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Research Article

Black Holes and Quantum Mechanics

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We look at black holes from different, novel perspectives.

1. Introduction

We will first show that black holes, generally, thought to be a general relativistic phenomena could also be understood without invoking general relativity at all. (Indeed, Laplace had anticipated these objects.)

We start by defining a black hole as an object at the surface of which the escape velocity equals the maximum possible velocity in the universe, namely, the velocity of light. We next use the well-known equation of Keplerian orbits [1]

$$\frac{1}{r} = \frac{GM}{L^2}\left(1 + e\cos\theta\right), \tag{1}$$

where L, the so-called impact parameter, is given by Rc, where R is the point of closest approach, in our case a point on the surface of the object, and c is the velocity of approach, in our case the velocity of light.

Choosing $\theta = 0$ and $e \approx 1$, we can deduce from (1)

$$R = \frac{2GM}{c^2}. \tag{2}$$

Equation (2) gives the Schwarzschild radius for a black hole and can be deduced from the full general relativity theory as well.

We will now use (2) to exhibit black holes at three different scales, the micro-, the macro-, and the cosmic scales.

2. Black Holes

Our starting point is the observation that a Planck mass, 10^{-5} gms at the Planck length 10^{-33} cms, satisfies (2) and as such a Schwarzschild black hole is. Rosen has used nonrelativistic quantum theory to show that such a particle is a mini universe [2].

We next come to stellar scales. It is well known that for an electron gas in a highly dense mass we have [3, 4]

$$K\left(\frac{\overline{M}^{4/3}}{\overline{R}^4} - \frac{\overline{M}^{2/3}}{\overline{R}^2}\right) = K'\frac{\overline{M}^2}{\overline{R}^4}, \tag{3}$$

where

$$\left(\frac{K}{K'}\right) = \left(\frac{27\pi}{64\alpha}\right)\left(\frac{\hbar c}{\gamma m_P^2}\right) \approx 10^{40}, \tag{4}$$

$$\overline{M} = \frac{9\pi}{8}\frac{M}{m_P}, \qquad \overline{R} = \frac{R}{\left(\hbar/m_e c\right)}, \tag{5}$$

M is the mass, R the radius of the body, m_P and m_e are the proton and electron masses, and $\hbar$ is the reduced Planck constant. From (3) and (4), it is easy to see that for $\overline{M} < 10^{60}$, there are highly condensed planet sized stars. (In fact these considerations lead to the Chandrasekhar limit in stellar theory.) We can also verify that for $\overline{M}$ approaching 10^{60} corresponding to a mass ~10^{36} gms, or roughly a hundred to a thousand times the solar mass, the radius R gets smaller and

smaller and would be ~10^8 cms, so as to satisfy (2) and give a black hole in broad agreement with theory and observation.

Finally for the universe as a whole, using only the theory of Newtonian gravitation, we had deduced [5]

$$R \sim \frac{2GM}{c^2}; \tag{6}$$

that is, (2) where this time $R \sim 10^{28}$ cms is the radius of the universe and $M \sim 10^{55}$ gms is the mass of the universe. (6) can be deduced alternatively from general relativistic considerations also as noted.

Equation (6) is the same as (2) and suggests that the universe itself is a black hole. (This will still be true if there is dark matter.)

It is remarkable that if we consider the universe to be a Schwarzschild black hole as suggested by (6), the time taken by a ray of light to traverse the universe, that is, from the horizon to the singularity, namely, $10^{-5}(M/M_0)$, equals the age of the universe ~10^{17} secs as shown elsewhere [5]. M_0 is the mass of the sum. We will deduce this result alternatively a little later.

3. Micro Black Holes

Attempts have been made to express elementary particles as tiny black holes by several authors, notably, Markov and Recami [6, 7]. These black holes do not reproduce charge or spin which are so essential.

Let us, instead, observe that if we treat an electron as a Kerr-Newman black hole, then we get the correct quantum mechanical $g = 2$ factor, but the horizon of the black hole becomes complex [4, 8]. Consider

$$r_+ = \frac{GM}{c^2} + \imath b, \qquad b \equiv \left(\frac{G^2M^2}{c^4} - \frac{GQ^2}{c^4} - a^2\right)^{1/2} \tag{7}$$

with G being the gravitational constant, M being the mass, and $a \equiv L/Mc$, L being the angular momentum. While (7) exhibits a naked singularity and as such has no physical meaning, we note that from the realm of quantum mechanics the position coordinate for a Dirac particle is given by

$$x = \left(c^2 p_1 H^{-1} t\right) + \frac{\imath}{2} c\hbar \left(\alpha_1 - c p_1 H^{-1}\right) H^{-1} \tag{8}$$

an expression that is very similar to (7). In the above, the various symbols have their usual meaning. In fact as was argued in detail [4], the imaginary parts of both (7) and (8) are the same, being of the order of the Compton wavelength.

It is at this stage that a proper physical interpretation begins to emerge. Dirac himself observed that to interpret (8) meaningfully it must be remembered that quantum mechanical measurements (unlike classical ones) are really averaged over the Compton scale. Within the scale there are the unphysical Zitterbewegung effects: for a point electron the velocity equals that of light.

Once such a minimum spacetime scale is invoked, then we have a noncommutative geometry as shown by Snyder [9, 10]

$$\begin{gathered}[x, y] = \left(\frac{\imath a^2}{\hbar}\right) L_z, \qquad [t, x] = \left(\frac{\imath a^2}{\hbar c}\right) M_x, \text{etc}, \\ [x, p_x] = \imath\hbar\left[1 + \left(\frac{a}{\hbar}\right)^2 p_x^2\right].\end{gathered} \tag{9}$$

The relations (9) are compatible with special relativity. Indeed, such minimum spacetime models were studied for several decades, precisely to overcome the divergences encountered in quantum field theory [4, 10–13].

All this is symptomatic of the fact that we cannot measure arbitrary small intervals of spacetime in quantum theory, as indeed argued by Dirac himself [14]. Indeed subsequently Salecker and Wigner argued that time within the Compton scale has no physical meaning [15] (and for a detailed discussion cf. [16]). Indeed this quantum mechanical feature explains what Misner et al. termed the greatest crisis of physics [8], namely, the singularity of the black hole. All this has been the matter of detailed study (cf. [16]).

4. Black Hole Thermodynamics

The author has approached this problem from the point of view of oscillations at the Planck scale [16]. Briefly, if there are N such oscillators with an amplitude Δx, then we have

$$R = \sqrt{N\Delta x^2}. \tag{10}$$

This leads to

$$R = \sqrt{N} l_P, \qquad M = \frac{m_P}{\sqrt{N}}, \tag{11}$$

where M is the arbitrary mass, R the extent, and l_P and m_P are the Planck length and Planck mass, respectively. We now use the fact that l_P is the Schwarzschild radius of the Planck mass as was shown by Rosen [2]. Substitution in the above gives us the Schwarzschild radius; that is (4)

$$R = \frac{2GM}{c^2}. \tag{12}$$

It can be immediately seen from (11) that

$$RM = l_P m_P. \tag{13}$$

It must be mentioned that the above is completely consistent with the mass and radius of an arbitrary black hole, including the universe itself.

From the theory of black hole thermodynamics we have as it is well known [17]

$$T = \frac{\hbar c^3}{8\pi k m G}, \tag{14}$$

namely, the Beckenstein temperature. Interestingly, (14) can be deduced alternatively from our above theory of oscillations

at the Planck scale. For this we use the following relations for a Schwarzschild black hole [17]

$$dM = TdS, \qquad S = \frac{kc}{4\hbar G}A, \tag{15}$$

where T is the Bekenstein temperature, S the entropy, and A is the area of the black hole. In our case, the mass $M = \sqrt{N}m_P$ and $A = Nl_P^2$, where N is arbitrary for an arbitrary black hole. This follows from (11). Whence,

$$T = \frac{dM}{dS} = \frac{4\hbar G}{kl_P^2 c}\frac{dM}{dN}. \tag{16}$$

If we use the fact that l_P is the Schwarzschild radius for the Planck mass m_P and use the expression for M, the above reduces to (14), the Bekenstein formula.

Equation (14) gives also the thermodynamic temperature of a Planck mass black hole. Further, in this theory as it is known [17],

$$\frac{dM}{dt} = -\frac{\beta}{M^2}, \tag{17}$$

with M being the mass. Before proceeding, we observe that we have deduced a string of N Planck oscillators, N arbitrary, form a Schwarzschild black hole of mass $\sqrt{N}m_P = M$. We can now deduce that

$$\begin{aligned} \frac{dM}{dt} &= \frac{m_P}{t_P}, \\ M &= \left(\frac{m_P}{t_P}\right)\cdot t, \end{aligned} \tag{18}$$

where t is the "Hawking-Bekenstein decay time." For the Planck mass, $M = m_P$, the decay time is the Planck time $t = t_P$. For the universe, the above gives the life time t as $\sim 10^{17}$ sec, the age of the universe again.

Further, we have also seen the emergence of the quantum of area [18] as it is evident from the N elementary Planck areas l_P^2 for the black hole (cf. also [18]).

It has also been argued that not only does the universe mimic a black hole but also the black hole is a two dimensional object [16, 19]. Indeed, the interior of a black hole is in any case inaccessible and the two dimensions follow from the area of the black hole which plays a central role in black hole thermodynamics. We have already seen that the area of the black hole is given by

$$A = Nl_p^2. \tag{19}$$

For these quantum gravity considerations, we have to deal with the quantum of area [16, 18]. In other words, we have to consider the black hole to be made up of N quanta of area. It is remarkable that we can get an opportunity to test these quantum gravity features in two-dimensional surfaces such as graphene.

That is, we could model a black hole as a "graphene" ball. Indeed, in the case of graphene as it is well known, and as the author deduced in 1995 [20, 21], this behaviour in two dimensions is given by

$$v_F\ \vec{\sigma}\cdot\vec{\nabla}\ \psi(r) = E\psi(r), \tag{20}$$

where $v_F \sim 10^6$ m/s is the Fermi velocity replacing c, the velocity of light, and $\psi(r)$ is a two-component wave function, $\vec{\sigma}$ and E, denoting the Pauli matrices and energy.

Though this resembles the neutrino equation, v_F is some three hundred times less than the velocity of light. However the author has argued that for a sufficiently large sheet of graphene, this would approximate the neutrino equation itself, that is, the usual Minkowski spacetime. From this point of view, a black hole can be simulated by a "graphene ball."

It may be mentioned that very recently Hawking has proposed rather shockingly that black holes may not have event horizons [22].

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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