

## Research Article

# Hubble Parameter Corrected Interactions in Cosmology

J. Sadeghi,<sup>1</sup> M. Khurshudyan,<sup>2</sup> M. Hakobyan,<sup>3,4</sup> and H. Farahani<sup>1</sup>

<sup>1</sup> Department of Physics, Mazandaran University, P.O. Box 47416-95447, Babolsar, Iran

<sup>2</sup> Department of Theoretical Physics, Yerevan State University, 1 Alex Manoogian, 0025 Yerevan, Armenia

<sup>3</sup> A.I. Alikhanyan National Science Laboratory, Alikhanian Brothers Street, Yerevan, Armenia

<sup>4</sup> Department of Nuclear Physics, Yerevan State University, Yerevan, Armenia

Correspondence should be addressed to H. Farahani; [h.farahani@umz.ac.ir](mailto:h.farahani@umz.ac.ir)

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We make steps in a new direction by considering fluids with EoS of more general form  $F(\rho, P) = 0$ . It is thought that there should be interaction between cosmic fluids, but this assumption for this stage carries only phenomenological character opening a room for different kinds of manipulations. In this paper we will consider a modification of an interaction  $Q$ , where we accept that interaction parameter  $b_1$  (order of unity) in  $Q = 3Hb_1\rho$  is time dependent and presented as a linear function of Hubble parameter  $H$  of the form  $b_0 + bH$ , where  $b$  and  $b_0$  are constants. We consider two different models including modified Chaplygin gas and polytropic gas which have bulk viscosity. Then, we investigate problem numerically and analyze behavior of different cosmological parameters concerning fluids and behavior of the universe.

## 1. Introduction

Experimental data interpretation claims that we have accelerated expansion for our universe. However this phenomenon can be understood as a theoretical model based consequence. In general relativity concepts of dark energy and dark matter were introduced by hand and it seems that they deal with the problem at intermediate level, because the considered number of models and articles is going to be behind reasonable limit. However, still the questions concerning the nature of dark energy and dark matter, about possible interactions and so forth, are open. Dark energy thought to be responsible to accelerate expansion. On theoretical and phenomenological levels scalar fields were considered as thought that scalar field can be a base of dark energy. One of them is a tachyonic scalar field. Concerning some fundamental problems, dynamical models of dark energy were proposed and considered from different corners. However, it is not the unique approach and the geometrical part of gravitational action was modified.

A set of observational data reveal, from the following picture of our universe which is called modern era in theoretical cosmology, that an expansion of our universe is accelerated [1–3]. Then, the density of matter is very much

less than critical density [4], the universe is flat, and the total energy density is very close to the critical [5]. Explanation of accelerated expansion of our universe takes two different ways and now they are developing and evaluating as different approaches; however there is not any natural restriction on possibilities of recombination of two approaches in one single approach. In that case we believe that joined approach will be more sufficient and rich with new and interesting physics. To explain recent observational data, which reveals accelerated expansion character of the universe, several models were proposed. One of the possible scenarios (general relativity framework) is the existence of a dark energy (73% of the energy of our universe) with negative pressure and positive energy density giving acceleration to the expansion [6, 7]. Concerning other components, dark matter occupies about 23% and usual baryonic matter occupies about 4%. Among different viewpoints concerning the nature of the dark component of the universe, we would like to mention the scalar field models; one of them is tachyonic field with its relativistic Lagrangian

$$L_{\text{TF}} = -V(\phi) \sqrt{1 - \partial_\mu \phi \partial^\mu \phi}, \quad (1)$$

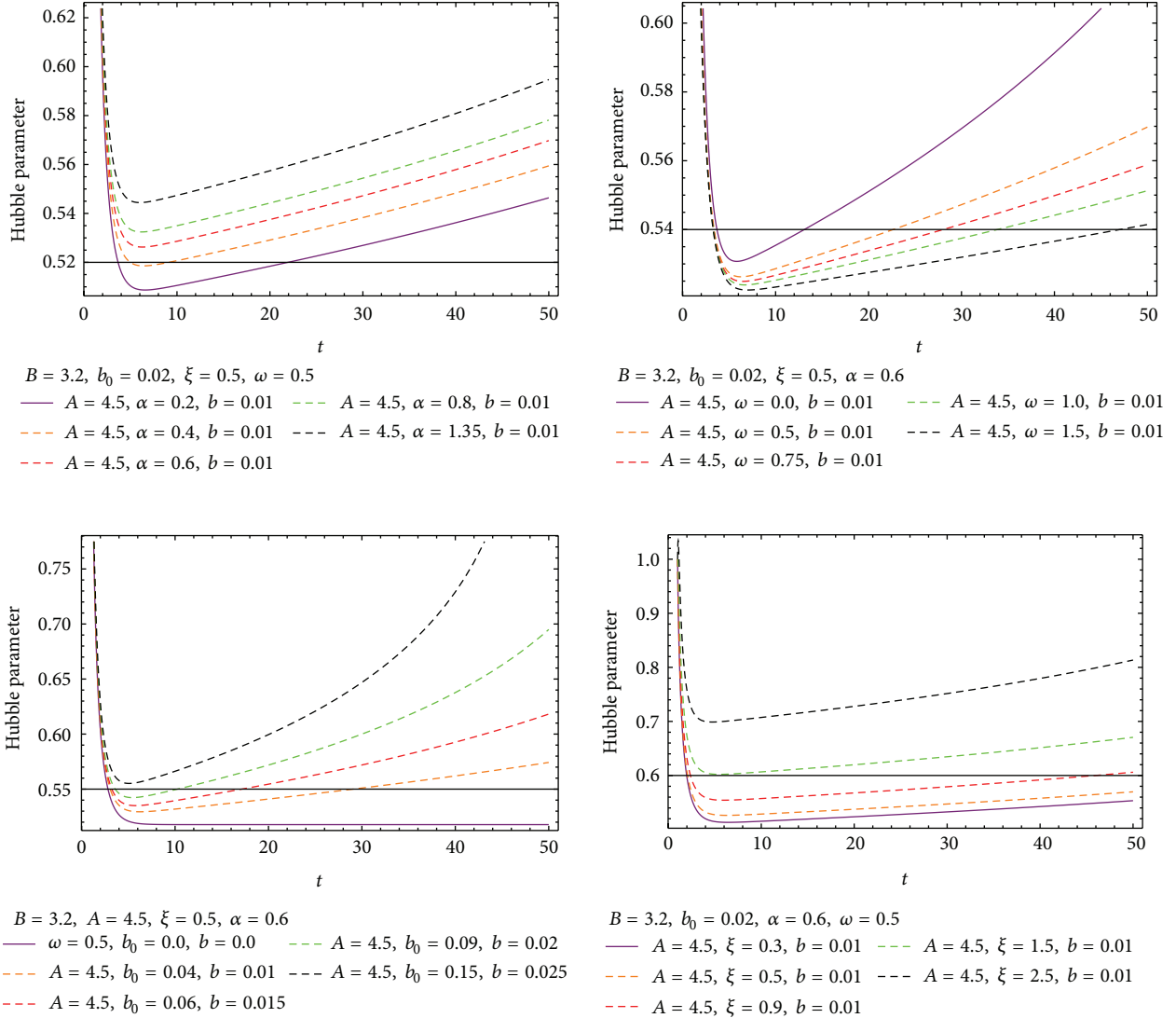


FIGURE 1: Behavior of Hubble parameter  $H$  against  $t$  for interacting barotropic fluid and viscous modified Chaplygin gas.

which captured a lot of attention (see, for instance, [8]). The stress energy tensor,

$$T^{ij} = \frac{\partial L}{\partial (\partial_i \phi)} \partial^j \phi - g^{ij} L, \quad (2)$$

gives the energy density and pressure as

$$\rho = \frac{V(\phi)}{\sqrt{1 - \partial_i \phi \partial^i \phi}}, \quad (3)$$

$$P = -V(\phi) \sqrt{1 - \partial_i \phi \partial^i \phi}.$$

A quintessence field [9] is another model based on scalar field with standard kinetic term, which is minimally coupled to gravity. In that case the action has a wrong sign kinetic term and the scalar field is called phantom [10]. Combination of the quintessence and the phantom is known as the quintom

model [11]. Extension of kinetic term in Lagrangian yields to a more general framework on field theoretic dark energy, which is called k-essence [12, 13]. A singular limit of k-essence is called cuscuton model [14]. This model has an infinite propagating speed for linear perturbations; however, causality is still valid. The most general form for a scalar field with second-order equation of motion is the galileon field which also could behave as dark energy [15].

Dark energy models based on idea of fluid are not less popular and are well studied. Fluids in cosmology are convenient, because, as practice teaches us, we can, for instance, different modifications in geometrical part of action encode in fluid part of field equations, giving illusion that in nature fluids with general form of EoS could be considered like to Chaplygin gas and its generalizations [16–22]. There are several models to describe dark energy such as the cosmological constant and its generalizations [23]. Among various models of dark energy, a new model of dark energy called Veneziano ghost dark (GD) energy, which is supposed

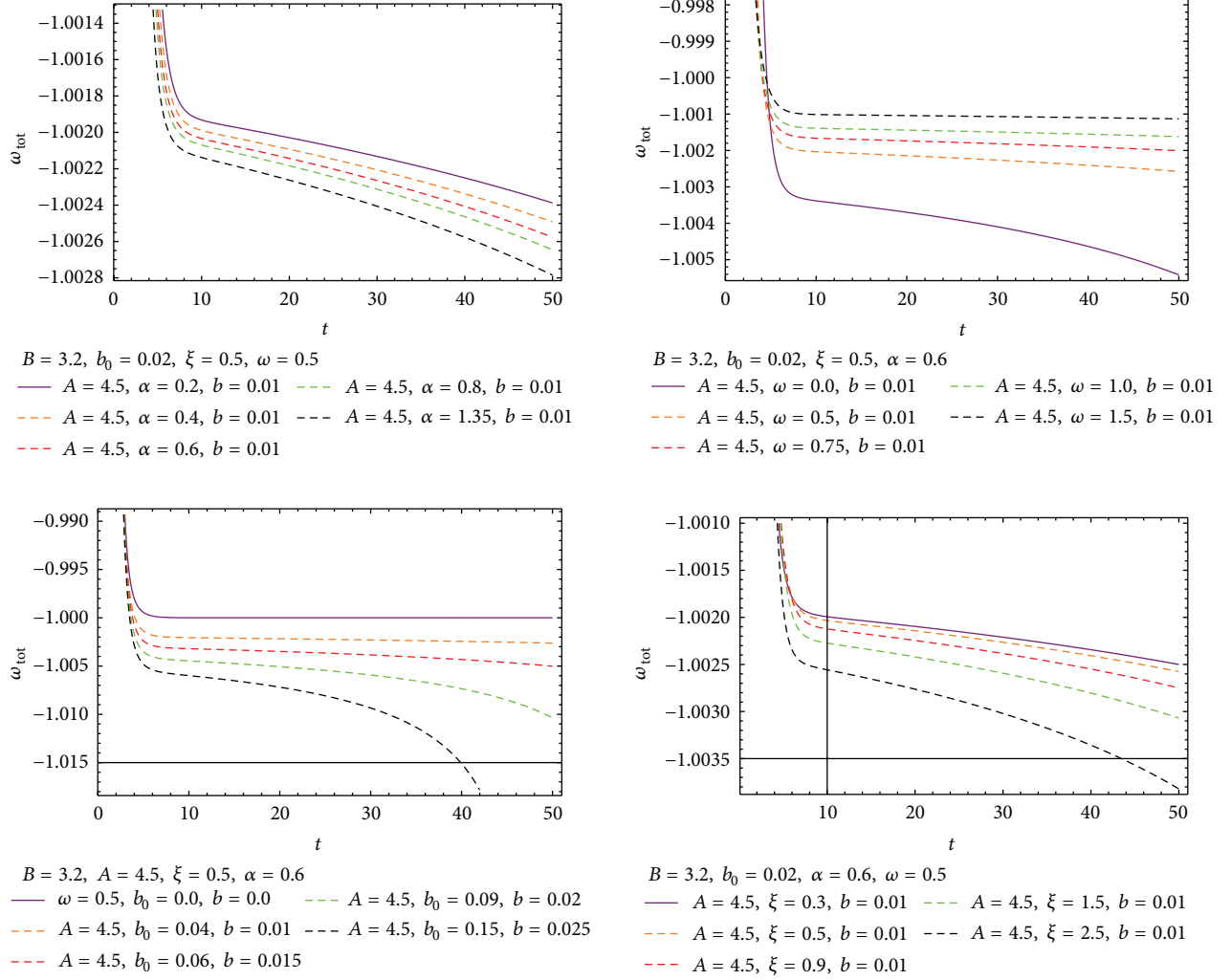


FIGURE 2: Behavior of EoS parameter  $\omega_{\text{tot}}$  against  $t$  for interacting barotropic fluid and viscous modified Chaplygin gas.

to exist to solve the  $U(1)_A$  problem in low-energy effective theory of QCD and has attracted a lot of interests in recent years [24–36]. Indeed, the contribution of the ghosts field to the vacuum energy in curved space or time-dependent background can be regarded as a possible candidate for the dark energy. It is completely decoupled from the physics sector. Veneziano ghost is unphysical in the QFT formulation in Minkowski space-time but exhibits important nontrivial physical effects in the expanding universe. It is hard to accept such linear behavior and it is thought that there should be some exponentially small corrections. However, it can be argued that the form of this behavior can be the result of the fact of the very complicated topological structure of strongly coupled QCD. This model has advantage compared to other models of dark energy, which can be explained by standard model and general relativity. Comparison with experimental data reveals that the current data is not favorite compared to the  $\Lambda$ CDM model, which is not conclusive, and future study of the problem is needed. Energy density of ghost dark energy may be read as

$$\rho_{\text{GD}} = \theta H, \quad (4)$$

where  $H$  is the Hubble parameter and  $\theta$  is the constant parameter of the model, which should be determined. The relation (4) is generalized by [37] as follows:

$$\rho_{\text{GD}} = \theta H + \vartheta H^2, \quad (5)$$

where  $\theta$  and  $\vartheta$  are the constant parameters of the model. Such kind of fluids could be named as a geometrical fluid, because it is clear that it contains information about geometry of the space-time and metric. Recently a model of varying ghost dark energy was proposed in [38] and extended to the case of interaction with variables  $\Lambda$  and  $G$  [28]. Unfortunately, the pure models are based on the energy density (4) and (5) may be ruled out. This has been recently shown in detail in [39] from the point of view of cosmic perturbations, but it was already indicated in the previous works [40, 41]. It brings us to consider some corrections such as viscosity and interaction to obtain valid model. Moreover, there is another problem with dark energy models of the forms (4) and (5) which is they do not have a  $\Lambda$ CDM limit. The absence of an additive term in the structure of the dark energy is highly problematic as the aforementioned works show. Irrespective

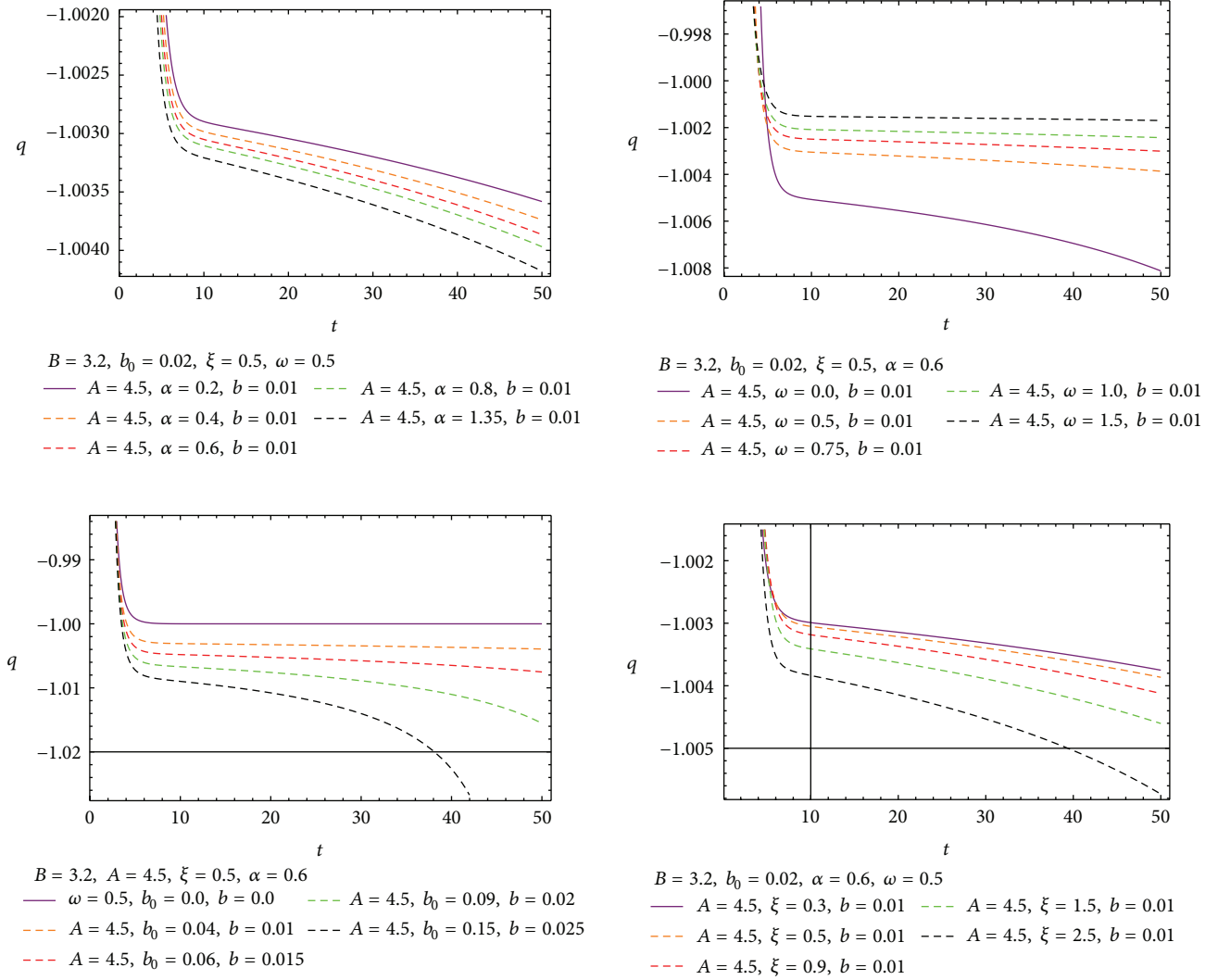


FIGURE 3: Behavior of deceleration parameter  $q$  against  $t$  for interacting barotropic fluid and viscous modified Chaplygin gas.

of the theoretical motivations for these models, the bare truth is that they are phenomenologically excluded. But there is also a fundamental motivation raising theoretical doubts on these models. The existence of linear terms in the Hubble rate is incompatible with the general covariance of the effective action of QFT in curved space-time. This is mentioned also in [39–41], but it is discussed in more detail in [42, 43], where it is also shown how to correctly generalize these models for the physics of the early universe by including only even the powers of the Hubble rate. However, other corrections like interaction term and viscosity may resolve above problems.

In this paper we would like to propose a modification in the interaction term  $Q = 3Hb_1\rho$  which by the general idea should exist between cosmic components. We will assume the interaction term as follows:

$$Q = 3H(b_0 + btH)\rho, \quad (6)$$

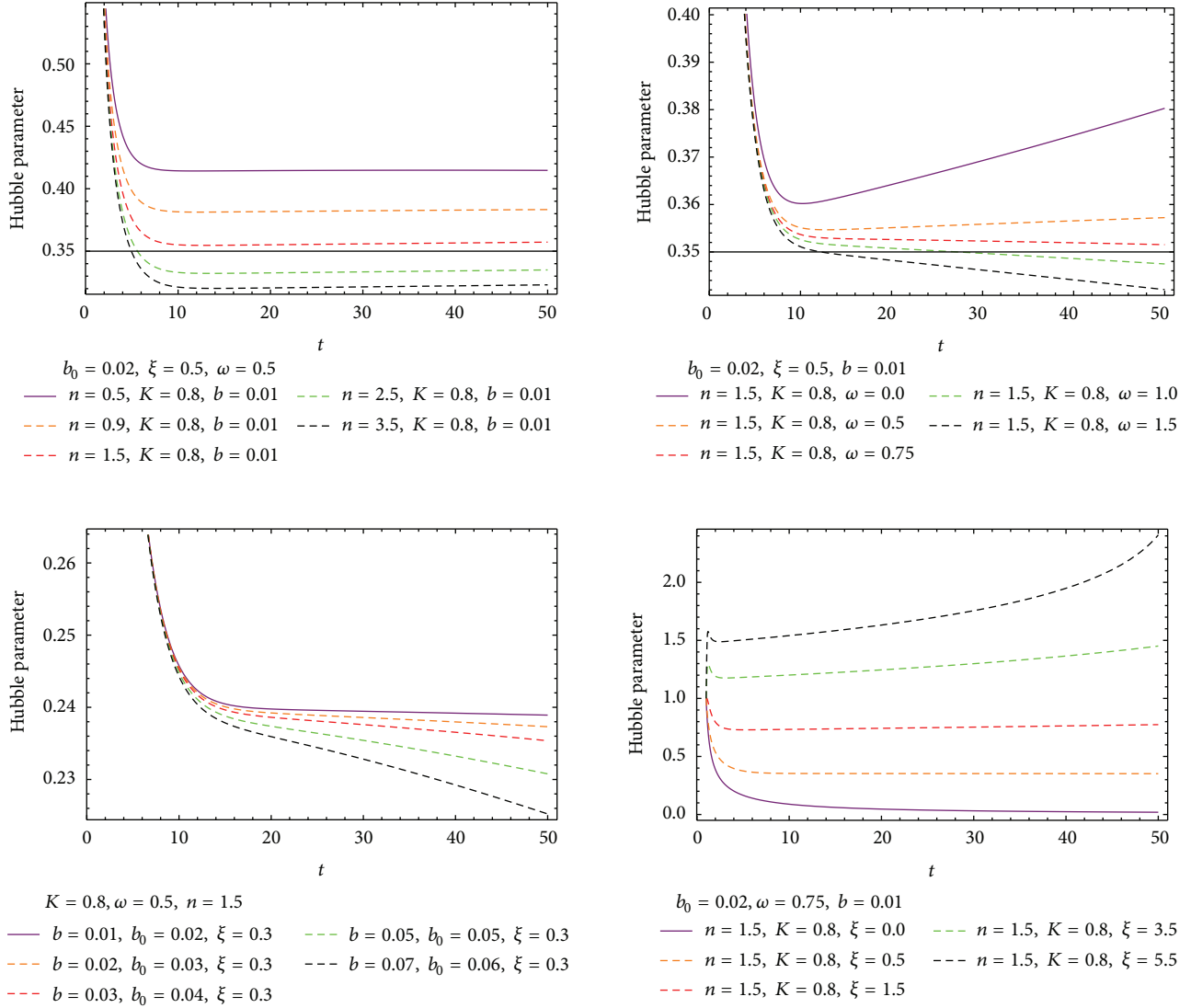
where  $b_0$  and  $b$  are constants. This assumption will bring us to the possibility that  $b_1$  is a function of time. Such assumption

already was considered in [44], while in [38] interacting varying ghost DE models was considered with time-dependent interaction term. Assumption was that  $b(t) = a(t)^\xi$ .

Due to the lack of information about dark energy and dark matter, usually the interaction terms are assumed to be proportional to the energy density, scale factor, Hubble parameter, and their derivative. In [44] the general time-dependent interactions are considered which prove that even very simple forms can alleviate the coincidence problem and lend the cosmic acceleration a transient character. This makes a good motivation to consider time-dependent interaction of the form (6).

Before to main formulation of our problem we would like to pay our attention to the question of interaction in cosmology between fluid components. Usually, three forms of  $Q$  are used:

$$\begin{aligned} Q &= 3Hb_1\rho_{de}, \\ Q &= 3Hb_1(\rho_{de} + \rho_{dm}), \\ Q &= 3Hb_1\rho_{dm}, \end{aligned} \quad (7)$$

FIGURE 4: Behavior of Hubble parameter  $H$  against  $t$  for interacting barotropic fluid and viscous polytropic gas.

where  $b_1$  is a coupling constant. From the thermodynamical view, it is argued that the second law of thermodynamics strongly favors that dark energy decays into dark matter, which implies that  $b$  is positive. These types of interactions are either positive or negative and cannot change sign. However, recently by using a model independent method to deal with the observational data, Cai and Su found that the sign of interaction  $Q$  in the dark sector changed in the redshift range of  $0.45 \leq z \leq 0.9$ . Hereafter, a sign-changeable interaction [45–48] was introduced as follows:

$$Q = q(\alpha\dot{\rho} + 3\beta H\rho), \quad (8)$$

where  $\alpha$  and  $\beta$  are dimensionless constants and the energy density  $\rho$  could be  $\rho_{\text{dm}}$ ,  $\rho_{\text{de}}$ , and  $\rho_{\text{tot}}$ .  $q$  is the deceleration parameter given by

$$q = -\frac{1}{H^2} \frac{\ddot{a}}{a} = -1 - \frac{\dot{H}}{H^2}. \quad (9)$$

This new type of interaction, where deceleration parameter  $q$  is a key ingredient, makes this type of interactions different from the ones considered in the literature and presented above, because it can change its sign when our universe changes from deceleration  $q > 0$  to acceleration  $q < 0$ .  $\gamma\dot{\rho}$  is introduced from the dimensional point of view. We would like also to stress a fact that by this way we import more information about the geometry of the universe into the interaction term. This fact means that we should consider more general forms for the interaction term. It is obvious that this splitting (as a mathematical act) can be done for any fluid with any number of components making a linear combination of pressure and energy density. From equations we see that unit of interaction  $Q$  should be  $\text{time}^{-1} \times \text{energy density}$ . Other types of interaction are of the form  $Q = \gamma\dot{\rho}$ , where for  $\rho$  we can say the same as in the previous case. Question of  $\text{time}^{-1}$  here was solved by taking derivative of energy density instead of using

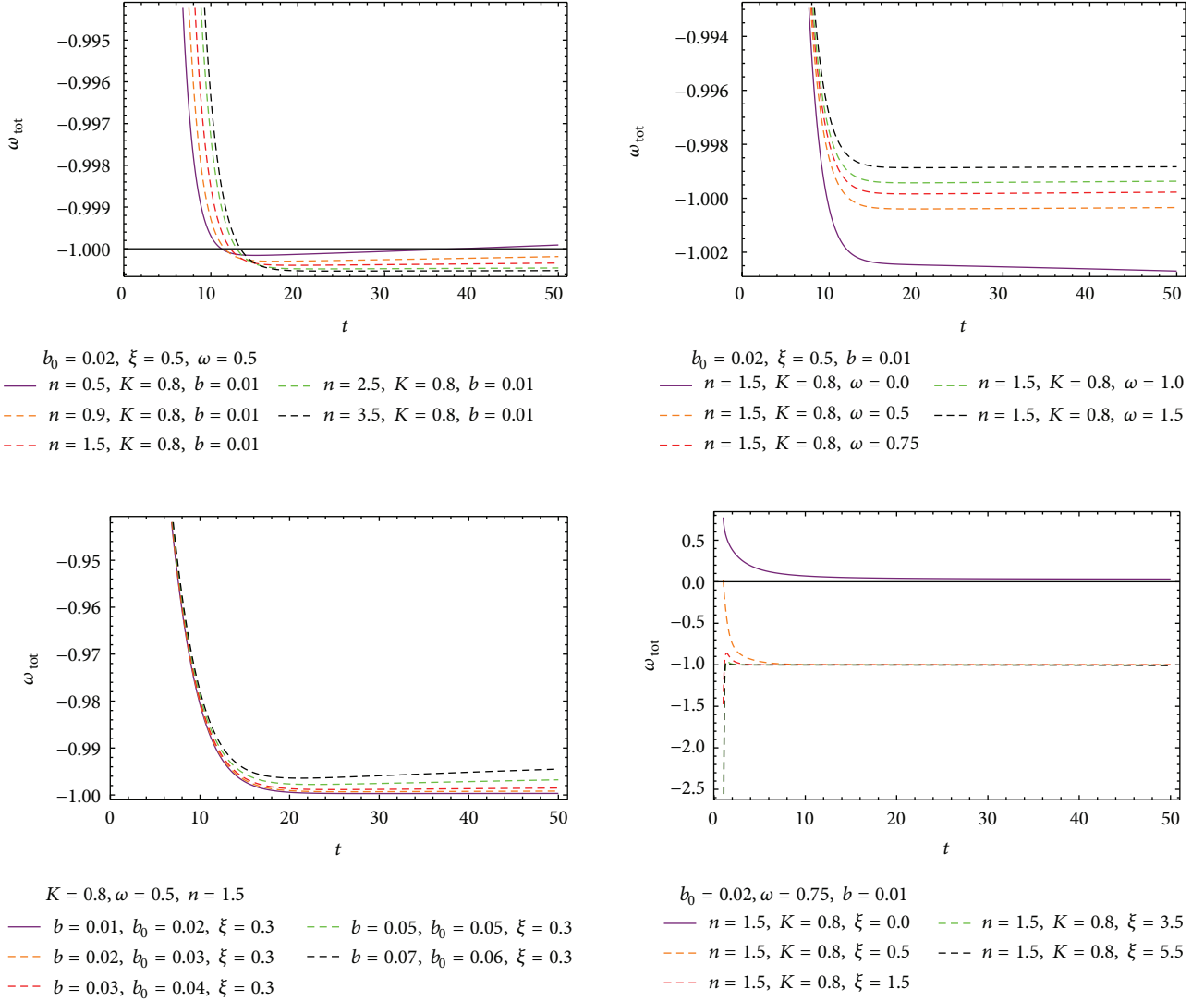


FIGURE 5: Behavior of EoS parameter  $\omega_{\text{tot}}$  against  $t$  for interacting barotropic fluid and viscous polytropic gas.

the Hubble parameter with time<sup>-1</sup> unit. The combination of these two types of interactions also was considered. In the framework of general relativity it is accepted that a dark energy can explain the present cosmic acceleration. Except cosmological constant there are many others candidates of dark energy. The property of dark energy is model dependent and to differentiate between different models of dark energy a sensitive diagnostic tool is needed. Hubble parameter  $H$  and deceleration parameter  $q$  are very important quantities which can describe the geometric properties of the universe. Since  $\dot{a} > 0$ , hence  $H > 0$  means the expansion of the universe. Also,  $\ddot{a} > 0$ , where  $q < 0$  indicates the accelerated expansion of the universe. Since, the various dark energy models give  $H > 0$  and  $q < 0$ , they cannot provide enough evidence to differentiate between the more accurate cosmological observational data and the more general models of dark energy. For this aim we need higher order of time derivative of scale factor and geometrical tool. Sahni et al. [49] proposed geometrical statefinder diagnostic tool, based

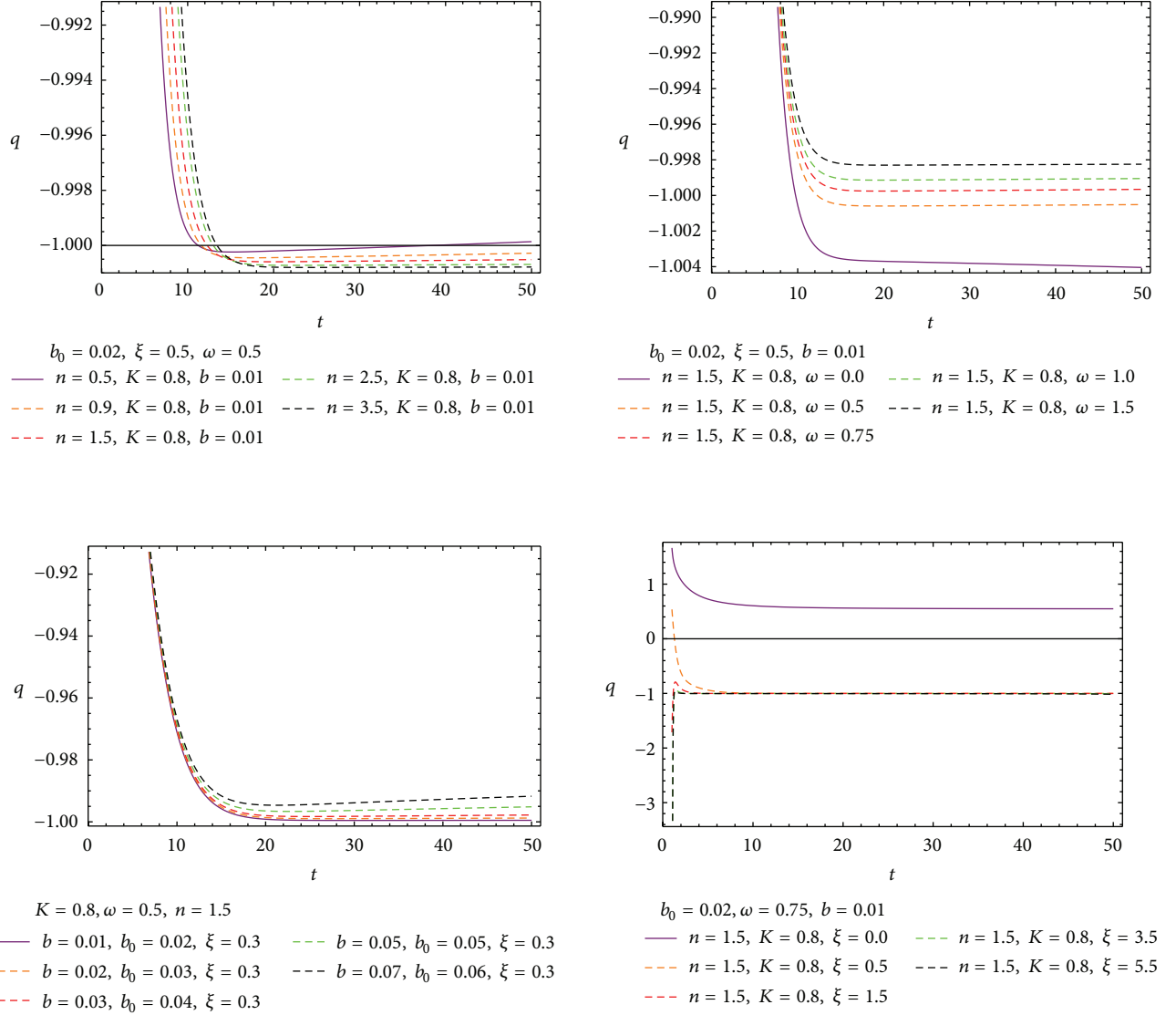
on dimensionless parameters  $(r, s)$  which are function of scale factor and its time derivative. These parameters are defined as

$$r = \frac{1}{H^3} \frac{\ddot{a}}{a}, \quad s = \frac{r - 1}{3(q - 1/2)}. \quad (10)$$

In stellar astrophysics, the polytropic gas model can explain the equation of state of degenerate white dwarfs, neutron stars, and also the equation of state of main sequence stars [50]. The idea of dark energy with polytropic gas equation of state has been investigated by Mukhopadhyay et al. in cosmology [51]. In addition to statefinder diagnostic tool, we used another analysis to discriminate between dark energy models which is  $\omega - \omega'$  analysis that has been used widely in the papers [52–64]. Subject of our interest is to consider two different models and study cosmological parameters.

As we know the viscous cosmology is an important theory to describe the evolution of the universe. It means that the presence of viscosity in the fluid introduces many interesting



FIGURE 6: Behavior of deceleration parameter  $q$  against  $t$  for interacting barotropic fluid and viscous polytropic gas.

pictures in the dynamics of homogeneous cosmological models, which is used to study the evolution of universe.

We consider that the composed models of a fluid consist of barotropic fluid  $P = \omega\rho$  coupled with

(1) viscous modified Chaplygin gas

$$P_{\text{VCG}} = A\rho_{\text{CG}} - \frac{B}{\rho_{\text{CG}}^\alpha} - 3\xi H \quad (11)$$

(2) and viscous polytropic gas

$$P_{\text{VPG}} = K\rho_{\text{PG}}^{1+(1/n)} - 3\xi H, \quad (12)$$

where  $K$  and  $n$  are the polytropic constant and polytropic index, respectively. The polytropic gas is a phenomenological model of dark energy. The polytropic gas model has a type III, where the singularity takes place at a characteristic scale factor  $a_s$ . Karami et al. investigated the interaction between dark energy and dark matter in polytropic gas scenario,

the phantom behavior of polytropic gas, reconstruction of  $f(T)$  gravity from the polytropic gas, and the correspondence between polytropic gas and agegraphic dark energy model [65–67]. The cosmological implications of polytropic gas dark energy model are also discussed in [68]. The evolution of deceleration parameter in the context of polytropic gas dark energy model represents the decelerated expansion at the early universe and accelerated phase later as expected. The polytropic gas model has also been studied from the viewpoint of statefinder analysis in [69].

There are several theoretical models to describe dark energy. Among them the model based on Chaplygin gas EoS and its extensions is interesting because of the possibility of dynamical analysis and solving some famous problems in cosmological constant model. Therefore, in order to construct a real model of our universe, we consider the modified Chaplygin (or polytropic) gas-like dark energy including viscosity and time-dependent interaction between components.

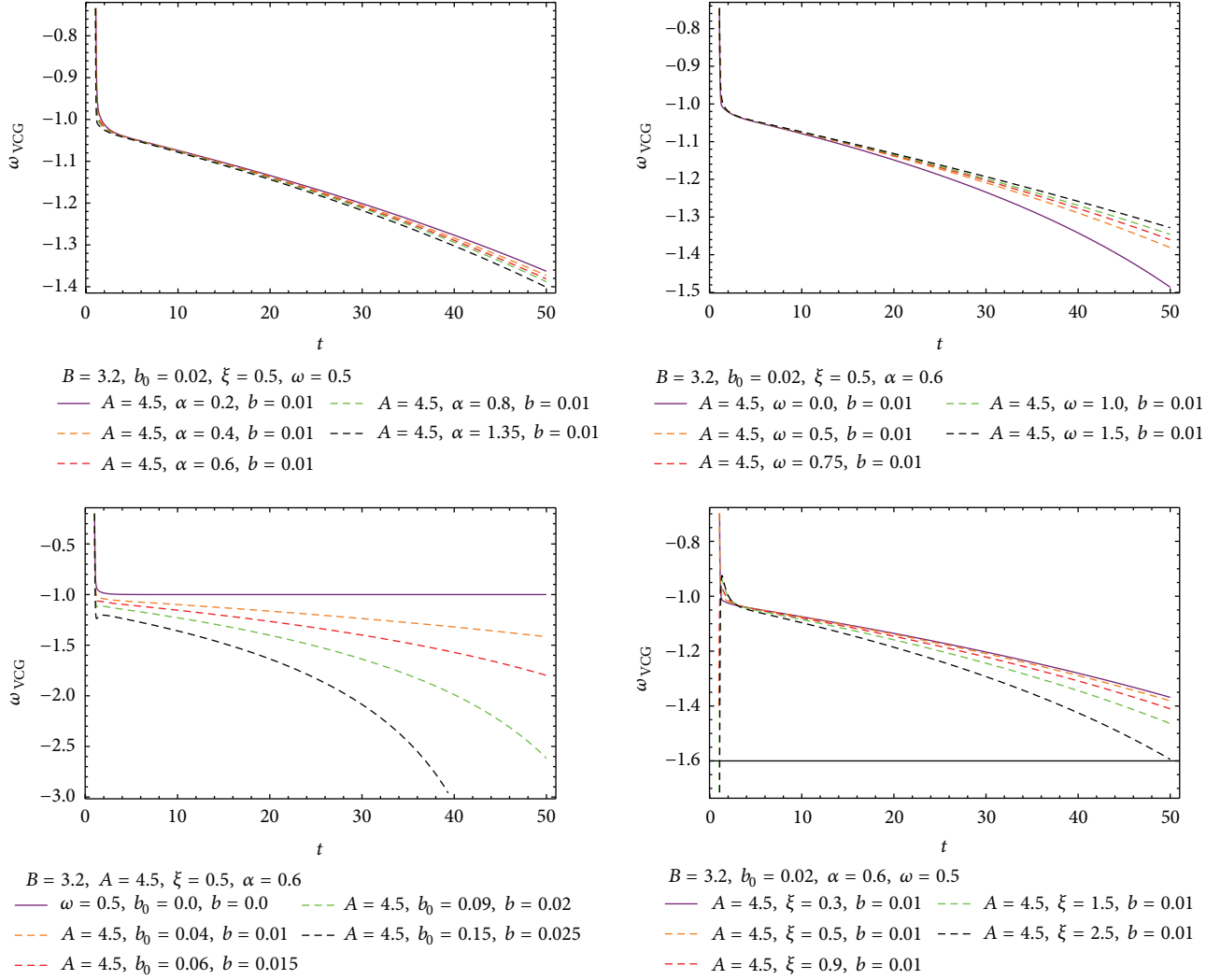


FIGURE 7: Behavior of EoS parameter of viscous Chaplygin  $\omega_{\text{VCG}}$  against  $t$  for interacting barotropic fluid and viscous Chaplygin gas.

Above points are strong theoretical motivation to consider a toy model of our universe which needs observational data for confirmation or rejection.

This paper is organized as follows. In the next section we will introduce the equations which govern our model. Then, we give numerical results corresponding to both models. In the discussion section we summarize our results. In Appendices A and B we analyze more quantities of both models.

## 2. The Field Equations and Models

The field equations that govern our model of consideration are

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R_{\alpha}^{\alpha} = T^{\mu\nu}. \quad (13)$$

By using the following FRW metric for a flat universe

$$ds^2 = -dt^2 + a(t)^2 (dr^2 + r^2 d\Omega^2), \quad (14)$$

field equations can be reduced to the following Friedmann equations:

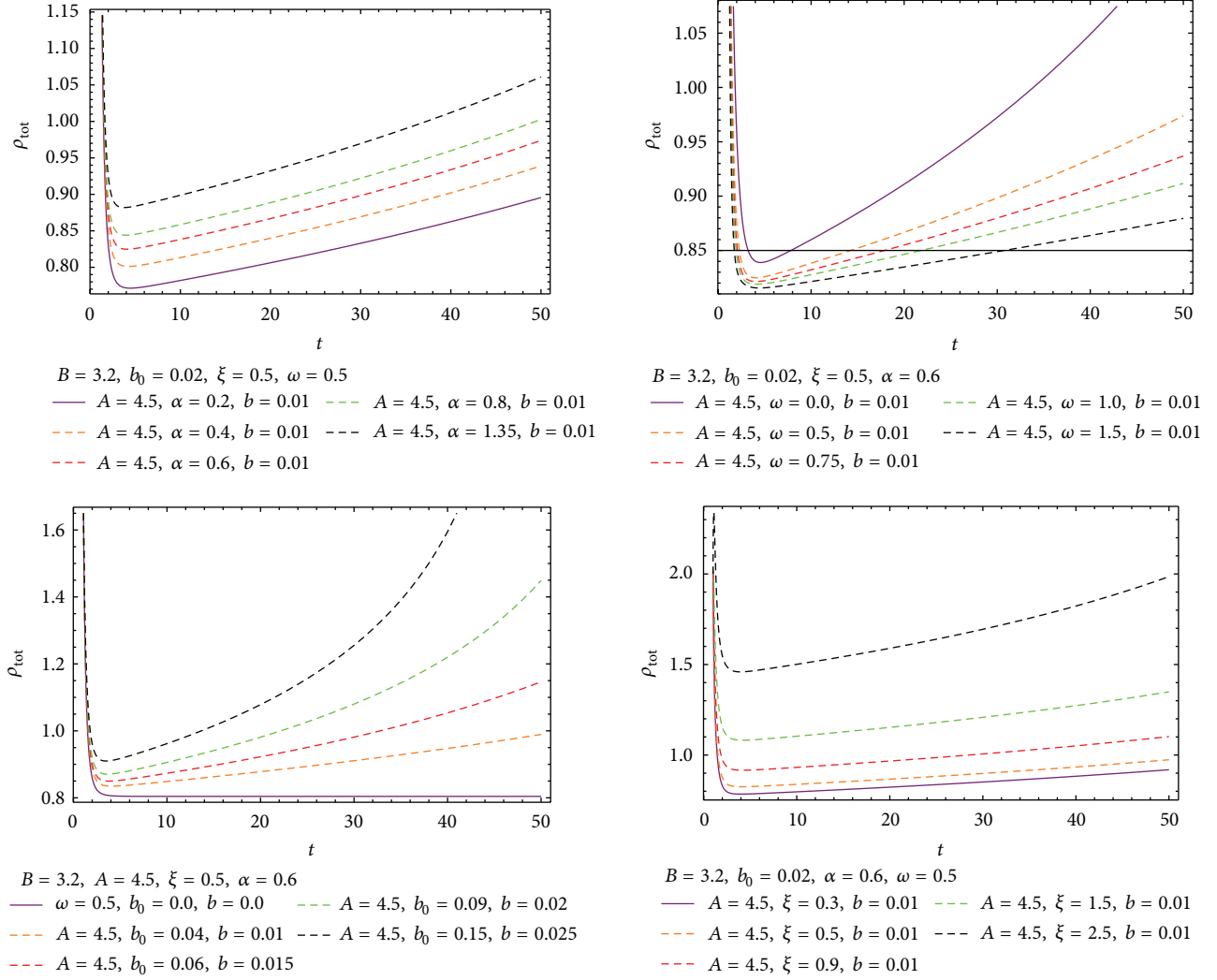
$$\begin{aligned} H^2 &= \frac{\dot{a}^2}{a^2} = \frac{\rho}{3}, \\ \dot{H} &= -\frac{1}{2}(\rho + P), \end{aligned} \quad (15)$$

where  $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$  and  $a(t)$  represents the scale factor. The  $\theta$  and  $\phi$  parameters are the usual azimuthal and polar angles of spherical coordinates, with  $0 \leq \theta \leq \pi$  and  $0 \leq \phi < 2\pi$ . The coordinates  $(t, r, \theta, \phi)$  are called comoving coordinates. Also  $\rho$  and  $p$  are total energy density and pressure, respectively.

Energy conservation  $T_{ij}^{;j} = 0$  is read as

$$\dot{\rho} + 3H(\rho + P) = 0. \quad (16)$$



FIGURE 8: Behavior of  $\rho_{\text{tot}}$  against  $t$  for interacting barotropic fluid and viscous Chaplygin gas.

In order to introduce an interaction between DE and DM, we should mathematically split (16) into the two following equations:

$$\dot{\rho}_{\text{DM}} + 3H(\rho_{\text{DM}} + P_{\text{DM}}) = Q, \quad (17)$$

$$\dot{\rho}_{\text{DE}} + 3H(\rho_{\text{DE}} + P_{\text{DE}}) = -Q. \quad (18)$$

For the barotropic fluid with  $P_{\text{DM}} = \omega \rho_{\text{DM}}$ , (17) will take the following form:

$$\dot{\rho}_b + 3H(1 + \omega - b_0 - btH)\rho_b = 3H(b_0 + btH)\rho_{\text{CG}}, \quad (19)$$

where the index  $b$  refers to DM and CG refers to dark energy. Dynamics of energy densities of Chaplygin and polytropic gases are read as

(1)

$$\begin{aligned} \dot{\rho}_{\text{CG}} + 3H(1 + A + b_0 + btH)\rho_{\text{CG}} - \frac{3HB}{\rho_{\text{CG}}^\alpha} \\ = -3h(b_0 + btH)\rho_b + 9H^2\xi, \end{aligned} \quad (20)$$

(2)

$$\begin{aligned} \dot{\rho}_{\text{PG}} + 3H(1 + K\rho_{\text{PG}}^{1/n} + b_0 + btH)\rho_{\text{PG}} \\ = -3h(b_0 + btH)\rho_b + 9H^2\xi. \end{aligned} \quad (21)$$

In the above equation, index PG refers to polytropic gases which serves as dark energy. Cosmological parameters of our interest are EoS parameters of each component  $\omega_i = P_i/\rho_i$  (index  $i$  refers to CG or PG), EoS parameter of composed fluid

$$\omega_{\text{tot}} = \frac{P_b + P_i}{\rho_b + \rho_i} \quad (22)$$

and deceleration parameter  $q$ , which can be written as

$$q = \frac{1}{2} \left( 1 + 3 \frac{P}{\rho} \right), \quad (23)$$

where  $P = P_b + P_i$  and  $\rho = \rho_b + \rho_i$ . Hereafter, index  $i$  means CG (modified Chaplygin gas which is usually written as MCG and PG for each model).

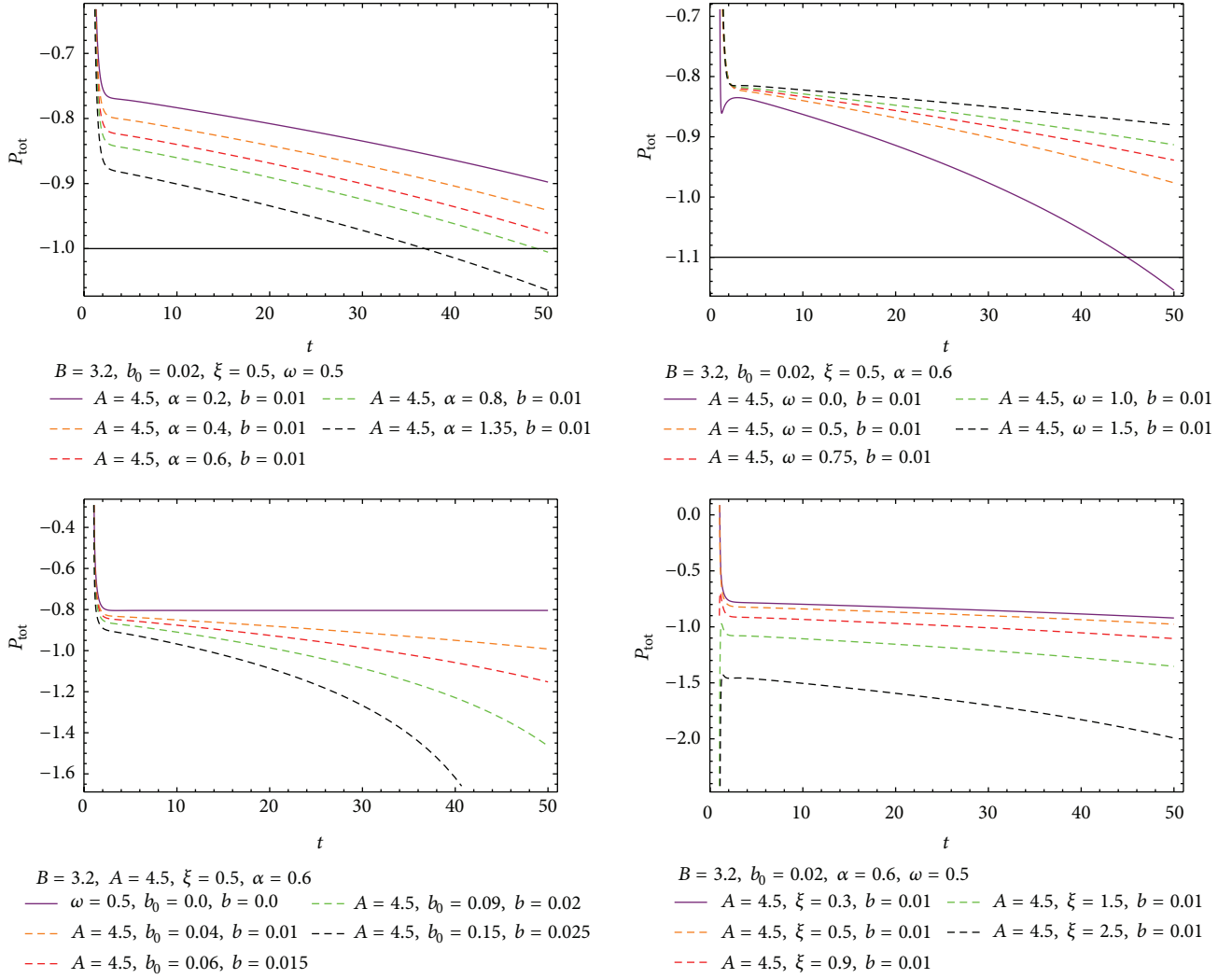


FIGURE 9: Behavior of  $P_{\text{tot}}$  against  $t$  for interacting barotropic fluid and viscous Chaplygin gas.

### 3. Numerical Results and Cosmological Parameters

**3.1. Model 1.** This model is based on differential equation (20) which yields to the following results.

Plots of Figure 1 show time evolution of Hubble expansion parameter in viscous modified Chaplygin gas model. In the first one we fixed all parameters and varied  $\alpha$ . We find that increasing  $\alpha$  increases Hubble parameter. Also it is clear from the first plot of Figure 1 that evolution of Hubble parameter corresponding to low values of  $\alpha$  is faster than that corresponding to higher values.

The second plot of Figure 1 shows behavior of Hubble expansion parameter with variation of  $\omega$  which shows that increasing  $\omega$  decreases Hubble parameter. Also it is clear from the second plot of Figure 1 (top right) that evolution of Hubble parameter corresponding to higher values of  $\omega$  is faster than that corresponding to lower values.

In the next plot of Figure 1 (dawn left) we fixed all parameters and varied interaction parameters  $b_0$  and  $b$ . We

find that increasing interaction parameters increases Hubble parameter. Also it is clear that evolution of Hubble parameter corresponding to low values of interaction parameters is faster than that corresponding to higher values.

Finally the last plot of Figure 1 shows the effect of viscosity. We find that increasing viscosity increases Hubble parameter. Also we find that evolution of Hubble parameter corresponding to low values of viscosity is faster than that corresponding to higher values. This plot has more agreement with observational data which tells that  $H_0 \approx 70$ , where  $H_0$  is the current value of Hubble parameter which is corresponding to late time behavior of the figures. This behavior coincides with observational data for small value of the viscous parameter.

Plots of Figure 2 deal with time variation of total equation of state parameter. We see sudden evolution at initial stage; then total equation of state parameter yields to approximately -1 as expected. We find from the first plot that increasing  $\alpha$  decreases  $\omega_{\text{tot}}$ .

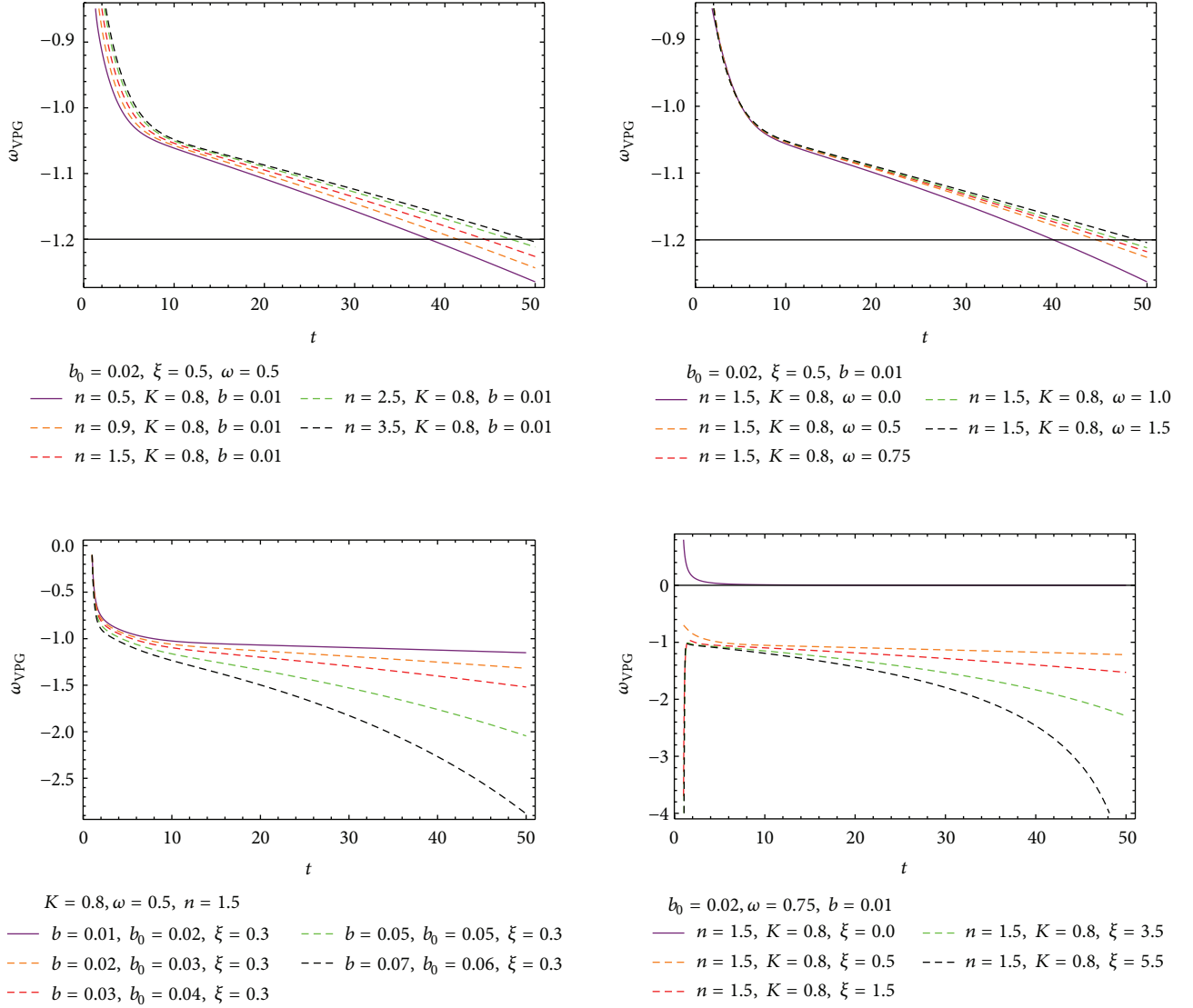


FIGURE 10: Behavior of EoS parameter of viscous polytropic  $\omega_{VPG}$  against  $t$  for interacting barotropic fluid and viscous polytropic gas.

From the second plot we find that increasing  $\omega$  increases  $\omega_{tot}$ . Also it is clear from this plot that evolution of total equation of state parameter corresponding to low values of  $\omega$  is faster than that corresponding to higher values.

In the third plot we can find variation of  $\omega_{tot}$  with interaction parameters and find that these parameters decrease value of  $\omega_{tot}$ . We can see that in the case without interaction ( $b_0 = b = 0$ ) the value of total equation of state parameter takes exactly  $-1$  with condition  $\omega_{tot} \geq -1$  which is quintessence-like universe. Then, presence of interaction terms changed  $\omega_{tot}$  to satisfy phantom-like universe  $\omega_{tot} \leq -1$ .

Finally we find that viscous coefficient decreases value of  $\omega_{tot}$ . If we assume that the infinitesimal value of viscous parameter, then  $\omega_{tot} \rightarrow -1$  is verified with phantom regime [70].

Observational data needs to have  $-1 \leq \omega \leq -1/3$  which is obtained by lower values of  $b$  and  $b_0$  or larger values of  $\omega$  which are illustrated in the second and third plots of Figure 2.

Plots of Figure 3 study behavior of  $q$  against  $t$  for interacting barotropic fluid and viscous modified Chaplygin gas. We see similar behavior with the plots of Figure 2. Hence, we can say that  $\alpha, b_0, b$ , and  $\xi$  decrease but  $\omega$  increases value of the deceleration parameter. This case may agree with  $\Lambda$ CDM model (where  $q \rightarrow -1$  is observed) by choosing small values of interaction constants and larger value of  $\omega$ .

**3.2. Model 2.** This model is based on differential equation (21) which yields to the following results.

Plots of Figure 4 show time evolution of Hubble expansion parameter in viscous polytropic gas model. We can see that the Hubble expansion parameter reduced suddenly at initial stage and take approximately constant value at the late time for appropriate parameters.

In the first plot of Figure 4 we fixed all parameters and varied  $n$ . We find that increasing  $n$  decreases Hubble parameter. Also it is clear from the first plot of Figure 4

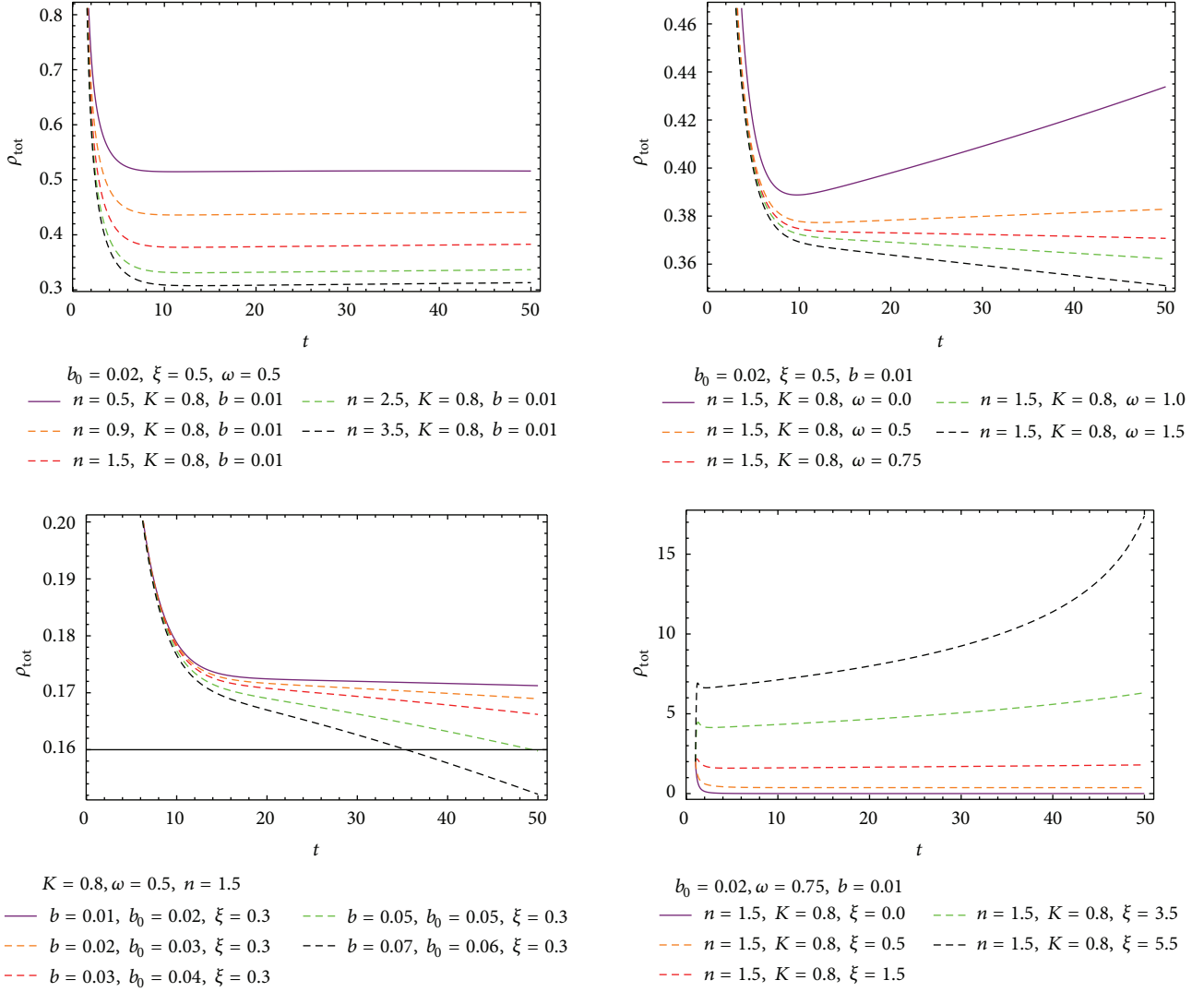


FIGURE 11: Behavior of  $\rho_{\text{tot}}$  against  $t$  for interacting barotropic fluid and viscous polytropic gas.

that evolution of Hubble parameter corresponding to higher values of  $n$  is faster than that corresponding to lower values.

The second plot of Figure 4 shows behavior of Hubble expansion parameter with variation of  $\omega$  which shows that increasing  $\omega$  decreases Hubble parameter. Also it is clear from the second plot of Figure 4 that evolution of Hubble parameter corresponding to higher values of  $\omega$  is faster than that corresponding to lower values. For the choice of  $n = 1.5$ ,  $K = 0.8$ ,  $b_0 = 0.02$ ,  $b = 0.01$ ,  $\xi = 0.5$ , and  $\omega = 0.5$ , the Hubble expansion parameter yields to constant value at the late time.

In the next plot of Figure 4 we fixed all parameters and varied the interaction parameters  $b_0$  and  $b$ . We find that increasing interaction parameters decreases Hubble parameter which is the opposite of the previous model. Also we can see that evolution of Hubble parameter corresponding to some values of interaction parameters is approximately the same.

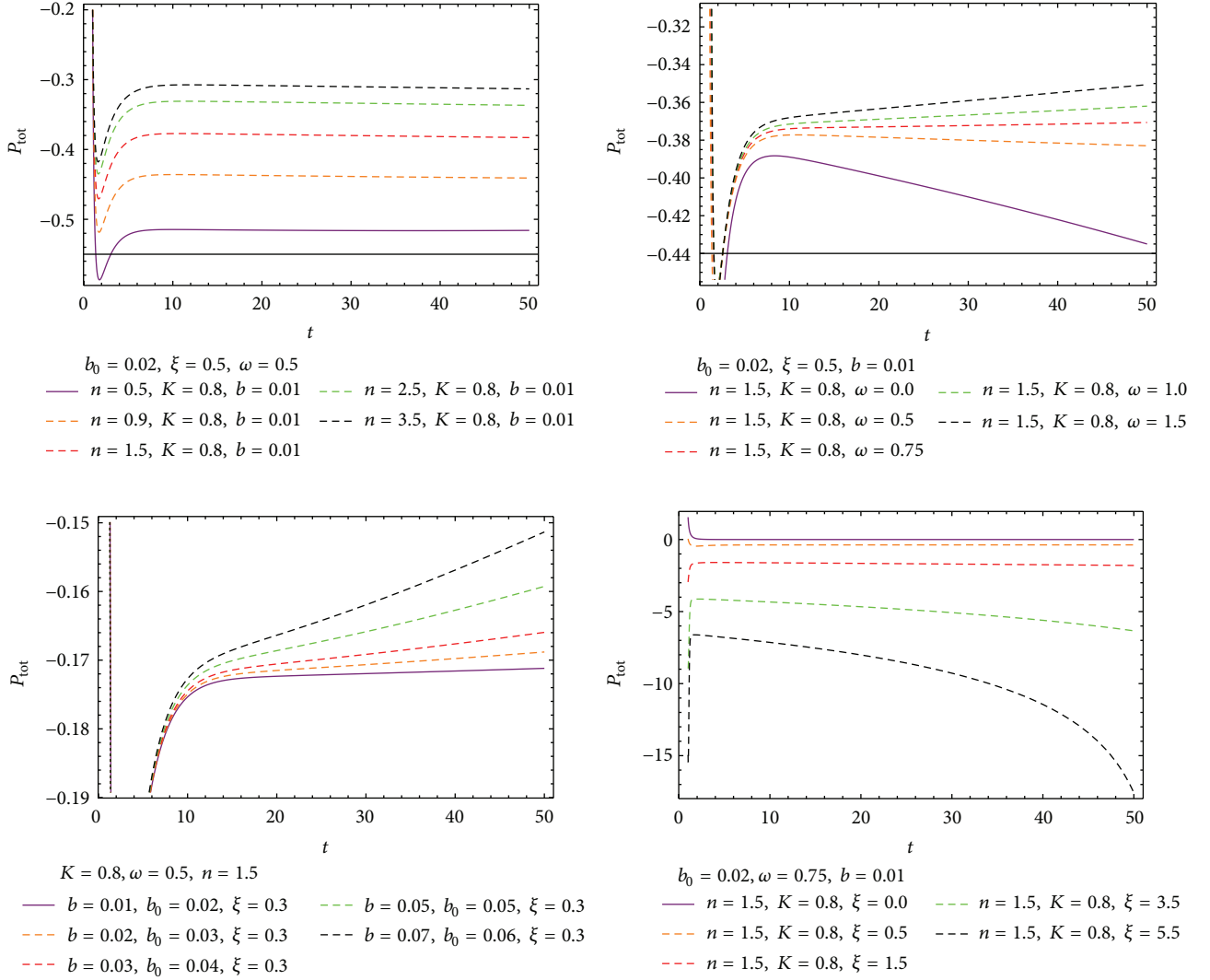
Finally the last plot of Figure 4 shows the effect of viscosity. We find that increasing viscosity increases Hubble

parameter. Also, we find that evolution of Hubble parameter corresponding to low values of viscosity is faster than that corresponding to higher values.

It seems that the value of the viscosity in the interval  $[0.5, 1.5]$  yields to more appropriate value of the current Hubble expansion parameter analogous to observational data. Plots of Figure 5 deal with time variation of total equation of state parameter. We see sudden evolution at initial stage; then total equation of state parameter yields to approximately  $-1$  as expected and similar to the previous model. We find from the first plot that increasing  $n$  decreases  $\omega_{\text{tot}}$  after sudden evolution.

From the second plot we find that increasing  $\omega$  increases  $\omega_{\text{tot}}$ . Also it is clear from this plot that revolution of total equation of state parameter corresponding to low values of  $\omega$  is faster than that corresponding to higher values which is similar to the previous model.

In the third plot we can find variation of  $\omega_{\text{tot}}$  with interaction parameters and find that these parameters increase

FIGURE 12: Behavior of  $P_{\text{tot}}$  against  $t$  for interacting barotropic fluid and viscous polytropic gas.

value of  $\omega_{\text{tot}}$ . We can see that in the case without interaction ( $b_0 = b = 0$ ) the value of total equation of state parameter takes the closest value to  $-1$ . Then, presence of interaction terms changed  $\omega_{\text{tot}}$  so we have  $\omega_{\text{tot}} \leq -1$  (phantom regime) in the case of interacting.

Finally we find that viscous coefficient decreases value of  $\omega_{\text{tot}}$ . So, in this case, presence of viscosity is necessary to have  $\omega_{\text{tot}} \rightarrow -1$ .

Comparing with observational data suggests that  $\omega = 0.75$  and  $\xi = 0.5$  are the best fitted values together with small values of interaction constants.

Plots of Figure 6 study behavior of deceleration parameter  $q$  against  $t$  for interacting barotropic fluid and viscous polytropic gas. We see similar behavior with the plots of Figure 5. Therefore, we can say that  $n$  and  $\xi$  decrease but  $\omega$ ,  $b_0$ , and  $b$  increase value of the deceleration parameter. This model also agrees with  $\Lambda$ CDM where  $q \rightarrow -1$ .

In Appendices A and B we study behavior of further cosmological parameters of both models such as energy density and pressure.

#### 4. Discussion

We considered two different models of viscous interacting cosmology with modified interaction term so it is depending on Hubble parameter and discussed numerically cosmological parameters of the models. In the first model we consider viscous modified Chaplygin gas which interacts with barotropic fluid. We obtained effect of interaction and viscous parameters on the cosmological quantities. We found that these parameters increase Hubble expansion parameter. If we neglect interaction parameters and viscosity, then evolution of Hubble parameter is faster than the case of interacting viscous cosmology. In the noninteracting case the Hubble parameter yields to constant after sudden reduction at initial stage. Also we studied equation of state parameters and found that interaction parameters and viscosity decrease value of EoS parameters. This situation is similar to deceleration parameter. In the noninteracting case, EoS and deceleration parameters yield to  $-1$  as expected. We then studied effect of these parameters on total density and pressure. We found

that both interaction parameters and viscosity increase value of total density but decrease value of total pressure. At the initial stage the total density suddenly decreased and yielded to a constant for noninteracting case, but it is increasing the function of time in presence of interaction term. We show that this model may agree with some observational data which say that  $H_0 \approx 70$  (in our scale  $H_0 \approx 0.7$ ) and  $q \rightarrow -1$ .

In the second model we consider viscous polytropic gas which interacts with barotropic fluid. Just before, we obtained effect of interaction and viscous parameters on the cosmological quantities. We found that interaction parameters decrease but viscosity increases Hubble expansion parameter. Behavior of interaction term in Hubble expansion parameter of this model is the opposite of previous model. If we neglect interaction parameters and viscosity, then evolution of Hubble parameter is faster than the case of interacting viscous cosmology. In the noninteracting or nonviscous cases the Hubble parameter yields to approximately a constant after sudden reduction at initial stage. Also we studied equation of state parameters and found that interaction parameters increase and viscosity decreases value of EoS parameters. EoS parameter yields to  $-1$  for the noninteracting case and yields to  $0$  for nonviscous case. The effect of interaction parameters on the deceleration parameter is similar to the EoS parameter but the deceleration parameter yields to approximately  $0.5$  for the nonviscous cosmology. Finally we studied effect of this parameter on total density and pressure. We found that interaction parameters decrease but viscosity increases value of total density. On the other hand interaction parameters increase total pressure but viscosity decreases one. This model also may agree with some observational data even more than the first model. In both models, the phantom regime is obtained by adding interaction and we have  $\omega_{\text{tot}} \leq -1$ . However further studies such as [71] are needed to confirm the viability of these models.

For the future work it is interesting to consider the effects of varying viscosity [72] on the cosmological parameters of present model.

## Appendices

### A. More Cosmological Parameters of Viscous Modified Chaplygin Gas

In this appendix we study equation of state parameter corresponding to viscous modified Chaplygin gas, total density, and pressure of the model numerically. Plots of Figure 7 show time evolution of  $\omega_{\text{VMCG}}$  with variation of  $\alpha$ ,  $\omega$ ,  $b_0$ ,  $b$ , and  $\xi$ . We find that  $\alpha$ ,  $b_0$ ,  $b$ , and  $\xi$  decrease value of equation of state parameter but  $\omega$  increased one. Then, plots of Figure 8 show that total density increases by  $\alpha$ ,  $b_0$ ,  $b$ , and  $\xi$  but pressure decreases with these parameters (see Figure 9).

### B. More Cosmological Parameters of Viscous Polytropic Gas

In this appendix we study equation of state parameter corresponding to viscous polytropic gas, total density, and

pressure of the model numerically. Plots of Figure 10 show time evolution of  $\omega_{\text{VPG}}$  with variation of  $n$ ,  $\omega$ ,  $b_0$ ,  $b$ , and  $\xi$ . We find that  $n$  and  $\omega$  increase value of equation of state parameter but  $b_0$ ,  $b$ , and  $\xi$  decreased one. Then, plots of Figure 11 show that total density increases by  $\xi$  and decreases by  $b_0$ ,  $b$ , and  $n$  but total pressure decreases with  $\xi$  and increases with other parameters (see Figure 12).

## Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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