# Study of $\chi_{c 0}(1 P) \rightarrow J / \psi \gamma$ and $\chi_{b 0}(1 P) \rightarrow \Upsilon(1 S) \gamma$ Decays via QCD Sum Rules 

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Received 19 February 2014; Revised 18 April 2014; Accepted 7 May 2014; Published 25 May 2014
Academic Editor: Alexey A. Petrov
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#### Abstract

We present the calculation of the form factor of the $\chi_{c 0}(1 P) \rightarrow J / \psi \gamma$ and $\chi_{b 0}(1 P) \rightarrow \Upsilon(1 S) \gamma$ decays in the frame work of QCD sum rules. We also find branching ratio $\mathscr{B}_{r}\left(\chi_{c 0}(1 P) \rightarrow J / \psi \gamma\right)=\left(1.36_{-0.6}^{+0.9}\right) \times 10^{-2}$ which is in agreement with the experimental data. Furthermore, we estimate the $\Gamma_{\text {tot }}\left(\chi_{b 0}(1 P)\right)=4.2_{-2.21}^{+3.18} \mathrm{MeV}$, where experimental bound for full width of $\chi_{b 0}$ is $\Gamma_{\text {tot }}\left(\chi_{b 0}(1 P)\right)<6 \mathrm{MeV}$.


## 1. Introduction

Heavy quarkonium states (like $b \bar{b}$ and $c \bar{c}$ ) and their decay modes offer a laboratory to study the strong interaction in the nonperturbative regime. Charmonium in particular has served as a calibration tool for the corresponding techniques and models [1, 2]. Heavy quarkonium states can have many bound states and decay channels used to study and determine different parameters of standard model (SM) and QCD from the theoretical perspective. In particular, the calculation of bottomonium masses [3], total widths, coupling constants [4-7], and branching ratio can serve as benchmarks for the low energy predictions of QCD. In addition, the theoretical calculations on the branching ratio of radiative decays of heavy quarkonium states are relatively clean with respect to the hadronic or semileptonic decays, and their comparison with experimental data could provide important insights into their nature and hyperfine interaction. In this regard, the decay width and branching ratio of radiative decays of several $c \bar{c}$ and $b \bar{b}$ states analysed by using an effective Lagrangian approach valid for heavy quarkonia $[8,9]$.

The QCD sum rules have been used for the radiative transitions in charmonium and bottomonium cases [10-15]. But exclusive $\chi_{c 0} \rightarrow J / \psi \gamma$ and $\chi_{b 0} \rightarrow \Upsilon \gamma$ decays have not
yet been studied in the framework of the QCD sum rules. The $\chi_{c 0} \rightarrow J / \psi \gamma$ decay studied by CLEO detector operating at CESR [16, 17]. The next experimental studies of exclusive $\chi_{c 0} \rightarrow J / \psi \gamma$ and $\chi_{b 0} \rightarrow \Upsilon \gamma$ decays are going to start with next operation of LHC.

We present the theoretical study on the form factor of exclusive $\chi_{c 0} \rightarrow J / \psi \gamma$ and $\chi_{b 0} \rightarrow \quad \Upsilon \gamma$ decays in the frame work of QCD sum rules. Note that in order to calculate the branching ratio we have to acquire information about the masses and decay constants of the participating particles. It is worth mentioning that the masses can be obtained either by means of the experimental results, that is, the particle data group or by the theoretical methods. The decay constants, on the other hand, can be calculated theoretically via different nonperturbative methods. In this respect, masses and decay constants and spectrum of heavy quarkonium states are calculated in the various approaches (see, e.g., $[1,18-25])$. Here, firstly, we calculate the form factor of $\chi_{c 0} \rightarrow J / \psi \gamma$ and $\chi_{b 0} \rightarrow \Upsilon \gamma$ decays in the framework of three-point QCD sum rules; secondly, we calculate the branching ratio of exclusive $\chi_{c 0} \rightarrow J / \psi \gamma$ and $\chi_{b 0} \rightarrow \Upsilon \gamma$ decays.

In Section 2, we introduce the QCD sum rules technique for the form factors of exclusive $\chi_{c 0} \rightarrow J / \psi \gamma$ and $\chi_{b 0} \rightarrow \Upsilon \gamma$
decays. Last section is devoted to the numerical analysis and discussion.

## 2. QCD Sum Rules for the Form Factors

The three-point correlation function associated with the $\chi_{c 0}(1 P) \rightarrow J / \psi \gamma$ and $\chi_{b 0}(1 P) \rightarrow \Upsilon(1 S) \gamma$ vertex is given by

$$
\begin{align*}
\Pi_{\mu \nu}=i^{2} \int & d^{4} x d^{4} y e^{-i p \cdot x+i p^{\prime} \cdot y}  \tag{1}\\
& \times\langle 0| \mathscr{T}\left(j_{\mu}^{V}(y) j_{\nu}^{e m}(x) \bar{j}^{S}(0)\right)|0\rangle
\end{align*}
$$

where $\mathscr{T}$ is the time ordering operator and $q$ is momentum of photon. Each meson and photon field can be described in terms of the quark field operators as follows:

$$
\begin{gather*}
j_{\mu}^{V}(y)=\overline{c(b)}(y) \gamma_{\mu} c(b)(y), \\
j^{S}(x)=\overline{c(b)}(x) c(b)(x)  \tag{2}\\
j_{\mu}^{e m}(x)=Q_{c(b)} \overline{c(b)}(x) \gamma_{\mu} c(b)(x)
\end{gather*}
$$

We calculate the correlation function equation (1) in two different methods. In phenomenological or physical approach, it can be evaluated in terms of hadronic parameters such as masses, decay constants, and form factors. In theoretical or QCD side, on the other hand, it is calculated in terms of QCD parameters, which are quark and gluon degrees of freedom, by the help of the operator product expansion (OPE) in deep Euclidean region. Equating the structure calculated in two different approaches of the same correlation function, we get a relation between hadronic parameters and QCD degrees of freedom. Finally, we apply double Borel transformation with respect to the momentum of initial and final mesons ( $p^{2}$ and $p^{\prime 2}$ ). This final operation suppresses the contribution of the higher states and continuum.
2.1. Phenomenological Side. We insert the complete sets of appropriate vector meson $(|V\rangle\langle V|)$ and scalar meson $(|S\rangle\langle S|)$ states (regarding the conservation of the quantum numbers of corresponding interpolating currents) inside correlation functions equation (1). Here, vector state is either $J / \psi$ or $\Upsilon$ and scalar state is $\chi_{c(b)}$ state. After integrating over the $x$ and $y$, we get the following result for the correlation function equation (1):

$$
\begin{equation*}
\Pi_{\mu \nu}=\frac{\langle 0| j^{s}(x)|S\rangle\langle S| j_{\nu}^{e m}(x)|V\rangle\langle V| \bar{j}_{\mu}^{V}|0\rangle}{\left(m_{V}^{2}-p^{\prime 2}\right)\left(m_{S}^{2}-p^{2}\right)}+\cdots, \tag{3}
\end{equation*}
$$

where $\cdots$ contains the contribution of the higher and continuum states with the same quantum numbers.

The matrix elements of the above equation are related to the hadronic parameters as follows:

$$
\begin{gathered}
\langle 0| j_{\mu}^{V}(x)|V\rangle=m_{V} f_{V} \epsilon_{\mu}^{\prime} \\
\langle S| \bar{j}^{S}|0\rangle=i m_{S} f_{S}^{*} \\
\langle S| j_{\nu}^{e m}(x)|V\rangle=e F\left(q^{2}=0\right)\left\{\left(p^{\prime} \cdot q\right) \epsilon_{v}^{\prime}-\left(q \cdot \epsilon^{\prime}\right) p_{\nu}^{\prime}\right\},
\end{gathered}
$$

where $F\left(q^{2}\right)$ is the form factor of transition and $\epsilon^{\prime}$ is the polarization vector associated with the vector meson. Using (4) in (3) and considering the summation over polarization vectors via

$$
\begin{gather*}
\epsilon_{\nu} \epsilon_{\theta}^{*}=-g_{\nu \theta} \\
\epsilon_{j}^{\prime} \epsilon_{\mu}^{\prime *}=-g_{j \mu}+\frac{p_{j}^{\prime} p_{\mu}^{\prime}}{m_{V}^{2}} \tag{5}
\end{gather*}
$$

the result of the physical side is as follows:

$$
\begin{equation*}
\Pi_{\mu \nu}=-\frac{e m_{V} f_{V} m_{S} f_{S}^{*}}{\left(m_{V}^{2}-p^{\prime 2}\right)\left(m_{S}^{2}-p^{2}\right)} F(0)\left(p^{\prime} \cdot q\right) g_{\mu \nu}+\cdots \tag{6}
\end{equation*}
$$

We are going to compare the coefficient of $g_{\mu \nu}$ structure for further calculation from different approaches of the correlation functions.
2.2. Theoretical (QCD) Side. Theoretical side consists of perturbative (bare loop; see Figure 1) and nonperturbative parts (the contributions of two gluon condensate diagrams, Figure 2). We calculate it in the deep Euclidean space ( $p^{2} \rightarrow$ $-\infty$ and $\left.p^{\prime 2} \rightarrow-\infty\right)$. We consider this side as

$$
\begin{equation*}
\Pi_{\mu \nu}\left(p^{\prime}, p\right)=\left(\Pi_{\mathrm{per}}+\Pi_{\text {nonper }}\right)\left(p^{\prime} \cdot q\right) g_{\mu v} \tag{7}
\end{equation*}
$$

2.2.1. Bare Loop. The perturbative part is a double dispersion integral as follows:

$$
\begin{align*}
\Pi_{\text {per }}= & -\frac{1}{4 \pi^{2}} \int d s^{\prime} \int d s \frac{\rho\left(s, s^{\prime}, q^{2}\right)}{\left(s-p^{2}\right)\left(s^{\prime}-p^{\prime 2}\right)}  \tag{8}\\
& + \text { subtraction terms }
\end{align*}
$$

where $\rho\left(s, s^{\prime}, q^{2}\right)$ is called spectral density. We aim to evaluate the spectral density with the help of the bare loop diagram in Figure 1. One of the generic methods to calculate this bare loop integral is the Cutkosky method, where the quark propagators of Feynman integrals are replaced by the Dirac delta functions:

$$
\begin{equation*}
\frac{1}{q^{2}-m^{2}} \longrightarrow(-2 \pi i) \delta\left(q^{2}-m^{2}\right) \tag{9}
\end{equation*}
$$

Then, using the Cutkosky method we get spectral density as

$$
\begin{equation*}
\rho\left(s, s^{\prime}, q^{2}\right)=\frac{2 m_{c(b)} N_{c}\left(-4 m_{c(b)}^{2}+q^{2}+s-s^{\prime}\right)}{3 \lambda^{1 / 2}\left(s, s^{\prime}, q^{2}\right)\left(q^{2}+s-s^{\prime}\right)} \tag{10}
\end{equation*}
$$

where $\lambda(a, b, c)=a^{2}+b^{2}+c^{2}-2 a c-2 b c-2 a b$ and $N_{c}=3$ is the color number. Note that, since three $\delta$ functions of integrand must vanish simultaneously, the physical regions in the $s-s^{\prime}$ plane must satisfy the following inequality:

$$
\begin{equation*}
-1 \leq f\left(s, s^{\prime}\right)=\frac{s\left(q^{2}+s-s^{\prime}\right)}{\lambda^{1 / 2}\left(m_{c(b)}^{2}, m_{c(b)}^{2}, s\right) \lambda^{1 / 2}\left(s, s^{\prime}, q^{2}\right)} \leq 1 \tag{11}
\end{equation*}
$$



Figure 1: The bare loop diagram for the $\chi_{c 0} \rightarrow J / \psi \gamma$ and $\chi_{b 0} \rightarrow \Upsilon \gamma$ decays.
2.2.2. Two Gluon Condensates. We consider the two gluon condensate diagrams. Note that we do not include the heavy quarks condensate diagrams, since the heavy quark contributions are exactly reducible to the gluon condensate [26]. Now, as a nonperturbative part, we must add contributions coming from the gluon condensates presented in Figures 2(a), 2(b), 2(c), 2(d), 2(e), and 2(f).

These diagrams are calculated in the Fock-Schwinger fixed-point gauge [27-29], where the vacuum gluon field is as follows:

$$
\begin{equation*}
A_{\mu}^{a}\left(k^{\prime}\right)=-\frac{i}{2}(2 \pi)^{4} G_{\rho \mu}^{a}(0) \frac{\partial}{\partial k_{\rho}^{\prime}} \delta^{(4)}\left(k^{\prime}\right) \tag{12}
\end{equation*}
$$

where $k^{\prime}$ is the gluon momentum and $A_{\mu}^{a}$ is the gluon field. In addition, the quark-gluon-quark vertex is used as

$$
\begin{equation*}
\Gamma_{i j \mu}^{a}=i g \gamma_{\mu}\left(\frac{\lambda^{a}}{2}\right)_{i j} \tag{13}
\end{equation*}
$$

We come across the following integrals in calculating the gluon condensate contributions [30, 31]:

$$
\begin{align*}
I_{0}[a, b, c]=\int & \frac{d^{4} k}{(2 \pi)^{4}} \\
& \times 1 \times\left(\left[k^{2}-m_{c(b)}^{2}\right]^{a}\left[(p+k)^{2}-m_{c(b)}^{2}\right]^{b}\right. \\
& \left.\times\left[\left(p^{\prime}+k\right)^{2}-m_{c(b)}^{2}\right]^{c}\right)^{-1}, \\
I_{\mu}[a, b, c]=\int \frac{d^{4} k}{(2 \pi)^{4}} & \times k_{\mu} \times\left(\left[k^{2}-m_{c(b)}^{2}\right]^{a}\left[(p+k)^{2}-m_{c(b)}^{2}\right]^{b}\right. \\
& \left.\times\left[\left(p^{\prime}+k\right)^{2}-m_{c(b)}^{2}\right]^{c}\right)^{-1}
\end{align*}
$$

where $k$ is the momentum of the spectator quark $m_{c(b)}$.
These integrals are calculated by shifting to the Euclidean space-time and using the Schwinger representation for the Euclidean propagator:

$$
\begin{equation*}
\frac{1}{\left(k^{2}+m^{2}\right)^{n}}=\frac{1}{\Gamma(n)} \int_{0}^{\infty} d \alpha \alpha^{n-1} e^{-\alpha\left(k^{2}+m^{2}\right)} . \tag{15}
\end{equation*}
$$

This kind of expression is very easy for the Borel transformation since

$$
\begin{equation*}
\mathscr{B}_{\hat{p}^{2}}\left(M^{2}\right) e^{-\alpha p^{2}}=\delta\left(\frac{1}{M^{2}}-\alpha\right), \tag{16}
\end{equation*}
$$

where $M$ is Borel parameter.
We perform integration over the loop momentum and over the two parameters which we use in the exponential representation of propagators [28]. As a final operation we apply double Borel transformations to $p^{2}$ and $p^{\prime 2}$. We get the Borel transformed form of the integrals in (14) as

$$
\begin{align*}
\widehat{I}_{0}(a, b, c)= & i \frac{(-1)^{a+b+c}}{16 \pi^{2} \Gamma(a) \Gamma(b) \Gamma(c)} \\
& \times\left(M_{1}^{2}\right)^{2-a-b}\left(M_{2}^{2}\right)^{2-a-c}  \tag{17}\\
& \times U_{0}(a+b+c-4,1-c-b), \\
\widehat{I}_{0_{\mu}}(a, b, c)= & \widehat{I}_{1}(a, b, c) p_{\mu}+\widehat{I}_{2}(a, b, c) p_{\mu}^{\prime}
\end{align*}
$$

where

$$
\begin{align*}
\widehat{I}_{1}(a, b, c)= & i \frac{(-1)^{a+b+c+1}}{16 \pi^{2} \Gamma(a) \Gamma(b) \Gamma(c)} \\
& \times\left(M_{1}^{2}\right)^{2-a-b}\left(M_{2}^{2}\right)^{3-a-c} \\
& \times U_{0}(a+b+c-5,1-c-b), \\
\widehat{I}_{2}(a, b, c)= & i \frac{(-1)^{a+b+c+1}}{16 \pi^{2} \Gamma(a) \Gamma(b) \Gamma(c)}  \tag{18}\\
& \times\left(M_{1}^{2}\right)^{3-a-b}\left(M_{2}^{2}\right)^{2-a-c} \\
& \times U_{0}(a+b+c-5,1-c-b),
\end{align*}
$$

and $M_{1}^{2}$ and $M_{2}^{2}$ are the Borel parameters. The function $\mathcal{U}_{0}(a, b)$ is as follows:

$$
\begin{align*}
\mathscr{U}_{0}(a, b)= & \int_{0}^{\infty} d y\left(y+M_{1}^{2}+M_{2}^{2}\right)^{a} y^{b} \\
& \times \exp \left[-\frac{B_{-1}}{y}-B_{0}-B_{1} y\right] \tag{19}
\end{align*}
$$

where

$$
\begin{align*}
B_{-1} & =\frac{1}{M_{1}^{2} M_{2}^{2}}\left[m_{c(b)}^{2}\left(M_{1}^{4}+M_{2}^{4}\right)+M_{2}^{2} M_{1}^{2}\left(2 m_{c(b)}^{2}-q^{2}\right)\right] \\
B_{0} & =\frac{2 m_{c(b)}^{2}}{M_{1}^{2} M_{2}^{2}}\left[M_{1}^{2}+M_{2}^{2}\right] \\
B_{1} & =\frac{m_{c(b)}^{2}}{M_{1}^{2} M_{2}^{2}} \tag{20}
\end{align*}
$$

where the circumflex of $\widehat{I}$ in the equations is used for the results after the double Borel transformation. As a result of


Figure 2: Two gluon condensate diagram as a radiative correction for the $\chi_{c 0} \rightarrow J / \psi \gamma$ and $\chi_{b 0} \rightarrow \Upsilon \gamma$ decays.
the lengthy calculations we obtain the following expressions for the two gluon condensate:

$$
\begin{align*}
\Pi_{\text {nonper }}= & -\frac{2 \pi \alpha_{s}\left\langle G^{2}\right\rangle}{3 m_{c(b)}} m_{c(b)}^{2} \\
\times & \left\{3 m _ { c ( b ) } ^ { 2 } \left[2 \left[\widehat{I}_{0}(1,4,1)+\widehat{I}_{0}(4,1,1)\right.\right.\right. \\
& \left.+\widehat{I}_{1}(1,1,4)+2 \widehat{I}_{1}(1,4,1)+2 \widehat{I}_{1}(4,1,1)\right] \\
& \left.+\widehat{I}_{0}(1,1,4)\right] \\
& -\widehat{I}_{0}(1,2,2)+6 \widehat{I}_{0}(1,3,1)+\widehat{I}_{0}(2,1,2)-2 \widehat{I}_{0}(2,2,1) \\
& -2 \widehat{I}_{1}(1,2,2)+6 \widehat{I}_{1}(1,3,1)+2 \widehat{I}_{1}(2,1,2) \\
& -6 \widehat{I}_{1}(2,2,1)+6 \widehat{I}_{1}(3,1,1)-3 \widehat{I}_{2}(1,1,3) \\
& \left.+6 \widehat{I}_{2}(1,3,1)\right\} . \tag{21}
\end{align*}
$$

Now, we can compare $g_{\mu \nu}$ coefficient of (6) and (7). Our result related to the sum rules for the corresponding form factor is as follows:

$$
\begin{align*}
F\left(q^{2}\right)= & \frac{e^{m_{S}^{2} / M^{2}} e^{m_{V}^{2} / M^{\prime 2}}}{f_{V} f_{S} m_{V} m_{S}} \\
& \times\left[\frac{1}{4 \pi^{2}} \int_{4 m_{c(b)}^{2}}^{s_{0}} d s \int_{4 m_{c(b)}^{2}}^{s_{0}^{\prime}} d s^{\prime} \rho\left(s, s^{\prime}, q^{2}\right) \theta\right. \\
& \left.\quad \times\left[1-\left(f\left(s, s^{\prime}\right)\right)^{2}\right] e^{-s / M^{2}} e^{-s^{\prime} / M^{\prime 2}}+\Pi_{\text {nonper }}\right] \tag{22}
\end{align*}
$$

Note that finally we have to set $q^{2}=0$ for the real photon.

## 3. Numerical Analysis

In this section we calculate the value of form factors and the branching ratios. We use $m_{c}=1.275 \pm 0.025 \mathrm{GeV}$ and $m_{b}=$ $4.18 \pm 0.03 \mathrm{GeV}$ [32], which correspond to the pole masses $m_{c}=1.65 \pm 0.07 \mathrm{GeV}$ and $m_{b}=4.78 \pm 0.06 \mathrm{GeV}$ [6]. Also, $m_{J / \psi}=3096.916 \pm 0.011 \mathrm{MeV}[32], m_{\chi_{c 0}}=3414.75 \pm 0.31 \mathrm{MeV}$ [32], $m_{\chi_{b 0}}=9859.44 \pm 0.42 \pm 0.3 \mathrm{MeV}$ [32], $m_{\Upsilon}=9460.30 \pm$ 0.26 MeV [32], $f_{\chi_{c 0}}=(343 \pm 112) \mathrm{MeV}$ [33], $f_{\chi_{b 0}}=(175 \pm$ 55) MeV [33], $f_{J / \psi}=(481 \pm 36) \mathrm{MeV}[3], f_{\Upsilon}=(746 \pm 62) \mathrm{MeV}$ [3], and the full width for $\chi_{c}: \Gamma_{\text {tot }}^{\chi_{c}}=10.4 \pm 0.6 \mathrm{MeV}$ [32] are used.

To do further numerical analyses we have to know the value or range of the auxiliary parameters of QCD sum rules. Those are the continuum thresholds ( $s_{0}$ and $s_{0}^{\prime}$ ) and the Borel mass parameters $\left(M^{2}\right.$ and $\left.M^{\prime 2}\right)$. The physical results are required to be either weakly dependent on or independent of the aforementioned parameters. Therefore, we must consider the working regions of these auxiliary parameters where the dependence of the form factors is weak. We also consider the working regions for the Borel mass parameters $M^{2}$ and $M^{\prime 2}$ in a way that both the contributions of the higher states and continuum are sufficiently suppressed and the contributions coming from higher dimensions operators can be ignored. With the aforementioned conditions, we find the stable region for the form factor in the following intervals: $5 \mathrm{GeV}^{2} \leq M^{2} \leq 12 \mathrm{GeV}^{2}$ and $5 \mathrm{GeV}^{2} \leq M^{\prime 2} \leq 10 \mathrm{GeV}^{2}$ for $\chi_{c 0} \rightarrow J / \psi \gamma$ decays (see also Figures 3 and 4). We also get $15 \mathrm{GeV}^{2} \leq M^{2} \leq 30 \mathrm{GeV}^{2}$ and $12 \mathrm{GeV}^{2} \leq M^{\prime 2} \leq 25 \mathrm{GeV}^{2}$ for $\chi_{b 0} \rightarrow \Upsilon \gamma$ decays.

The continuum thresholds, $s_{0}$ and $s_{0}^{\prime}$, are fixed by the mass of the corresponding ground-state hadron. Note that they must not be greater than the energy of the first excited states with the same quantum numbers. In our numerical calculations the following regions for the continuum thresholds in


Figure 3: The dependence of the $F\left(q^{2}=-10\right)$ on $M^{\prime 2}$ for three different values of $s_{0}=13.8,14.54,16.11, s_{0}^{\prime}=11.56,12.25,13.69$, and $M^{2}=8$ for the $\chi_{c 0} \rightarrow J / \psi \gamma$ decays. The green, blue, and purple lines are for minimum, central, and maximum values of $s_{0}$ and $s_{0}^{\prime}$.
$s$ and $s^{\prime}$ channels are used: $\left(m_{S}+0.3\right)^{2} \leq s_{0} \leq\left(m_{S}+0.7\right)^{2}$ and $\left(m_{V}+0.3\right)^{2} \leq s_{0}^{\prime} \leq\left(m_{V}+0.7\right)^{2}$ for $s$ and $s^{\prime}$ channels, respectively. Here, $m_{S}$ is the mass of either $\chi_{c 0}$ or $\chi_{b 0}$ meson and $m_{V}$ is the mass of either $J / \psi$ or $\Upsilon$ meson. We studied the dependence of the form factor $\left(F\left(q^{2}\right)\right.$ ) on continuum thresholds (see Figures 3 and 4). The figures show the weak dependence of the form factor for the chosen intervals.

Note that we follow the standard procedure in the QCD sum rules, where the continuum thresholds are supposed to be independent of Borel mass parameters and $q^{2}$. However, this assumption is not free of uncertainties (see, e.g., [34]).

Considering the large negative $q^{2}$ enables us to evaluate the correlation function by means of OPE that gives better convergence property; therefore, we use the extrapolation of the form factor from negative $q^{2}$ into the physical region (positive $q^{2}$ region).

The best fit curve for

$$
\begin{equation*}
F\left(q^{2}\right)=a e^{-b q^{2}}+c \tag{23}
\end{equation*}
$$

is employed for the negative $q^{2}$ region and fit is extrapolated for the positive $q^{2}$ region (see Figure 5). The values $a=0.83 \pm$ $0.23, b=0.2 \pm 0.02$, and $c=0.01 \pm 0.003$ for $\chi_{c 0} \rightarrow J / \psi \gamma$ and $a=0.412 \pm 0.14, b=0.2 \pm 0.016$, and $c=0.0084 \pm 0.003$ for $\chi_{b 0} \rightarrow \Upsilon \gamma$ decays are obtained via (23).

Using $q^{2}=0$ in (23), we obtain the $F(0)=0.83 \pm$ $0.23 \mathrm{GeV}^{-1}$ and the $F(0)=0.41 \pm 0.14 \mathrm{GeV}^{-1}$ for $\chi_{c 0} \rightarrow J / \psi \gamma$ and $\chi_{b 0} \rightarrow \Upsilon \gamma$ decays, respectively. It is worth mentioning that firstly, the contributions of the two gluon condensate in the value of the $F(0)$ is about $3 \%$. Secondly, roughly $80 \%$ of the errors in our numerical calculation arise from the variation continuum thresholds, Borel mass parameter in the given intervals, and uncertainties of the input parameters and the remaining $20 \%$ occurs as a result of the quark masses when one proceeds from the $\overline{M S}$ to the pole-scheme mass parameters, the input parameters.


Figure 4: The dependence of the $F\left(q^{2}=-10\right)$ on $M^{2}$ for three different values of $s_{0}=13.8,14.54,16.11, s_{0}^{\prime}=11.56,12.25,13.69$, and $M^{\prime 2}=6$ for the $\chi_{c 0} \rightarrow J / \psi \gamma$ decays. The green, blue, and purple lines are for minimum, central, and maximum values of $s_{0}$ and $s_{0}^{\prime}$.


Figure 5: The dependence of the fit function (23) and the form factor (22) on $q^{2}$ for values of $s_{0}=14.5, s_{0}^{\prime}=12.25, M^{2}=8$, and $M^{\prime 2}=6$ for the $\chi_{c 0} \rightarrow J / \psi \gamma$ decays. The blue line is (22).

The matrix element for the decay of $\chi_{c 0} \rightarrow J / \psi \gamma$ and $\chi_{b 0} \rightarrow \Upsilon \gamma$ is as follows:

$$
\begin{equation*}
M=e F\left(q^{2}=0\right)\left\{\left(p^{\prime} \cdot q\right) \epsilon^{\prime} \cdot \epsilon-\left(q \cdot \epsilon^{\prime}\right)\left(p^{\prime} \cdot \epsilon\right)\right\} \tag{24}
\end{equation*}
$$

where $p^{\prime}$ and $\epsilon^{\prime}$ are the momentum and polarization of final state vector meson, that is, either $J / \psi$ or $\Upsilon$ mesons, $\epsilon$ is the polarization of real photon, and $p$ is the momentum of initial scalar meson.

Using this matrix element, we get

$$
\begin{equation*}
\Gamma=\frac{|\vec{p}|}{8 \pi m_{S}^{2}}|M|^{2}=\frac{\alpha}{8} F^{2}(0) m_{S}^{3}\left(1-\frac{m_{V}^{2}}{m_{S}^{2}}\right)^{3} \tag{25}
\end{equation*}
$$

where $m_{S}$ is mass of either $\chi_{c 0}$ or $\chi_{b 0}$ meson and $m_{V}$ is either mass of $J / \psi$ or $\Upsilon$ meson. The decay width for $\chi_{c 0} \rightarrow J / \psi \gamma$ decays is

$$
\begin{equation*}
\Gamma\left(\chi_{c 0} \longrightarrow \frac{J}{\psi \gamma}\right)=\left(14.2_{-6.1}^{+9.3}\right) \times 10^{-5} \mathrm{GeV} \tag{26}
\end{equation*}
$$

The branching ratio of $\chi_{c 0} \rightarrow J / \psi \gamma$ can be evaluated with (26) and using the experimental total width that is

$$
\begin{equation*}
\mathscr{B}_{r}\left(\chi_{c 0} \longrightarrow \frac{J}{\psi \gamma}\right)=\left(1.36_{-0.6}^{+0.9}\right) \times 10^{-2} \tag{27}
\end{equation*}
$$

This result is in good agreement with the experimental measurement [32] and the result of the potential model given in $[8,9]$, which are

$$
\begin{align*}
& \mathscr{B}_{r}\left(\chi_{c 0} \longrightarrow \frac{J}{\psi \gamma}\right)=(1.17 \pm 0.08) \times 10^{-2},  \tag{28}\\
& \mathscr{B}_{r}\left(\chi_{c 0} \longrightarrow \frac{J}{\psi \gamma}\right)=(1.28 \pm 0.11) \times 10^{-2}, \tag{29}
\end{align*}
$$

respectively.
We get $F(0)=0.41 \pm 0.13 \mathrm{GeV}^{-1}$ for $\chi_{b 0} \rightarrow \Upsilon \gamma$ decays. Using this value we calculate the decay width as follows:

$$
\begin{equation*}
\Gamma\left(\chi_{b 0} \longrightarrow \Upsilon \gamma\right)=\left(7.4_{-3.9}^{+5.6}\right) \times 10^{-5} \mathrm{GeV} \tag{30}
\end{equation*}
$$

This decay width and the measured branching ratio $\mathscr{B}_{r}\left(\chi_{b} \rightarrow\right.$ $\Upsilon(\gamma)=(1.76 \pm 0.30) \times 10^{-2}$ [17] allow us to evaluate the total width of $\chi_{b 0}$. We estimate that the full width $\Gamma_{\text {tot }}\left(\chi_{b 0}(1 P)\right)=4.2_{-2.21}^{+3.18} \mathrm{MeV}$, which is consistent with the experimental results that indicate the full width $\Gamma_{\text {tot }}<6 \mathrm{MeV}$ [17].

## Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

## Acknowledgments

The author is grateful to T. M. Aliev and K. Azizi for their useful discussions.

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