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Research Article

Hawking Radiation-Quasi-Normal Modes Correspondence and Effective States for Nonextremal Reissner-Nordström Black Holes

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It is known that the nonstrictly thermal character of the Hawking radiation spectrum harmonizes Hawking radiation with black hole (BH) quasi-normal modes (QNM). This paramount issue has been recently analyzed in the framework of both Schwarzschild BHs (SBH) and Kerr BHs (KBH). In this assignment, we generalize the analysis to the framework of *nonextremal* Reissner-Nordström BHs (RNBH). Such a generalization is important because in both Schwarzschild and Kerr BHs an absorbed (emitted) particle has only mass. Instead, in RNBH the particle has charge as well as mass. In doing so, we expose that, for the RNBH, QNMs can be naturally interpreted in terms of quantum levels for both particle emission and absorption. Conjointly, we generalize some concepts concerning the RNBH's "effective states."

1. Introduction

A RNBH of mass M is identical to a SBH of mass M except that a RNBH has the *nonzero* charge quantity Q. In this paper, we are interested in RNBHs with the *nonextremal* constraint $M > Q$ [1]. The quantity Q is the physical mechanism for the RNBH's *dual* horizons from (1) in [1]:

$$r_{\pm} = R_{\pm_{\text{RNBH}}}(M, Q) = M \pm \sqrt{M^2 - Q^2}, \tag{1}$$

because the RNBH outer (event) horizon radius $R_{+_{\text{RNBH}}}(M, Q)$ and the *RNBH inner (Cauchy) horizon radius* $R_{-_{\text{RNBH}}}(M, Q)$ are clearly functions of both M and Q, not just M, as in the well known case of the *SBH horizon radius*

$$r_s = R_{\text{SBH}}(M) = 2M. \tag{2}$$

Energy conservation plays a fundamental role in BH radiance [2] because the emission or absorption of Hawking quanta with mass m and energy-frequency ω causes a BH of mass M to undergo a transition between *discrete* energy spectrum levels [3–7], where

$$E = m = \omega = \Delta M \tag{3}$$

for $G = c = k_B = \hbar = 1/4\pi\epsilon_0 = 1$ (Planck units). Given that emission and absorption are *reverse* processes for the quantized energy spectrum conservation [3–7], we consider this pair of transitions as being *equal in magnitude* but *opposite in direction* from the neutral radius perspective of $r_0 = (r_+ + r_-)/2$.

It is known that the countable character of successive emissions of Hawking quanta which is a consequence of the nonstrictly thermal character of the Hawking radiation spectrum (see [3–12]) generates a natural correspondence between Hawking radiation and BH QNMs [3–7]. Moreover, it has also been shown that QNMs can be naturally interpreted in terms of quantum levels, where the emission or absorption of a particle is interpreted as a transition between two distinct levels on the *discrete* energy spectrum [3–7]. The thermal spectrum correction is an imperative adjustment to the physical interpretation of BH QNMs because these results are important to realize the underlying unitary quantum gravity theory [3–7]. Hod's intriguing works [13, 14] suggested that BH QNMs carry principle information regarding a BH's horizon area quantization. Hod's influential conjecture was later refined and clarified by Maggiore [15]. Moreover, it is also believed that QNMs delve into the microstructure of spacetime [16].

To make sense of the state space for the energy spectrum states and the underlying BH perturbation field states, an *effective framework* based on the *nonstrictly thermal* behavior of Hawking's framework began to emerge [3–7]. In the midst of this superceding BH effective framework [3–7], the BH effective state concept was originally introduced for KBHs in [6] and subsequently applied through Hawking's periodicity arguments [17, 18] to the BH tunneling mechanism's nonstrictly black body spectrum [7]. The effective state is meaningful to BH physics and thermodynamics research because one needs additional features and knowledge to consider in future experiments and observations.

In this paper, our objective is to apply the nonstrictly thermal BH effective framework of [3–7] to nonextremal RNBHs. Thus, upon recalling that a RNBH of mass M is identical to a SBH of mass M except that a RNBH has the charge Q, we prepare for our BH QNM investigation by reviewing relevant portions of the SBH effective framework [3–7] for quantities related to SBH states and transitions in Section 2. Then in Section 3, we launch our RNBH QNM exploration by introducing a RNBH effective framework for quantities pertaining to RNBH states and transitions. Finally, we conclude with a brief comparison between the fundamental SBH and RNBH results in Section 4 followed by the recapitulation in Section 5.

2. Schwarzschild Black Hole Framework: Background and Review

2.1. Schwarzschild Black Hole States and Transitions. Here, we recall some quantities that characterize the SBH.

First, consider a SBH of initial mass M, when the SBH emits or absorbs a quantum of energy-frequency ω (for particle mass m and SBH mass change ΔM, such that $m = \omega = \Delta M$) to achieve a final mass of $M-\omega$ or $M+\omega$, respectively, for the SBH mass-energy transition between states in state space. Thus, we follow [3–5], where the *SBH initial and final horizon area* are

$$\begin{aligned} A_{\mathrm{SBH}}(M) &= 16\pi M^2 = 4\pi R_{\mathrm{SBH}}^2(M), \\ A_{\mathrm{SBH}}(M\pm\omega) &= 16\pi (M\pm\omega)^2 = 4\pi R_{\mathrm{SBH}}^2(M\pm\omega), \end{aligned} \tag{4}$$

respectively, for the *SBH area quanta number*

$$N_{\mathrm{SBH}}(M,\omega) = \frac{A_{\mathrm{SBH}}(M)}{\left|\Delta A_{\mathrm{SBH}}(M,\omega)\right|}, \tag{5}$$

such that the *SBH horizon area change* for the corresponding mass change ΔM is

$$\begin{aligned} \Delta A_{\mathrm{SBH}}(M,\omega) &= A_{\mathrm{SBH}}(M\pm\omega) - A_{\mathrm{SBH}}(M) \\ &= 32\pi M\omega + O\left(\omega^2\right) \sim 32\pi M \Delta M \\ &= 32\pi M \Delta E, \end{aligned} \tag{6}$$

because the transition's minus $(-)$ and plus $(+)$ signs depend on emission and absorption, respectively. Next, in [3–5], the *Bekenstein-Hawking SBH initial and final entropy* are

$$\begin{aligned} S_{\mathrm{SBH}}(M) &= \frac{A_{\mathrm{SBH}}(M)}{4}, \\ S_{\mathrm{SBH}}(M\pm\omega) &= \frac{A_{\mathrm{SBH}}(M\pm\omega)}{4}, \end{aligned} \tag{7}$$

respectively, where the corresponding *SBH entropy change* is

$$\Delta S_{\mathrm{SBH}}(M,\omega) = \frac{\Delta A_{\mathrm{SBH}}(M,\omega)}{4}. \tag{8}$$

Subsequently, the *SBH initial and final total entropy* are [3–5]

$$\begin{aligned} S_{\mathrm{SBH-total}}(M) &= S_{\mathrm{SBH}}(M) - \ln S_{\mathrm{SBH}}(M) \\ &\quad + \frac{3}{2A_{\mathrm{SBH}}(M)}, \\ S_{\mathrm{SBH-total}}(M\pm\omega) &= S_{\mathrm{SBH}}(M\pm\omega) - \ln S_{\mathrm{SBH}}(M\pm\omega) \\ &\quad + \frac{3}{2A_{\mathrm{SBH}}(M\pm\omega)}, \end{aligned} \tag{9}$$

respectively. Additionally, the *SBH initial and final Hawking temperature* are [3–5]

$$\begin{aligned} T_{H_{\mathrm{SBH}}}(M) &= \frac{1}{8\pi M}, \\ T_{H_{\mathrm{SBH}}}(M\pm\omega) &= \frac{1}{8\pi(M\pm\omega)}, \end{aligned} \tag{10}$$

respectively. Therefore, the quantum transition's *SBH emission tunneling rate* is [3–5]

$$\begin{aligned} \Gamma_{\mathrm{SBH}}(M,\omega) &\sim \exp\left[-8\pi M\omega\left(1-\frac{\omega}{2M}\right)\right] \\ &\sim \exp\left[-\frac{\omega}{T_{H_{\mathrm{SBH}}}(M)}\left(1-\frac{\omega}{R_{\mathrm{SBH}}(M)}\right)\right] \\ &\sim \exp\left[+\Delta S_{\mathrm{SBH}}(M,\omega)\right]. \end{aligned} \tag{11}$$

2.2. Schwarzschild Black Hole Effective States and Transitions. Here, we recall some effective quantities that characterize the SBH.

Given that M is the mass state *before* and $M \pm \omega$ is the mass state *after* the quantum transition, the *SBH effective mass* and *SBH effective horizon* are, respectively, identified in [3–5] as

$$\begin{gathered} M_E(M,\omega) = \frac{M + (M \pm \omega)}{2} = M \pm \frac{\omega}{2}, \\ R_{E_{\text{SBH}}}(M,\omega) = 2M_E(M,\omega), \end{gathered} \tag{12}$$

which are *average quantities* between the two states *before* and *after* the process [3–5]. Consequently, using (4) and (12) we define the *SBH effective horizon area* as

$$\begin{aligned} A_{E_{\text{SBH}}}(M,\omega) &\equiv \frac{A_{\text{SBH}}(M) + A_{\text{SBH}}(M \pm \omega)}{2} \\ &= 16\pi M_E^2(M,\omega) = 4\pi R_{E_{\text{SBH}}}^2(M,\omega), \end{aligned} \tag{13}$$

which is the average of the SBH's initial and final horizon areas. Subsequently, utilizing (7), the Bekenstein-Hawking *SBH effective entropy* is defined as

$$S_{E_{\text{SBH}}}(M,\omega) \equiv \frac{S_{\text{SBH}}(M) + S_{\text{SBH}}(M \pm \omega)}{2}, \tag{14}$$

and consequently employs (13) and (14) to define the *SBH effective total entropy* as

$$\begin{aligned} &S_{E_{\text{SBH-total}}}(M,\omega) \\ &\quad \equiv S_{E_{\text{SBH}}}(M,\omega) - \ln S_{E_{\text{SBH}}}(M,\omega) + \frac{3}{2A_{E_{\text{SBH}}}(M,\omega)}. \end{aligned} \tag{15}$$

Thus, employing (3) and (10), the *SBH effective temperature* is [3–5]

$$\begin{aligned} T_{E_{\text{SBH}}}(M,\omega) &= \left(\frac{T_{H_{\text{SBH}}}^{-1}(M) + T_{H_{\text{SBH}}}^{-1}(M \pm \omega)}{2}\right)^{-1} \\ &= \left(8\pi\left[\frac{M + M \pm \omega}{2}\right]\right)^{-1} \\ &= \frac{1}{4\pi(2M \pm \omega)} = \frac{1}{8\pi M_E(M,\omega)}, \end{aligned} \tag{16}$$

which is the inverse of the average value of the inverses of the initial and final Hawking temperatures. Consequently, (16) lets one rewrite (11) to define the *SBH effective emission tunneling rate* (in the Boltzmann-like form) as [3–5]

$$\begin{aligned} \Gamma_{E_{\text{SBH}}}(M,\omega) &\sim \exp\left[-\frac{\omega}{T_{E_{\text{SBH}}}(M,\omega)}\right] \\ &= \exp\left[+\Delta S_{E_{\text{SBH}}}(M,\omega)\right], \end{aligned} \tag{17}$$

such that (14) defines the *SBH effective entropy change* as

$$\Delta S_{E_{\text{SBH}}}(M,\omega) = S_{\text{SBH}}(M \pm \omega) - S_{\text{SBH}}(M) = \frac{\Delta A_{E_{\text{SBH}}}(M,\omega)}{4} \tag{18}$$

because the *SBH effective horizon area change* is

$$\Delta A_{E_{\text{SBH}}}(M,\omega) = 16\pi M_E(M,\omega)\,\omega \tag{19}$$

and the *SBH effective area quanta number* is

$$N_{E_{\text{SBH}}}(M,\omega) = \frac{A_{E_{\text{SBH}}}(M,\omega)}{\Delta A_{E_{\text{SBH}}}(M,\omega)}. \tag{20}$$

2.3. Effective Application of Quasi-Normal Modes to the Schwarzschild Black Hole. Here, we recall how the SBH perturbation field QNM states can be applied to the SBH effective framework.

The quasi-normal frequencies (QNFs) are typically labeled as ω_{nl}, where l is the angular momentum quantum number [3–5, 15, 19]. Thus, for each l, such that $l \geq 2$ for gravitational perturbations, there is a countable sequence of QNMs labeled by the overtone number n, which is a natural number [3–5, 15].

Now $|\omega_n|$ is the damped harmonic oscillator's proper frequency that is defined as [3–5, 15]

$$|\omega_n| = (\omega_0)_n = \sqrt{\omega_{n_{\mathbb{R}}}^2 + \omega_{n_{\mathbb{I}}}^2}. \tag{21}$$

Maggiore [15] articulated that the establishment $|\omega_n| = \omega_{n_{\mathbb{R}}}$ is only correct for the very long-lived and lowly excited QNMs approximation $|\omega_n| \gg \omega_{n_{\mathbb{I}}}$, whereas for a lot of BH QNMs, such as those that are highly excited, the opposite limit is correct [3–5, 15]. Therefore, the ω parameter in (12)–(20) is substituted for the $|\omega_n|$ parameter [3–5] because we wish to employ BH QNFs. When n is large, the SBH QNFs become independent of l and thereby exhibit the nonstrictly thermal structure [3–5]

$$\begin{aligned} \omega_n &= \ln 3 \times T_{E_{\text{SBH}}}(M,|\omega_n|) + 2\pi i\left(n + \frac{1}{2}\right) \times T_{E_{\text{SBH}}}(M,|\omega_n|) \\ &\quad + \mathcal{O}\left(n^{-1/2}\right) = \frac{\ln 3}{4\pi\left[2M - |\omega_n|\right]} + \frac{2\pi i}{4\pi\left[2M - |\omega_n|\right]} \\ &\quad \times \left(n + \frac{1}{2}\right) + \mathcal{O}\left(n^{-1/2}\right) = \frac{\ln 3}{8\pi M_E(M,|\omega_n|)} \\ &\quad + \frac{2\pi(n + 1/2)}{8\pi M_E(M,|\omega_n|)} i + \mathcal{O}\left(n^{-1/2}\right), \end{aligned} \tag{22}$$

where

$$\begin{aligned} m_n &\equiv \omega_{n_{\mathbb{R}}} = \frac{\ln 3}{8\pi M_E(M,|\omega_n|)}, \\ p_n &\equiv \omega_{n_{\mathbb{I}}} = \frac{2\pi}{8\pi M_E(M,|\omega_n|)}\left(n + \frac{1}{2}\right). \end{aligned} \tag{23}$$

Thus, when referring to highly excited QNMs one gets $|\omega_n| \approx p_n$ [3–5], where the quantized levels differ from [15] because they are not equally spaced in exact form. Therefore, according to [3–5], we have

$$|\omega_n| = \frac{\sqrt{(\ln 3)^2 + 4\pi^2(n+1/2)^2}}{8\pi M_E(M,|\omega_n|)} = T_{E_{\mathrm{SBH}}}(M,|\omega_n|)\sqrt{(\ln 3)^2 + 4\pi^2\left(n+\frac{1}{2}\right)^2}, \quad (24)$$

which is solved to yield

$$|\omega_n| = M - \sqrt{M^2 - \frac{\sqrt{(\ln 3)^2 + 4\pi^2(n+1/2)^2}}{4\pi}} \quad (25)$$

when we obey $|\omega_n| < M$ because a BH cannot emit more energy than its total mass.

3. Reissner-Nordström Black Hole Framework: An Introduction

We note that for this framework we consider the *RNBH event horizon* features, which are derived from the $R_{+_{\mathrm{RNBH}}}(M,Q)$ in (1).

3.1. Reissner-Nordström Black Hole States and Transitions. Here, we recall some quantities that characterize the RNBH.

First, consider a RNBH of initial mass M and initial charge Q. Using (1), we define the *RNBH initial event horizon area* as

$$A_{+_{\mathrm{RNBH}}}(M,Q) = 4\pi\left(M+\sqrt{M^2-Q^2}\right)^2 = 4\pi R^2_{+_{\mathrm{RNBH}}}(M,Q), \quad (26)$$

the *Bekenstein-Hawking RNBH initial entropy* as

$$S_{+_{\mathrm{RNBH}}}(M,Q) = \frac{A_{+_{\mathrm{RNBH}}}(M,Q)}{4}, \quad (27)$$

and the *RNBH initial electrostatic potential* as

$$\Phi_+(M,Q) = \frac{Q}{4\pi R_{+_{\mathrm{RNBH}}}(M,Q)} = \frac{Q}{4\pi\left(M+\sqrt{M^2-Q^2}\right)}. \quad (28)$$

Consequently, (17) of [2] identifies the *RNBH initial Hawking temperature* as

$$T_{+H_{\mathrm{RNBH}}}(M,Q) = \frac{\sqrt{M^2-Q^2}}{2\pi\left(M+\sqrt{M^2-Q^2}\right)^2} = \frac{R_{+_{\mathrm{RNBH}}}(M,Q) - R_{-_{\mathrm{RNBH}}}(M,Q)}{A_{+_{\mathrm{RNBH}}}(M,Q)}. \quad (29)$$

Second, consider when the RNBH emits or absorbs a quantum of energy-frequency ω with charge q to achieve a final mass of $M-\omega$ or $M+\omega$ and a final charge of $Q-q$ or $Q+q$, respectively, for the RNBH mass-energy transition between states in state space. For this, all we need to do is replace the RNBH's mass and charge parameters in (26) and (29). Thus, (26) establishes the *RNBH final event horizon area* as

$$A_{+_{\mathrm{RNBH}}}(M\pm\omega,Q\pm q) = 4\pi R^2_{+_{\mathrm{RNBH}}}(M\pm\omega,Q\pm q) = 4\pi\left((M\pm\omega)+\sqrt{(M\pm\omega)^2-(Q\pm q)^2}\right)^2. \quad (30)$$

Equation (27) presents the *Bekenstein-Hawking RNBH final entropy* as

$$S_{+_{\mathrm{RNBH}}}(M\pm\omega,Q\pm q) = \frac{A_{+_{\mathrm{RNBH}}}(M\pm\omega,Q\pm q)}{4}, \quad (31)$$

and (28) defines the *RNBH final electrostatic potential* as

$$\Phi_+(M\pm\omega,Q\pm q) = \frac{Q}{4\pi R_{+_{\mathrm{RNBH}}}(M\pm\omega,Q\pm q)} = \frac{Q}{4\pi\left((M\pm\omega)+\sqrt{(M\pm\omega)^2-(Q\pm q)^2}\right)} \quad (32)$$

for usage in (29) of [20], where it is proposed that the *RNBH adiabatic invariant* is

$$I_{+_{\mathrm{RNBH}}}(M,\omega,Q,q) = \int\frac{\omega - \Phi_+(M\pm\omega,Q)\,q}{\omega} = \int\frac{\Delta M - \Phi_+(M\pm\Delta M,Q)\,\Delta Q}{\Delta M} \quad (33)$$

because $\Delta Q = q$. Hence, (29) identifies the *RNBH final Hawking temperature* as

$$T_{+H_{\mathrm{RNBH}}}(M\pm\omega,Q\pm q) = \frac{R_{+_{\mathrm{RNBH}}}(M\pm\omega,Q\pm q) - R_{-_{\mathrm{RNBH}}}(M\pm\omega,Q\pm q)}{A_{+_{\mathrm{RNBH}}}(M\pm\omega,Q\pm q)} = \frac{\sqrt{(M\pm\omega)^2-(Q\pm q)^2}}{2\pi\left((M\pm\omega)+\sqrt{(M\pm\omega)^2-(Q\pm q)^2}\right)^2}. \quad (34)$$

Next, upon generalizing (16) in [2] and the work [21], we define the *RNBH tunneling rate* as

$$\Gamma_{+_{\mathrm{RNBH}}}(M,\omega,Q,q) \sim \exp\left[-4\pi\left(2\omega\left(M\pm\frac{\omega}{2}\right) - (M\pm\omega)\sqrt{(M\pm\omega)^2-(Q\pm q)^2} + M\sqrt{M^2-Q^2}\right)\right] \sim \exp\left[\Delta S_{+_{\mathrm{RNBH}}}(M,\omega,Q,q)\right], \quad (35)$$

where we utilize (30) to define the *Bekenstein-Hawking RNBH entropy change* as

$$\Delta S_{+_{\mathrm{RNBH}}}(M,\omega,Q,q)=\frac{\Delta A_{+_{\mathrm{RNBH}}}(M,\omega,Q,q)}{4}, \tag{36}$$

such that the *RNBH event horizon area change* is

$$\begin{aligned}\Delta A_{+_{\mathrm{RNBH}}}(M,\omega,Q,q)&=A_{+_{\mathrm{RNBH}}}(M\pm\omega,Q\pm q)\\&\quad-A_{+_{\mathrm{RNBH}}}(M,Q)\end{aligned} \tag{37}$$

so we can define the *RNBH event horizon area quanta number* as

$$N_{+_{\mathrm{RNBH}}}(M,\omega,Q,q)=\frac{A_{+_{\mathrm{RNBH}}}(M,Q)}{\left|\Delta A_{+_{\mathrm{RNBH}}}(M,\omega,Q,q)\right|}. \tag{38}$$

3.2. Reissner-Nordström Black Hole Effective States and Transitions. Here, we define some effective quantities that characterize the RNBH.

The *RNBH effective mass* is equivalent to the SBH effective mass component of (12), which is

$$M_E(M,\omega)\equiv\frac{M+(M\pm\omega)}{2}. \tag{39}$$

Next, we define the *RNBH effective charge* as

$$Q_E(Q,q)\equiv\frac{Q+(Q\pm q)}{2}, \tag{40}$$

which is the average of the RNBH's initial charge Q and final charge $Q\pm q$. From this, (1), (39), and (40) are used to define the corresponding *RNBH effective event horizon* and *RNBH effective Cauchy horizon* as

$$\begin{aligned}r_{\pm E}\equiv R_{\pm E_{\mathrm{RNBH}}}(M,\omega,Q,q)&\equiv M_E(M,\omega)\\&\quad\pm\sqrt{M_E^2(M,\omega)-Q_E^2(Q,q)},\end{aligned} \tag{41}$$

with respect to the energy conservation and pair production neutrality of (39). Next, we employ (26), (39), and (41) to define the *RNBH effective event horizon area* as

$$\begin{aligned}&A_{+E_{\mathrm{RNBH}}}(M,\omega,Q,q)\\&\quad\equiv4\pi R^2_{+E_{\mathrm{RNBH}}}(M,\omega,Q,q)\\&\quad\equiv4\pi\left(M_E(M,\omega)+\sqrt{M_E^2(M,\omega)-Q_E^2(Q,q)}\right)^2,\end{aligned} \tag{42}$$

which is then used to define the *RNBH effective entropy* as

$$S_{+E_{\mathrm{RNBH}}}(M,\omega,Q,q)\equiv\frac{A_{+E_{\mathrm{RNBH}}}(M,\omega,Q,q)}{4}. \tag{43}$$

Afterwards, we use (28) and (42) to define the *RNBH effective electrostatic potential* as

$$\begin{aligned}&\Phi_{+E}(M,\omega,Q,q)\\&\quad\equiv\frac{Q_E(Q,q)}{4\pi R_{+E_{\mathrm{RNBH}}}(M,\omega,Q,q)}\\&\quad\equiv\frac{Q_E(Q,q)}{4\pi\left(M_E(M,\omega)+\sqrt{M_E^2(M,\omega)-Q_E^2(Q,q)}\right)}\end{aligned} \tag{44}$$

so we can utilize the $T_{E_{\mathrm{SBH}}}(M,\omega)$ in (16) along with (39), (40), and (44) to define the *RNBH effective adiabatic invariant* as

$$\begin{aligned}&I_{+E_{\mathrm{RNBH}}}(M,\omega,Q,q)\\&\quad\equiv\int\frac{dM_E(M,\omega)-\Phi_{+E}(M,\omega,Q,q)\,dQ_E(Q,q)}{T_{E_{\mathrm{SBH}}}(M,\omega)}.\end{aligned} \tag{45}$$

At this point, (16) and (35) let us introduce and define the *RNBH effective temperature* as

$$\begin{aligned}&T_{+E_{\mathrm{RNBH}}}(M,\omega,Q,q)\\&\quad\equiv\frac{\sqrt{(M\pm\omega/2)^2-(Q\pm q/2)^2}}{2\pi\left[(M\pm\omega/2)+\sqrt{(M\pm\omega/2)^2-(Q\pm q/2)^2}\right]^2}\\&\quad\equiv\frac{\sqrt{M_E^2(M,\omega)-Q_E^2(Q,q)}}{2\pi\left(M_E(M,\omega)+\sqrt{M_E^2(M,\omega)-Q_E^2(Q,q)}\right)^2}\\&\quad\equiv\frac{R_{+E_{\mathrm{RNBH}}}(M,\omega,Q,q)-R_{-E_{\mathrm{RNBH}}}(M,\omega,Q,q)}{A_{+E_{\mathrm{RNBH}}}(M,\omega,Q,q)},\end{aligned} \tag{46}$$

which authorizes us to exercise (36) and (46) to rewrite (35) to define the RNBH effective tunneling rate as

$$\begin{aligned}\Gamma_{+E_{\mathrm{RNBH}}}(M,\omega,Q,q)&\sim\exp\left[\frac{\pm\omega}{T_{+E_{\mathrm{RNBH}}}(M,\omega,Q,q)}\right]\\&\sim\exp\left[\Delta S_{+_{\mathrm{RNBH}}}(M,\omega,Q,q)\right],\end{aligned} \tag{47}$$

such that the *RNBH effective entropy change* is defined as

$$\Delta S_{+E_{\mathrm{RNBH}}}(M,\omega,Q,q)\equiv\frac{\Delta A_{+E_{\mathrm{RNBH}}}(M,\omega,Q,q)}{4} \tag{48}$$

for the *RNBH effective event horizon area change*

$$\Delta A_{+E_{\mathrm{RNBH}}}(M,\omega,Q,q)\equiv\frac{2\omega q+Q^3\pi}{\left(M^2-Q^2\right)^{3/2}} \tag{49}$$

and the *RNBH effective event horizon area quanta number*

$$N_{+E_{\mathrm{RNBH}}}(M,\omega,Q,q)\equiv\frac{A_{+E_{\mathrm{RNBH}}}(M,\omega,Q,q)}{\left|\Delta A_{+E_{\mathrm{RNBH}}}(M,\omega,Q,q)\right|}. \tag{50}$$

4. Effective Application of Quasi-Normal Modes to the Reissner-Nordström Black Hole

Here, we explain how the RNBH perturbation field QNM states can be applied to the RNBH effective framework.

Similarly to SBH QNFs, the RNBH QNFs become independent of l for large n [22]. Thus, for large n, we have two families of the QNM:

$$\omega_n = \ln 3 \times T_{+H_{\mathrm{SBH}}}(M,Q) - 2\pi\left(n+\frac{1}{2}\right) i \times T_{+H_{\mathrm{SBH}}}(M,Q) + \frac{qQ}{R_{+_{\mathrm{SBH}}}(M,Q)}, \tag{51}$$

$$\begin{aligned}\omega_n &= \ln 2 \times T_{+H_{\mathrm{RNBH}}}(M,Q) - 2\pi\left(n+\frac{1}{2}\right) i \times T_{+H_{\mathrm{RNBH}}}(M,Q) \\ &\quad + \frac{qQ}{R_{+_{\mathrm{RNBH}}}(M,Q)} = \frac{\ln 2\sqrt{M^2-Q^2}}{2\pi\left(M+\sqrt{M^2-Q^2}\right)^2} \\ &\quad - \frac{(n+1/2)\sqrt{M^2-Q^2}}{\left(M+\sqrt{M^2-Q^2}\right)^2} i + \frac{qQ}{R_{+_{\mathrm{RNBH}}}(M,Q)}.\end{aligned} \tag{52}$$

Now the approximation of (51) and (52) is only relevant under the assumption that the BH radiation spectrum is strictly thermal [3–5] because they both use the Hawking temperature $T_{+H_{\mathrm{RNBH}}}$ in (29). Hence, to operate in compliance with [3–5] and thereby account for the thermal spectrum deviation of (35), we opt to select the (52) case and upgrade it by effectively replacing its $T_{H_{\mathrm{RNBH}}}$ in (29) with the $T_{+E_{\mathrm{RNBH}}}$ in (46). Therefore, the corrected expression for the RNBH QNFs of (52) which encodes the nonstrictly thermal behavior of the radiation spectrum is defined as

$$\begin{aligned}\omega_n &\equiv \ln 2 \times T_{+E_{\mathrm{RNBH}}}(M,|\omega_n|,Q,q) \\ &\quad - 2\pi\left(n+\frac{1}{2}\right) i \times T_{+E_{\mathrm{RNBH}}}(M,|\omega_n|,Q,q) \\ &\quad + \frac{qQ_E(Q,q)}{R_{+E_{\mathrm{RNBH}}}(M,|\omega_n|,Q,q)} \\ &\equiv \frac{\ln 2\sqrt{M_E^2(M,|\omega_n|)-Q_E^2(Q,q)}}{2\pi\left(M_E(M,|\omega_n|)+\sqrt{M_E^2(M,|\omega_n|)-Q_E^2(Q,q)}\right)^2} \\ &\quad - \frac{(n+1/2)\sqrt{M_E^2(M,|\omega_n|)-Q_E^2(Q,q)}}{\left(M_E(M,|\omega_n|)+\sqrt{M_E^2(M,|\omega_n|)-Q_E^2(Q,q)}\right)^2} i \\ &\quad + \frac{qQ_E(Q,q)}{R_{+E_{\mathrm{RNBH}}}(M,|\omega_n|,Q,q)}.\end{aligned} \tag{53}$$

From (39), (41), and (46) we define the effective quantities associated with the QNMs as

$$M_E(M,|\omega_n|) \equiv \frac{M+(M-|\omega_n|)}{2}, \tag{54}$$

$$\begin{aligned}r_{\pm E} &\equiv R_{\pm E_{\mathrm{RNBH}}}(M,|\omega_n|,Q,q) = \\ &= M_E(M,|\omega_n|) \pm \sqrt{M_E^2(M,|\omega_n|)-Q_E^2(Q,q)},\end{aligned} \tag{55}$$

$$\begin{aligned}&T_{+E_{\mathrm{RNBH}}}(M,|\omega_n|,Q,q) \\ &\equiv \frac{\sqrt{(M-|\omega_n|/2)^2-(Q-q/2)^2}}{2\pi\left[(M-|\omega_n|/2)+\sqrt{(M-|\omega_n|/2)^2-(Q-q/2)^2}\right]^2} \\ &= \frac{\sqrt{M_E^2(M,|\omega_n|)-Q_E^2(Q,q)}}{2\pi\left(M_E(M,|\omega_n|)+\sqrt{M_E^2(M,|\omega_n|)-Q_E^2(Q,q)}\right)^2} \\ &= \frac{R_{+E_{\mathrm{RNBH}}}(M,|\omega_n|,Q,q)-R_{-E_{\mathrm{RNBH}}}(M,|\omega_n|,Q,q)}{A_{+E_{\mathrm{RNBH}}}(M,|\omega_n|,Q,q)},\end{aligned} \tag{56}$$

respectively, for the quantum overtone number n in (53). Hence, (53) lets us rewrite the SBH case of (23) to present the RNBH case

$$\begin{aligned}m_n &\equiv \ln 2 \times T_{+E_{\mathrm{RNBH}}}(M,|\omega_n|,Q,q) \\ &\quad + \frac{eQ_E(Q,q)}{R_{+E_{\mathrm{RNBH}}}(M,|\omega_n|,Q,q)} \\ &= \frac{\ln 2\sqrt{M_E^2(M,|\omega_n|)-Q_E^2(Q,q)}}{2\pi\left(M_E(M,|\omega_n|)+\sqrt{M_E^2(M,|\omega_n|)-Q_E^2(Q,q)}\right)^2} \\ &\quad + \frac{qQ_E(Q,q)}{R_{+E_{\mathrm{RNBH}}}(M,|\omega_n|,Q,q)},\end{aligned} \tag{57}$$

$$\begin{aligned}p_n &\equiv -2\pi\left(n+\frac{1}{2}\right) \times T_{+E_{\mathrm{RNBH}}}(M,|\omega_n|,Q,q) \\ &= -\frac{(n+1/2)\sqrt{M_E^2(M,|\omega_n|)-Q_E^2(Q,q)}}{\left(M_E(M,|\omega_n|)+\sqrt{M_E^2(M,|\omega_n|)-Q_E^2(Q,q)}\right)^2}.\end{aligned}$$

Thus, we recall that if $|\omega_n| \approx p_n$, then we are referring to highly excited QNMs [3–5]. Therefore, the SBH case of (24) becomes the RNBH case

$$\begin{aligned} |\omega_n| &\equiv \frac{\sqrt{M_E^2(M,|\omega_n|) - Q_E^2(Q,q)}\sqrt{(\ln 2)^2 - 4\pi^2(n+1/2)^2}}{2\pi\left(M_E(M,|\omega_n|) + \sqrt{M_E^2(M,|\omega_n|) - Q_E^2(Q,q)}\right)^2} \\ &+ \frac{qQ_E(Q,q)}{R_{+E_{\mathrm{RNBH}}}(M,|\omega_n|,Q,q)} = T_{+E_{\mathrm{RNBH}}}(M,|\omega_n|,Q,q) \\ &\times \sqrt{(\ln 2)^2 - 4\pi^2\left(n+\frac{1}{2}\right)^2} + \frac{qQ_E(Q,q)}{R_{+E_{\mathrm{RNBH}}}(M,|\omega_n|,Q,q)}. \end{aligned} \tag{58}$$

Hence, upon considering (40) and (54), one can rewrite (58) as

$$\begin{aligned} |\omega_n| \equiv{}& \frac{\sqrt{(M-|\omega_n|/2)^2 - (Q-q/2)^2}\sqrt{(\ln 2)^2 - 4\pi^2(n+1/2)^2}}{2\pi\left[(M-|\omega_n|/2) + \sqrt{(M-|\omega_n|/2)^2 - (Q-q/2)^2}\right]^2} \\ &+ \frac{q(Q-q/2)}{(M-|\omega_n|/2) + \sqrt{(M-|\omega_n|/2)^2 - (Q-q/2)^2}}, \end{aligned} \tag{59}$$

where the solution of (59) in terms of $|\omega_n|$ will be the answer of $|\omega_n|$. Therefore, given a quantum transition between the levels n and $n-1$, we define $|\Delta\omega_{n,n-1}| \equiv |\omega_n - \omega_{n-1}|$ where (41)–(45) are rewritten as

$$\begin{aligned} r_{\pm E} &\equiv R_{\pm E_{\mathrm{RNBH}}}(M,|\Delta\omega_{n,n-1}|,Q,q) \\ &\equiv M_E(M,|\Delta\omega_{n,n-1}|) \\ &\quad \pm \sqrt{M_E^2(M,|\Delta\omega_{n,n-1}|) - Q_E^2(Q,q)}, \\ A_{+E_{\mathrm{RNBH}}}&(M,|\Delta\omega_{n,n-1}|,Q,q) \\ &\equiv 4\pi R^2_{+E_{\mathrm{RNBH}}}(M,|\Delta\omega_{n,n-1}|,Q,q) \\ &\equiv 4\pi\Big(M_E(M,|\Delta\omega_{n,n-1}|) \\ &\quad + \sqrt{M_E^2(M,|\Delta\omega_{n,n-1}|) - Q_E^2(Q,q)}\Big)^2, \\ S_{+E_{\mathrm{RNBH}}}&(M,|\Delta\omega_{n,n-1}|,Q,q) \equiv \frac{A_{+E_{\mathrm{RNBH}}}(M,|\Delta\omega_{n,n-1}|,Q,q)}{4}, \\ \Phi_{+E}&(M,|\Delta\omega_{n,n-1}|,Q,q) \\ &\equiv \frac{Q_E(Q,q)}{4\pi R_{+E_{\mathrm{RNBH}}}(M,|\Delta\omega_{n,n-1}|,Q,q)} \\ &\equiv \frac{Q_E(Q,q)}{4\pi\left(M_E(M,|\Delta\omega_{n,n-1}|) + \sqrt{M_E^2(M,|\Delta\omega_{n,n-1}|) - Q_E^2(Q,q)}\right)}, \\ I_{+E_{\mathrm{RNBH}}}&(M,|\Delta\omega_{n,n-1}|,Q,q) \\ &= \int \frac{dM_E(M,|\Delta\omega_{n,n-1}|) - \Phi_{+E}(M,|\Delta\omega_{n,n-1}|,Q,q)\,dQ_E(Q,q)}{T_{E_{\mathrm{SBH}}}(M,|\Delta\omega_{n,n-1}|)}, \end{aligned} \tag{60}$$

and (47)–(50) become

$$\begin{aligned} \Gamma_{+E_{\mathrm{RNBH}}}&(M,|\Delta\omega_{n,n-1}|,Q,q) \\ &\sim \exp\left[\frac{\pm|\Delta\omega_{n,n-1}|}{T_{+E_{\mathrm{RNBH}}}(M,|\Delta\omega_{n,n-1}|,Q,q)}\right] \\ &\sim \exp\left[\Delta S_{+_{\mathrm{RNBH}}}(M,|\Delta\omega_{n,n-1}|,Q,q)\right], \\ \Delta S_{+E_{\mathrm{RNBH}}}&(M,|\Delta\omega_{n,n-1}|,Q,q) \\ &\equiv \frac{\Delta A_{+E_{\mathrm{RNBH}}}(M,|\Delta\omega_{n,n-1}|,Q,q)}{4}, \\ \Delta A_{+E_{\mathrm{RNBH}}}&(M,|\Delta\omega_{n,n-1}|,Q,q) \equiv \frac{2|\Delta\omega_{n,n-1}|q + \pi Q^3}{(M^2-Q^2)^{3/2}}, \\ N_{+E_{\mathrm{RNBH}}}&(M,|\Delta\omega_{n,n-1}|,Q,q) \\ &\equiv \frac{A_{+E_{\mathrm{RNBH}}}(M,|\Delta\omega_{n,n-1}|,Q,q)}{|\Delta A_{+E_{\mathrm{RNBH}}}(M,|\Delta\omega_{n,n-1}|,Q,q)|}, \end{aligned} \tag{61}$$

respectively.

5. A Brief Comparison

Here, we will show that the SBH results of Section 2 are in fundamental agreement with the RNBH results of Section 3 for small Q, where we recall that the RNBH of mass M is identical to a SBH of mass M except that a RNBH has the nonzero charge quantity Q.

First, for small Q, the SBH's $T_{E_{\mathrm{SBH}}}(M,\omega)$ of (16) is related to the RNBH's $T_{+E_{\mathrm{RNBH}}}(M,\omega,Q,q)$ of (46) as

$$\begin{aligned} T_{+E_{\mathrm{RNBH}}}&(M,\omega,Q,q) \\ &\equiv T_{E_{\mathrm{SBH}}}(M,\omega) - \frac{3q^2Q^2}{8(2m\pm\omega)^5\pi} + \mathcal{O}\left(Q^4,q^4\right). \end{aligned} \tag{62}$$

Second, for small Q, the SBH's $A_{E_{\text{SBH}}}(M,\omega)$ of (13) complies with the RNBH's $A_{+E_{\text{RNBH}}}(M,\omega,Q,q)$ of (42) as

$$A_{+E_{\text{RNBH}}}(M,\omega,Q,q) \equiv A_{E_{\text{SBH}}}(M,\omega) - 8\pi Q^2 + \mathcal{O}\left(Q^4\right). \tag{63}$$

Third, for small Q, the SBH's $S_{E_{\text{SBH}}}(M,\omega)$ of (14) corresponds with the RNBH's $S_{+E_{\text{RNBH}}}(M,\omega,Q,q)$ of (43) as

$$S_{+E_{\text{RNBH}}}(M,\omega,Q,q) \equiv S_{E_{\text{SBH}}}(M,\omega) - 2\pi Q^2 + \mathcal{O}\left(Q^4\right). \tag{64}$$

Fourth, for small Q, the SBH's QNF $|\omega_n|$ of (24) is consistent with the RNBH's QNF $|\omega_n|$ of (58) and (59) as

$$\begin{aligned}|\omega_n| &\equiv \frac{\sqrt{\ln 2^2 - 4\pi^2(n+1/2)^2}}{4\left(2M-|\omega_n|\right)\pi} + \frac{qQ}{2M-|\omega_n|}\\ &= \Big(\Big(3\left(16\pi M^2 - 16\pi|\omega_n|M + 4\pi|\omega_n|^2\right.\\ &\qquad \left.+\sqrt{-(\ln 2+\pi+2\pi n)(-\ln 2+\pi+2\pi n)}\right)\Big)\\ &\quad \times\left(8(2M-|\omega_n|)^5\pi\right)^{-1}\Big)Q^2q^2\\ &= \frac{q^2}{4M-|\omega_n|} + \mathcal{O}\left(Q^4,q^4\right),\end{aligned} \tag{65}$$

which can be applied to (62)-(63) by replacing the ω parameter with the pertinent $|\omega_n|$. Hence, (62)–(65) indicate that in general the SBH results of Section 2 are fundamentally consistent with the RNBH results of Section 3 for small Q. Moreover, in (65) for large n, the result is consistent with the SBH because ln 2 is negligible, but for small n there is an argument between scientists regarding ln 2 and ln 3 because these refer to the two distinct QNM families of (51) and (52).

Here, we provide the physical answer of (65) for the case of emission by using the fact that Q is small, so the term which includes Q^2 is also very small and therefore negligible:

$$\begin{aligned}(\omega_0)_n &\equiv |\omega_n| \approx M\\ &\quad - \sqrt{M^2 + \frac{q^2}{2} - Qq - \frac{1}{4\pi}\sqrt{\ln 2^2 - 4\pi^2\left(n+\frac{1}{2}\right)^2}}.\end{aligned} \tag{66}$$

Thus, by setting $(\omega_0)_n \equiv |\omega_n|$ we obtain

$$\begin{aligned}\Delta M_n &\equiv -\Delta\omega_{n,n-1} = (\omega_0)_{n-1} - (\omega_0)_n\\ &\equiv \sqrt{M^2 + \frac{q^2}{2} - Qq - \frac{1}{4\pi}\sqrt{(\ln 2)^2 + 4\pi^2\left(n+\frac{1}{2}\right)^2}}\\ &\quad - \sqrt{M^2 + \frac{q^2}{2} - Qq - \frac{1}{4\pi}\sqrt{(\ln 2)^2 + 4\pi^2\left(n-\frac{1}{2}\right)^2}}\end{aligned} \tag{67}$$

for an emission involving quantum levels n and $n-1$, which becomes

$$\begin{aligned}\Delta M_n &\approx \sqrt{M^2 + \frac{q^2}{2} - Qq - \frac{1}{2}\left(n+\frac{1}{2}\right)}\\ &\quad - \sqrt{M^2 + \frac{q^2}{2} - Qq - \frac{1}{2}\left(n-\frac{1}{2}\right)}\end{aligned} \tag{68}$$

for large n.

6. Conclusion Remarks

We began our paper by summarizing some basic similarities and differences between SBHs and RNBHs in terms of charge and horizon radii. Moreover, we briefly explored the Parikh-Wilczek statement that explains how energy conservation and pair production [2, 23] are fundamentally related to such BHs. For a BH's discrete energy spectrum, the emission or absorption of a particle yields a transition between two distinct levels, where particle emission and absorption are reverse processes [3–7]. For this, we touched on the important issue that the nonstrictly thermal character of Hawking's radiation spectrum generates a natural correspondence between Hawking's radiation and BH QNMs, because these structures exemplify features of the BH's energy spectrum [3–5], which has been recently generalized to the emerging concept of a BH's effective state [6, 7].

Next, we prepared for our nonextremal RNBH QNM investigation by first reviewing relevant portions of the SBH effective framework [3–5] in Section 2. There, we listed the noneffective and effective quantities for SBH states and transitions, with direct application to the QNM characterization and framework of [3–5]. Subsequently, in Section 3, we identified some existing noneffective quantities and introduced new effective quantities for RNBH states and transitions so we could apply the BH framework of [3–5] to implement a RNBH framework. These results are crucial because the effective quantities in [3–5] have been achieved for the stable four-dimensional RNBH solution in Einstein's general relativity—now effective frameworks exist for the SBH, KBH, and (nonextremal) RNBH solutions.

Ultimately, the RNBH effective quantities permitted us to utilize both the KBH's effective state concept [6, 7] and the BH QNMs [3–5] to construct a foundation for the RNBH's effective state in this developing BH effective framework. The RNBH effective state concept is meaningful because, as scientists who wish to demystify the BH paradigm, we need additional features and knowledge to consider in future experiments and observations.

Finally, we stress that the nonstrictly thermal behavior of the Hawking radiation spectrum has been recently used to construct two very intriguing proposals to solve the BH information loss paradox. The first one received the First Award in the 2013 Gravity Research Foundation Essay Competition [12]. The latter won the Community Rating at the 2013 FQXi Essay Contest—It from Bit or Bit from It [24]. We are working to extend this second approach to the RNBH framework [25].

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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References

[1] B. Wang, R. K. Su, P. K. N. Yu, and E. C. M. Young, "Can a nonextremal Reissner-Nordström black hole become extremal by assimilating an infalling charged particle and shell?" *Physical Review D*, vol. 57, no. 8, pp. 5284–5286, 1998.

[2] M. K. Parikh and F. Wilczek, "Hawking radiation as tunneling," *Physical Review Letters*, vol. 85, no. 24, pp. 5042–5045, 2000.

[3] C. Corda, "Effective temperature for black holes," *Journal of High Energy Physics*, vol. 2011, p. 101, 2011.

[4] C. Corda, "Black hole quantum spectrum," *The European Physical Journal C*, vol. 73, p. 2665, 2013.

[5] C. Corda, "Effective temperature, hawking radiation and quasinormal modes," *International Journal of Modern Physics D*, vol. 21, no. 11, Article ID 1242023, 2012.

[6] C. Corda, S. H. Hendi, R. Katebi, and N. O. Schmidt, "Effective state, Hawking radiation and quasi-normal modes for Kerr black holes," *Journal of High Energy Physics*, vol. 2013, p. 8, 2013.

[7] C. Corda, "Non-strictly black body spectrum from the tunnelling mechanism," *Annals of Physics*, vol. 337, pp. 49–54, 2013.

[8] B. Zhang, Q.-Y. Cai, L. You, and M. S. Zhan, "Hidden messenger revealed in Hawking radiation: a resolution to the paradox of black hole information loss," *Physics Letters B*, vol. 675, no. 1, pp. 98–101, 2009.

[9] B. Zhang, Q.-Y. Cai, M. S. Zhan, and L. You, "Entropy is conserved in Hawking radiation as tunneling: a revisit of the black hole information loss paradox," *Annals of Physics*, vol. 326, no. 2, pp. 350–363, 2011.

[10] B. Zhang, Q.-Y. Cai, M. S. Zhan, and L. You, "Noncommutative information is revealed from Hawking radiation as tunneling," *EPL (Europhysics Letters)*, vol. 94, no. 2, Article ID 20002, 2011.

[11] B. Zhang, Q.-Y. Cai, M. S. Zhan, and L. You, "Comment on 'What the information loss is not'," http://arxiv.org/abs/1210.2048. In press.

[12] B. Zhang, Q. Y. Cai, M. S. Zhan, and L. You, "Information conservation is fundamental: recovering the lost information in Hawking radiation," *International Journal of Modern Physics D*, vol. 22, Article ID 1341014, 8 pages, 2013.

[13] S. Hod, "Gravitation, the quantum and Bohr's correspondence principle," *General Relativity and Gravitation*, vol. 31, no. 11, pp. 1639–1644, 1999.

[14] S. Hod, "Bohr's correspondence principle and the area spectrum of quantum black holes," *Physical Review Letters*, vol. 81, no. 20, pp. 4293–4296, 1998.

[15] M. Maggiore, "Physical interpretation of the spectrum of black hole quasinormal modes," *Physical Review Letters*, vol. 100, no. 14, Article ID 141301, 2008.

[16] M. Casals and A. C. Ottewill, "Branch cut and quasinormal modes at large imaginary frequency in Schwarzschild spacetime ," *Physical Review D*, vol. 86, no. 2, Article ID 024021, 2012.

[17] R. Banerjee and B. R. Majhi, "Quantum tunneling and trace anomaly," *Physics Letters B*, vol. 674, no. 3, pp. 218–222, 2009.

[18] S. W. Hawking, "The path integral approach to quantum gravity," in *General Relativity: An Einstein Centenary Survey*, S. W. Hawking and W. Israel, Eds., Cambridge University Press, Cambridge, UK, 1979.

[19] T. Padmanabhan, "Quasi-normal modes: a simple derivation of the level spacing of the frequencies," *Classical and Quantum Gravity*, vol. 21, no. 1, p. L1, 2004.

[20] A. Lopez-Ortega, "Area spectrum of the d-dimensional Reissner-Nordström black hole in the small charge limit ," *Classical and Quantum Gravity*, vol. 28, no. 3, Article ID 035009, 2011.

[21] J. Zhang and Z. Zhao, "Hawking radiation of charged particles via tunneling from the Reissner-Nordström black hole," *Journal of High Energy Physics*, vol. 10, p. 55, 2005.

[22] S. Hod, "Quasinormal spectrum and quantization of charged black holes," *Classical and Quantum Gravity*, vol. 23, no. 4, p. L23, 2006.

[23] M. K. Parikh, "A secret tunnel through the horizon," *General Relativity and Gravitation*, vol. 36, no. 11, pp. 2419–2422, 2004.

[24] C. Corda, "Time-Dependent Schrodinger Equation for Black Hole Evaporation: no Information Loss," http://fqxi.org/community/forum/topic/1856.

[25] C. Corda, S. H. Hendi, R. Katebi, and N. O. Schmidt, inpreparation.