \frac{S_W^2}{C_W} - \frac{g\beta_5}{2\sqrt{3}} \right) Z_\mu^0 \right. \\
&\quad \left. + \frac{g}{2\sqrt{3}} \left(\frac{1}{t} T_W - \beta_6 \right) Z_\mu^{\prime 0} \right], \\
g_x \bar{\ell}_R \gamma^\mu \ell_R B_\mu &= \bar{\ell}_R \gamma^\mu \ell_R \left[-g S_W A_\mu + \left(g T_W^2 C_W + g_x \beta_1 \right) Z_\mu^0 \right. \\
&\quad \left. - g_x \left(\frac{1}{\sqrt{3}} T_W - \beta_2 \right) Z_\mu^{\prime 0} \right],
\end{aligned} \tag{4.19}$$

and therefore

$$\bar{\ell} \gamma^\mu \ell A_\mu \propto -g S_W, \tag{4.20}$$

$$\bar{\ell}_L \gamma^\mu \ell_L K_{R\mu}^{\prime 0} \propto 0 \equiv g_{k'L}^\ell = g_{k'R}^\ell, \tag{4.21}$$

$$\bar{\ell}_L \gamma^\mu \ell_L Z_\mu^0 \propto \frac{1}{2} g (-1 + T_W^2) C_W + \eta_3 \equiv g_{zL}^\ell,$$

$$\bar{\ell}_R \gamma^\mu \ell_R Z_\mu^0 \propto g T_W^2 C_W + \eta_5 \equiv g_{zR}^\ell, \tag{4.22}$$

$$\bar{\ell}_L \gamma^\mu \ell_L Z_\mu^{\prime 0} \propto \frac{1}{6\sqrt{3}t} (3g - 2tg_x) T_W + \eta_4 \equiv g_{z'L}^\ell,$$

$$\bar{\ell}_R \gamma^\mu \ell_R Z_\mu^{\prime 0} \propto -\frac{g_x}{\sqrt{3}} T_W + \eta_6 \equiv g_{z'R}^\ell, \tag{4.23}$$

where

$$\begin{aligned}
\eta_3 &= \frac{(-1 + 2C_W^2) g_x \vartheta_2^2}{8tV^2 C_W^5}, \\
\eta_4 &= \frac{\left(gt^3 (1 + 2C_W^2) \right)^2 - 12gt^3 S_W^2 C_W^2 - 6g_x S_W^4}{24\sqrt{3}t^2 V^2 C_W^5 S_W}, \\
\eta_5 &= -\frac{g_x \vartheta_2^2}{4tV^2 C_W^3} T_W^2, \\
\eta_6 &= -\frac{\sqrt{3} g_x \vartheta_2^2}{4t^2 V^2 C_W^2} T_W^3,
\end{aligned} \tag{4.24}$$

and, again, $t = g_x/g$. We note that by (4.20) we can make the association $gS_W = |e|$. Then for $f = e$, (4.15)–(4.17) and (4.21)–(4.23) lead to

$$\begin{aligned} \mathcal{L}_{\text{eff}-e}^{\text{NC}} &\approx - \sum_{P=L,R} \frac{1}{2} \left(g_{z'P}^e \frac{G_{\nu z'}}{M_{z'}^2} + g_{zP}^e \frac{G_{\nu z}}{M_z^2} + g_{k'P}^e \frac{G_{\nu k'}}{M_{k'}^2} \right) n_e \bar{\nu}_{\ell L} \gamma_0 \nu_{\ell L} \\ &\approx - \left\{ \left[\frac{T_W^4}{144t^2 g_x^2 V^2} (3g - 2tg_x)^2 + \frac{T_W^2}{8V^2} (1 - T_W^2) \right. \right. \\ &\quad \left. \left. + \frac{1}{2} \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2 \vartheta_2^2} \right) (1 - 2C_W^2) \right]_L \right. \\ &\quad \left. + \left[\frac{T_W^4 (2tg_x - 3g)}{24tg_x V^2} - \frac{T_W^4}{4V^2} + \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2 \vartheta_2^2} \right) S_W^2 \right]_R \right\} n_e \bar{\nu}_{\ell L} \gamma_0 \nu_{\ell L}. \end{aligned} \quad (4.25)$$

Since intermediate neutral bosons in (4.9) do not distinguish between different lepton flavors, the interaction through NC with electron is described by the following effective potential:

$$\begin{aligned} V_{\text{NC}}^e &= V_{\text{NC}}^\mu = V_{\text{NC}}^\tau = V_{\text{NC}}^\ell \\ &= V_{\text{NC}}^{\ell L} + V_{\text{NC}}^{\ell R}, \end{aligned} \quad (4.26)$$

where

$$\begin{aligned} V_{\text{NC}}^{\ell L} &= \left[\frac{T_W^4}{144t^2 g_x^2 V^2} (3g - 2tg_x)^2 + \frac{T_W^2}{8V^2} (1 - T_W^2) \right. \\ &\quad \left. + \frac{1}{2} \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{V^2 \vartheta_2^2} \right) (1 - 2C_W^2) \right] n_e, \\ V_{\text{NC}}^{\ell R} &= \left[\frac{T_W^4 (2tg_x - 3g)}{24tg_x V^2} - \frac{T_W^4}{4V^2} + \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2 \vartheta_2^2} \right) S_W^2 \right] n_e, \end{aligned} \quad (4.27)$$

and index ℓ refers to neutrino flavor. We note that the potential through CC comes from interactions of electron neutrinos with left-handed electrons, while the effective potential through NC comes from left- and right-handed electrons.

Considering both NC and CC, we can write the effective potential felt by neutrinos as $V^\ell = V^{\ell L} + V^{\ell R}$, where

$$\begin{aligned} V^{\ell L} &= \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2 \vartheta_2^2} \right) \delta_{e\ell} n_e + V_{\text{NC}}^{\ell L}, \\ V^{\ell R} &= V_{\text{NC}}^{\ell R}. \end{aligned} \quad (4.28)$$

Comparing with SM expression for such potential:

$$V_{\text{NC}}^\ell = -\sqrt{2}G_F \left(\frac{1}{2} - 2S_W^2 \right) n_e, \quad V_{\text{CC}}^e = \sqrt{2}G_F n_e, \quad (4.29)$$

we can find that

$$\begin{aligned}
V^{\ell L} &= V^{\ell L} + \left[\frac{T_W^4}{144t^2 g_x^2 V^2} (3g - 2tg_x)^2 + \frac{T_W^2}{8V^2} (1 - T_W^2) \right] n_e, \\
V^{\ell R} &= V_{\text{NC}}^{\ell R} + \left[\frac{T_W^4 (2tg_x - 3g)}{24tg_x V^2} - \frac{T_W^4}{4V^2} \right] n_e,
\end{aligned} \tag{4.30}$$

where we adopt in what follow, the convention that V denotes SM-like part of the model; thus, the new terms beyond SM [] can be associated with the parameters ε 's in NSI [63]. So, in the approximation $(\vartheta_i/V)^n \approx 0$, for $n > 2$, we obtain

$$\varepsilon_{\ell\ell}^{eL} \approx \frac{(1 - 2S_W^2)\vartheta_2^2}{8V^2 C_W^4}, \tag{4.31}$$

$$\varepsilon_{\ell\ell}^{eR} \approx -\frac{S_W^2(1 + 2S_W^2)\vartheta_2^2}{4V^2 C_W^4}. \tag{4.32}$$

We note that on limit $V \rightarrow \infty$, we recover SM. The NSIs are a subleading interaction, as expected. By (4.31) and (4.32), we obtain $\varepsilon_{\ell\ell}^{eR} \approx -2S_W^2 \varepsilon_{\ell\ell}^{eL} - (\vartheta_2^2/V^2)T_W^4$.

4.2.2. Quarks Sector

For the quarks of the first family, the Lagrangian density in (3.33) describes the interactions with gauge bosons $W_{3\mu}$, $W_{8\mu}$, and B_μ ; then, by (3.24) and (3.25) we obtain the following interactions for up quarks:

$$\begin{aligned}
-\frac{g_x}{3} \bar{u}_L \gamma^\mu u_L B_\mu &= \bar{u}_L \gamma^\mu u_L \left[\frac{g}{3} S_W A_\mu - \frac{g_x}{3} \left(\frac{1}{t} T_W^2 C_W + \beta_1 \right) Z_\mu^0 \right. \\
&\quad \left. + \frac{g_x}{3} \left(\frac{1}{\sqrt{3}} T_W - \beta_2 \right) Z_\mu^{\prime 0} \right], \\
\frac{g}{2} \bar{u}_L \gamma^\mu u_L W_3^\mu &= \bar{u}_L \gamma^\mu u_L \left[\frac{g}{2} S_W A_\mu - \frac{g\vartheta_1}{2V} K_{R\mu}^{\prime 0} + \frac{g(C_W + \beta_3)}{2} Z_\mu^0 + \frac{g\beta_4}{2} Z_\mu^{\prime 0} \right], \\
\frac{-g}{2\sqrt{3}} \bar{u}_L \gamma^\mu u_L W_8^\mu &= \bar{u}_L \gamma^\mu u_L \left[\frac{-g}{6} S_W A_\mu - \frac{g\vartheta_1}{2V} K_{R\mu}^{\prime 0} \right. \\
&\quad \left. + \frac{g}{2\sqrt{3}} \left(\frac{1}{\sqrt{3}} T_W S_W - \beta_5 \right) Z_\mu^0 + \frac{g}{2\sqrt{3}} \left(\frac{1}{t} T_W - \beta_6 \right) Z_\mu^{\prime 0} \right], \\
-\frac{2g_x}{3} \bar{u}_R \gamma^\mu u_R B_\mu &= \bar{u}_R \gamma^\mu u_R \left[\frac{2g}{3} S_W A_\mu - \frac{2g_x}{3} \left(\frac{1}{t} T_W^2 C_W + \beta_1 \right) Z_\mu^0 \right. \\
&\quad \left. + \frac{2g_x}{3} \left(\frac{1}{t} T_W - \beta_6 \right) Z_\mu^{\prime 0} \right].
\end{aligned} \tag{4.33}$$

The couplings quark-quark-boson for the first family are given by

$$\bar{u}_L \gamma^\mu u_L A_\mu \propto \frac{2}{3} g S_W, \quad (4.34)$$

$$\bar{u}_R \gamma^\mu u_R A_\mu \propto \frac{2}{3} g S_W, \quad (4.35)$$

$$\bar{u}_L \gamma^\mu u_L K_{R\mu}^{\prime 0} \propto -\frac{g \vartheta_1}{V} \equiv g_{k'L}^{\prime \mu},$$

$$\bar{u}_R \gamma^\mu u_R K_{R\mu}^{\prime 0} \propto 0 \equiv g_{k'R}^{\prime \mu},$$

$$\bar{u}_L \gamma^\mu u_L Z_\mu^0 \propto \frac{1}{6} g (3 - T_W^2) C_W + \zeta_1 \equiv g_{zL}^{\prime \mu}, \quad (4.36)$$

$$\bar{u}_R \gamma^\mu u_R Z_\mu^0 \propto -\frac{2}{3} g T_W^2 C_W + \zeta_3 \equiv g_{zR}^{\prime \mu},$$

$$\bar{u}_L \gamma^\mu u_L Z_\mu^{\prime 0} \propto \frac{1}{6\sqrt{3}t} (3g + 2tg_x) T_W \equiv g_{z'L}^{\prime \mu},$$

$$\bar{u}_R \gamma^\mu u_R Z_\mu^{\prime 0} \propto \frac{2}{3\sqrt{3}} g_x T_W + \zeta_4 \equiv g_{z'R}^{\prime \mu},$$

where

$$\begin{aligned} \zeta_1 &= \frac{g_x (-12C_W^4 \vartheta_1^2 + (1 + 2C_W^2) \vartheta_2^2)}{24tV^2 C_W^5}, \\ \zeta_2 &= \frac{12gt^3 C_W^4 (1 - 4C_W^2) \vartheta_1^2 + (gt^3 (1 - 2C_W^2 - 8C_W^4) + 6g_x S_W^4) \vartheta_2^2}{24\sqrt{3}t^2 V^2 C_W^5 S_W}, \\ \zeta_3 &= \frac{g S_W^2 \vartheta_2^2}{6 C_W^5 V^2}, \\ \zeta_4 &= \frac{g_x S_W^3 \vartheta_2^2}{2\sqrt{3}t^2 V^2 C_W^5}. \end{aligned} \quad (4.37)$$

We note that (4.34) and (4.35) reflect the fact that quarks interact electrically through photons with coupling constant $Q_f \sin \theta_W$, as in SM. The effective Lagrangian at low energies for neutrino interaction with quarks up through neutral currents are given by (4.10) with $f = u$:

$$\begin{aligned} \mathcal{L}_{\text{quark}, u}^{\text{NC}} &\approx -\frac{1}{2} \sum_{P=L,R} \left(g_{z'P}^{\prime \mu} \frac{G_{\nu z'}}{M_{z'}^2} + g_{zP}^{\prime \mu} \frac{G_{\nu z}}{M_z^2} + g_{k'P}^{\prime \mu} \frac{G_{\nu k'}}{M_{k'}^2} \right) n_u \bar{\nu}_{\ell L} \gamma_0 \nu_{\ell L} \\ &\approx - \left\{ \left[\frac{1}{24V^2} (3 + T_W^4) + \frac{T_W^4}{144t^4 V^2} (9 - 4t^4) \right. \right. \\ &\quad \left. \left. + \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2 \vartheta_2^2} \right) \left(\frac{1}{2} - \frac{2}{3} S_W^2 \right) - \frac{\vartheta_1^2}{4V^2 \vartheta_2^2} \right]_L \right. \\ &\quad \left. + \left[\frac{T_W^4}{6V^2} + \frac{T_W^4 (3g - 2tg_x)}{36tg_x V^2} - \frac{2}{3} \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2 \vartheta_2^2} \right) S_W^2 \right]_R \right\} n_u \bar{\nu}_{\ell L} \gamma_0 \nu_{\ell L}, \end{aligned} \quad (4.38)$$

where n_u is the up quarks average density.

SM predictions, using result of (4.7), can be written as

$$V_{\text{NC}}^u = V_{\text{NC}}^{uL} + V_{\text{NC}}^{uR} = \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2\vartheta_2^2} \right) \left(\frac{1}{2} - \frac{4}{3}S_W^2 \right) n_u, \quad (4.39)$$

where

$$\begin{aligned} V_{\text{NC}}^{uL} &= \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2\vartheta_2^2} \right) \left(\frac{1}{2} - \frac{2}{3}S_W^2 \right) n_u, \\ V_{\text{NC}}^{uR} &= -\frac{2}{3} \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2\vartheta_2^2} \right) S_W^2 n_u. \end{aligned} \quad (4.40)$$

By comparison, we obtain

$$\begin{aligned} V_{\text{NC}}^{uL} &\approx V_{\text{NC}}^{uL} + \left[\frac{1}{24V^2} (3 + T_W^4) + \frac{T_W^4}{144t^4V^2} (9 - 4t^4) - \frac{\vartheta_1^2}{4V^2\vartheta_2^2} \right] n_u, \\ V_{\text{NC}}^{uR} &\approx V_{\text{NC}}^{uR} + \left[\frac{T_W^4}{6V^2} + \frac{T_W^4(3g - 2tg_x)}{36tg_xV^2} \right] n_u. \end{aligned} \quad (4.41)$$

Then we can say that $\varepsilon_{\ell\ell}^u = \varepsilon_{\ell\ell}^{uL} + \varepsilon_{\ell\ell}^{uR}$, where

$$\begin{aligned} \varepsilon_{\ell\ell}^{uL} &\approx -\frac{\vartheta_1^2}{2V^2} + \frac{\vartheta_2^2}{24V^2C_W^4} (9 - 8S_W^2), \\ \varepsilon_{\ell\ell}^{uR} &\approx \frac{\vartheta_2^2}{6V^2} \frac{S_W^2}{C_W^4}. \end{aligned} \quad (4.42)$$

Again, we obtain universal NSI, as for the electrons. We note that $\varepsilon_{\ell\ell}^{uL} = -(\vartheta_1^2/2V^2) + (3\vartheta_2^2/8V^2C_W^4) - 2\varepsilon_{\ell\ell}^{uR}$ and in the limit $V \rightarrow \infty$ we recover SM.

For down quarks by (3.33) and (3.24), we obtain that

$$\begin{aligned} -\frac{g_x}{3} \bar{d}_L \gamma^\mu d_L B_\mu &= \bar{d}_L \gamma^\mu d_L \left[\frac{g}{3} S_W A_\mu - \frac{g_x}{3} \left(\frac{1}{t} T_W^2 C_W + \beta_1 \right) Z_\mu^0 \right. \\ &\quad \left. + \frac{g_x}{3} \left(\frac{1}{\sqrt{3}} T_W - \beta_2 \right) Z_\mu^{\prime 0} \right], \\ -\frac{g}{2} \bar{d}_L \gamma^\mu d_L W_3^\mu &= \bar{d}_L \gamma^\mu d_L \left[-\frac{g S_W}{2} A_\mu + \frac{g\vartheta_1}{2V} K_{R\mu}^{\prime 0} - \frac{g(C_W + \beta_3)}{2} Z_\mu^0 - \frac{g\beta_4}{2} Z_\mu^{\prime 0} \right], \\ \frac{-g}{2\sqrt{3}} \bar{d}_L \gamma^\mu d_L W_8^\mu &= \bar{d}_L \gamma^\mu d_L \left[\frac{-g S_W}{6} A_\mu + \frac{g}{2\sqrt{3}} \left(\frac{1}{\sqrt{3}} T_W S_W - \beta_5 \right) Z_\mu^0 \right. \\ &\quad \left. + -\frac{g\vartheta_1}{2V} K_{R\mu}^{\prime 0} + \frac{g}{2\sqrt{3}} \left(\frac{1}{t} T_W - \beta_6 \right) Z_\mu^{\prime 0} \right], \end{aligned}$$

$$\begin{aligned} \frac{g_x}{3} \bar{d}_R \gamma^\mu d_R B_\mu = \bar{d}_L \gamma^\mu d_L \left[-\frac{g S_W}{3} A_\mu + \frac{g_x}{3} \left(\frac{1}{t} T_W^2 C_W + \beta_1 \right) Z_\mu^0 \right. \\ \left. + \frac{g_x}{3} \left(-\frac{1}{\sqrt{3}} T_W + \beta_2 \right) Z_\mu^{\prime 0} \right]. \end{aligned} \quad (4.43)$$

$$\begin{aligned} \bar{d}_L \gamma^\mu d_L A_\mu &\alpha -\frac{1}{3} g S_W, \\ \bar{d}_R \gamma^\mu d_R A_\mu &\alpha -\frac{1}{3} g S_W, \\ \bar{d}_L \gamma^\mu d_L K_{R\mu}^{\prime 0} &\alpha 0 \equiv g_{k'L}^d, \\ \bar{d}_R \gamma^\mu d_R K_{R\mu}^{\prime 0} &\alpha 0 \equiv g_{k'R'}^d, \\ \bar{d}_L \gamma^\mu d_L Z_\mu^0 &\alpha -\frac{1}{6} g (3 + T_W^2) C_W + \zeta_5 \equiv g_{z'L}^d, \\ \bar{d}_R \gamma^\mu d_R Z_\mu^0 &\alpha \frac{g}{3} T_W^2 C_W + \zeta_7 \equiv g_{z'R'}^d, \\ \bar{d}_L \gamma^\mu d_L Z_\mu^{\prime 0} &\alpha \frac{1}{6\sqrt{3}t} (3g + 2tg_x) T_W + \zeta_6 \equiv g_{z'L}^d, \\ \bar{u}_R \gamma^\mu u_R Z_\mu^{\prime 0} &\alpha -\frac{1}{3\sqrt{3}} g_x T_W + \zeta_8 \equiv g_{z'R'}^d \end{aligned} \quad (4.44)$$

where

$$\begin{aligned} \zeta_5 &= \frac{g\vartheta_2^2}{24V^2 C_W^5} (3 - 2S_W^2), \\ \zeta_6 &= \frac{(-1 + 3C_W^2 + 6C_W^4 - 8C_W^6)}{24\sqrt{3}V^2 C_W^5 S_W^3}, \\ \zeta_7 &= -\frac{gS_W^2 \vartheta_2^2}{12V^2 C_W^5}, \\ \zeta_8 &= -\frac{g_x S_W^3 \vartheta_2^2}{4\sqrt{3}t^2 V^2 C_W^5}. \end{aligned} \quad (4.45)$$

Then by (4.10) for $f = d$, we obtain the following effective Lagrangian for NC:

$$\begin{aligned} \mathcal{L}_{\text{quark}, d}^{\text{NC}} &\approx -\left(g_{z'V}^d \frac{G_{\nu z'}}{M_{z'}^2} + g_{zV}^d \frac{G_{\nu z}}{M_z^2} + g_{k'V}^d \frac{G_{\nu k'}}{M_{k'}^2} \right) n_d \bar{\nu}_{\ell L} \gamma_0 \nu_{\ell L} \\ &\approx -\left\{ \left[\frac{(3S_W^2 - 2S_W^4)}{24V^2 C_W^4} + \frac{(9 - 4t^4)}{144t^4 V^2} T_W^4 \right. \right. \\ &\quad \left. \left. + \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2 \vartheta_2^2} \right) \left(-\frac{1}{2} + \frac{1}{3} S_W^2 \right) \right]_L \right. \\ &\quad \left. + \left[-\frac{S_W^2}{24V^2 C_W^4} + \frac{1}{3} \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2 \vartheta_2^2} \right) S_W^2 \right]_R \right\} n_d \bar{\nu}_{\ell L} \gamma_0 \nu_{\ell L}, \end{aligned} \quad (4.46)$$

and the effective potential felt by neutrinos when crossing a medium composed by a density n_d of *down* quarks is $V_{\text{NC}}^d = V_{\text{NC}}^{dL} + V_{\text{NC}}^{dR}$, where

$$V_{\text{NC}}^{dL} \approx \left[\frac{(3S_W^2 - 2S_W^4)}{24V^2 C_W^4} + \frac{(9 - 4t^4)}{144t^4 V^2} T_W^4 + \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2 \vartheta_2^2} \right) \left(-\frac{1}{2} + \frac{1}{3} S_W^2 \right) \right] n_d, \quad (4.47)$$

$$V_{\text{NC}}^{dR} \approx \left[-\frac{S_W^2}{24V^2 C_W^4} + \frac{1}{3} \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2 \vartheta_2^2} \right) S_W^2 \right] n_d. \quad (4.48)$$

Then we can easily see that in SM the NC effective potential for neutrinos in a *d*-quark medium, using result of (4.7), will be given by

$$V_{\text{NC}}^d = V_{\text{NC}}^{dL} + V_{\text{NC}}^{dR} \approx -\left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2 \vartheta_2^2} \right) \left(\frac{1}{2} - \frac{2}{3} S_W^2 \right) n_d, \quad (4.49)$$

$$V_{\text{NC}}^{dL} = \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2 \vartheta_2^2} \right) \left(-\frac{1}{2} + \frac{1}{3} S_W^2 \right) n_d,$$

$$V_{\text{NC}}^{dR} = \frac{1}{3} \left(\frac{1}{2\vartheta_2^2} - \frac{\vartheta_1^2}{2V^2 \vartheta_2^2} \right) S_W^2 n_d. \quad (4.50)$$

Then from (4.47)–(4.50), we obtain

$$V_{\text{NC}}^{dL} \approx V_{\text{NC}}^{dL} + \left[\frac{(3S_W^2 - 2S_W^4)}{24V^2 C_W^4} + \frac{(9 - 4t^4)}{144t^4 V^2} T_W^4 \right] n_d, \quad (4.51)$$

$$V_{\text{NC}}^{dR} \approx V_{\text{NC}}^{dR} - \frac{S_W^2}{24V^2 C_W^4} n_d,$$

and neglecting terms of order $(\vartheta_i/V)^n$, for $n > 2$, we obtain that $\varepsilon_{\ell\ell}^d = \varepsilon_{\ell\ell}^{dL} + \varepsilon_{\ell\ell}^{dR}$, where

$$\varepsilon_{\ell\ell}^{dL} \approx \frac{\vartheta_2^2}{24V^2 C_W^4} (3 - 2S_W^2), \quad (4.52)$$

$$\varepsilon_{\ell\ell}^{dR} \approx -\frac{S_W^2 \vartheta_2^2}{12V^2 C_W^4}. \quad (4.53)$$

Then we obtain $\varepsilon_{\ell\ell}^{dL} \approx (\vartheta_2^2/8V^2 C_W^4) + \varepsilon_{\ell\ell}^{dR}$. Note that again in limit $V \rightarrow \infty$ we recover the SM.

5. Results

In last sections we saw that in 331 model we chose, all NSI parameters are universal and diagonal and will not affect oscillation experiments. However, measurements of cross-section will be sensitive to such parameters, through modifications on g_i^α [51]. We will now compare

Table 1: Values for NSI in 331 model and experimental limits taken of the strongest constraints on these parameters are given in [18, 19, 21, 22].

	331 Model	Exp. 90% C.L.
$\varepsilon_{\ell\ell}^{eL} \approx \frac{(1 - 2S_W^2)\vartheta_2^2}{8V^2 C_W^4}$	$0.114 \left(\frac{\vartheta_2^2}{V^2} \right)$	$-0.14 < \varepsilon_{ee}^{eL} < 0.09$ $-0.033 < \varepsilon_{\mu\mu}^{eL} < 0.055$
$\varepsilon_{\ell\ell}^{eR} \approx -2S_W^2 \varepsilon_{\ell\ell}^{eL} - \frac{\vartheta_2^2}{V^2} T_W^4$	$-0.143 \left(\frac{\vartheta_2^2}{V^2} \right)$	$-0.6 < \varepsilon_{\tau\tau}^{eL} < 0.4$ $-0.03 < \varepsilon_{ee}^{eR} < 0.18$ $-0.040 < \varepsilon_{\mu\mu}^{eR} < 0.053$
$\varepsilon_{\ell\ell}^{uL} \approx -\frac{\vartheta_1^2}{2V^2} + \frac{\vartheta_2^2}{24V^2 C_W^4} (9 - 8S_W^2)$	$0.50 \left(\frac{\vartheta_2^2 - \vartheta_1^2}{V^2} \right)$	$-0.4 < \varepsilon_{\tau\tau}^{eR} < 0.6$ $-1 < \varepsilon_{ee}^{uL} < 0.3$ $ \varepsilon_{\mu\mu}^{uL} < 0.003$
$\varepsilon_{\ell\ell}^{uR} \approx \frac{\vartheta_2^2}{6V^2} \frac{S_W^2}{C_W^4}$	$0.065 \left(\frac{\vartheta_2^2}{V^2} \right)$	$ \varepsilon_{\tau\tau}^{uL} < 1.4$ $-0.4 < \varepsilon_{ee}^{uR} < 0.7$ $-0.008 < \varepsilon_{\mu\mu}^{uR} < 0.003$
$\varepsilon_{\ell\ell}^{dL} \approx \frac{\vartheta_2^2}{24V^2 C_W^4} (3 - 2S_W^2)$	$0.179 \left(\frac{\vartheta_2^2}{V^2} \right)$	$ \varepsilon_{\tau\tau}^{uR} < 3$ $-0.3 < \varepsilon_{ee}^{dL} < 0.3$ $ \varepsilon_{\mu\mu}^{dL} < 0.003$
$\varepsilon_{\ell\ell}^{dR} \approx -\frac{S_W^2 \vartheta_2^2}{12V^2 C_W^4}$	$-0.033 \left(\frac{\vartheta_2^2}{V^2} \right)$	$ \varepsilon_{\tau\tau}^{dL} < 1.1$ $-0.6 < \varepsilon_{ee}^{dR} < 0.5$ $-0.008 < \varepsilon_{\mu\mu}^{dR} < 0.015$
		$ \varepsilon_{\tau\tau}^{dR} < 6$

our results with those obtained in cross-section measurements. We will assume $\sin^2\theta_W = 0.23149(13)$.

In Table 1 we can see that constrains in $\varepsilon_{\ell\ell}^{eP}$ lead to $V^2 > 4.7\vartheta_2^2$, while the constrains in $\varepsilon_{\ell\ell}^{uR}$ lead to $V^2 > 21.7\vartheta_2^2$, and the constrains in $\varepsilon_{\ell\ell}^{dP}$ ($|\varepsilon_{\mu\mu}^{dL}| < 0.003$) lead to $V^2 > 60\vartheta_2^2$. If ϑ_2 has its maximum value of 174.105 GeV, then $V \gtrsim 1.3$ TeV. We note also that by $|\varepsilon_{\mu\mu}^{uL}| < 0.003$ we obtain $|\vartheta_2^2 - \vartheta_1^2| < 0.006 V^2$; then, for $V \sim 1.3$ TeV and $\vartheta_2 = 174$ GeV, we obtain $142 \text{ GeV} < \vartheta_1 < 201 \text{ GeV}$. We therefore cannot predict any hierarchy to the VEV's ϑ_1 and ϑ_2 . Based on those results, we obtain the following inferior limits for the new gauge bosons masses:

$$\begin{aligned}
 M_{K_I} = M_{Z'} &> 610 \text{ GeV}, \\
 M_{K'} &> 613 \text{ GeV}, \\
 M_{K_R} &> 740 \text{ GeV}.
 \end{aligned}
 \tag{5.1}$$

6. Conclusion

We presented in this work a procedure to show that models with extended gauge symmetries $SU(3)_C \times SU(3)_L \times U(1)_X$ can lead to neutrino nonstandard interactions, respecting the Standard Model Gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$, without spoiling the available

experimental data and reproducing the known phenomenology at low energies. We also have shown that with an assumption about a mass hierarchy for the Higgs triplets VEV's we could qualitatively address the mass hierarchy problem in standard model. Finally we obtained limits for the triplets VEV's based on limits for NSI in cross-section experiments.

We believe that the class of model presented here is an interesting theoretical possibility to look for new physics beyond SM. We restrained our work to a simple scenario, but flavor-changing interactions can be naturally introduced in the model, leading to new constraints on NSI.

Acknowledgments

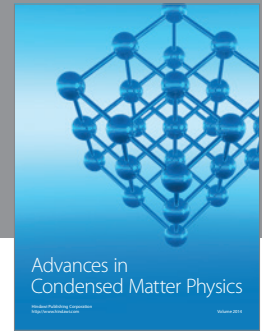
The authors would like to thank Alex Dias and Marcelo Guzzo for valuable discussions. One of them (M. Medina) would like to thank CNPq for financial support.

References

- [1] B. T. Cleveland, T. Daily, R. Davis et al., "Measurement of the solar electron neutrino flux with the homestake chlorine detector," *Astrophysical Journal*, vol. 496, no. 1, pp. 505–526, 1998.
- [2] K. S. Hirata, K. Inoue, T. Ishida et al., "Observation of a small atmospheric $\nu\mu/\nu e$ ratio in Kamiokande," *Physics Letters Section B*, vol. 280, no. 1-2, pp. 146–152, 1992.
- [3] S. Fukuda, Y. Fukuda, M. Ishitsuka et al., "Constraints on neutrino oscillations using 1258 days of super-kamiokande solar neutrino data," *Physical Review Letters*, vol. 86, no. 25, pp. 5656–5660, 2001.
- [4] Q. R. Ahmad, R. C. Allen, T. C. Andersen et al., "Direct evidence for neutrino flavor transformation from neutral-current interactions in the Sudbury Neutrino Observatory," *Physical Review Letters*, vol. 89, no. 1, Article ID 011301, pp. 1–6, 2002.
- [5] C. Arpesella, G. Bellini, J. Benziger et al., "First real time detection of ^7Be solar neutrinos by Borexino," *Physics Letters Section B*, vol. 658, no. 4, pp. 101–108, 2008.
- [6] R. Becker-Szendy, C. B. Bratton, D. Casper et al., "Electron- and muon-neutrino content of the atmospheric flux," *Physical Review D*, vol. 46, no. 9, pp. 3720–3724, 1992.
- [7] Y. Fukuda, T. Hayakawa, E. Ichihara et al., "Measurement of the flux and zenith-angle distribution of upward throughgoing muons by Super-Kamiokande," *Physical Review Letters*, vol. 82, no. 13, pp. 2644–2648, 1999.
- [8] S. Fukuda, Y. Fukuda, M. Ishitsuka et al., "Tau neutrinos favored over sterile neutrinos in atmospheric muon neutrino oscillations," *Physical Review Letters*, vol. 85, no. 19, pp. 3999–4003, 2000.
- [9] Y. Fukuda, T. Hayakawa, K. Inoue et al., "Atmospheric $\nu\mu/\nu e$ ratio in the multi-GeV energy range," *Physics Letters Section B*, vol. 335, no. 2, pp. 237–245, 1994.
- [10] W. W. M. Allison, G. J. Alner, D. S. Ayres et al., "Neutrino oscillation effects in Soudan 2 upward-stopping muons," *Physical Review D*, vol. 72, no. 5, Article ID 052005, pp. 1–12, 2005.
- [11] M. Ambrosio, R. Antolini, C. Aramo et al., "Measurement of the atmospheric neutrino-induced upgoing muon flux using MACRO," *Physics Letters Section B*, vol. 434, no. 3-4, pp. 451–457, 1998.
- [12] S. Abe, T. Ebihara, S. Enomoto et al., "Precision measurement of neutrino oscillation parameters with KamLAND," *Physical Review Letters*, vol. 100, no. 22, 2008.
- [13] M. Apollonio, A. Baldini, C. Bemporad et al., "Limits on neutrino oscillations from the CHOOZ experiment," *Physics Letters Section B*, vol. 466, no. 2-4, pp. 415–430, 1999.
- [14] M. Apollonio et al., "Search for neutrino oscillations on a long baseline at the CHOOZ nuclear power station," *The European Physical Journal C*, vol. 27, pp. 331–374, 2003.
- [15] E. Aliu, S. Andringa, S. Aoki et al., "Evidence for muon neutrino oscillation in an accelerator-based experiment," *Physical Review Letters*, vol. 94, no. 8, Article ID 081802, 2005.
- [16] P. Adamson, C. Andreopoulos, K. E. Arms et al., "Measurement of neutrino oscillations with the MINOS detectors in the NuMI beam," *Physical Review Letters*, vol. 101, no. 13, Article ID 131802, 2008.
- [17] Y. Abe et al., "Indication for the disappearance of reactor electron antineutrinos in the Double Chooz experiment," In press, <http://arxiv.org/abs/1112.6353>.

- [18] S. Davidson, C. Peña-Garay, N. Rius, and A. Santamaria, "Present and future bounds on non-standard neutrino interactions," *Journal of High Energy Physics*, vol. 7, no. 3, pp. 225–257, 2003.
- [19] J. Barranco, O. G. Miranda, C. A. Moura, and J. W. F. Valle, "Constraining nonstandard neutrino-electron interactions," *Physical Review D*, vol. 77, no. 9, Article ID 093014, 2008.
- [20] F. J. Escrihuela, M. A. Tórtola, O. G. Miranda, and J. W. F. Valle, "Global constraints on muon-neutrino non-standard interactions," *Physical Review D*, vol. 83, Article ID 093002, 2011.
- [21] J. Barranco, O. G. Miranda, C. A. Moura, and J. W. F. Valle, "Constraining nonstandard interactions in $\nu e e$ or $\nu^- e e$ scattering," *Physical Review D*, vol. 73, no. 11, Article ID 113001, 2006.
- [22] F. J. Escrihuela, M. A. Tórtola, O. G. Miranda, and J. W. F. Valle, "Constraining nonstandard neutrino-quark interactions with solar, reactor, and accelerator data," *Physical Review D*, vol. 80, no. 10, Article ID 105009, 2009.
- [23] M. B. Gavela, D. Hernandez, T. Ota, and W. Winter, "Large gauge invariant nonstandard neutrino interactions," *Physical Review D*, vol. 79, no. 1, Article ID 013007, 2009.
- [24] S. Antusch, J. P. Baumann, and E. Fernández-Martínez, "Non-standard neutrino interactions with matter from physics beyond the Standard Model," *Nuclear Physics B*, vol. 810, no. 1-2, pp. 369–388, 2009.
- [25] J. Schechter and J. W. F. Valle, "Neutrino masses in SU(2) U(1) theories," *Physical Review D*, vol. 22, no. 9, pp. 2227–2235, 1980.
- [26] L. J. Hall, V. A. Kostelecky, and S. Raby, "New flavor violations in supergravity models," *Nuclear Physics, Section B*, vol. 267, no. 2, pp. 415–432, 1986.
- [27] J. W. F. Valle, "Resonant oscillations of massless neutrinos in matter," *Physics Letters B*, vol. 199, no. 3, pp. 432–436, 1987.
- [28] Y. Grossman, "Non-standard neutrino interactions and neutrino oscillation experiments," *Physics Letters Section B*, vol. 359, no. 1-2, pp. 141–147, 1995.
- [29] S. Bergmann, M. M. Guzzo, P. C. De Holanda, P. I. Krastev, and H. Nunokawa, "Status of the solution to the solar neutrino problem based on nonstandard neutrino interactions," *Physical Review D*, vol. 62, no. 7, Article ID 073001, pp. 1–16, 2000.
- [30] R. N. Mohapatra, S. Antusch, K. S. Babu et al., "Theory of neutrinos: a white paper," *Reports on Progress in Physics*, vol. 70, no. 11, pp. 1757–1867, 2007.
- [31] J. C. Pati and A. Salam, "Lepton number as the fourth "color"," *Physical Review D*, vol. 10, no. 1, pp. 275–289, 1974.
- [32] H. Georgi, "Towards a grand unified theory of flavor," *Nuclear Physics, Section B*, vol. 156, no. 1, pp. 126–134, 1979.
- [33] F. Wilczek and A. Zee, "Families from spinors," *Physical Review D*, vol. 25, no. 2, pp. 553–565, 1982.
- [34] A. H. Galeana, R. E. Martínez, W. A. Ponce, and A. Zepeda, "Unification of forces and flavors for three families," *Physical Review D*, vol. 44, no. 7, pp. 2166–2178, 1991.
- [35] S. L. Adler, "Axial-vector vertex in spinor electrodynamics," *Physical Review*, vol. 177, no. 5, pp. 2426–2438, 1969.
- [36] H. Georgi and S. L. Glashow, "Gauge theories without anomalies," *Physical Review D*, vol. 6, no. 2, pp. 429–431, 1972.
- [37] S. Okubo, "Gauge groups without triangular anomaly," *Physical Review D*, vol. 16, no. 12, pp. 3528–3534, 1977.
- [38] F. Pisano and V. Pleitez, "SU(3)U(1) model for electroweak interactions," *Physical Review D*, vol. 46, no. 1, pp. 410–417, 1992.
- [39] R. Foot, O. F. Hernández, F. Pisano, and V. Pleitez, "Lepton masses in an SU(3)L-U(1)N gauge model," *Physical Review D*, vol. 47, no. 9, pp. 4158–4161, 1993.
- [40] P. H. Frampton, "Chiral dilepton model and the flavor question," *Physical Review Letters*, vol. 69, no. 20, pp. 2889–2891, 1992.
- [41] W. A. Ponce, Y. Giraldo, and L. A. Sánchez, "Minimal scalar sector of 3-3-1 models without exotic electric charges," *Physical Review D*, vol. 67, no. 7, Article ID 075001, 2003.
- [42] R. A. Diaz, R. Martínez, and F. Ochoa, "Scalar sector of the SU(3) \times SU(3)L \times U(1) X model," *Physical Review D*, vol. 69, no. 9, Article ID 095009, 2004.
- [43] P. V. Dong, H. N. Long, D. T. Nhung, and D. V. Soa, "SU(3)C SU(3)L U(1)X model with two Higgs triplets," *Physical Review D*, vol. 73, no. 3, Article ID 035004, pp. 1–11, 2006.
- [44] P. V. Dong and H. N. Long, "Higgs-gauge boson interactions in the economical 3-3-1 model," *Physical Review D*, vol. 73, Article ID 075005, 15 pages, 2006.
- [45] R. Martínez, W. A. Ponce, and L. A. Sánchez, "SU(3)c \times SU(3)L \times U(1)X as an SU(6) \times U(1)X subgroup," *Physical Review D*, vol. 65, Article ID 055013, 11 pages, 2002.

- [46] W. A. Ponce, J. B. Flórez, and L. A. Sánchez, "Analysis of $SU(3)_c \otimes SU(3)_l \otimes U(1)_X$ local gauge theory," *International Journal of Modern Physics A*, vol. 17, no. 5, pp. 643–659, 2002.
- [47] R. N. Mohapatra, S. Antusch, K. S. Babu et al., "Theory of neutrinos: a white paper," *Reports on Progress in Physics*, vol. 70, no. 11, pp. 1757–1867, 2007.
- [48] N. Fornengo, M. Maltoni, R. T. Bayo, and J. W.F. Valle, "Probing neutrino nonstandard interactions with atmospheric neutrino data," *Physical Review D*, vol. 65, no. 1, Article ID 013010, 2002.
- [49] M. Blennow, D. Meloni, T. Ohlsson, F. Terranova, and M. Westerberg, "Non-standard interactions using the OPERA experiment," *European Physical Journal C*, vol. 56, no. 4, pp. 529–536, 2008.
- [50] D. Y. Bardin, S. M. Bilenky, and B. Pontecorvo, "On the $\nu_e + e \rightarrow \nu_e + e$ process," *Physics Letters B*, vol. 32, no. 1, pp. 68–70, 1970.
- [51] A. Bolaños, O. G. Miranda, A. Palazzo, M. A. Tórtola, and J. W.F. Valle, "Probing nonstandard neutrino-electron interactions with solar and reactor neutrinos," *Physical Review D*, vol. 79, no. 11, Article ID 113012, 2009.
- [52] T. P. Cheng, *Gauge Theory of Elementary Particle Physics*, Oxford University Press, New York, NY, USA, 1986.
- [53] S. L. Alder, "Axial-vector vertex in spinor electrodynamics," *Physical Review*, vol. 177, pp. 2426–2438, 1969.
- [54] H. Georgi, *Lie Algebras in Particles Physics*, Benjamin, 1982.
- [55] H. Georgi, *Lie Algebras in Particle Physics*, vol. 54, Benjamin/Cummings Publishing Co, Reading, Mass, USA, 1982.
- [56] A. G. Dias, tese de Doutorado, IFUSP, 2005.
- [57] H. D. Politzer, "Reliable perturbative results for strong interactions?" *Physical Review Letters*, vol. 30, no. 26, pp. 1346–1349, 1973.
- [58] D. J. Gross and F. Wilczek, "Ultraviolet behavior of non-abelian gauge theories," *Physical Review Letters*, vol. 30, no. 26, pp. 1343–1346, 1973.
- [59] D. Van Soa, T. Inami, and H. N. Long, "Bilepton production in $e^- \gamma$ collisions," *European Physical Journal C*, vol. 34, no. 3, pp. 285–289, 2004.
- [60] L. Wolfenstein, "Neutrino oscillations in matter," *Physical Review D*, vol. 17, no. 9, pp. 2369–2374, 1978.
- [61] P. B. Pal, "Particle physics confronts the solar neutrino problem," *International Journal of Modern Physics A*, vol. 7, p. 5387, 1992.
- [62] F. Mandl and G. Shaw, *Quantum Field Theory*, Wiley and Sons, New York, NY, USA, 1984.
- [63] S. Pakvasa and J. W. F. Valle, "Neutrino properties before and after KamLAND," *Proceedings of the Indian National Science Academy A*, vol. 70, pp. 189–222, 2004.



Hindawi

Submit your manuscripts at
<http://www.hindawi.com>

