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Research Article

Influence of Neutral Currents on Electron and Gamma Polarizations in the Process $e + N \rightarrow e' + N + \gamma$

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The differential cross section of electron inelastic scattering by nuclei followed by γ radiation is calculated using the multipole decomposition of the hadronic currents and by taking into account the longitudinal polarization of the initial electron and the circular polarization of the γ radiation. We performed the analysis of the angular and energy dependence of the degree of electron and photon polarization which can yield information on values of weak neutral currents parameters.

1. Introduction

For many years lepton scattering by nucleon and nuclei has been playing a key role in the determination of the nuclear electromagnetic form factors and in testing the standard model of electroweak interactions [1–4]. During the last decade there has been a considerable effort to measure parity-violating asymmetries in electron scattering [5–7]. These asymmetries can improve the precision of the determination of the mixing angle θ_W [8–11]. Electron scattering has also been used for the determination of the vector strange-quark matrix element [11–16].

In this work we study the influence of weak neutral currents parameters on the spin asymmetry coefficient of electron and photon polarization in the electron inelastic scattering by nuclei followed by gamma radiation. This process is illustrated by the following equation:

$$e^- + (A, Z) \rightarrow e^{-\prime} + (A, Z)^* \quad \hookrightarrow (A, Z) + \gamma_{\mathrm{RL}} \tag{1}$$

2. Differential Cross Section

In the first order perturbation theory the square of matrix element of the process (1) can be expressed as [17]

$$\sum_{M_i M_f} \left| M_{fi} \right|^2 = \frac{1}{2J_i + 1} \times \sum_{M_i M_f} \left| \sum_{M_n} \left\langle J_f M_f \right| H_\gamma \left(\Omega_\gamma, \sigma \right) \left| J_n M_n \right\rangle \times \left\langle J_n M_n \right| H_{ee'} \left(\vec{q}, s_e \right) \left| J_i M_i \right\rangle \right|^2, \tag{2}$$

where M_{ni} and M_{fn} are the matrix elements of $|J_i M_i\rangle \rightarrow |J_n M_n\rangle$ and $|J_n M_n\rangle \rightarrow |J_f M_f\rangle$ transitions. J_i, J_n, and J_f are, respectively, the spins of the initial, intermediate (excited), and the final states of the nucleus. Let us consider

$$H_\gamma \left(\Omega_\gamma, \sigma \right) = -\frac{2\pi e}{\sqrt{\omega \Omega}} \sum_{J_1 \geq 1} (-i)^{J_1} \left[J_1 \right] \left(\sigma \widehat{T}^m_{J_1 \sigma} - \widehat{T}^e_{J_1 \sigma} \right), \tag{3}$$

the interaction Hamiltonian for the emission of gamma rays with polarization σ in the direction Ω_γ. Here $[J_1] = (2J_1+1)^{1/2}$. The magnetic and electric multipole operators $\widehat{T}^m_{J_1\sigma}$ and $\widehat{T}^e_{J_1\sigma}$ are defined in a system of coordinates $X'Y'Z'$ where z-axis is oriented along the momentum $\vec{k}$ of the photon (Figure 3).

In order to define all the multipole operators in a system of coordinates where z-axis is oriented along the transferred momentum $\vec{q}$ we perform a rotation of the system of coordinates $X'Y'Z'$ by angles φ_γ and θ_γ. So we obtain the relation [18]:

$$\widehat{T}^{m\,(e)}_{J_1\sigma}(k) = \sum_{M_1} \widehat{T}^{m(e)}_{J_1M_1} D^{J_1}_{M_1\sigma}\left(\varphi_\gamma, \theta_\gamma, \varphi_\gamma\right). \tag{4}$$

By using (3) and (4) and applying the Wigner-Eckart theorem the matrix element of the Hamiltonian H_γ can take the following form:

$$\begin{aligned}
&\left\langle J_f M_f \right| H_\gamma\left(\Omega_\gamma, \sigma\right) \left| J_n M_n \right\rangle \\
&= -\frac{2\pi e}{\sqrt{\omega\Omega}} \\
&\quad \times \sum_{\substack{J_1 \geq 1 \\ M_1}} (-i)^{J_1} [J_1] D^{J_1}_{M_1\sigma}\left(\varphi_\gamma, \theta_\gamma, \varphi_\gamma\right) \\
&\qquad \times \begin{pmatrix} J_f & J_1 & J_n \\ -M_f & M_1 & M_n \end{pmatrix} \left\langle J_f \right| \left| \sigma \widehat{T}^m_{J_1\sigma} - \widehat{T}^e_{J_1\sigma} \right| \left| J_n \right\rangle .
\end{aligned} \tag{5}$$

So the matrix element for the nuclear transition from the initial state $|J_iM_i\rangle$ to the intermediate state $|J_nM_n\rangle$ is given by

$$M_{ni} = \left\langle J_n M_n \right| \widehat{H}_{ee'} \left| J_i M_i \right\rangle = C_\gamma \ell^\gamma_\mu J^\gamma_\mu + C_W \ell^Z_\mu \mathcal{J}^Z_\mu, \tag{6}$$

where

$$\ell^Z_\mu = \overline{u}\left(p'\right) \gamma_\mu \left(a_V + a_A \gamma_5\right) u\left(p\right), \tag{7}$$

$$\mathcal{J}_\mu\left(q\right) = \left\langle J_n M_n \right| \int d\vec{x} \exp\left(-i\vec{q}\vec{x}\right) \widehat{\mathcal{J}}^h_\mu\left(\vec{x}\right) \left| J_i M_i \right\rangle \tag{8}$$

are the leptonic and hadronic currents. $C_\gamma = 4\pi\alpha/q^2_\mu$ and $C_W = -G_F/\sqrt{2}$ are related, respectively, to the electromagnetic and weak interactions. $q_\mu = (p'-p)_\mu$ is the transferred 4-momentum, α is the fine structure constant, and G_F is the Fermi coupling constant for weak interaction. The constants a_V and a_A take specific values depending on the process. In the Weinberg-Salam model, for electron or muon scattering processes we have according to [19] $a_V = -1 + 4\sin^2\theta_W$ and $a_A = -1$.

The density of hadronic current $\widehat{\mathcal{J}}^h_\mu$ is composed by vector and axial-vector currents with isoscalar ($\tau = 0$) and isovector ($\tau = 1$) components [18]:

$$\widehat{\mathcal{J}}^h_\mu = \beta^{(\tau)}_V \left(\widehat{J}_\mu\right)_{\tau M_\tau} + \beta^{(\tau)}_A \left(\widehat{J}^5_\mu\right)_{\tau M_\tau}, \tag{9}$$

where $\beta^{(\tau)}_V$ and $\beta^{(\tau)}_A$ are the linking constants whose values depend on the process considered. For the electromagnetic interactions $M_\tau = 0$, $\beta^{(\tau)}_V = 1$, and $\beta^{(\tau)}_A = 0$ ($\tau = 0, 1$) and the hadronic current is given by

$$J^\gamma_\mu = \left(J_\mu\right)_{00} + \left(J_\mu\right)_{10}. \tag{10}$$

In the case of processes with weak neutral currents, $M_\tau = 0$ ($\tau = 0, 1$) so the hadronic current is given by

$$\begin{aligned}
J^Z_\mu = {} & \beta^{(0)}_V \left(J_\mu\right)_{00} + \beta^{(0)}_A \left(J^5_\mu\right)_{00} \\
& + \beta^{(1)}_V \left(J_\mu\right)_{10} + \beta^{(1)}_A \left(J^5_\mu\right)_{10}.
\end{aligned} \tag{11}$$

In the Weinberg-Salam model, for scattering processes according to [19] we have

$$\begin{aligned}
\beta^{(0)}_V &= -2\sin^2\theta_W \qquad \beta^{(0)}_A = 0, \\
\beta^{(1)}_V &= 1 - 2\sin^2\theta_W \qquad \beta^{(1)}_A = 1,
\end{aligned} \tag{12}$$

where θ_W is the Weinberg angle.

The square of the matrix element (7) for electron electroweak scattering by nuclei takes the following form:

$$\left|M_{ni}\right|^2 = C^2_\gamma \left\{\left(\ell^{\gamma *}_\mu \ell^\gamma_\nu\right)\left(J^{\gamma *}_\mu J^\gamma_\nu\right) + 2\rho\,\mathrm{Re}\left(\ell^{\gamma *}_\mu \ell^Z_\nu J^{\gamma *}_\mu J^Z_\nu\right)\right\}, \tag{13}$$

where $\rho = C_W/C_\gamma$.

In (13) we neglect the term proportional to ρ^2. In the ultrarelativistic longitudinally polarized electrons case we obtain the relation

$$\ell^{\gamma *}_\mu \ell^Z_\nu = \left(a_V - a_A s_e\right) \ell^{\gamma *}_\mu \ell^\gamma_\nu, \tag{14}$$

where the tensor $\ell^{\gamma *}_\mu \ell^\gamma_\nu$ is given by

$$\ell^{\gamma *}_\mu \ell^\gamma_\nu = \frac{\delta_\mu}{2EE'} \left\{\delta_{\mu\nu}\left(pp'\right) - p_\mu p'_\nu - p_\nu p'_\mu - s_e \varepsilon_{\mu\nu\alpha\beta} p_\alpha p'_\beta\right\}. \tag{15}$$

Here E and p (E' and p') are the energy and the 4-momentum of the initial (final) electron; $s_e = \pm 1$ is the helicity of the initial electron. The factor δ_μ takes the following values:

$$\delta_\mu = \begin{cases} -1 & \text{for } \mu = 1, 2, 3 \\ +1 & \text{for } \mu = 4. \end{cases} \tag{16}$$

The differential cross section, calculated by multipole decomposition of matrix elements [19, 20] and by taking into

account the circular polarization of the emitted photon, is given by the following formula:

$$\begin{aligned}
&\frac{d\sigma}{d\Omega_{e'}d\Omega_{\gamma}} \\
&= \frac{\Gamma_{\gamma}^{n\to f}}{\Gamma_{\text{Total}}^{n\to f}} \\
&\times \sum_{\tau\tau'} \begin{pmatrix} \tau_n & \tau & \tau_i \\ -M_{\tau_n} & 0 & M_{\tau_i} \end{pmatrix} \\
&\qquad \times \begin{pmatrix} \tau_n & \tau' & \tau_i \\ -M_{\tau_n} & 0 & M_{\tau_i} \end{pmatrix} 2\alpha\sigma_{ee'} W_0^t(q_\gamma)_{fn} \\
&\times \left\{ 1 + \frac{1}{R_0^0(q)_{ni}} \right. \\
&\quad \times \sum_{\substack{L\geq 2 \\ \text{even}}} f_L^{(n\to f+\gamma)} \left(P_L\left(\cos\theta_\gamma\right) R_L^0 \right. \\
&\qquad\qquad + P_L^1\left(\cos\theta_\gamma\right)\cos\varphi_\gamma R_L^1 \\
&\qquad\qquad \left. + P_L^2\left(\cos\theta_\gamma\right)\cos 2\varphi_\gamma R_L^2 \right) \\
&\quad + \frac{s_\gamma}{R_0^0(q)_{ni}} \sum_{\substack{L\geq 1 \\ \text{odd}}} f_L^{(n\to f+\gamma)} \left(P_L\left(\cos\theta_\gamma\right) \widetilde{R}_L^0 \right. \\
&\qquad\qquad \left. + P_L^1\left(\cos\theta_\gamma\right)\cos\varphi_\gamma \widetilde{R}_L^1 \right) \\
&\quad \left. + \frac{s_\gamma}{R_0^0(q)_{ni}} \sum_{\substack{L\geq 3 \\ \text{odd}}} f_L^{(n\to f+\gamma)} P_L^2\left(\cos\theta_\gamma\right)\cos 2\varphi_\gamma \widetilde{R}_L^2 \right\},
\end{aligned} \tag{17}$$

where $\Gamma_{\gamma}^{n\to f}$ and $\Gamma_{\text{total}}^{n\to f}$ indicate, respectively, the relative and total disintegration widths of the excited nucleus state and $P_L^m(\cos\theta_\gamma)$ is Legendre polynomial. The angles θ_γ and φ_γ determine the photon direction, s_γ is photon helicity, and $\sigma_{ee'}$ is the differential cross section of the electron scattering by unpolarized nuclei given by

$$\sigma_{ee'} = 4\pi\sigma_M R_0^0(q)_{ni} f_{\text{rec}}^{-1}, \tag{18}$$

where σ_M and f_{rec} are, respectively, the Mott cross section and the nuclear correction factor given by

$$\sigma_M = \left(\frac{Z\alpha\cos(\theta/2)}{2E\sin^2(\theta/2)} \right)^2, \qquad f_{\text{rec}} = 1 + \frac{2E}{M}\sin^2\frac{\theta}{2}, \tag{19}$$

where M is the mass of the nucleus and Z is the atomic number.

The functions R_L^m and $\widetilde{R}_L^m$ are given by the following formulae:

$$\begin{aligned}
R_L^0 &= X_1\left(v_\ell W_L^\ell + v_t W_L^t\right) + X_2 v_{t'} W_L^a, \\
R_L^1 &= X_2 v_{ac} W_L^{ac} + X_3 v_{t\ell'} W_L^{a\ell} + X_4 v_{t\ell} W_L^{t\ell}, \\
R_L^2 &= X_1 v_{tt} W_L^{tt}, \\
\widetilde{R}_L^0 &= X_1 v_{t'} W_L^{t'} + X_2\left(v_t W_L^{a'} + v_{\ell'} W_L^{\ell'}\right), \\
\widetilde{R}_L^1 &= X_2 v_{ac'} W_L^{ac'} + X_3 v_{t\ell} W_L^{a\ell'} + X_4 v_{t\ell'} W_L^{t\ell'}, \\
\widetilde{R}_L^2 &= X_2 v_{tt} W_L^{tt'}, \\
X_1 &= 1 + 2\rho\beta_V^{(\tau')}\left(a_V - a_A s_e\right), \\
X_2 &= 2\rho\beta_A^{(\tau')}\left(a_V - a_A s_e\right), \\
X_3 &= 2\rho\beta_A^{(\tau)}\left(a_V - a_A s_e\right), \\
X_4 &= \left(1 + \rho\left(\beta_V^{(\tau)} + \beta_V^{(\tau')}\right)\left(a_V - a_A s_e\right)\right).
\end{aligned} \tag{20}$$

Here v_i $(i = \ell, t, \ldots, ac')$ are the leptonic functions and W_L^i are the hadronic functions. The expressions of the hadronic and the leptonic functions by taking into account the longitudinal polarization of electrons are given in the Appendix. The quantum number L is defined by the relation $0 \leq L \leq 2J_n$. The functions $f_L^{(n\to f+\gamma)}$ are defined as follows:

$$f_L^{(n\to f+\gamma)} = \begin{cases} \dfrac{W_L^t\left(q_\gamma\right)}{W_0^t\left(q_\gamma\right)} & \text{for } L \text{ even} \\ \dfrac{W_L^{t'}\left(q_\gamma\right)}{W_0^t\left(q_\gamma\right)} & \text{for } L \text{ odd,} \end{cases} \tag{21}$$

where

$$\begin{aligned}
W_L^t\left(q_\gamma\right) &= -(-)^{J_f+J_n} \\
&\quad \times \sum_{J_1'J_1} \sqrt{(2L+1)(2J_1+1)(2J_1'+1)} \\
&\qquad \times \begin{pmatrix} J_1' & J_1 & L \\ 1 & -1 & 0 \end{pmatrix} \begin{Bmatrix} J_1' & J_1 & L \\ J_n & J_n & J_f \end{Bmatrix} \\
&\quad \times \left\{ P_{J_1'+J_1}^{+}\left(F_{EJ_1'}F_{EJ_1} + F_{MJ_1'}F_{MJ_1}\right) \right. \\
&\qquad \left. + P_{J_1'+J_1}^{-}\left(F_{EJ_1'}F_{MJ_1} - F_{MJ_1'}F_{EJ_1}\right) \right\}.
\end{aligned} \tag{22}$$

Hence $q_\gamma = E_\gamma$ is the gamma transition energy.

Quantum numbers J_1 and J_1' are given by

$$\begin{gathered}\left|J_n - J_f\right| \le J_1 \le J_n + J_f, \qquad \left|J_n - J_f\right| \le J_1' \le J_n + J_f, \\ W_L^{t'}\left(q_\gamma\right) = W_L^t\left(q_\gamma\right) \quad \text{for } L \text{ odd}, \\ P_{J_1'+J_1}^{+} = \frac{1}{2}(-1)^{(1/2)(J_1'-J_1)}\left(1 + (-1)^{J_1'+J_1}\right), \\ P_{J_1'+J_1}^{-} = \frac{1}{2}(-1)^{(1/2)(J_1'-J_1+1)}\left(1 - (-1)^{J_1'+J_1}\right).\end{gathered} \tag{23}$$

3. Study of the Transitions $J_i = 0 \to J_n = 1 \to J_f = 0$

As an example of transitions let us consider the following process:

$$\begin{gathered}e^- + {}^{12}C(0^+0) \to e^{-\prime} + {}^{12}C^*(1^+1) \\ \hookrightarrow {}^{12}C(0^+0) + \gamma_{\mathrm{RL}}\end{gathered} \tag{24}$$

So the quantum numbers L, J, and J' are defined as

$$\begin{gathered}0 \le L \le 2J_n \Longrightarrow 0 \le L \le 2; \\ \left|J_i - J_n\right| \le J \le J_i + J_n \Longrightarrow J = 1; \\ \left|J_i - J_n\right| \le J' \le J_i + J_n \Longrightarrow J' = 1.\end{gathered} \tag{25}$$

The differential cross section of the process (24) which takes into account the longitudinal polarization of the electron and the circular polarization of the photon is given by the formula

$$\begin{aligned}\frac{d\sigma}{d\Omega_{e'}d\Omega_\gamma} &= \Sigma_0 \Big\{R_0^0 + f_2^{(n\to f+\gamma)}\Big[P_2\left(\cos\theta_\gamma\right)R_2^0 + P_2^1\left(\cos\theta_\gamma\right)\cos\varphi_\gamma R_2^1 \\ &\qquad + P_2^2\left(\cos\theta_\gamma\right)\cos 2\varphi_\gamma R_2^2\Big] \\ &\quad + s_\gamma f_1^{(n\to f+\gamma)}\Big[P_1\left(\cos\theta_\gamma\right)\widetilde{R}_1^0 \\ &\qquad + P_1^1\left(\cos\theta_\gamma\right)\cos\varphi_\gamma \widetilde{R}_1^1\Big]\Big\},\end{aligned} \tag{26}$$

where

$$\begin{aligned}\Sigma_0 &= \frac{1}{\sqrt{3}}F_{M1}^2\left(q_\gamma\right)\sigma_{ee'}, \qquad f_1^{(n\to f+\gamma)} = -\frac{\sqrt{3}}{\sqrt{2}}, \\ f_2^{(n\to f+\gamma)} &= \frac{1}{\sqrt{2}}, \\ R_0^0(q) &= \frac{1}{3\sqrt{3}}F_{M1} \\ &\quad \times\Big[v_t F_{M1}\left(1 + 2\rho\beta_V^{(1)}\left(a_V - a_A s_e\right)\right) \\ &\qquad + 2\,v_{t'}F_{E1}^5\rho\beta_A^{(1)}\left(a_V - a_A s_e\right)\Big], \\ R_2^0(q) &= \frac{1}{\sqrt{2}}R_0^0(q), \\ R_2^1(q) &= \frac{1}{3\sqrt{3}}v_{ac}\,F_{M1}\rho\beta_A^{(1)}\left(a_V - a_A s_e\right)\left(F_{C1}^5 - \frac{q_0}{q}F_{L1}^5\right), \\ R_2^2(q) &= -\frac{1}{6\sqrt{6}}v_{tt}F_{M1}^2\left(1 + 2\rho\beta_V^{(1)}\left(a_V - a_A s_e\right)\right), \\ \widetilde{R}_1^0(q) &= -\frac{1}{3\sqrt{2}}F_{M1} \\ &\quad \times\Big[v_{t'}F_{M1}\left(1 + 2\rho\beta_V^{(1)}\left(a_V - a_A s_e\right)\right) \\ &\qquad + 2v_t F_{E1}^5\rho\beta_A^{(1)}\left(a_V - a_A s_e\right)\Big], \\ \widetilde{R}_1^1(q) &= -\frac{1}{3}v_{ac'}F_{M1}\rho\beta_A^{(1)}\left(a_V - a_A s_e\right)\left(F_{C1}^5 - \frac{q_0}{q}F_{L1}^5\right).\end{aligned} \tag{27}$$

The matrix elements F_{M1}, F_{C1}^5, F_{L1}^5, and F_{E1}^5, calculated in the shell model with a harmonic oscillator potential, are given by [21]:

$$\begin{aligned}F_{M1} &= \frac{\psi}{6\sqrt{\pi}}\frac{q}{M}e^{-y}\left(F_1 - \mu\left(2 - y\right)\right); \\ F_{C1}^5 &= -\frac{\psi}{3\sqrt{2\pi}}\frac{q}{M}e^{-y}\left(\frac{3}{2}F_A - \left(1 - y\right)\left(q_0F_p - 2MF_T\right)\right); \\ F_{L1}^5 &= -\frac{\sqrt{2}\psi}{3\sqrt{\pi}}e^{-y}\left(1 - y\right)\left(F_A - \frac{q^2}{2M}F_p\right); \\ F_{E1}^5 &= -\frac{\psi}{3\sqrt{\pi}}e^{-y}F_A\left(2 - y\right),\end{aligned} \tag{28}$$

where $\psi = -0.003$ [19], $\mu = F_1 + 2M_nF_2$, $q_0 = -\Delta E$ is the energy of the transition, and $y = (bq/2)^2$, where b is the parameter of the harmonic oscillator.

Writing explicitly the electron and photon polarization, the differential cross section (26) takes the following form:

$$\frac{d\sigma}{d\Omega_{e'}d\Omega_\gamma} = \frac{1}{3\sqrt{3}}F_{M1}\Sigma_0\left(\Sigma + s_e\Delta + s_\gamma\Delta' + s_e s_\gamma\Delta''\right), \tag{29}$$

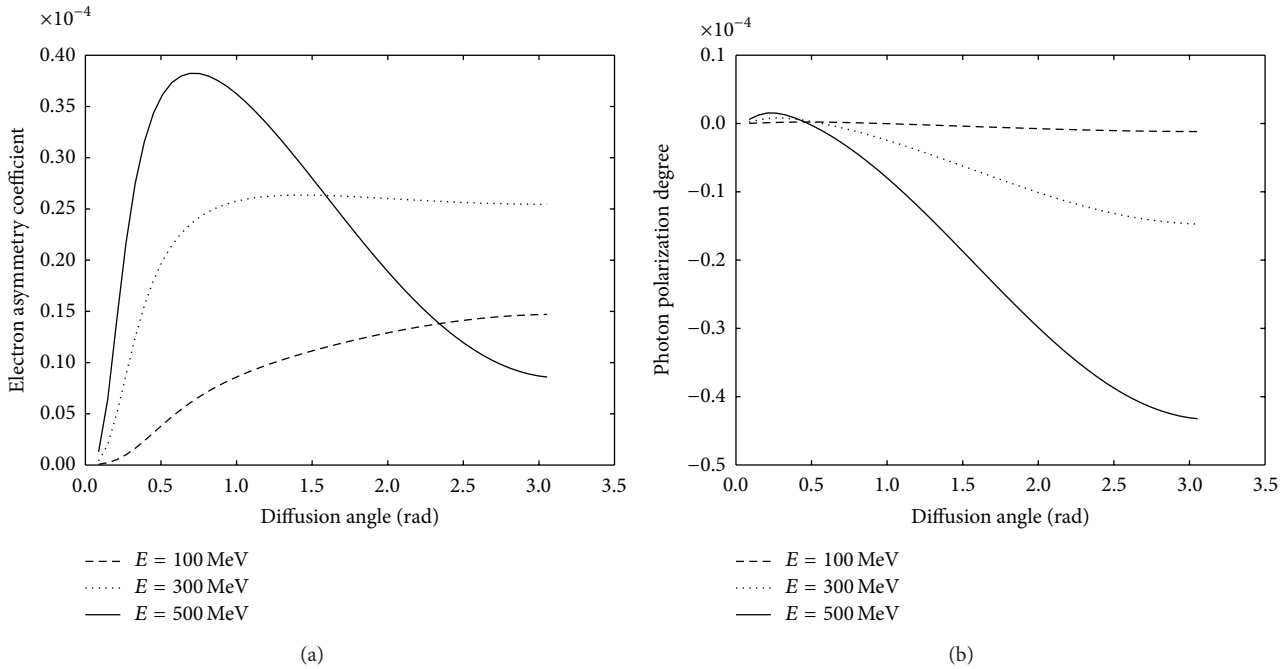

Figure 1: (a) Asymmetry coefficient for the electron. (b) Photon polarization degree.

where

$$\begin{aligned}
\Sigma = {} & \left(1 + \frac{1}{2}P_2\left(\cos\theta_\gamma\right)\right) \\
& \times \left(v_t F_{M1}\left(1 + 2\rho\beta_V^{(1)}a_V\right) - 2\tilde{v}_t F_{E1}^5 \rho\beta_A^{(1)}a_A\right) \\
& - \frac{1}{\sqrt{2}}P_2^1\left(\cos\theta_\gamma\right)\cos\varphi_\gamma \\
& \times \tilde{v}_{ac}\rho\beta_A^{(1)}a_A\left(F_{C1}^5 - \frac{q_0}{q}F_{L1}^5\right) \\
& - \frac{1}{4}P_2^2\left(\cos\theta_\gamma\right)\cos 2\varphi_\gamma v_{tt}F_{M1}\left(1 + 2\rho\beta_V^{(1)}a_V\right), \\
\Delta = {} & \rho\left\{\left(2 + P_2\left(\cos\theta_\gamma\right)\right)\left(\tilde{v}_t F_{E1}^5\beta_A^{(1)}a_V - v_t F_{M1}\beta_V^{(1)}a_A\right)\right. \\
& + \frac{1}{\sqrt{2}}P_2^1\left(\cos\theta_\gamma\right)\cos\varphi_\gamma \\
& \times \tilde{v}_{ac}\beta_A^{(1)}a_V\left(F_{C1}^5 - \frac{q_0}{q}F_{L1}^5\right) \\
& \left. + \frac{1}{2}P_2^2\left(\cos\theta_\gamma\right)\cos 2\varphi_\gamma v_{tt}F_{M1}\beta_V^{(1)}a_A\right\}, \\
\Delta' = {} & \rho\left\{\frac{3}{2}P_1\left(\cos\theta_\gamma\right)\left(v_t F_{E1}^5\beta_A^{(1)}a_V - \tilde{v}_t F_{M1}\beta_V^{(1)}a_A\right)\right. \\
& + \sqrt{3}P_1^1\left(\cos\theta_\gamma\right)\cos\varphi_\gamma \\
& \left. \times v_{ac'}\beta_A^{(1)}a_V\left(F_{C1}^5 - \frac{q_0}{q}F_{L1}^5\right)\right\}, \\
\Delta'' = {} & \frac{3}{4}P_1\left(\cos\theta_\gamma\right)\left(\tilde{v}_t F_{E1}^5\left(1 + 2\rho\beta_V^{(1)}a_V\right) - 2v_t F_{M1}\rho\beta_A^{(1)}a_A\right) \\
& - \sqrt{3}P_1^1\left(\cos\theta_\gamma\right)\cos\varphi_\gamma v_{ac'}\rho\beta_A^{(1)}a_A\left(F_{C1}^5 - \frac{q_0}{q}F_{L1}^5\right).
\end{aligned} \tag{30}$$

Let us consider now two coefficients of asymmetry which can be experimentally determined, that is, the electron polarization ratio and the photon circular polarization degree:

$$A_{R,L} = \frac{d\sigma\left(s_e = +1\right) - d\sigma\left(s_e = -1\right)}{d\sigma\left(s_e = +1\right) + d\sigma\left(s_e = -1\right)}, \tag{31}$$

$$B_{R,L} = \frac{d\sigma\left(s_\gamma = +1\right) - d\sigma\left(s_\gamma = -1\right)}{d\sigma\left(s_\gamma = +1\right) + d\sigma\left(s_\gamma = -1\right)}. \tag{32}$$

Assuming that the photon is emitted in the direction of the incident electron we have

$$\cos\theta_\gamma = \frac{E'\cos\theta - E}{q}, \qquad \varphi_\gamma = 0, \qquad \sin\theta_\gamma = \frac{E'\sin\theta}{q}, \tag{33}$$

and after averaging on the photon polarization, the asymmetry coefficient for the electron defined by (31) is given by

$$A_{R,L} = \frac{\Delta}{\Sigma}. \tag{34}$$

Averaging the differential cross section (29) on the electron polarization, the photon polarization degree defined by (32) takes the form

$$B_{R,L} = \frac{\Delta'}{\Sigma}. \tag{35}$$

Figures 1(a) and 1(b) show the dependence on the diffusion angle θ of the asymmetry coefficient for electron and photon

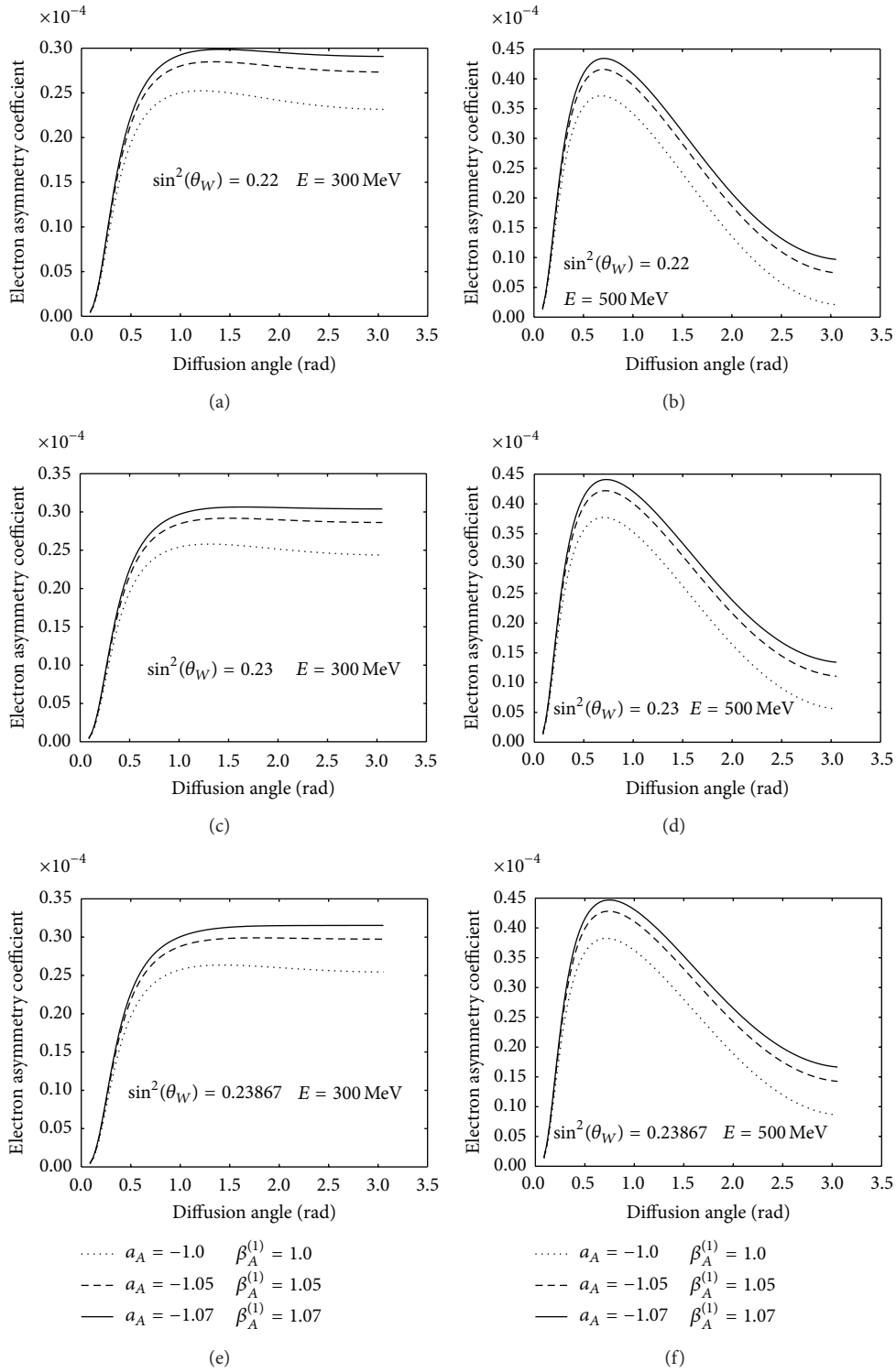

Figure 2: Asymmetry coefficient for the electron.

polarization degrees for three values of the energy for incident electron. Calculation is done by using $\sin^2\theta_W = 0.23125$ from [8] and formulas (12) relative to the coupling constants of standard model.

Figure 1(a) shows for $E = 300\,\text{MeV}$ and $E = 500\,\text{MeV}$ variation of asymmetry coefficient. This variation is greater for electron scattering around value of angle $\theta = 45^\circ$. However, the photon degree of polarization undergoes greater variation for large values of scattering angle.

Figures 2(a)–2(f) represent the angular dependence of asymmetry coefficient for electron with energy values $E = 300$ and $E = 500\,\text{MeV}$ and for different values of parameters

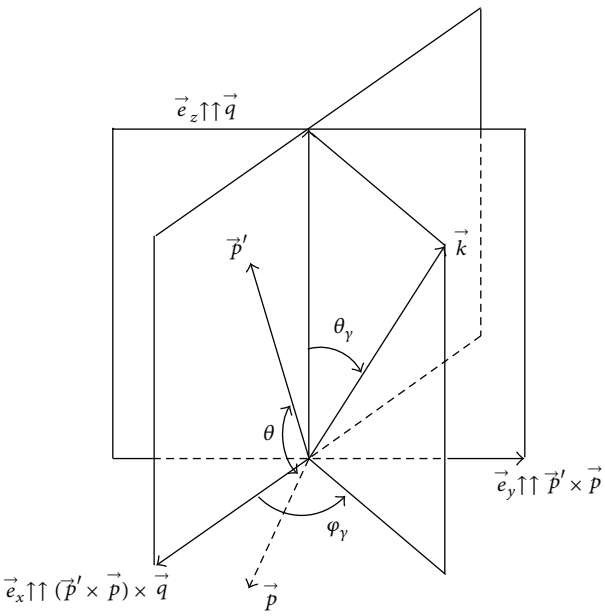

Figure 3: System of coordinates.

a_A and $\beta_A^{(1)}$. Electron asymmetry coefficient is very sensitive to change of these parameters, but it could be sensitive to the change of $\sin^2\theta_W$ which depend on these parameters in the standard model.

4. Conclusion

The differential cross section of the inelastic electron scattering by nuclei is calculated and the expressions of the asymmetries coefficients are obtained; their angular-energy dependence is analyzed and some results are carried out. The experimental study of the electron scattering processes accompanied by gamma radiation can play an important role in research on the parameters of the weak neutral currents.

Appendix

By taking into account the longitudinal polarization of electrons we obtain for the leptonic functions the following expressions:

$$v_\ell = \left(\frac{q_\mu^2}{q^2}\right)^2, \qquad v_t = \frac{1}{2}\frac{q_\mu^2}{q^2} + \tan^2\frac{\theta}{2}, \qquad v_{tt} = -\frac{1}{2}\frac{q_\mu^2}{q^2},$$

$$v_{t\ell} = \frac{1}{\sqrt{2}}\frac{q_\mu^2}{q^2}\left(\frac{q_\mu^2}{q^2} + \tan^2\frac{\theta}{2}\right)^{1/2}, \qquad v_{t\ell'} = \frac{1}{\sqrt{2}} s_e \frac{q_\mu^2}{q^2}\tan\frac{\theta}{2},$$

$$v_{\ell'} = \frac{q_\mu^2}{q^2}, \qquad v_{t'} = -s_e\frac{1}{\sqrt{2}}\left(\frac{q_\mu^2}{q^2} + \tan^2\frac{\theta}{2}\right)^{1/2}\tan\frac{\theta}{2} = s_e\widetilde{v}_t,$$

$$v_{ac} = s_e\tan\frac{\theta}{2} = s_e\widetilde{v}_{ac}, \qquad v_{ac'} = \left(\frac{q_\mu^2}{q^2} + \tan^2\frac{\theta}{2}\right)^{1/2}. \tag{A.1}$$

The hadronic functions are given by the following formulae:

$$\begin{aligned}
W_L^\ell(q) &= \sum_{J'J} A_{0;0}^{(L)} P_{J'+J}^{+} F_{CJ'} F_{CJ},\\
W_L^t(q) &= -\sum_{J'J} A_{1;-1}^{(L)} \left\{P_{J'+J}^{+}\left(F_{EJ'}F_{EJ} + F_{MJ'}F_{MJ}\right)\right.\\
&\qquad \left. - P_{J'+j}^{-}\left(F_{EJ'}F_{MJ} - F_{MJ'}F_{EJ}\right)\right\},\\
W_L^{tt}(q) &= -\sum_{J'J} A_{1;1}^{(L)} \left\{P_{J'+J}^{+}\left(F_{EJ'}F_{EJ} - F_{MJ'}F_{MJ}\right)\right.\\
&\qquad \left. + P_{J'+j}^{-}\left(F_{EJ'}F_{MJ} + F_{MJ'}F_{EJ}\right)\right\},\\
W_L^{t\ell} &= 2\sqrt{2}\sum_{J'J} A_{0;1}^{(L)}\, F_{CJ'}\left(P_{J'+J}^{+}F_{EJ} + P_{J'+J}^{-}F_{MJ}\right),\\
W_L^{a}(q) &= -\sum_{J'J} A_{1;-1}^{(L)} \left\{P_{J'+J}^{+}\left(F_{EJ'}^5F_{MJ} + F_{MJ'}^5F_{EJ}\right)\right.\\
&\qquad \left. + P_{J'+j}^{-}\left(F_{EJ'}^5F_{EJ} + F_{MJ'}^5F_{MJ}\right)\right\},\\
W_L^{a\ell} &= 2\sqrt{2}\sum_{J'J} A_{0;1}^{(L)} F_{CJ'}\left(P_{J'+J}^{+}F_{MJ}^5 + P_{J'+J}^{-}F_{EJ}^5\right),\\
W_L^{ac} &= \sum_{J'J} A_{0;1}^{(L)}\left(F_{CJ'}^5 - \frac{q_0}{q}F_{LJ'}^5\right)\left(P_{J'+J}^{+}F_{MJ} - P_{J'+J}^{-}F_{EJ}\right),\\
W_L^{\ell'} &= \sum_{J'J} A_{0;0}^{(L)} P_{J'+J}^{-}\left(F_{CJ'}^5 - \frac{q_0}{q}F_{LJ'}^5\right)F_{CJ},\\
W_L^{tt'}(q) &= \sum_{J'J} A_{1;1}^{(L)} \left\{P_{J'+J}^{+}\left(F_{EJ'}^5F_{MJ} - F_{MJ'}^5F_{EJ}\right)\right.\\
&\qquad \left. - P_{J'+j}^{-}\left(F_{EJ'}^5F_{EJ} + F_{MJ'}^5F_{MJ}\right)\right\},\\
W_L^{t'}(q) &= W_L^t(q); \qquad W_L^{t\ell'}(q) = -W_L^{t\ell}(q);\\
W_L^{a'}(q) &= W_L^a(q);\\
W_L^{a\ell'}(q) &= -W_L^{a\ell}(q); \qquad W_L^{ac'}(q) = -W_L^{ac}(q).
\end{aligned} \tag{A.2}$$

The coefficients $A_{m'm}^{(L)}$ and $P_{J'+J}^{\pm}$ are given by the following formulae:

$$\begin{aligned}
A_{m'm}^{(L)} &= (-1)^{J_i+J_n}\left[J'\right][J][L]\left(\frac{(L+M)!}{(L-M)!}\right)^{1/2}\\
&\quad \times \begin{pmatrix} J' & J & L\\ m' & m & M\end{pmatrix}\begin{Bmatrix} J' & J & L\\ J_n & J_n & J_i\end{Bmatrix},\\
P_{J'+J}^{+} &= \frac{1}{2}(-1)^{(1/2)(J'-J)}\left(1 + (-1)^{J'+J}\right),\\
P_{J'+J}^{-} &= \frac{1}{2}(-1)^{(1/2)(J'-J+1)}\left(1 - (-1)^{J'+J}\right).
\end{aligned} \tag{A.3}$$

The functions F_{CJ}, F_{LJ}, F_{MJ}, and F_{EJ} (F^5_{CJ}, F^5_{LJ}, F^5_{MJ}, and F^5_{EJ}) are the matrix elements of the multipole vector (axial-vector) Coulomb, longitudinal, transverse magnetic, and transverse electric operators defined as follows:

$$\begin{aligned} F_{CJ} &= \left\langle J_f \left\| \widehat{M}^{\text{coul}}_J \right\| J_i \right\rangle; \qquad F_{LJ} = \left\langle J_f \left\| \widehat{L}_J \right\| J_i \right\rangle; \\ F_{EJ} &= \left\langle J_f \left\| \widehat{T}^{\text{el}}_J \right\| J_i \right\rangle; \qquad F_{MJ} = \left\langle J_f \left\| i\widehat{T}^{m}_J \right\| J_i \right\rangle; \\ F^5_{CJ} &= \left\langle J_f \left\| i\widehat{M}^{\text{coul5}}_J \right\| J_i \right\rangle; \qquad F^5_{LJ} = \left\langle J_f \left\| i\widehat{L}^5_J \right\| J_i \right\rangle; \\ F^5_{EJ} &= \left\langle J_f \left\| i\widehat{T}^{\text{el5}}_J \right\| J_i \right\rangle; \qquad F^5_{MJ} = \left\langle J_f \left\| \widehat{T}^{m5}_J \right\| J_i \right\rangle. \end{aligned} \tag{A.4}$$

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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