Research Article

Reconstructions of Einstein-Aether Gravity from Ordinary and Entropy-Corrected Versions of Holographic and New Agegraphic Dark Energy Models

Ujjal Debnath

Department of Mathematics, Indian Institute of Engineering Science and Technology, Shibpur, Howrah 711 103, India

Correspondence should be addressed to Ujjal Debnath; ujjaldebnath@yahoo.com

Here we briefly discuss the Einstein-Aether gravity theory by modification of Einstein-Hilbert action. We find the modified Friedmann equations and then from the equations we find the effective density and pressure for Einstein-Aether gravity sector. These can be treated as if dark energy provided some restrictions on the free function $F(K)$, where K is proportional to H^2. Assuming two types of the power law solutions of the scale factor, we have reconstructed the unknown function $F(K)$ from HDE and NADE and their entropy-corrected versions (ECHDE and ECNADE). We also obtain the EoS parameter for Einstein-Aether gravity dark energy. For HDE and NADE, we have shown that the type I scale factor generates the quintessence scenario only and type II scale factor generates phantom scenario. But for ECHDE and ECNADE, both types of scale factors can accommodate the transition from quintessence to phantom stages; that is, phantom crossing is possible for entropy-corrected terms of HDE and NADE models. Finally, we show that the models are classically unstable.

1. Introduction

The type I a Supernovae and Cosmic Microwave Background (CMB) [1, 2] observations indicate that our universe is presently accelerating. This acceleration is caused by some unknown matter which has the property that positive energy density and negative pressure are dubbed as "dark energy" (DE). Observations indicate that dark energy occupies about 70% of the total energy of the universe, and the contribution of dark matter is ~26%. Recent WMAP data analysis [3, 4] also give us the confirmation of this acceleration. Although a long-time debate has been done on this well-reputed and interesting issue of modern cosmology, we still have little knowledge about DE. The most appealing and simplest candidate for DE is the cosmological constant Λ. Over the past decade there have been many theoretical models for mimicking the dark energy behaviors, such as ΛCDM, containing a mixture of cosmological constant Λ and cold dark matter (CDM). However, two problems arise from this scenario, namely, "fine-tuning" and the "cosmic coincidence" problems. In order to solve these two problems, many dynamical DE theoretical models have been proposed [5–9]. The scalar field or quintessence [10, 11] is one of the most favored candidate of DE which produces sufficient negative pressure to drive acceleration.

In recent times, considerable interest has been stimulated in explaining the observed dark energy by the holographic dark energy (HDE) model [12–14]. An approach to the problem of this dark energy arises from the holographic principle [15]. For an effective field theory in a box size L with UV cutoff Λ_c, the entropy is given by $L^3 \Lambda_c^3$. Taking the whole universe into account the largest IR cutoff L is chosen by saturating the inequality so that we get the holographic dark energy density as [13, 16] in the form $\rho_\Lambda = 3c^2 M_p^2 L^{-2}$ where c is a numerical constant and $M_p \equiv 1/\sqrt{8\pi G}$ is the reduced Planck mass. The most natural choice of L is the Hubble horizon H^{-1}. However, Hsu [17] and Li [16] pointed out that in this case the EoS

of holographic dark energy is zero and the expansion of the universe cannot be accelerated. The next choice of L is the particle horizon. Unfortunately, in this case, the EoS of HDE is always larger than $-1/3$ and the expansion of the universe also cannot be accelerated. Finally, Li [16] found out that L might be the future event horizon of the universe. On the basis of the holographic principle, several others have studied holographic model for dark energy [18–20]. Obviously, in the derivation of HDE, the black hole (BH) entropy S_{BH} plays an important role. Usually, we know that $S_{BH} = A/4G$, where $A(\sim L^2)$ is the area of BH horizon. Due to thermal equilibrium fluctuation, quantum fluctuation, or mass and charge fluctuations, the BH entropy-area relation has been modified in loop quantum gravity (LQG) in the form [21, 22] $S_{BH} = A/4G + \xi \ln(A/4G) + \zeta$, where ξ and ζ are dimensionless constants of order unity. Recently, motivated by this corrected entropy-area relation in the setup of LQG, the energy density of the entropy-corrected HDE (ECHDE) was proposed by Wei [23] and also details were discussed in [24].

Based on principle of quantum gravity, Cai [25] proposed a new dark energy model based on the energy density. The energy density of metric fluctuations of Minkowski-space-time is given by $\rho_\Lambda \sim M_p^2/\tilde{t}^2$ [26, 27]. As the most natural choice, the time scale $\tilde{t}$ is chosen to be the age of our universe, $T = \int_0^a (da/aH)$, where a is the scale factor of our universe and H is the Hubble parameter. Therefore, the dark energy is called agegraphic dark energy (ADE) [25]. The energy density of agegraphic dark energy is given by $\rho_\Lambda = 3\alpha^2 M_p^2 T^{-2}$, where the numerical factor $3\alpha^2$ is introduced to parameterize some uncertainties, such as the species of quantum fields in the universe. Since the original ADE model suffers from the difficulty to describe the matter-dominated epoch, so Wei and Cai [28] have chosen the time scale $\tilde{t}$ to be the conformal time η instead of T, which is defined by $dt = ad\eta$ (where t is the cosmic time); the energy density is obtained as $\rho_\Lambda = 3\alpha^2 M_p^2 \eta^{-2}$, which is called new agegraphic dark energy (NADE) model [28]. It was found that the coincidence problem could be solved naturally in the NADE model. The ADE models have given rise to a lot of interest recently and have been examined and studied in details in [28, 29]. Recently, very similar to the ECHDE model, the energy density of the entropy-corrected NADE (ECNADE) was proposed by Wei [23] and investigated in details for acceleration of the universe [30].

Another approach to explore the accelerated expansion of the universe is the modified theories of gravity. In this case cosmic acceleration would arise not from dark energy as a substance but rather from the dynamics of modified gravity [31]. Modified gravity constitutes an interesting dynamical alternative to ΛCDM cosmology. The simplest modified gravity is DGP brane-world model [32]. The other alternative is $f(R)$ gravity [33] where the Einstein-Hilbert action has been modified. Other modified gravity includes $f(T)$ gravity, $f(G)$ gravity, Gauss-Bonnet gravity, Horava-Lifshitz gravity, Brans-Dicke gravity, and so forth, [34–40].

Here we have assumed another type of modified gravity developed by Jacobson and Mattingly [41, 42], known as

Einstein-Aether theory. Zlosnik et al. [43, 44] has proposed the generalization of Einstein-Aether theory. These years a lot of work has been done in generalized Einstein-aether theories [45–51]. In the generalized Einstein-Aether theories by taking a special form of the Lagrangian density of Aether field, the possibility of Einstein-Aether theory as an alternative to dark energy model is discussed in detail, that is, taking a special Aether field as a dark energy candidate and it has been investigated the observational constraints for Aether field [52, 53]. Meng et al. [52, 53] have shown that only Einstein-Aether gravity may be generating dark energy, which caused the acceleration of the universe.

Recently the reconstruction procedure or correspondences between various dark energy models became a very challenging subject in cosmological phenomena. Correspondence between different DE models, reconstruction of DE/gravity, and their cosmological implications have been discussed by several authors [54–62]. We shall reconstruct Einstein-Aether gravity model with HDE and NADE separately. For this purpose, we first briefly discuss the Einstein-Aether gravity theory by modification of Einstein-Hilbert action in Section 2. We find the modified Friedmann equations and then from the equations we find the effective density and pressure for Einstein-Aether gravity sector. These can be treated as if dark energy provided some restrictions on the free function $F(K)$. Assuming two types of the power law solutions of the scale factor, we can reconstruct the unknown function $F(K)$ from HDE and NADE and their entropy-corrected versions (ECHDE and ECNADE) in Section 3. Finally, we give some cosmological implications of the reconstructed models in Section 4.

2. Einstein-Aether Gravity Theory and Modified Friedmann Equations

Einstein-Aether theory is the extension of general relativity (GR) that couples with the metric and incorporates a dynamical unit timelike vector field (i.e., Aether). The action of the Einstein-Aether gravity theory with the normal Einstein-Hilbert part action can be written in the form [43, 52]

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} + \mathcal{L}_{EA} + \mathcal{L}_m \right], \quad (1)$$

where $\mathcal{L}_{EA}$ is the Lagrangian density for vector field and $\mathcal{L}_m$ denotes the Lagrangian density for matter field. The Lagrangian density for the vector field [43, 52] is given by

$$\mathcal{L}_{EA} = \frac{M^2}{16\pi G} F(K) + \frac{1}{16\pi G} \lambda \left(A^a A_a + 1 \right), \quad (2)$$

$$K = M^{-2} K_{cd}^{ab} \nabla_a A^c \nabla_b A^d, \quad (3)$$

$$K_{cd}^{ab} = c_1 g^{ab} g_{cd} + c_2 \delta_c^a \delta_d^b + c_3 \delta_d^a \delta_c^b, \quad (4)$$

where c_i are dimensionless constants, M is the coupling constant, λ is a Lagrangian multiplier, A^a is a contravariant

vector, and $F(K)$ is an arbitrary function of K. From (1), we get the Einstein's field equations

$$G_{ab} = T_{ab}^{\text{EA}} + 8\pi G T_{ab}^m, \tag{5}$$

$$\nabla_a \left(F' J_b^a \right) = 2\lambda A_b, \tag{6}$$

where

$$F' = \frac{dF}{dK}, \qquad J_b^a = 2K_{bc}^{ad} \nabla_d A^c. \tag{7}$$

Here T_{ab}^m is the energy momentum tensor for matter and T_{ab}^{EA} is the energy momentum tensor for the vector field given as follows:

$$T_{ab}^m = (\rho + p) u_a u_b + p g_{ab}, \tag{8}$$

where ρ and p are, respectively, the energy density and pressure of matter and $u_a = (1, 0, 0, 0)$ is the fluid 4-velocity vector and

$$T_{ab}^{\text{EA}} = \frac{1}{2} \nabla_d \left[\left(J_{(a}{}^d A_{b)} - J^d{}_{(a} A_{b)} - J_{(ab)} A^d \right) F' \right]$$
$$- Y_{(ab)} F' + \frac{1}{2} g_{ab} M^2 F + \lambda A_a A_b \tag{9}$$

with

$$Y_{ab} = -c_1 \left[\left(\nabla_d A_a \right) \left(\nabla^d A_b \right) - \left(\nabla_a A_d \right) \left(\nabla_b A^d \right) \right], \tag{10}$$

where the subscript (ab) means symmetric with respect to the indices and $A^a = (1, 0, 0, 0)$ is time-like unit vector.

We consider the Friedmann-Robertson-Walker (FRW) metric of the universe as

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right) \right], \tag{11}$$

where $k\ (= 0, \pm 1)$ is the curvature scalar and $a(t)$ is the scale factor. From (3) and (4), we get

$$K = \frac{3\beta H^2}{M^2}, \tag{12}$$

where β is constant. From (5), we get the modified Friedmann equation for Einstein-Aether gravity as follows:

$$\beta \left(-F' + \frac{F}{2K} \right) H^2 + \left(H^2 + \frac{k}{a^2} \right) = \frac{8\pi G}{3} \rho,$$
$$\beta \frac{d}{dt} \left(HF' \right) + \left(-2\dot{H} + \frac{2k}{a^2} \right) = 8\pi G (\rho + p), \tag{13}$$

where $H\ (=\dot{a}/a)$ is Hubble parameter. Also the conservation equation is given by

$$\dot{\rho} + 3\frac{\dot{a}}{a} (\rho + p) = 0. \tag{14}$$

Let ρ_{EA} and p_{EA} be the effective energy density and pressure governed by the Einstein-Aether gravity; then we may write (13) in the following form:

$$\left(H^2 + \frac{k}{a^2} \right) = \frac{8\pi G}{3} \rho + \frac{1}{3} \rho_{\text{EA}},$$
$$\left(-2\dot{H} + \frac{2k}{a^2} \right) = 8\pi G (\rho + p) + (\rho_{\text{EA}} + p_{\text{EA}}), \tag{15}$$

and hence we obtain

$$\rho_{\text{EA}} = 3\beta H^2 \left(F' - \frac{F}{2K} \right),$$
$$p_{\text{EA}} = -3\beta H^2 \left(F' - \frac{F}{2K} \right) - \beta \left(\dot{H} F' + H \dot{F}' \right). \tag{16}$$

The equation of state (EoS) parameter due to the Einstein-Aether contribution is given by

$$w_{\text{EA}} = \frac{p_{\text{EA}}}{\rho_{\text{EA}}} = -1 - \frac{\left(\dot{H} F' + H \dot{F}' \right)}{3H^2 \left(F' - F/2K \right)}. \tag{17}$$

Since density is always positive, so $\rho_{\text{EA}} > 0$ implies $F' > F/2K$, where we assume $\beta > 0$. The effective density and pressure governed by Einstein-Aether gravity generate dark energy if $\rho_{\text{EA}} + 3p_{\text{EA}} < 0$, which provides the condition $2H^2(F' - F/2K) > -(\dot{H} F' + H \dot{F}')$.

3. Reconstruction of Einstein-Aether Gravity Model

Since Einstein-Aether theory is the modified gravity theory and this may generate dark energy. So this gravity can be corresponding with other well-known dark energy models. For this purpose, we consider the dark energy models which are holographic dark energy (HDE) and new agegraphic dark energy (NADE) and their entropy-corrected versions (ECHDE and ECNADE). The Einstein-Aether gravity sector contains a function $F(K)$; so equating the density with the dark energy models, we can find $F(K)$, so $F(K)$ can be constructed from the dark energy models. For reconstruction of $F(K)$ in terms of K, we need to know the form of scale factor $a(t)$. For this purpose, we assume two types of power law forms of $a(t)$ for providing the acceleration of the universe and we reconstruct the Einstein-Aether gravity according to the HDE, ECHDE, NADE, and ECNADE models.

(i) *Type I.* Consider $a(t) = a_0 t^m$, $m > 0$, where the constant a_0 represents the present-day value of the scale factor [63]. With this choice of scale factor we obtain H, $\dot{H}$, and K in the following form:

$$H = \frac{m}{t}, \qquad \dot{H} = -\frac{m}{t^2}, \qquad K = \frac{3\beta m^2}{M^2 t^2}. \tag{18}$$
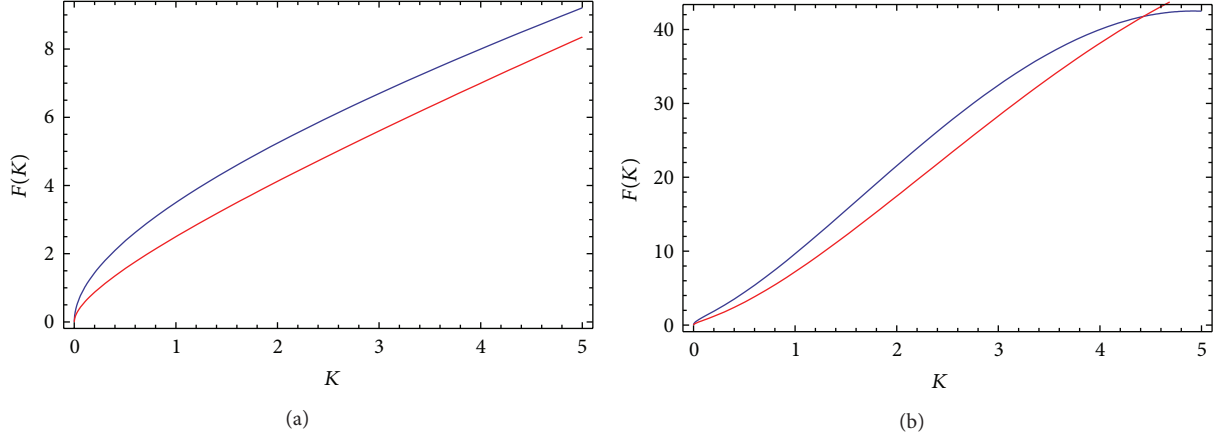
(a)

(b)

FIGURE 1: ((a) and (b)) The variations of $F(K)$ against K for HDE and ECHDE models. Blue line represents type I model and red line represents type II model.

We see that $\dot{H} < 0$, so this model corresponds to only quintessence dominated universe (not phantom) and hence the scale factor may be called the quintessence scale factor.

(ii) *Type II.* Consider $a(t) = a_0(t_s - t)^{-n}$, $t < t_s$, $n > 0$, where the constant a_0 represents the present-day value of the scale factor and t_s is the probable future singularity finite time [63]:

$$H = \frac{n}{t_s - t}, \qquad \dot{H} = \frac{n}{(t_s - t)^2}, \qquad K = \frac{3\beta n^2}{M^2(t_s - t)^2}. \tag{19}$$

We see that $\dot{H} > 0$, so this model corresponds to only phantom dominated universe (not quintessence) and hence the scale factor may be called the phantom scale factor.

3.1. Reconstruction from Holographic Dark Energy (HDE) Model. We now suggest the correspondence between the holographic dark energy scenario and Einstein-Aether dark energy model. The holographic dark energy (HDE) density can be written as [55, 64, 65]

$$\rho_\Lambda = 3c^2 L^{-2}, \qquad L = R_h, \tag{20}$$

where c is a constant and R_h represents the future event horizon. From observation, the best fit value of c is $0.818^{+0.113}_{-0.097}$. For type I scale factor, R_h is defined as

$$R_h = a \int_t^\infty \frac{dt}{a} = \frac{t}{m-1}, \qquad m > 1. \tag{21}$$

For type II scale factor, R_h is defined as

$$R_h = a \int_t^{t_s} \frac{dt}{a} = \frac{t_s - t}{n+1}. \tag{22}$$

For type I scale factor, equating the energy densities (i.e., $\rho_{EA} = \rho_\Lambda$), we get the reconstructed equation

$$\frac{dF}{dK} - \frac{F}{2K} = \frac{c^2(m-1)^2}{\beta m^2}, \tag{23}$$

which immediately gives the solution

$$F(K) = \frac{2c^2(m-1)^2}{\beta m^2} K + A_1 \sqrt{K}. \tag{24}$$

For type II scale factor, we also get similar solution

$$F(K) = \frac{2c^2(n+1)^2}{\beta n^2} K + A_2 \sqrt{K}. \tag{25}$$

Here A_1 and A_2 are integration constants. For these solutions, the EoS for Einstein-Aether gravity can be obtained as $w_{EA} = -1 + 2/3m$ for type I model and $w_{EA} = -1 - 2/3n$ for type II model. We see that $-1 < w_{EA} < -1/3$ (i.e., quintessence) if $m > 1$ for type I model and for $n > 0$, $w_{EA} < -1$ (phantom), for type II model. The graphs of $F(K)$ with respect to K has been drawn in Figure 1(a) for both type I and type II models. From the figure, we see that the reconstruction function $F(K)$ increases as K increases for HDE model.

3.2. Reconstruction from Entropy-Corrected Holographic Dark Energy (ECHDE) Model. Using the corrected entropy-area relation, the energy density of the ECHDE can be written as [23]

$$\rho_\Lambda = \frac{3c^2}{R_h^2} + \frac{\xi}{R_h^4} \ln\left(R_h^2\right) + \frac{\zeta}{R_h^4}, \tag{26}$$

where ξ and ζ are constants. For type I scale factor, equating the energy densities (i.e., $\rho_{EA} = \rho_\Lambda$), we get the reconstructed equation

$$\frac{dF}{dK} - \frac{F}{2K} = \frac{c^2(m-1)^2}{\beta m^2}$$

$$+ \frac{(m-1)^4 M^2}{(3\beta m^2)^2} K \left[\zeta + \xi \ln\left(\frac{3\beta m^2}{M^2(m-1)^2 K}\right) \right], \tag{27}$$

which immediately gives the solution

$$
F(K) = \frac{2c^2(m-1)^2}{\beta m^2}K + \frac{(m-1)^4 M^2}{9(3\beta m^2)^2}K^2
$$
$$
\times \left[3\zeta + 2\xi + 2\xi \ln\left(\frac{3\beta m^2}{M^2(m-1)^2 K} \right) \right] + B_1 \sqrt{K}.
$$
(28)

Here B_1 is integration constant. In this case, using (17), the EoS parameter for Einstein-Aether gravity can be written as

$$
w_{\text{EA}} = -1 + \frac{4}{3m}
$$
$$
\times \left[\left(27c^2 t^2 + (m-1)^2 \right. \right.
$$
$$
\times \left\{ 9\zeta - \xi + 12\xi \ln\left(\frac{t}{m-1} \right) \right\} \right)
$$
$$
\times \left(54c^2 t^2 + (m-1)^2 \right.
$$
$$
\left. \times \left\{ 9\zeta + 2\xi + 12\xi \ln\left(\frac{t}{m-1} \right) \right\} \right)^{-1} \right],
$$
$$
m > 1.
$$
(29)

For type II scale factor, we also get the similar solution

$$
F(K) = \frac{2c^2(n+1)^2}{\beta n^2}K + \frac{(n+1)^4 M^2}{9(3\beta n^2)^2}K^2
$$
$$
\times \left[3\zeta + 2\xi + 2\xi \ln\left(\frac{3\beta n^2}{M^2(n+1)^2 K} \right) \right] + B_2 \sqrt{K}.
$$
(30)

Here B_2 is integration constant. In this case, using (17), the EoS parameter for Einstein-Aether gravity can be written as

$$
w_{\text{EA}} = -1 - \frac{4}{3n}
$$
$$
\times \left[\left(27c^2 (t_s - t)^2 + (n+1)^2 \right. \right.
$$
$$
\times \left\{ 9\zeta - \xi + 12\xi \ln\left(\frac{(t_s - t)}{n+1} \right) \right\} \right)
$$
$$
\times \left(54c^2 (t_s - t)^2 + (n+1)^2 \right.
$$
$$
\left. \times \left\{ 9\zeta + 2\xi + 12\xi \ln\left(\frac{(t_s - t)}{n+1} \right) \right\} \right)^{-1} \right].
$$
(31)

The graphs of $F(K)$ with respect to K has been drawn in Figure 1(b) for both type I and type II models. From the

figure, we see that the reconstruction function $F(K)$ increases as K increases for ECHDE model. The EoS parameter w_{EA} against time t has been drawn in Figure 3(a) for both type I and type II models. We see that Einstein-Aether EoS parameter w_{EA} gives the transition from $w_{\text{EA}} > -1$ to $w_{\text{EA}} < -1$ stages for both type I and type II models. So for type I and type II models, when we assume ECHDE, the Einstein-Aether DE interpolates from quintessence era to phantom stage. So phantom crossing is possible for these models. Thus we conclude that for both type I and type II models, entropy-corrected terms ($\xi \neq 0, \zeta \neq 0$) generate the phantom crossing.

3.3. Reconstruction from New Agegraphic Dark Energy (NADE) Model. The energy density of the new agegraphic dark energy (NADE) model is given by [28]

$$
\rho_\Lambda = \frac{3\alpha^2}{\eta^2},
$$
(32)

where $\eta = \int (dt/a(t))$ is the conformal time and the numerical factor $3\alpha^2$ serves to parameterize some uncertainties, which include the effect of curved spacetime, some species of quantum fields in the universe, and so forth. From observation, the best fit value of α is $2.716^{+0.111}_{-0.109}$.

For type I model, the conformal time is

$$
\eta = \int_0^t \frac{dt}{a} = \frac{t^{1-m}}{a_0(1-m)}, \quad m < 1.
$$
(33)

For type II model, the conformal time is

$$
\eta = \int_t^{t_s} \frac{dt}{a} = \frac{(t_s - t)^{n+1}}{a_0(n+1)}.
$$
(34)

For type I model, equating the energy densities (i.e., $\rho_{\text{EA}} = \rho_\Lambda$), we get the reconstructed equation

$$
\frac{dF}{dK} - \frac{F}{2K} = \frac{a_0^2 \alpha^2 (1-m)^2}{\beta m^2} \left(\frac{3\beta m^2}{M^2} \right)^m \frac{1}{K^m},
$$
(35)

which immediately gives the solution

$$
F(K) = \frac{2a_0^2 \alpha^2 (1-m)^2}{\beta m^2 (1-2m)} \left(\frac{3\beta m^2}{M^2} \right)^m K^{1-m} + C_1 \sqrt{K}.
$$
(36)

For type II model, we get the similar reconstructed equation, which gives the similar solution

$$
F(K) = \frac{2a_0^2 \alpha^2 (n+1)^2}{\beta n^2 (2n+1)} \left(\frac{3\beta n^2}{M^2} \right)^{-n} K^{n+1} + C_2 \sqrt{K},
$$
(37)

where C_1 and C_2 are integration constants. For these solutions, the EoS for Einstein-Aether gravity can be obtained as $w_{\text{EA}} = -5/3 + 2/3m$ for type I model and $w_{\text{EA}} = -5/3 - 2/3n$ for type II model. We see that for $1/2 < m < 1$, we have $-1 < w_{\text{EA}} < -1/3$, that is, quintessence model for type I model and $w_{\text{EA}} < -5/3 < -1$ (phantom divide) for all $n > 0$ for type II model. The graphs of $F(K)$ with respect to K has been drawn in Figure 2(a) for both type I and type II models. From the figure, we see that the reconstruction function $F(K)$ increases as K increases for NADE model.
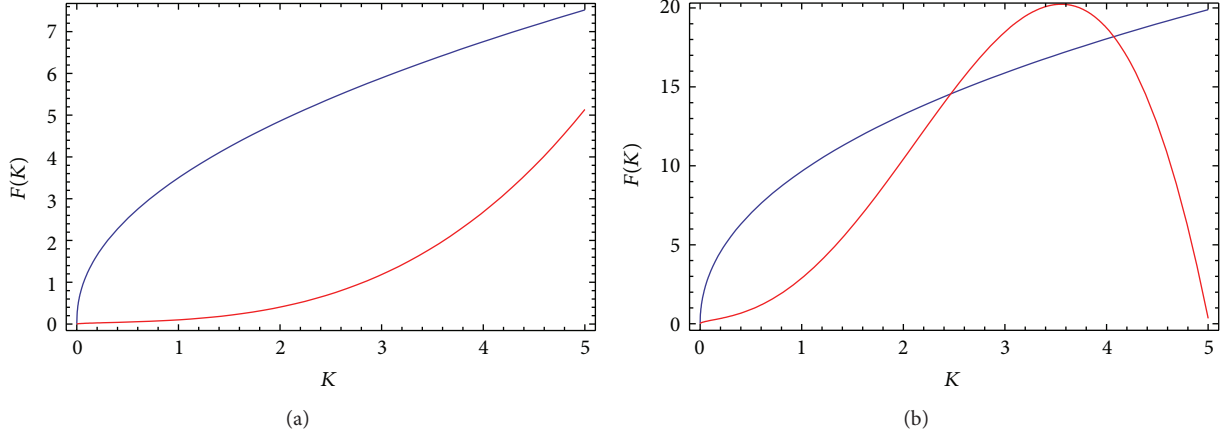
(a)

(b)

FIGURE 2: ((a) and (b)) The variations of $F(K)$ against K for NADE and ECNADE models. Blue line represents type I model and red line represents type II model.

3.4. Reconstruction from Entropy-Corrected New Agegraphic Dark Energy (ECNADE) Model.

Using the corrected entropy-area relation, the energy density of the ECNADE can be written as [23]

$$\rho_\Lambda = \frac{3\alpha^2}{\eta^2} + \frac{\xi}{\eta^4} \ln\left(\eta^2\right) + \frac{\zeta}{\eta^4}, \tag{38}$$

where ξ and ζ are constants. Here we have replaced R_h by conformal time η in (26). For type I model, equating the energy densities (i.e., $\rho_{EA} = \rho_\Lambda$), we get the reconstructed equation

$$\frac{dF}{dK} - \frac{F}{2K} = \frac{a_0^2\alpha^2(1-m)^2}{\beta m^2}\left(\frac{3\beta m^2}{M^2}\right)^m K^{-m}$$

$$+ \frac{a_0^4(1-m)^4}{3\beta m^2}\left(\frac{3\beta m^2}{M^2}\right)^{2m-1} K^{1-2m}$$

$$\times \left[\zeta + \xi \ln\left\{\left(\frac{3\beta m^2}{M^2}\right)^{1-m} \frac{K^{m-1}}{a_0^2(1-m)^2}\right\}\right], \tag{39}$$

which immediately gives the solution

$$F(K) = \frac{2a_0^2\alpha^2(1-m)^2}{\beta m^2(1-2m)}\left(\frac{3\beta m^2}{M^2}\right)^m K^{1-m}$$

$$+ D_1\sqrt{K} + \frac{2a_0^4(1-m)^4}{3\beta m^2(3-4m)^2}\left(\frac{3\beta m^2}{M^2}\right)^{2m-1} K^{2-2m}$$

$$\times \left[(3-4m)\zeta + 2(1-m)\xi\right.$$

$$\left. + (3-4m)\xi \ln\left\{\left(\frac{3\beta m^2}{M^2}\right)^{1-m} \frac{K^{m-1}}{a_0^2(1-m)^2}\right\}\right]. \tag{40}$$

Here D_1 is integration constant. In this case, using (17), the EoS parameter for Einstein-Aether gravity can be written as

$$w_{EA} = -1 + \frac{2(1-m)}{3m}$$

$$\times \left[\left(a_0^2\beta m^2(2\zeta - \xi) + \alpha^2 M^2\left(\frac{3\beta m^2}{M^2}\right)^m t^{2-2m}\right.\right.$$

$$\left. + 4a_0^2\beta m^2\xi \ln\left(\frac{t^{1-m}}{a_0(1-m)}\right)\right)$$

$$\times \left(a_0^2\beta m^2\zeta + \alpha^2 M^2\left(\frac{3\beta m^2}{M^2}\right)^m t^{2-2m}\right.$$

$$\left.\left. + 2a_0^2\beta m^2\xi \ln\left(\frac{t^{1-m}}{a_0(1-m)}\right)\right)^{-1}\right],$$

$$m < 1. \tag{41}$$

For type II model, we get the similar reconstructed equation, which gives the similar solution

$$F(K) = \frac{2a_0^2\alpha^2(1+n)^2}{\beta n^2(1+2n)}\left(\frac{3\beta n^2}{M^2}\right)^{-n} K^{1+n}$$

$$+ D_2\sqrt{K} + \frac{2a_0^4(1+n)^4}{3\beta n^2(3+4n)^2}\left(\frac{3\beta n^2}{M^2}\right)^{-2n-1} K^{2+2n}$$

$$\times \left[(3+4n)\zeta + 2(1+n)\xi\right.$$

$$\left. + (3+4n)\xi \ln\left\{\left(\frac{3\beta n^2}{M^2}\right)^{1+n} \frac{K^{-n-1}}{a_0^2(1+n)^2}\right\}\right]. \tag{42}$$
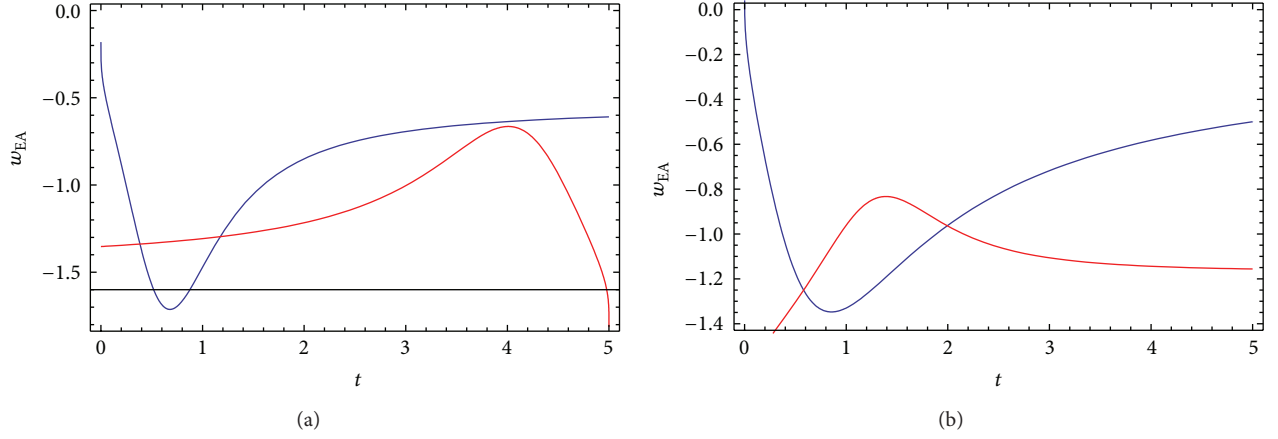
(a)

(b)

FIGURE 3: ((a) and (b)) The variations of w_{EA} against t for ECHDE and ECNADE models. Blue line represents type I model and red line represents type II model.

Here D_2 is integration constant. In this case, using (17), the EoS parameter for Einstein-Aether gravity can be written as

$$
\begin{aligned}
w_{EA} = &-1 - \frac{2(1+n)}{3n} \\
&\times \left[\left(a_0^2 \beta n^2 (2\zeta - \xi) + \alpha^2 M^2 \left(\frac{3\beta n^2}{M^2} \right)^{-n} t^{2+2n} \right.\right. \\
&\left. + 4a_0^2 \beta n^2 \xi \ln \left(\frac{t^{1+n}}{a_0 (1+n)} \right) \right) \\
&\times \left(a_0^2 \beta n^2 \zeta + \alpha^2 M^2 \left(\frac{3\beta n^2}{M^2} \right)^{-n} t^{2+2n} \right. \\
&\left.\left. + 2a_0^2 \beta n^2 \xi \ln \left(\frac{t^{1+n}}{a_0 (1+n)} \right) \right)^{-1} \right].
\end{aligned}
$$

(43)

The graphs of $F(K)$ with respect to K has been drawn in Figure 2(b) for both type I and type II models. From the figure, we see that the reconstruction function $F(K)$ increases as K increases for type I model for ECNADE. But for type II model, $F(K)$ first increases with positive value up to a certain value of K and then it sharply decreases from positive value to negative value for increasing K. The EoS parameter w_{EA} against time t has been drawn in Figure 3(b) for both type I and type II models. We see that Einstein-Aether EoS parameter w_{EA} gives the transition from $w_{EA} > -1$ to $w_{EA} < -1$ stages for both type I and type II models. So for type I and type II models, when we assume ECNADE, the Einstein-Aether DE interpolates from quintessence era to phantom stage. So phantom crossing is possible for these models. Thus we conclude that for both type I and type II models, entropy-corrected terms ($\xi \neq 0, \zeta \neq 0$) generate the phantom crossing.

4. Discussions and Concluding Remarks

In this work, we have assumed the Einstein-Aether gravity theory by modification of Einstein-Hilbert action in FRW universe. We find the modified Friedmann equations and then from the equations we find the effective density and pressure for Einstein-Aether gravity sector. These can be treated as if dark energy provided some restrictions on the free function $F(K)$, where K is proportional to H^2. Assuming two types of the power law forms of the scale factor, we have reconstructed the unknown function $F(K)$ from HDE and NADE and their entropy-corrected versions (ECHDE and ECNADE). From Figures 1 and 2, we observed that the function $F(K)$ increases with positive sign for increasing K for type I and type II models when we assumed HDE, ECHDE, and NADE. But for type II model in ECNADE, $F(K)$ first increases with positive value up to a certain value of K and then it sharply decreases from positive value to negative value for increasing K. From the reconstructions of all the models, we obtain $F(K) \rightarrow 0$ as $K \rightarrow 0$. We also obtain the EoS parameter for Einstein-Aether gravity dark energy. For HDE and NADE, we have shown that the type I scale factor generates the quintessence scenario ($m > 1$ for HDE and $1/2 < m < 1$ for NADE) only and type II scale factor generates phantom scenario ($n > 0$). So these models cannot generate phantom crossing. But for ECHDE and ECNADE, the EoS parameter w_{EA} in terms of time t has been drawn in Figures 3(a) and 3(b). For ECHDE and ECNADE, both types of scale factors can accommodate the transition from quintessence to phantom stages; that is, phantom crossing is possible for these models. Hence we conclude that phantom crossing happens for entropy-corrected terms ($\xi \neq 0, \zeta \neq 0$) of HDE and NADE models. Also we have observed that the forms of the constructed function $F(K)$ and the EoS parameter w_{EA} are similar for type I and type II models of the scale factor.

We may also assume the scale factor in de Sitter space time as in the form $a(t) = a_0 e^{Ht}$, where H is constant, which can describe the early-time inflation of the universe. In this
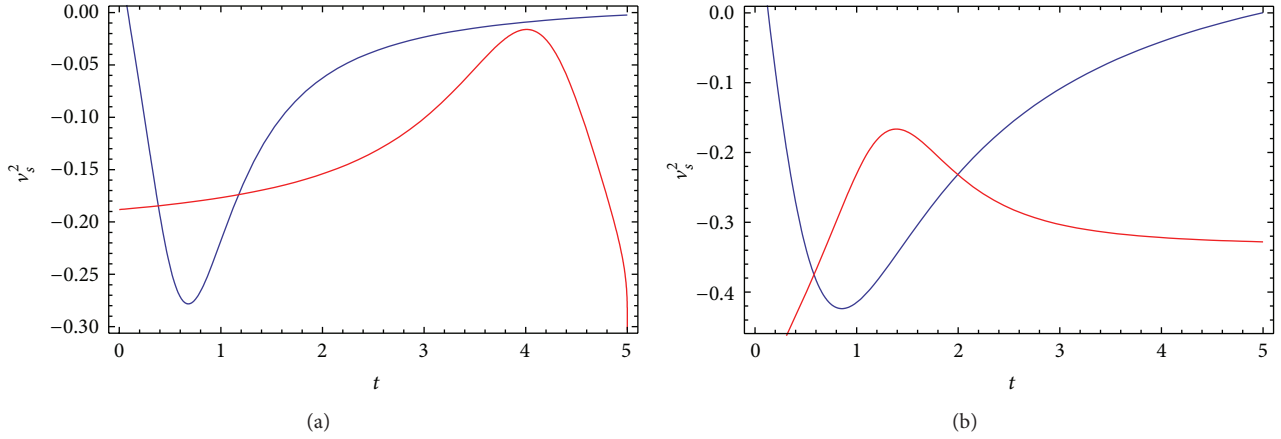
FIGURE 4: ((a) and (b)) The variations of v_s^2 against t for ECHDE and ECNADE models. Blue line represents type I model and red line represents type II model.

case we shall get $K = 3\beta H^2/M^2$, which is constant and hence $F(K)$ must be a constant function. So the reconstruction is not possible for de Sitter space. For this choice of scale factor, we must have $\dot{H} = 0$ and from (17), we get $w_{EA} = -1$, which behaves like the cosmological constant. For de Sitter space $H = $ constant implies the Ricci scalar $R = $ constant which also implies $f(R)$ must be a constant function. But they have obtained $f(R)$ in terms of R, so the right-hand side of $f(R)$ must be constant. So for HDE, NADE, ECHDE, and ECNADE models, the $f(R)$ cannot be reconstructed in terms of R. So their reconstruction analysis in de Sitter space is not correct.

Next we want to examine the stability of the Einstein-Aether gravity model. For this purpose we need to verify the sign of the square speed of sound which is defined by $v_s^2 = \partial p_{EA}/\rho_{EA} = \dot{p}_{EA}/\dot{\rho}_{EA}$. If $v_s^2 > 0$, the model is stable and $v_s^2 < 0$ implies the model is classically unstable. Some authors [61, 62, 66–69] have shown that HDE, ADE, NADE, Chaplygin gas, holographic Chaplygin, holographic $f(T)$, holographic $f(G)$, new agegraphic $f(T)$, and new agegraphic $f(G)$ models are classically unstable because square speed of sound is negative throughout the evolution of the universe. In our reconstructing Einstein-Aether gravity model from HDE, NADE, ECHDE, and ECNADE, we have to examine the signs of v_s^2. For HDE model, we find $v_s^2 = -1 + 2/3m$ for type I model and $v_s^2 = -1 - 2/3n$ for type II model. Since $m > 1$ and $n > 0$, so we obtain $v_s^2 < 0$ for type I and type II models. Also For NADE model, we find $v_s^2 = -5/3 + 2/3m$ for type I model and $v_s^2 = -5/3 - 2/3n$ for type II model. Since $1/2 < m < 1$ and $n > 0$, so we get $v_s^2 < 0$ for type I and type II models. Hence we conclude that the Einstein-Aether gravity for HDE and NADE models are classically unstable. For ECHDE and ECNADE models, the expressions of p_{EA} and ρ_{EA} are complicated, so we draw the graphs of v_s^2 for these models in Figures 4(a) and 4(b), respectively, for type I and type II models. From the figures, we observe that $v_s^2 < 0$ for Einstein-Aether gravity in ECHDE and ECNADE models. So all the models are classically unstable at present and future stages of the FRW universe.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

References

[1] S. Perlmutter, G. Aldering, M. Della Valle et al., "Discovery of a supernova explosion at half the age of the Universe," *Nature*, vol. 391, no. 6662, pp. 51–54, 1998.

[2] A. G. Riess, A. V. Filippenko, P. Challis et al., "Observational evidence from supernovae for an accelerating universe and a cosmological constant," *The Astronomical Journal*, vol. 116, no. 3, p. 1009, 1998.

[3] S. Briddle, "Precision cosmology? Not just yet," *Science*, vol. 299, p. 1532, 2003.

[4] D. N. Spergel, L. Verde, and H. V. Peiris, "First-Year *Wilkinson Microwave Anisotropy Probe* (WMAP)* observations: determination of cosmological parameters," *The Astrophysical Journal Supplement Series*, vol. 148, no. 1, article 175, 2003.

[5] E. J. Copeland, M. Sami, and S. Tsujikawa, "Dynamics of dark energy," *International Journal of Modern Physics. D*, vol. 15, no. 11, pp. 1753–1935, 2006.

[6] C. Armendariz-Picon, V. Mukhanov, and P. J. Steinhardt, "Dynamical solution to the problem of a small cosmological constant and late-time cosmic acceleration," *Physical Review Letters*, vol. 85, no. 21, pp. 4438–4441, 2000.

[7] A. Sen, "Tachyon matter," *Journal of High Energy Physics*, vol. 2002, article 065, 2002.

[8] B. Feng, X. Wang, and X. Zhang, "Dark energy constraints from the cosmic age and supernova," *Physics Letters B*, vol. 607, no. 1-2, pp. 35–41, 2005.

[9] A. Kamenshchik, U. Moschella, and V. Pasquier, "An alternative to quintessence," *Physics Letters B: Nuclear, Elementary Particle and High-Energy Physics*, vol. 511, no. 2–4, pp. 265–268, 2001.

[10] P. J. E. Peebles and B. Ratra, "Cosmology with a time-variable cosmological "constant"," *The Astrophysical Journal*, vol. 325, pp. L17–L20, 1988.

[11] R. R. Caldwell, R. Dave, and P. J. Steinhardt, "Cosmological imprint of an energy component with general equation of state," *Physical Review Letters*, vol. 80, no. 8, pp. 1582–1585, 1998.

[12] K. Enqvist, S. Hannested, and M. S. Sloth, "Searching for a holographic connection between dark energy and the low/CMB multipoles," *Journal of Cosmology and Astroparticle Physics*, vol. 2005, article 004, 2005.

[13] X. Zhang, "Statefinder diagnostic for holographic dark energy model," *International Journal of Modern Physics D*, vol. 14, no. 9, pp. 1597–1606, 2005.

[14] D. Pavon and W. Zimdahl, "Holographic dark energy and present cosmic acceleration," in *A Century of Relativity Physics: ERE 2005; XXVIII Spanish Relativity Meeting*, vol. 841 of *AIP Conference Proceedings*, 2006.

[15] W. Fischler and L. Susskind, "Holography and cosmology," http://arxiv.org/abs/hep-th/9806039.

[16] M. Li, "A model of holographic dark energy," *Physics Letters B*, vol. 603, no. 1-2, pp. 1–5, 2004.

[17] S. D. H. Hsu, "Entropy bounds and dark energy," *Physics Letters B*, vol. 594, no. 1-2, pp. 13–16, 2004.

[18] Y. Gong, "Extended holographic dark energy," *Physical Review D*, vol. 70, Article ID 064029, 2004.

[19] A. G. Cohen, D. B. Kaplan, and A. E. Nelson, "Effective field theory, black holes, and the cosmological constant," *Physical Review Letters*, vol. 82, no. 25, pp. 4971–4974, 1999.

[20] H. Wei and S. N. Zhang, "Age problem in the holographic dark energy model," *Physical Review D*, vol. 76, no. 6, Article ID 063003, 7 pages, 2007.

[21] R. Banerjee and B. R. Majhi, "Quantum tunneling and back reaction," *Physics Letters B*, vol. 662, no. 1, pp. 62–65, 2008.

[22] S. K. Modak, "Corrected entropy of BTZ black hole in tunneling approach," *Physics Letters B*, vol. 671, no. 1, pp. 167–173, 2009.

[23] H. Wei, "Entropy-corrected holographic dark energy," *Communications in Theoretical Physics*, vol. 52, no. 4, pp. 743–749, 2009.

[24] M. Jamil and M. U. Farooq, "Interacting holographic dark energy with logarithmic correction," *Journal of Cosmology and Astroparticle Physics*, vol. 2010, no. 3, p. 001, 2010.

[25] R.-G. Cai, "A dark energy model characterized by the age of the universe," *Physics Letters B*, vol. 657, no. 4-5, pp. 228–231, 2007.

[26] M. Maziashvili, "Space-time in light of Karolyhazy uncertainty relation," *International Journal of Modern Physics D*, vol. 16, no. 9, p. 1531, 2007.

[27] M. Maziashvili, "Cosmological implications of Károlyházy uncertainty relation," *Physics Letters B*, vol. 652, no. 4, pp. 165–168, 2007.

[28] H. Wei and R. G. Cai, "A new model of agegraphic dark energy," *Physics Letters B*, vol. 660, no. 3, pp. 113–117, 2008.

[29] Y. S. Myung and M. G. Seo, "Origin of holographic dark energy models," *Physics Letters B*, vol. 671, no. 4-5, pp. 435–439, 2009.

[30] M. U. Farooq, M. Jamil, and M. A. Rashid, "Interacting entropy-corrected new agegraphic K-essence, tachyon and dilaton scalar field models in non-flat universe," *International Journal of Theoretical Physics*, vol. 49, no. 9, pp. 2278–2287, 2010.

[31] S. Tsujikawa, "Modified gravity models of dark energy," in *Lectures on Cosmology*, vol. 800 of *Lecture Notes in Physics*, pp. 99–145, Springer, Berlin, Germany, 2010.

[32] G. R. Dvali, G. Gabadadze, and M. Porrati, "Metastable gravitons and infinite volume extra dimensions," *Physics Letters Section B: Nuclear, Elementary Particle and High-Energy Physics*, vol. 484, no. 1-2, pp. 112–118, 2000.

[33] A. de Felice and T. Tsujikawa, " $f(R)$ theories," *Living Reviews in Relativity*, vol. 13, article 3, 2010.

[34] M. C. B. Abdalla, S. Nojiri, and S. D. Odintsov, "Consistent modified gravity: dark energy, acceleration and the absence of cosmic doomsday," *Classical and Quantum Gravity*, vol. 22, no. 5, pp. L35–L42, 2005.

[35] E. V. Linder, "Einstein's other gravity and the acceleration of the universe," *Physical Review D*, vol. 81, no. 12, Article ID 127301, 3 pages, 2010.

[36] K. K. Yerzhanov, S. R. Myrzakul, I. I. Kulnazarov, and R. Myrzakulov, "Accelerating cosmology in $F(T)$ gravity with scalar field," http://arxiv.org/abs/1006.3879.

[37] S. Nojiri and S. D. Odintsov, "Modified Gauss–Bonnet theory as gravitational alternative for dark energy," *Physics Letters B*, vol. 631, no. 1-2, pp. 1–6, 2005.

[38] I. Antoniadis, J. Rizos, and K. Tamvakis, "Singularity-free cosmological solutions of the superstring effective action," *Nuclear Physics B*, vol. 415, no. 2, pp. 497–514, 1994.

[39] P. Hořava, "Membranes at quantum criticality," *Journal of High Energy Physics*, vol. 2009, article 020, 2009.

[40] C. Brans and R. H. Dicke, "Mach's principle and a relativistic theory of gravitation," *Physical Review*, vol. 124, no. 3, pp. 925–935, 1961.

[41] T. Jacobson and D. Mattingly, "Gravity with a dynamical preferred frame," *Physical Review D*, vol. 64, no. 2, 2001.

[42] T. Jacobson and D. Mattingly, "Einstein-aether waves," *Physical Review D*, vol. 70, Article ID 024003, 2004.

[43] T. G. Zlosnik, P. G. Ferreira, and G. D. Starkman, "Modifying gravity with the aether: an alternative to dark matter," *Physical Review D*, vol. 75, no. 4, Article ID 044017, 2007.

[44] T. G. Zlosnik, P. G. Ferreira, and G. D. Starkman, "Growth of structure in theories with a dynamical preferred frame," *Physical Review D*, vol. 77, no. 8, Article ID 084010, 2008.

[45] D. Garfinkle and T. Jacobson, "A positive-energy theorem for einstein-aether and hořava gravity," *Physical Review Letters*, vol. 107, Article ID 191102, 2011.

[46] E. V. Linder and R. J. Scherrer, "Aetherizing Lambda: barotropic fluids as dark energy," *Physical Review D*, vol. 80, Article ID 023008, 2009.

[47] J. D. Barrow, "Some inflationary Einstein-Aether cosmologies," *Physical Review D*, vol. 85, Article ID 047503, 2012.

[48] J. Zuntz, T. G. Zlosnik, F. Bourliot, P. G. Ferreira, and G. D. Starkman, "Vector field models of modified gravity and the dark sector," *Physical Review D: Particles, Fields, Gravitation and Cosmology*, vol. 81, no. 10, Article ID 104015, 2010.

[49] B. Li, D. F. Mota, and J. D. Barrow, "Detecting a Lorentz-violating field in cosmology," *Physical Review D*, vol. 77, no. 2, 2008.

[50] M. Gasperini, "Singularity prevention and broken Lorentz symmetry," *Classical and Quantum Gravity*, vol. 4, no. 2, p. 485, 1987.

[51] M. Gasperini, "Repulsive gravity in the very early universe," *General Relativity and Gravitation*, vol. 30, no. 12, pp. 1703–1709, 1998.

[52] X. Meng and X. Du, "Einstein-aether theory as an alternative to dark energy model?" *Physics Letters B*, vol. 710, no. 4-5, pp. 493–499, 2012.

[53] M. Xin-He and D. Xiao-Long, "A specific case of generalized einstein-aether theories," *Communications in Theoretical Physics*, vol. 57, no. 2, article 227, 2012.

[54] M. H. Daouda, M. E. Rodrigues, and M. J. S. Houndjo, "Static anisotropic solutions in $f(T)$ theory," *European Physical Journal C*, vol. 72, article 1890, 2012.

[55] M. J. S. Houndjo and O. F. Piattella, "Reconstructing $f(R,T)$ gravity from holographic dark energy," *International Journal of Modern Physics D*, vol. 21, no. 3, Article ID 1250024, 2012.

[56] M. R. Setare, "Holographic modified gravity," *International Journal of Modern Physics D*, vol. 17, no. 12, p. 2219, 2008.

[57] M. R. Setare and E. N. Saridakis, "Correspondence between holographic and Gauss-Bonnet dark energy models," *Physics Letters B*, vol. 670, no. 1, pp. 1–4, 2008.

[58] M. R. Setare and M. Jamil, "Correspondence between entropy-corrected holographic and Gauss-Bonnet dark-energy models," *EPL*, vol. 92, no. 4, Article ID 49003, 2010.

[59] U. Debnath and M. Jamil, "Correspondence between DBI-essence and modified Chaplygin gas and the generalized second law of thermodynamics," *Astrophysics and Space Science*, vol. 335, no. 2, pp. 545–552, 2011.

[60] S. Chottopadhyay and U. Debnath, "Correspondence between Ricci and other dark energies," *International Journal of Theoretical Physics*, vol. 50, no. 2, pp. 315–324, 2011.

[61] Abdul Jawad, S. Chattopadhyay, and A. Pasqua, "Reconstruction of $f(G)$ gravity with the new agegraphic dark-energy model," *The European Physical Journal Plus*, vol. 128, article 88, 2013.

[62] A. Jawad, A. Pasqua, and S. Chattopadhyay, "Correspondence between $f(G)$ gravity and holographic dark energy via power-law solution," *Astrophysics and Space Science*, vol. 344, no. 2, pp. 489–494, 2013.

[63] S. Nojiri and S. D. Odintsov, "Introduction to modified gravity and gravitational alternative for dark energy," *International Journal of Geometric Methods in Modern Physics*, vol. 4, no. 1, pp. 115–145, 2007.

[64] X. Wu and Z.-H. Zhu, "Reconstructing $f(R)$ theory according to holographic dark energy," *Physics Letters B*, vol. 660, no. 4, pp. 293–298, 2008.

[65] D. Pavón and W. Zimdahl, "Holographic dark energy and cosmic coincidence," *Physics Letters B*, vol. 628, no. 3-4, pp. 206–210, 2005.

[66] K. Y. Kim, H. W. Lee, and Y. S. Myung, "Instability of agegraphic dark energy models," *Physics Letters B*, vol. 660, no. 3, pp. 118–124, 2008.

[67] Y. S. Myung, "Instability of holographic dark energy models," *Physics Letters B*, vol. 652, no. 5-6, pp. 223–227, 2007.

[68] M. Sharif and A. Jawad, "Cosmological evolution of interacting new holographic dark energy in non-flat universe," *The European Physical Journal C*, vol. 72, pp. 2097–2098, 2012.

[69] M. R. Setare, "Interacting holographic generalized Chaplygin gas model," *Physics Letters B*, vol. 654, no. 1-2, pp. 1–6, 2007.

Journal of
Gravity

Journal of
Photonics

The Scientific World Journal

Journal of
Soft Matter

Advances in
Condensed Matter Physics

Journal of
Aerodynamics

Journal of
Fluids

Journal of
Computational Methods in Physics

Advances in
High Energy Physics

Journal of
Thermodynamics

Physics Research International

Advances in
Astronomy

International Journal of
Statistical Mechanics

International Journal of
Optics

Hindawi

Submit your manuscripts at
http://www.hindawi.com

Journal of
Solid State Physics

Journal of
Astrophysics

International Journal of
Superconductivity

Journal of
Biophysics

Journal of
Atomic and Molecular Physics