Hindawi Publishing Corporation
Advances in High Energy Physics
Volume 2014, Article ID 252795, 7 pages
http://dx.doi.org/10.1155/2014/252795

Research Article

Semileptonic Transition of Tensor $\chi_{c2}(1P)$ to D_s Meson

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Received 13 May 2014; Revised 17 July 2014; Accepted 17 July 2014; Published 13 October 2014

Academic Editor: Kingman Cheung

Taking into account the two-gluon condensate corrections, the transition form factors of the semileptonic $\chi_{c2} \rightarrow D_s\bar{\ell}\nu(\ell = e, \mu)$ decay channel are calculated via three-point QCD sum rules. These form factors are used to estimate the decay width of the transition under consideration in both electron and muon channels. The obtained results can be used both in direct search for such decay channels at charm factories and in analysis of the B_c meson decay at LHC.

1. Introduction

Charmonium physics lies in the boundary region between perturbative and nonperturbative QCD. Hence, the study of the exclusive decay of charmonium such as χ_{cJ} states provides essential tools to test the perturbative and nonperturbative aspects of QCD. Recently, interest in charmonium spectroscopy has been renewed with the discovery of numerous charmonium and charmonium-like states. Compared to other states, we have very limited information about the charmonium χ_{cJ} $(J = 0, 1, 2)$ states and their decay properties. More experimental data and theoretical results on exclusive decay of P-wave charmonia are needed to better understand the decay dynamics of these states. The production and decay mechanisms of the χ_{cJ} states are actively being studied. Even though these states are not directly produced in e^+e^- collisions, they are produced abundantly in $\psi(3686)$ $E1$ transitions. The large $\psi(3686)$ data sample taken with BESIII supplies a good opportunity for a detailed study of χ_{cJ} states [1].

The χ_{c2} meson, whose inclusive decay is the subject of the present study, is a tensor $c\bar{c}$ bound state with quantum numbers $J^{PC} = 2^{++}$. This meson specially attracts the interest of experimentalists for testing the predictions of perturbative QCD in the laboratory [2–4]. The first observation of χ_{c2} was reported in B-decay at CLEO experiment in 2001 [5]. The measurements of the same collaboration on the two-photon decay rates of the even-parity, scalar $\chi_{b(c0)}$, and tensor $\chi_{b(c2)}$ states [6] were motivation to investigate the properties of these mesons and their radiative and strong decay (for a list see [7]) both theoretically and experimentally. Compared to their hadronic and radiative decay, the semileptonic decay of such states has not been studied more. As the semileptonic channels contribute significantly to the total decay width, more theoretical and experimental studies on these transitions are needed. In this respect, we analyze the semileptonic $\chi_{c2} \rightarrow D_s\bar{\ell}\nu$ transition with $(\ell = e, \mu)$ via three-point QCD sum rules in the present work. Taking into account the two-gluon condensate contribution, we calculate the transition form factors associated with this channel. The fit function of form factors is then used to estimate the corresponding decay width in both e and μ channels. Our results can be used in direct search for the semileptonic decay of the χ_{c2} meson at charm factories. It is expected that the semileptonic $B_c \rightarrow \chi_{c2} l\bar{\nu}$ decay has considerable contributions to the total decay width of the B_c meson [8]; hence, our results on the semileptonic decay of χ_{c2} state can also be used in analysis of the B_c meson decay at LHC.

The plan of this paper is as follows. In the next section, using an appropriate three-point correlation function, we derive QCD sum rules for the form factors defining the semileptonic $\chi_{c2} \rightarrow D_s\bar{\ell}\nu$ transitions. The last section is devoted to the numerical analysis of the form factors, determination of their behavior in terms of transferred momentum squared, estimation of the decay width of the transitions under consideration, and concluding remarks.

2. QCD Sum Rules for Transition Form Factor

QCD sum rule method has been a useful and successful nonperturbative tool to describe physical parameters of hadrons [9]. In this model, the hadronic parameters of the ground-state hadrons are extracted via equating the following two-alternative representations through a dispersion relation: the first is the operator product expansion (OPE) of the Borel-transformed correlation function of the two relevant currents and the second is the expression of the same correlation function calculated in terms of hadronic degrees of freedom.

One of the most efficient tools to do quantitative analysis of the semileptonic decay is based on their low-energy effective Hamiltonian. The effective Hamiltonian for the $\chi_{c2} \to D_s\bar{\ell}\nu$ decay, which is based on the three-level $c \to s$ transition at quark level, can be written as

$$\mathcal{H}_{\text{eff}}\left(c \longrightarrow s\bar{\ell}\nu_\ell\right) = \frac{G_F}{\sqrt{2}} V_{cs}\bar{c}\gamma_\mu\left(1-\gamma_5\right)s\bar{l}\gamma^\mu\left(1-\gamma_5\right)\nu, \quad (1)$$

where G_F is the Fermi coupling constant and V_{cs} is an element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. By sandwiching the effective Hamiltonian between the initial and final states we obtain the following matrix elements for the vector and axial-vector parts of the transition current $j_\mu^{\text{tr}} = \bar{c}\gamma_\mu(1-\gamma_5)s$, parameterized in terms of form factors,

$$\begin{aligned} \left\langle D_s\left(p'\right) \mid j_\mu^{\text{tr},V} \mid \chi_{c2}\left(p,\varepsilon\right)\right\rangle &= h\left(q^2\right)\epsilon_{\mu\nu\theta\eta}\epsilon^{\nu\lambda}P_\lambda P^\theta q^\eta, \\ \left\langle D_s\left(p'\right) \mid j_\mu^{\text{tr},A} \mid \chi_{c2}\left(p,\varepsilon\right)\right\rangle & \\ &= i\left\{K\left(q^2\right)\epsilon_{\mu\nu}P^\nu + \epsilon_{\theta\eta}P^\theta P^\eta\left[P_\mu b_+\left(q^2\right) + q_\mu b_-\left(q^2\right)\right]\right\}, \end{aligned} \quad (2)$$

where $h(q^2)$, $K(q^2)$, $b_+(q^2)$, and $b_-(q^2)$ are transition form factors, $\epsilon_{\theta\eta}$ is the polarization tensor of the χ_{c2} meson, $P_\mu = (p+p')_\mu$, and $q_\mu = (p-p')_\mu$. Our main task in the following is to calculate these transition form factors via QCD sum rule technique. For this aim, we consider the following three-point correlation function:

$$\begin{aligned} \Pi_{\mu\alpha\beta}\left(p,p',q\right) &= i^2\int d^4x e^{-ip\cdot x} \\ &\times \int d^4y e^{ip'\cdot y}\left\langle 0 \mid \mathcal{T} \mid j^{D_s}(y)\, j_\mu^{\text{tr},V(A)}(0)\, j_{\alpha\beta}^{\dagger\chi_{c2}}(x) \mid 0\right\rangle, \end{aligned} \quad (3)$$

where $\mathcal{T}$ is the time-ordered operator. To proceed we need to define the interpolating currents of the initial and final mesonic states. These interpolating fields can be written as

$$\begin{aligned} j^{D_s}(y) &= \bar{c}(y)\, i\gamma_5 s(y), \\ j_{\alpha\beta}^{\chi_{c2}}(x) &= \frac{i}{2}\left[\bar{c}(x)\gamma_\alpha \overleftrightarrow{\mathcal{D}}_\beta(x) c(x) + \bar{c}(x)\gamma_\beta \overleftrightarrow{\mathcal{D}}_\alpha(x) c(x)\right], \end{aligned} \quad (4)$$

where the two-side derivative $\overleftrightarrow{\mathcal{D}}_\beta(x)$ is defined as

$$\overleftrightarrow{\mathcal{D}}_\beta(x) = \frac{1}{2}\left[\overrightarrow{\mathcal{D}}_\beta(x) - \overleftarrow{\mathcal{D}}_\beta(x)\right], \quad (5)$$

with

$$\begin{aligned} \overrightarrow{\mathcal{D}}_\beta(x) &= \overrightarrow{\partial}_\beta(x) - i\frac{g}{2}\lambda^a A_\beta^a(x), \\ \overleftarrow{\mathcal{D}}_\beta(x) &= \overleftarrow{\partial}_\beta(x) + i\frac{g}{2}\lambda^a A_\beta^a(x). \end{aligned} \quad (6)$$

Here, λ^a are the Gell-Mann matrices and $A_\beta^a(x)$ are the external gluon fields. Considering the Fock-Schwinger gauge ($x^\beta A_\beta^a(x) = 0$), these external fields are expressed in terms of the gluon field strength tensor in the following way:

$$\begin{aligned} A_\beta^a(x) &= \int_0^1 d\alpha\alpha x_\beta G_{\beta\nu}^a(\alpha x) \\ &= \frac{1}{2}x_\beta G_{\beta\nu}^a(0) + \frac{1}{3}x_\eta x_\beta \mathcal{D}_\eta G_{\beta\nu}^a(0) + \cdots. \end{aligned} \quad (7)$$

In order to calculate the hadronic side of the aforementioned correlation function, we will insert appropriate complete sets of intermediate states with the same quantum numbers as the mentioned interpolating fields into (3). After performing integrals over four x and y, we get

$$\begin{aligned} \Pi_{\mu\alpha\beta}^{\text{HAD}}\left(p,p',q\right) &= \left\langle 0 \mid j^{D_s}(0) \mid D_s\left(p'\right)\right\rangle \\ &\times \left\langle D_s\left(p'\right) \mid j_\mu^{\text{tr},V(A)}(0) \mid \chi_{c2}\left(p,\varepsilon\right)\right\rangle \\ &\times \left\langle \chi_{c2}\left(p,\varepsilon\right) \mid j_{\alpha\beta}^{\dagger\chi_{c2}}(0) \mid 0\right\rangle \\ &\times \left(\left(p'^2 - m_{D_s}^2\right)\left(p^2 - m_{\chi_{c2}}^2\right)\right)^{-1} + \cdots, \end{aligned} \quad (8)$$

where $\cdots$ symbolizes the contribution of higher states and the continuum. To proceed, we need to know the matrix elements $\langle 0 \mid j^{D_s}(0) \mid D_s(p')\rangle$ and $\langle \chi_{c2}(p,\varepsilon) \mid j_{\alpha\beta}^{\dagger\chi_{c2}}(0) \mid 0\rangle$, which are defined in terms of the decay constants, masses, and polarization tensor of the initial state

$$\begin{aligned} \left\langle 0 \mid j^{D_s}(0) \mid D_s\left(p'\right)\right\rangle &= i\frac{f_{D_s} m_{D_s}^2}{m_c + m_s}, \\ \left\langle \chi_{c2}\left(p,\varepsilon\right) \mid j_{\alpha\beta}^{\dagger\chi_{c2}}(0) \mid 0\right\rangle &= f_{\chi_{c2}} m_{\chi_{c2}}^3 \varepsilon_{\alpha\beta}^{*\lambda}, \end{aligned} \quad (9)$$

where $f_{\chi_{c2}}$ and f_{D_s} are leptonic decay constants of χ_{c2} and D_s mesons, respectively. Combining all matrix elements given in (2) and (9) in (8), the final representation of the correlation function for the hadronic side is obtained as

$$\begin{aligned}\Pi^{\mathrm{HAD}}_{\mu\alpha\beta}\left(p,p',q\right) &= \frac{f_{\chi_{c2}} f_{D_s} m_{\chi_{c2}} m_{D_s}^2}{8\left(m_c+m_s\right)\left(p'^2-m_{D_s}^2\right)\left(p^2-m_{\chi_{c2}}^2\right)} \\ &\times \left\{\frac{2}{3}\left[-\Delta K\left(q^2\right)-\Delta' b_-\left(q^2\right)\right] q_\mu g_{\beta\alpha} \right. \\ &+ \frac{2}{3}\left[\left(-\Delta+4m_{\chi_{c2}}^2\right)K\left(q^2\right)-\Delta' b_+\left(q^2\right)\right]P_\mu g_{\beta\alpha} \\ &- i\left(\Delta-4m_{\chi_{c2}}^2\right)h\left(q^2\right)\varepsilon_{\lambda\nu\beta\mu}P_\lambda P_\alpha q_\nu \\ &\left. + \Delta K\left(q^2\right) q_\alpha g_{\beta\mu} + \text{other structures}\right\} + \cdots,\end{aligned} \tag{10}$$

where

$$\begin{aligned}\Delta &= m_{D_s}^2 + 3m_{\chi_{c2}}^2 - q^2, \\ \Delta' &= m_{D_s}^4 - 2m_{D_s}^2\left(m_{\chi_{c2}}^2+q^2\right)+\left(m_{\chi_{c2}}^2-q^2\right)^2,\end{aligned} \tag{11}$$

and we have held only the structures which we are going to choose in order to find the corresponding form factors. Note that, for obtaining the above representation, we have performed summation over the polarization tensor using

$$\sum_\lambda \varepsilon_{\mu\nu}^{\lambda}\varepsilon_{\alpha\beta}^{*\lambda} = \frac{1}{2}\eta_{\mu\alpha}\eta_{\nu\beta}+\frac{1}{2}\eta_{\mu\beta}\eta_{\nu\alpha}-\frac{1}{3}\eta_{\mu\nu}\eta_{\alpha\beta}, \tag{12}$$

where

$$\eta_{\mu\nu} = -g_{\mu\nu}+\frac{p_\mu p_\nu}{m_{\chi_{c2}}^2}. \tag{13}$$

In OPE side, the correlation function is calculated in deep Euclidean region (see, for instance, [10, 11]). Placing the explicit forms of the interpolating currents into the correlation function and contracting out all quark pairs via Wick's theorem, we obtain

$$\begin{aligned}\Pi^{\mathrm{OPE}}_{\mu\alpha\beta}\left(p,p',q\right) &= \frac{i^3}{4}\int d^4x \\ &\times \int d^4y e^{-ip\cdot x}e^{ip'\cdot y} \\ &\times \left\{\mathrm{Tr}\left[\gamma_5 S_s^{kj}\left(y\right)\gamma_\mu\left(1-\gamma_5\right)S_c^{ji}\left(-x\right)\right.\right. \\ &\left.\left.\times \gamma_\alpha \overleftrightarrow{\mathscr{D}}_\beta\left(x\right)S_c^{ik}\left(x-y\right)\right]+\left[\beta\longleftrightarrow\alpha\right]\right\},\end{aligned} \tag{14}$$

where S_c and S_q are the heavy and light quark propagator, respectively. They are given by [12]

$$\begin{aligned}S_c^{ij}\left(x\right) &= \frac{i}{\left(2\pi\right)^4} \\ &\times \int d^4k e^{-ik\cdot x} \\ &\times \left\{\frac{\delta_{ij}}{\not{k}-m_c} - \frac{g_s G_{ij}^{\alpha\beta}}{4}\frac{\sigma_{\alpha\beta}\left(\not{k}+m_c\right)+\left(\not{k}+m_c\right)\sigma_{\alpha\beta}}{\left(k^2-m_c^2\right)^2}\right. \\ &\left. + \frac{\pi^2}{3}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle \delta_{ij} m_c \frac{k^2+m_c\not{k}}{\left(k^2-m_c^2\right)^4}+\cdots\right\}, \\ S_s^{ij}\left(x\right) &= i\frac{\not{x}}{2\pi^2x^4}\delta_{ij}-\frac{m_s}{4\pi^2x^2}\delta_{ij}-\frac{\langle\bar{s}s\rangle}{12}\left(1-i\frac{m_s}{4}\not{x}\right)\delta_{ij} \\ &- \frac{x^2}{192}m_0^2\langle\bar{s}s\rangle\left(1-i\frac{m_s}{6}\not{x}\right)\delta_{ij} \\ &- i\frac{ig_s}{32\pi^2x^2}G_{\mu\nu}^{ij}\left(\not{x}\sigma^{\mu\nu}+\sigma^{\mu\nu}\not{x}\right)+\cdots.\end{aligned} \tag{15}$$

Despite being very small compared to the perturbative part, we include the contribution coming from the gluon condensate terms as nonperturbative effects. The correlation function in OPE side is also written as

$$\begin{aligned}\Pi^{\mathrm{OPE}}_{\mu\alpha\beta}\left(p,p',q\right) = &\left(\Pi_1^{\mathrm{pert}}\left(q^2\right)+\Pi_1^{\text{non-pert}}\left(q^2\right)\right)q_\alpha g_{\beta\mu} \\ &+ \left(\Pi_2^{\mathrm{pert}}\left(q^2\right)+\Pi_2^{\text{non-pert}}\left(q^2\right)\right)q_\mu g_{\beta\alpha} \\ &+ \left(\Pi_3^{\mathrm{pert}}\left(q^2\right)+\Pi_3^{\text{non-pert}}\left(q^2\right)\right)P_\mu g_{\beta\alpha} \\ &+ \left(\Pi_4^{\mathrm{pert}}\left(q^2\right)+\Pi_4^{\text{non-pert}}\left(q^2\right)\right)\varepsilon_{\lambda\nu\beta\mu}P_\lambda P_\alpha q_\nu \\ &+ \text{other structures},\end{aligned} \tag{16}$$

where $\Pi_i^{\mathrm{pert}}(q^2)$ with $i = 1, 2, 3, 4$ are the perturbative parts of the coefficients of the selected structures. They are expressed in terms of double dispersion integrals as

$$\Pi_i^{\mathrm{pert}}\left(q^2\right) = \int ds\int ds'\frac{\rho_i\left(s,s',q^2\right)}{\left(s-p^2\right)\left(s'-p'^2\right)} + \text{subtracted terms}, \tag{17}$$

where the spectral densities $\rho_i(s,s',q^2)$ are obtained by taking the imaginary parts of the Π_i^{pert} functions; that is, $\rho_i(s,s',q^2) = (1/\pi)\,\mathrm{Im}[\Pi_i^{\mathrm{pert}}]$. Replacing the explicit expressions of the above propagators into (14) and performing integrals over four x and y we find the spectral densities corresponding to four different Dirac structures as

$$\begin{aligned}
\rho_1\left(s,s',q^2\right) &= \frac{3}{32\pi^2}\int_0^1 dx\int_0^{1-x} dy\left[m_s\left(4x+2y-3\right)\right. \\
&\qquad\left.+m_c\left(8x+4y-5\right)\right], \\
\rho_2\left(s,s',q^2\right) &= \frac{3}{16\pi^2}\int_0^1 dx \\
&\quad\times\int_0^{1-x} dy\left[-m_c-m_s\left(4x+2y-3\right)\right], \\
\rho_3\left(s,s',q^2\right) &= \frac{3}{16\pi^2}\int_0^1 dx\int_0^{1-x} dy\left[-m_c-m_s+2m_s y\right], \\
\rho_4\left(s,s',q^2\right) &= 0.
\end{aligned} \tag{18}$$

From a similar way we also calculate the functions $\Pi_i^{\text{non-pert}}(q^2)$.

The QCD sum rules for the form factors are obtained by matching the coefficients of the same structures from both sides of the correlation function. To suppress the contributions of the higher states and continuum, the double Borel transformation with respect to quantities p^2 and p'^2 is applied to both sides of the obtained sum rules according to the following rule:

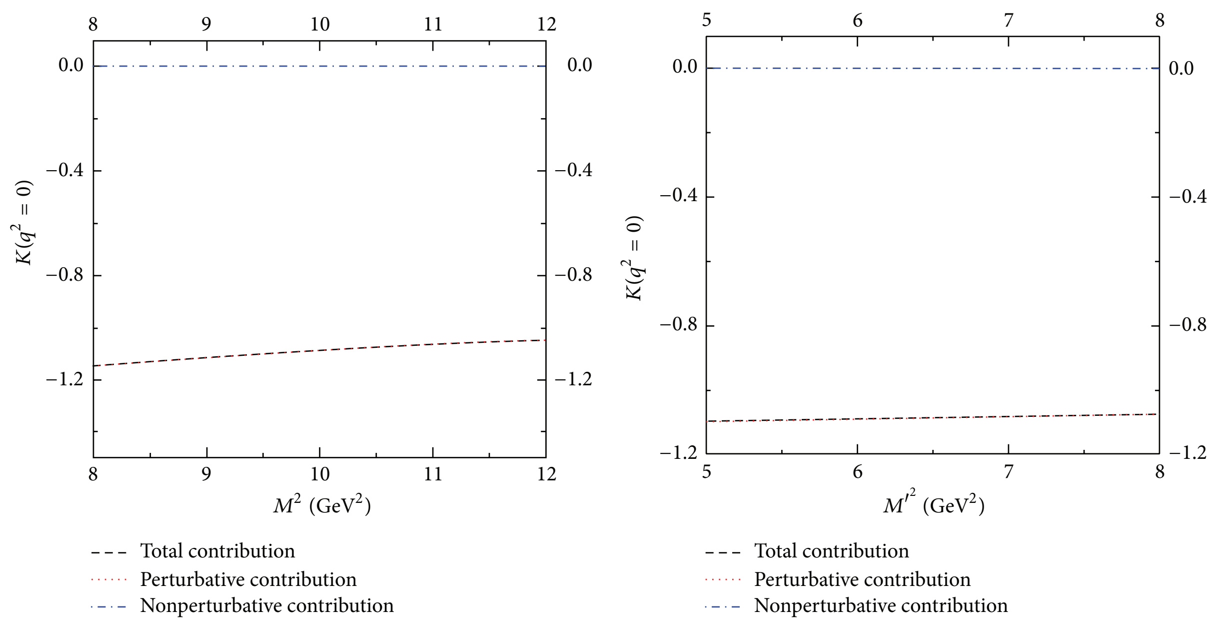
$$\begin{aligned}
&\widehat{\mathscr{B}}_{M^2}\widehat{\mathscr{B}}_{M'^2}\frac{1}{\left(p^2-m_1^2\right)^a}\frac{1}{\left(p'^2-m_2^2\right)^b} \\
&\quad\longrightarrow(-1)^{a+b}\frac{\left(M^2\right)^{a-1}\left(M'^2\right)^{b-1}}{\Gamma(a)\Gamma(b)}e^{-m_1^2/M^2}e^{-m_2^2/M'^2},
\end{aligned} \tag{19}$$

where M^2 and M'^2 are the Borel mass parameters. To further suppress the contributions of the higher state and continuum, we perform continuum subtraction and use the quark-hadron duality assumption. As a result, we get the following sum rules for the form factors:

$$\begin{aligned}
&K\left(q^2\right) \\
&\quad=\frac{8\left(m_c+m_s\right)}{f_{\chi_{c2}}f_{D_s}m_{D_s}^2 m_{\chi_{c2}}\Delta}e^{m_{\chi_{c2}}^2/M^2}e^{m_{D_s}^2/M'^2} \\
&\quad\quad\times\left\{\int_{4m_c^2}^{s_0}ds\right. \\
&\quad\quad\quad\times\int_{(m_c+m_s)^2}^{s_0'}ds'e^{-s/M^2}e^{-s'/M'^2}\rho_1\left(s,s',q^2\right) \\
&\quad\quad\quad\quad\left.\times\,\theta\left[L\left(s,s',q^2\right)\right]+\widehat{\mathscr{B}}_{M^2}\widehat{\mathscr{B}}_{M'^2}\Pi_1^{\text{non-pert}}\right\}, \\
&b_-\left(q^2\right) \\
&\quad=-\frac{12\left(m_c+m_s\right)}{f_{\chi_{c2}}f_{D_s}m_{D_s}^2 m_{\chi_{c2}}\Delta'}e^{m_{\chi_{c2}}^2/M^2}e^{m_{D_s}^2/M'^2} \\
&\quad\quad\times\left\{\int_{4m_c^2}^{s_0}ds\right. \\
&\quad\quad\quad\times\int_{(m_c+m_s)^2}^{s_0'}ds'e^{-s/M^2}e^{-s'/M'^2}\rho_2\left(s,s',q^2\right) \\
&\quad\quad\quad\quad\left.\times\,\theta\left[L\left(s,s',q^2\right)\right]+\widehat{\mathscr{B}}_{M^2}\widehat{\mathscr{B}}_{M'^2}\Pi_2^{\text{non-pert}}\right\} \\
&\quad\quad-\frac{\Delta}{\Delta'}K\left(q^2\right), \\
&b_+\left(q^2\right) \\
&\quad=-\frac{12\left(m_c+m_s\right)}{f_{\chi_{c2}}f_{D_s}m_{D_s}^2 m_{\chi_{c2}}\Delta'}e^{m_{\chi_{c2}}^2/M^2}e^{m_{D_s}^2/M'^2} \\
&\quad\quad\times\left\{\int_{4m_c^2}^{s_0}ds\right. \\
&\quad\quad\quad\times\int_{(m_c+m_s)^2}^{s_0'}ds'e^{-s/M^2}e^{-s'/M'^2}\rho_3\left(s,s',q^2\right) \\
&\quad\quad\quad\quad\left.\times\,\theta\left[L\left(s,s',q^2\right)\right]+\widehat{\mathscr{B}}_{M^2}\widehat{\mathscr{B}}_{M'^2}\Pi_3^{\text{non-pert}}\right\} \\
&\quad\quad+\frac{-\Delta+4m_{\chi_{c2}}^2}{\Delta'}K\left(q^2\right), \\
&h\left(q^2\right) \\
&\quad=-\frac{8\left(m_c+m_s\right)}{f_{\chi_{c2}}f_{D_s}m_{D_s}^2 m_{\chi_{c2}}\left(\Delta-4m_{\chi_{c2}}^2\right)}e^{m_{\chi_{c2}}^2/M^2}e^{m_{D_s}^2/M'^2} \\
&\quad\quad\times\left\{\int_{4m_c^2}^{s_0}ds\right. \\
&\quad\quad\quad\times\int_{(m_c+m_s)^2}^{s_0'}ds'e^{-s/M^2}e^{-s'/M'^2}\rho_4\left(s,s',q^2\right) \\
&\quad\quad\quad\quad\left.\times\,\theta\left[L\left(s,s',q^2\right)\right]+\widehat{\mathscr{B}}_{M^2}\widehat{\mathscr{B}}_{M'^2}\Pi_4^{\text{non-pert}}\right\},
\end{aligned} \tag{20}$$

where s_0 and s_0' are continuum thresholds in the initial and final channels, respectively. The function $L(s,s',q^2)$ is given by

$$\begin{aligned}
L\left(s,s',q^2\right) &= s'y\left(1-x-y\right)-m_c^2\left(x+y\right) \\
&\quad+q^2x\left(1-x-y\right)+sxy.
\end{aligned} \tag{21}$$



(a) (b)

Figure 1: (a) $K(q^2 = 0)$ as a function of the Borel mass parameter M^2 at $M'^2 = 6.5\,\mathrm{GeV}^2$. (b) $K(q^2 = 0)$ as a function of the Borel mass parameter M'^2 at $M^2 = 10\,\mathrm{GeV}^2$.

The functions $\widehat{\mathscr{B}}_{M^2}\widehat{\mathscr{B}}_{M'^2}\Pi_i^{\text{non-pert}}$ are very lengthy; hence we do not present their explicit expressions here. We should stress that the contributions of the light quark condensates are eliminated by applying the double Borel transformations with respect to the initial and final momenta; hence, in the $\widehat{\mathscr{B}}_{M^2}\widehat{\mathscr{B}}_{M'^2}\Pi_i^{\text{non-pert}}$ functions we only consider the two-gluon condensate contributions.

3. Numerical Results and Discussion

In this section, we present our numerical results for the form factors of the semileptonic $\chi_{c2} \rightarrow D_s\bar{\ell}\nu$ transition whose sum rules have been found in the previous section. For this aim, we use the following input parameters: $m_c = (1.275 \pm 0.025)\,\mathrm{GeV}$, $m_s = 95 \pm 5\,\mathrm{MeV}$ [7], $G_F = 1.17 \times 10^{-5}\,\mathrm{GeV}^{-2}$, $\langle \alpha_s G^2/\pi \rangle = (0.012 \pm 0.004)\,\mathrm{GeV}^4$, $\langle \bar{s}s(1\,\mathrm{GeV})\rangle = -0.8(0.24 \pm 0.01)^3\,\mathrm{GeV}^3$ [13, 14], $m_0^2(1\,\mathrm{GeV}) = (0.8 \pm 0.2)\,\mathrm{GeV}^2$ [15, 16], $f_{D_s} = 245 \pm 15.7 \pm 4.5\,\mathrm{MeV}$ [17], and $f_{\chi_{c2}} = 0.0111 \pm 0.0062$ [18].

The sum rules for the form factors also contain four auxiliary parameters: two Borel mass parameters M^2 and M'^2 as well as two continuum thresholds s_0 and s_0'. According to the criteria of the method the physical quantities such as form factors should be independent of these parameters. Hence, we will look for regions such that the dependence of form factors on these helping parameters is weak. The continuum thresholds s_0 and s_0' are not totally arbitrary, but they depend on the energy of the first excited states with the same quantum numbers as the interpolating currents of the initial and final channels, respectively. Our numerical calculations reveal that, in the intervals $13\,\mathrm{GeV}^2 \leq s_0 \leq 15\,\mathrm{GeV}^2$ and $4.5\,\mathrm{GeV}^2 \leq s_0' \leq 6\,\mathrm{GeV}^2$, the form factors demonstrate weak dependence on continuum thresholds.

The working region for the Borel mass parameters is determined with the requirements that not only the higher states and continuum contributions are suppressed but also the contributions of the higher order operators are small; that is, the sum rules are convergent. From these restrictions, the working regions for the Borel parameters are found to be $8\,\mathrm{GeV}^2 \leq M^2 \leq 12\,\mathrm{GeV}^2$ and $5\,\mathrm{GeV}^2 \leq M'^2 \leq 8\,\mathrm{GeV}^2$. Note that the above regions for the continuum thresholds are obtained according to the standard criteria of the QCD sum rules; that is, the continuum thresholds are independent of Borel mass parameters. However, some recent works (see, for instance, [19]) show that the standard criteria do not lead to correct results and the continuum thresholds should be taken as functions of Borel masses. Following [19], we will add an extra 15% systematic error to the uncertainties of the form factors.

As an example, we depict the dependence of form factor $K(q^2)$ on M^2 and M'^2 in Figure 1 at $q^2 = 0$. From this figure we observe that the form factor $K(q^2)$ demonstrates a good stability with respect to the variations of Borel mass parameters in their working regions. It is also clear that the perturbative part exceeds the nonperturbative one substantially and constitutes approximately the whole contribution.

Having determined the working regions for the auxiliary parameters, we proceed to find the behaviors of the form factors in terms of q^2. The sum rules for the form factors are truncated at some points below the perturbative cut; so to extend our results to the full physical region, we look for a parameterization of the form factors such that its results coincide with the results of sum rules at reliable region. Our

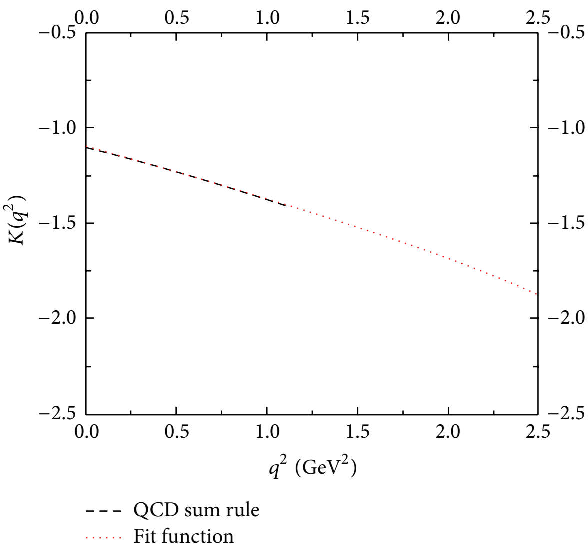


FIGURE 2: $K(q^2)$ as a function of q^2 at $M^2 = 10\,\mathrm{GeV}^2$ and $M'^2 = 6.5\,\mathrm{GeV}^2$.

analysis shows that the form factors are well fitted to the function

$$f\left(q^2\right) = f_0 \exp\left[c_1 \frac{q^2}{m_{\rm fit}^2} + c_2\left(\frac{q^2}{m_{\rm fit}^2}\right)^2\right], \qquad (22)$$

where the values of the parameters, f_0, c_1, c_2, and $m_{\rm fit}^2$, obtained using $M^2 = 10\,\mathrm{GeV}^2$ and $M'^2 = 6.5\,\mathrm{GeV}^2$ for $\chi_{c2} \rightarrow D_s\bar{\ell}\nu$ transition, are presented in Table 1. The errors appearing in the results belong to the uncertainties in the input parameters, those coming from determination of the working regions of the auxiliary parameters and the previously discussed systematic uncertainties. As an example we depict the dependence of the form factor $K(q^2)$ on q^2 at $M^2 = 10\,\mathrm{GeV}^2$ and $M'^2 = 6.5\,\mathrm{GeV}^2$ in Figure 2, which shows a good fitting of the sum rules results to those obtained from the above fit function.

Our final purpose in this section is to obtain the decay width of the $\chi_{c2} \rightarrow D_s\bar{\ell}\nu$ transition. The differential decay width for this transition is obtained as

$$\begin{aligned}
\frac{d\Gamma}{dq^2} &= \frac{G_F^2 V_{cs}^2}{2^{10} 3^2 m_{\chi_{c2}}^7 \pi^3 q^6}\left(m_\ell^2 - q^2\right)^2 \Delta'^{3/2} \\
&\times \Big\{ \left|b_-\left(q^2\right)\right|^2 \Delta' m_\ell^2 q^4 + \left|b_+\left(q^2\right)\right|^2 \\
&\quad \times \Delta' \left[\left(m_{D_s}^2 - m_{\chi_{c2}}^2\right)^2 m_\ell^2 + \left(m_{D_s}^2 - m_{\chi_{c2}}^2\right)^2 q^2 \right. \\
&\qquad \left. - 2\left(m_{D_s}^2 + m_{\chi_{c2}}^2\right) q^4 + q^6\right] \\
&+ 2\,\mathrm{Re}\left[K\left(q^2\right) b_+^*\left(q^2\right)\right] \\
&\times \Delta' \left[-q^4 + m_{D_s}^2\left(m_\ell^2 + q^2\right) - m_{\chi_{c2}}^2\left(m_\ell^2 + q^2\right)\right] \\
&- 2\,\mathrm{Re}\left[b_-\left(q^2\right) b_+^*\left(q^2\right)\right] \Delta' m_\ell^2 q^2 \left(m_{D_s}^2 - m_{\chi_{c2}}^2\right) \\
&+ \left|K\left(q^2\right)\right|^2 \\
&\times \left[m_{D_s}^4\left(m_\ell^2 + q^2\right) + m_{\chi_{c2}}^4\left(m_\ell^2 + q^2\right) \right. \\
&\quad + q^4\left(m_\ell^2 + q^2\right) - 2 m_{D_s}^2 \\
&\qquad \left. \times \left(m_{\chi_{c2}}^2 + q^2\right)\left(m_\ell^2 + q^2\right) + m_{\chi_{c2}}^2 q^2 \left(m_\ell^2 + 5q^2\right)\right] \\
&+ 3\left|h\left(q^2\right)\right|^2 \Delta' m_{\chi_{c2}}^2 q^2 \left(m_\ell^2 + q^2\right) \\
&- 2\,\mathrm{Re}\left[K\left(q^2\right) b_-^*\left(q^2\right)\right] \Delta' m_\ell^2 q^2 \Big\}.
\end{aligned} \qquad (23)$$

TABLE 1: Parameters appearing in the fit function of the form factors.

	f_0	c_1	c_2	$m_{\rm fit}^2$
$K(q^2)$	$-(1.09 \pm 0.38)$	2.90	−0.68	12.65
$b_-(q^2)$	$2.05 \pm 0.72\,\mathrm{GeV}^{-2}$	7.01	21.86	12.65
$b_+(q^2)$	$2.42 \pm 0.85\,\mathrm{GeV}^{-2}$	5.63	129.51	12.65
$h(q^2)$	$-(4.42 \pm 1.55) \times 10^{-9}\,\mathrm{GeV}^{-2}$	−1.91	1.11	12.65

TABLE 2: Numerical results of decay width for different lepton channels.

	Γ (GeV)
$\chi_{c2} \rightarrow D_s\bar{\mu}\nu_\mu$	$(1.60 \pm 0.67) \times 10^{-11}$
$\chi_{c2} \rightarrow D_s\bar{e}\nu_e$	$(1.62 \pm 0.68) \times 10^{-11}$

After performing integration over q^2 in (23) in the interval $m_\ell^2 \leq q^2 \leq (m_{\chi_{c2}} - m_{D_s})^2$, we obtain the decay widths in both e and μ channels as presented in Table 2. Considering the developments in experimental side we hope that it will be possible to study such decay channels in the experiment in near future. Comparison of future data with theoretical calculations will help us get useful information on the structure of χ_{c2} tensor meson as well as the perturbative and nonperturbative aspects of QCD. The obtained results in this work can also be used in the analysis of the B_c meson decay at LHC as the $B_c \rightarrow \chi_{c2}$ is expected to have a considerable contribution.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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