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*Research Article*

# Dynamics of Interacting Tachyonic Teleparallel Dark Energy

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We consider a tachyon scalar field which is nonminimally coupled to gravity in the framework of teleparallel gravity. We analyze the phase-space of the model, known as tachyonic teleparallel dark energy, in the presence of an interaction between dark energy and background matter. We find that although there exist some late-time accelerated attractor solutions, there is no scaling attractor. So, unfortunately interacting tachyonic teleparallel dark energy cannot alleviate the coincidence problem.

## 1. Introduction

Recent astrophysical observations including Supernova Ia [1, 2], large scale structure [3, 4], the baryon acoustic oscillations [5], and cosmic microwave background radiation [6–9] have indicated that our universe is experiencing an accelerating phase of expansion.

There are two ways to explain the current cosmic expansion. The first one is to add extra terms in the gravitational Lagrangian or modify gravity on large scales and the second one is to introduce an unknown energy component, dubbed as dark energy with negative pressure.

The simplest modification of general relativity is the so-called $f(R)$ gravity in which one generalizes the Einstein-Hilbert action to a general function of the Ricci scalar $R$ (for reviews see [10–13]). In the other side the simplest candidate of dark energy is the vacuum energy or the cosmological constant $\Lambda$ with a constant equation of state parameter $\omega = -1$. However, the cosmological constant suffers from serious problems such as huge amount of fine tuning required for its magnitude and lack of dynamics [14–17]. Due to these problems numerous dynamical dark energy models have been proposed. Quintessence [18–22], phantom [23–29], tachyon [30, 31], and the combination of quintessence and phantom in a unified model named quintom [32–38] are well-known examples of dark energy models (for reviews on dark energy models, see [39, 40]).

Although usually the scalar fields are minimally coupled to gravity, there are compelling reasons (e.g., quantum corrections and renormalizability of the scalar field theory in curved space) to include an explicit nonminimal coupling between dark energy and gravity in the action [41–48]. Also, a possible coupling between dark energy and dark matter can be included in generalized versions of the aforementioned models [49, 50]. Because of this possibility, various forms of interacting dark energy models have been constructed hitherto [51–65].

Furthermore, an equivalent form of classical gravity is the so-called teleparallel gravity, in which, instead of using the torsionless Levi-Civita connection, one uses the curvatureless Weitzenbock one. In this theory, the dynamical variables are a set of four tetrad (or vierbein) fields that form the pseudoorthogonal bases for the tangent space at each point of spacetime [66–68]. The teleparallel Lagrangian density $T$ can be constructed from torsion tensor and only differs with the Ricci scalar by a total divergence. Thus, apart from some conceptual differences, general relativity and teleparallel gravity are dynamically equivalent theories and indistinguishable from general relativity at the level of field equations [67, 68]. Recently, $f(R)$ inspired teleparallel gravity, the so-called $f(T)$ gravity, has attracted much attention [69–83].

Teleparallel dark energy is a recently proposed scenario in which a nonminimal coupling between quintessence and gravity in the framework of teleparallel gravity was considered [84, 85]. This theory has a rich structure and its dynamics was studied in [86–88]. Tachyonic teleparallel dark energy is a generalization of teleparallel dark energy by inserting a noncanonical scalar field instead of quintessence in the action [89]. Phase-space analysis of this model has been investigated in [90]. In this paper we study the dynamics of interacting tachyonic teleparallel dark energy. We find stable solutions of the model which are late-time accelerated attractors and correspond to dark energy dominated solutions. Thus, coincidence problem cannot be solved in interacting tachyonic teleparallel dark energy model.

An outline of the present letter is as follows. In Section 2 we briefly review teleparallel gravity. In Section 3 we present tachyonic teleparallel dark energy and obtain energy density, pressure, and equation of state of the model. In Section 4 we study dynamics of the model in a system of autonomous differential equations, find its fixed points, and study their stabilities. Section 5 is devoted to conclusions.

## 2. Teleparallel Gravity

Let us start with a brief review of the key ingredients of teleparallel gravity [66–72]. In this theory, the metric tensor is obtained from the dual vierbein as

$$g_{\mu\nu} = \eta_{ij} e^i_\mu e^j_\nu, \tag{1}$$

where Latin $i$, $j$ are indices running over 0, 1, 2, 3 for the tangent space at each point $x^\mu$ of the manifold and Greek $\mu$ and $\nu$ are coordinate indices on the manifold, taking the values 0, 1, 2, 3. Furthermore, the torsion tensor of the Weitzenbock connection $\Gamma^\rho_{\nu\mu}$ [91] reads

$$T^\rho_{\mu\nu} = \Gamma^\rho_{\nu\mu} - \Gamma^\rho_{\mu\nu} = e^\rho_i \left(\partial_\mu e^i_\nu - \partial_\nu e^i_\mu\right). \tag{2}$$

In the present formalism all the information concerning the gravitational field is included in the torsion tensor $T^\rho_{\mu\nu}$. The corresponding teleparallel Lagrangian can be constructed from this torsion tensor under the assumptions of invariance under general coordinate transformation, global Lorentz transformations, and the parity operation, along with requiring the Lagrangian density to be second order in the torsion tensor [92]. The starting action in a universe governed by teleparallel gravity is

$$S = \int d^4 x e \left[\frac{T}{2\kappa^2} + \mathscr{L}_m\right], \tag{3}$$

where $e = \det(e^i_\mu) = \sqrt{-g}$ and $T$ is the torsion scalar given by

$$\mathscr{L} = T = \frac{1}{4} T^{\rho\mu\nu} T_{\rho\mu\nu} + \frac{1}{2} T^{\rho\mu\nu} T_{\nu\mu\rho} - T^\rho_{\rho\mu} T^{\nu\mu}_{\nu}. \tag{4}$$

Variation with respect to the vierbein fields yields to the equations of motion which are exactly the same as those of general relativity for every geometry choice. Since the action (3) in a Friedmann-Robertson-Walker (FRW) background is equivalent to a matter domination universe in the framework of general relativity, hence it cannot be accelerated. So, we should generalize action (3) either by replacing $T$ with $f(T)$ [69–83] or by adding a scalar field responsible for dark energy in teleparallel gravity. On the other hand, one can easily find that dark energy in the framework of teleparallel gravity is completely identical to the one in the framework of general relativity, and hence there is nothing new. Recently Geng et al. [84] have proposed to modify action (3) by including a nonminimal coupling between quintessence and gravity in the framework of teleparallel gravity and named it teleparallel dark energy. Here we generalize teleparallel dark energy model by replacing canonical scalar field (quintessence) by a noncanonical scalar field. The noncanonical scalar field is tachyon field and the model has been called tachyonic teleparallel dark energy.

## 3. Interacting Tachyonic Teleparallel Dark Energy

The action of teleparallel dark energy with a nonminimal coupling between tachyon field and teleparallel gravity reads

$$S = \int d^4 x e \left[\frac{T}{2\kappa^2} + \xi f(\varphi) T - V(\varphi)\sqrt{1 - 2X} + \mathscr{L}_m\right], \tag{5}$$

where $\xi$ is a dimensionless constant and $f(\varphi)$ is an arbitrary function of scalar field and it is responsible for nonminimal coupling between dark energy and gravity. $V(\varphi)$ is the tachyonic potential and $X = (1/2)\partial_\mu\varphi\partial^\mu\varphi$ (we use the metric signature $(+,-,-,-)$). We should emphasize that in torsion formulation of general relativity the only scalar is the torsion scalar and hence the nonminimal coupling will be between $T$ and tachyon field in analogy with the standard nonminimal tachyon cosmology in general relativity where the scalar field couples to the Ricci scalar.

Phase-space analysis of tachyon field in the framework of general relativity has been studied in several papers (see, e.g., [93–95]). To obtain a closed autonomous system of ordinary differential equations out of the cosmological field equations [96] has proposed the following transformation of the tachyon field:

$$\varphi \longrightarrow \phi = \int d\varphi \sqrt{V(\varphi)} \Longleftrightarrow \partial\varphi = \frac{\partial\phi}{\sqrt{V(\phi)}}. \tag{6}$$

It is shown that the preceding transformation can help us study tachyon dynamics for a wide class of self-interaction potentials beyond the inverse square one. It is revealed that for power law potentials the late-time attractor is always the de Sitter solution while for *sinh*-like potentials the late-time attractor can be either the inflationary tachyon dominated solution or the matter scaling phase.

In this paper we also use the field redefinition (6) to study the dynamics of the interacting tachyon field nonminimally coupled with gravity in the framework of teleparallel gravity. Therefore, our starting action is obtained using (6) in (5):

$$S = \int d^4 x e \left[\frac{T}{2\kappa^2} + \xi f(\phi) T - V(\phi)\sqrt{1 - \frac{2X}{V(\phi)}} + \mathscr{L}_m\right]. \tag{7}$$

Also we consider a possible interaction between dark energy and dark matter because there is no physical argument to exclude the interaction between them.

In a spatially flat FRW space-time,

$$ds^2 = dt^2 - a^2(t)\left(dr^2 + r^2 d\Omega^2\right), \quad (8)$$

and a vierbein choice of the form $e^i_\mu = \text{diag}(1, a, a, a)$, the equation of motion of the scalar field can be obtained by variation of the action (7) with respect to $\phi$:

$$\ddot{\phi} + 3\mu^{-2}H\dot{\phi} + \left(1 - \frac{3X}{V(\phi)}\right)V_{,\phi} + 6\xi\mu^{-3}H^2 f_{,\phi} = -\frac{Q}{\dot{\phi}}, \quad (9)$$

with $Q$ being a general interaction coupling term between dark energy and dark matter. Furthermore, the effective energy density and pressure of tachyonic dark energy read

$$\begin{aligned}\rho_\phi &= \mu V(\phi) - 6\xi H^2 f(\phi),\\ P_\phi &= -\mu^{-1}V(\phi) + 2\xi\left(3H^2 + 2\dot{H}\right)f(\phi) + 4\xi H f_{,\phi}\dot{\phi},\end{aligned} \quad (10)$$

where $f_{,\phi} = df/d\phi$ and $\mu = 1/\sqrt{1 - 2X/V}$.

In (9) and (10) we have used the useful relation,

$$T = -6H^2, \quad (11)$$

which simply arises from the calculation of (4) for the FRW metric (8). The scalar field evolution (9) expresses the continuity equation for the field and matter as follows:

$$\begin{aligned}\dot{\rho}_\phi + 3H\left(1 + \omega_\phi\right)\rho_\phi &= -Q,\\ \dot{\rho}_m + 3H\left(1 + \omega_m\right)\rho_m &= Q,\end{aligned} \quad (12)$$

where $\omega_\phi = P_\phi/\rho_\phi$ is the equation of state parameter of dark energy which is attributed to the scalar field $\phi$. The barotropic index is defined by $\gamma \equiv 1 + \omega_m$ with $0 < \gamma < 2$.

In FRW background (8) the corresponding Friedmann equations are given by

$$\begin{aligned}H^2 &= \frac{\kappa^2}{3}\left(\rho_\phi + \rho_m\right),\\ \dot{H} &= -\frac{\kappa^2}{2}\left(\rho_\phi + P_\phi + \rho_m + P_m\right),\end{aligned} \quad (13)$$

where $H = \dot{a}/a$ is the Hubble parameter and a dot stands for the derivative with respect to the cosmic time $t$. In these equations, $\rho_m$ and $P_m$ are the matter energy density and the pressure, respectively.

Tachyonic teleparallel dark energy has been studied in [89]. It is shown that such a scenario can realize phantom divide crossing during its evolution and so it exhibits very interesting cosmological behavior. Therefore, it seems necessary to perform a phase-space analysis of such theory. In phase-space analysis we investigate late-time solutions that are independent from the initial conditions and the specific universe evolution.

## 4. Phase-Space Analysis

In order to study the phase-space and stability of the model (7), we should translate the evolution equations in the language of the autonomous dynamical system $X' = f(X)$ [97–100], where $X$ is the column vector constituted by suitable auxiliary variables, $f(X)$ is the corresponding column vector of the autonomous equations, and prime denotes derivative with respect to the logarithm of the scale factor $N = \ln a$. The critical points $X_c$ are extracted from $X' = 0$ and in order to determine the stability properties of these critical points we expand around $X_c$, setting $X = X_c + U$ with $U$ the perturbations of the variables considered as column vector. Thus, up to the first order we acquire $U' = M \cdot U$, where the matrix $M$ contains the coefficients of the perturbation equations. For each critical point, the eigenvalues of $M$ determine its type and stability.

Now, let us transform the cosmological equations into an autonomous dynamical system. To this end following, for example, [101–106] we introduce the following auxiliary variables:

$$x \equiv \frac{\dot{\phi}}{\sqrt{V}}, \qquad y \equiv \frac{\kappa\sqrt{V}}{\sqrt{3}H}, \qquad u \equiv \kappa\sqrt{f}. \quad (14)$$

The auxiliary variables allow us to straightforwardly obtain the density parameter of dark energy and dark matter:

$$\Omega_{\text{DE}} \equiv \frac{\kappa^2\rho_{\text{DE}}}{3H^2} = \mu y^2 - 2\xi u^2, \quad (15)$$

$$\Omega_m \equiv \frac{\kappa^2\rho_m}{3H^2} = 1 - \Omega_{\text{DE}}, \quad (16)$$

while the equation of state of the field reads

$$\begin{aligned}\omega_{\text{DE}} \equiv \omega_\phi &= \frac{P_\phi}{\rho_\phi}\\ &= \frac{-\mu^{-1}y^2 + 2\xi u\left[\left(2\sqrt{3}/3\right)\alpha x y + u\left(1 - (2/3)s\right)\right]}{\mu y^2 - 2\xi u^2},\end{aligned} \quad (17)$$

where $\alpha \equiv f_{,\phi}/\sqrt{f}$ and

$$s = -\frac{\dot{H}}{H^2} = \frac{4\sqrt{3}\alpha\xi u x y + 3\mu\left(x^2 - \gamma\right)y^2}{2\left(2\xi u^2 + 1\right)} + \frac{3\gamma}{2}. \quad (18)$$

Other quantities with great physical significance, namely, the total equation of state parameter and the deceleration parameter, are given by

$$\begin{aligned}\omega_{\text{tot}} &\equiv \frac{P_\phi + P_m}{\rho_\phi + \rho_m}\\ &= \mu y^2\left(x^2 - \gamma\right) + 2\xi u\left[\frac{2\sqrt{3}}{3}\alpha x y + u\left(\gamma - \frac{2}{3}s\right)\right] + \gamma - 1,\end{aligned}$$

Table 1: Location and existence conditions of the critical points and the corresponding values of the dark energy equation of state parameter $\omega_{DE}$, of the total equation of state parameter $\omega_{tot}$, and of the deceleration parameter $q$.

| Label | Location of $(x_c, y_c, u_c)$ | $\omega_{DE}$ | $\omega_{tot}$ | $q$ | Existence |
|---|---|---|---|---|---|
| $A$ | $0, 1, 0$ | $-1$ | $-1$ | $-1$ | $\lambda = 0$ |
| $B$ | $0, -1, 0$ | $-1$ | $-1$ | $-1$ | $\lambda = 0$ |
| $C$ | $0, \sqrt{\frac{\alpha}{\lambda^2}\left(\alpha\xi + \sqrt{\xi\left(\alpha^2\xi - 2\lambda^2\right)}\right)}, \frac{\alpha\xi + \sqrt{\xi(\alpha^2\xi - 2\lambda^2)}}{2\xi\lambda}$ | $-1$ | $-1$ | $-1$ | $\xi \geq \frac{2\lambda^2}{\alpha^2}$ and $\frac{\lambda}{\alpha} > 0$ or $\xi < 0, \quad \alpha > 0$ and $\lambda < 0$ |
| $D$ | $0, \sqrt{\frac{\alpha}{\lambda^2}\left(\alpha\xi - \sqrt{\xi\left(\alpha^2\xi - 2\lambda^2\right)}\right)}, \frac{\alpha\xi - \sqrt{\xi(\alpha^2\xi - 2\lambda^2)}}{2\xi\lambda}$ | $-1$ | $-1$ | $-1$ | $\xi \geq \frac{2\lambda^2}{\alpha^2}$ and $\frac{\lambda}{\alpha} > 0$ or $\xi < 0, \quad \alpha < 0$ and $\lambda > 0$ |

$$q \equiv -1 - \frac{\dot{H}}{H^2} = \frac{1}{2} + \frac{3}{2}\omega_{tot} = \frac{3}{2}\mu y^2\left(x^2 - \gamma\right) + \xi u\left[2\sqrt{3}\alpha x y + u\left(3\gamma - 2s\right)\right] + \frac{3\gamma}{2} - 1. \tag{19}$$

We mention that relations (19) are always valid, that is, independently of the specific state of the system (they are valid in the whole phase-space and not only at the critical points).

With the help of the auxiliary variables (14), the equations of motion (9) and (13) can be rewritten as a dynamical system; namely,

$$\begin{aligned} x' &= \frac{\sqrt{3}}{2}\left[\lambda x^2 y + \lambda\left(2 - 3x^2\right)y - 4\alpha\xi u\mu^{-3}y^{-1} - 2\sqrt{3}x\mu^{-2}\right] - \widehat{Q}, \\ y' &= \left(-\frac{\sqrt{3}}{2}\lambda x y + s\right)y, \\ u' &= \frac{\sqrt{3}\alpha x y}{2}, \end{aligned} \tag{20}$$

where $\widehat{Q} = Q/\dot{\phi}H\sqrt{V(\phi)}$ and $\lambda \equiv -V_{,\phi}/\kappa V$. Equations (20) can be an autonomous system when interaction term $Q$ is chosen to be a suitable form. From now on we assume the nonminimal coupling function to be $f(\phi) \propto \phi^2$ such that $\alpha$ is a constant. On the other hand the usual assumption in the literatures is to consider an exponential potential of the form $V = V_0 e^{-k\lambda\phi}$ [107–110]. Such a potential leads a constant $\lambda$. In fact $\lambda = -V_{,\phi}/kV \simeq$ const is valid for arbitrary but nearly flat potentials [111, 112]. Note that an exponential potential of the form $V = V_0 e^{-k\lambda\phi}$ is equivalent to the inverse square potential $V(\phi) \propto \phi^{-2}$ in terms of untransformed field $\phi$. Moreover, since the densities of dark energy and dark matter are nearly equal today there may be some coupling or interaction between them. Thus, various forms of interacting dark energy models [113–115] have been constructed in order to fulfil the observational requirements. In these models different forms of the coupling between dark energy and dark matter were proposed. Here we consider the most familiar interaction term extensively considered in the literature $Q = \beta\kappa\rho_m\dot{\phi}$ where $\beta$ is a constant [107, 108, 116]. Note that an interaction between dark matter and dark energy is a reasonable assumption but there is no reason to consider that baryonic matter is also coupled.

Using the above mentioned points we have a three-dimensional autonomous system as follows:

$$\begin{aligned} x' &= \frac{\sqrt{3}}{2}\left[\lambda x^2 y + \lambda\left(2 - 3x^2\right)y - 4\alpha\xi u\mu^{-3}y^{-1} - 2\sqrt{3}x\mu^{-2}\right] - \sqrt{3}\beta y^{-1}\Omega_m, \\ y' &= \left(-\frac{\sqrt{3}}{2}\lambda x y + s\right)y, \\ u' &= \frac{\sqrt{3}\alpha x y}{2}. \end{aligned} \tag{21}$$

Now, let us proceed to the phase-space analysis. The critical or fixed points $(x_c, y_c, u_c)$ of the autonomous system (21) are obtained by setting the left hand sides of the equations to zero, namely, by imposing the conditions $x' = y' = u' = 0$. One should note that we are interested in solutions which imply expanding solution, nonnegative dimensionless energy density parameters for the different species.

After some algebraic calculus, we find four critical points $(A, B, C, D)$ presented in Table 1. In the same table we have provided the existence conditions of each point and the values of $\omega_{DE}$, $\omega_{tot}$, and $q$ which can be used to discuss whether there exists acceleration phase or not. To study the stability of the critical points we substitute linear perturbations $x \rightarrow x_c + \delta x$, $y \rightarrow y_c + \delta y$, and $u \rightarrow u_c + \delta u$ about the critical point $(x_c, y_c, u_c)$ into the autonomous system (21). The

Table 2: The eigenvalues of the linearization matrices corresponding to the critical points. Here $B^+ = (\alpha\xi + \sqrt{\xi(\alpha^2\xi - 2\lambda^2)})$ and $B^- = (\alpha\xi - \sqrt{\xi(\alpha^2\xi - 2\lambda^2)})$.

| Label | Matrix $M$ | Eigenvalues |
|---|---|---|
| $A$ | $\begin{pmatrix} -3 & \sqrt{3}\beta\Omega_m & -2\sqrt{3}\alpha\xi \\ 0 & -3\gamma & 0 \\ \frac{\sqrt{3}}{2}\alpha & 0 & 0 \end{pmatrix}$ | $-\frac{3}{2} + \frac{1}{2}\sqrt{9 - 12\alpha^2\xi}$, $-\frac{3}{2} - \frac{1}{2}\sqrt{9 - 12\alpha^2\xi}$, $-3\gamma$ |
| $B$ | $\begin{pmatrix} -3 & \sqrt{3}\beta\Omega_m & 2\sqrt{3}\alpha\xi \\ 0 & -3\gamma & 0 \\ -\frac{\sqrt{3}}{2}\alpha & 0 & 0 \end{pmatrix}$ | $-\frac{3}{2} + \frac{1}{2}\sqrt{9 - 12\alpha^2\xi}$, $-\frac{3}{2} - \frac{1}{2}\sqrt{9 - 12\alpha^2\xi}$, $-3\gamma$ |
| $C$ | $\begin{pmatrix} -3 & 2\sqrt{3}\lambda + \frac{\sqrt{3}\beta\lambda^2\Omega_m}{\alpha B^+} & -2\sqrt{3}\xi\lambda\sqrt{\frac{\alpha}{B^+}} \\ \frac{\sqrt{3}\alpha B^+}{2\lambda} & -3\gamma & 3\gamma\sqrt{\frac{B^+}{\alpha}} \\ \frac{\sqrt{3}\alpha\sqrt{\alpha B^+}}{2\lambda} & 0 & 0 \end{pmatrix}$ | $-\frac{3}{2} + \frac{1}{2}\sqrt{9 + 12\alpha\sqrt{\xi(\alpha^2\xi - 2\lambda^2)} + 6\beta\Omega_m\lambda}$, $-\frac{3}{2} - \frac{1}{2}\sqrt{9 + 12\alpha\sqrt{\xi(\alpha^2\xi - 2\lambda^2)} + 6\beta\Omega_m\lambda}$, $-3\gamma$ |
| $D$ | $\begin{pmatrix} -3 & 2\sqrt{3}\lambda + \frac{\sqrt{3}\beta\lambda^2\Omega_m}{\alpha B^-} & 2\sqrt{3}\xi\lambda\sqrt{\frac{\alpha}{B^-}} \\ \frac{\sqrt{3}\alpha B^-}{2\lambda} & -3\gamma & -3\gamma\sqrt{\frac{B^-}{\alpha}} \\ -\frac{\sqrt{3}\alpha\sqrt{\alpha B^-}}{2\lambda} & 0 & 0 \end{pmatrix}$ | $-\frac{3}{2} + \frac{1}{2}\sqrt{9 - 12\alpha\sqrt{\xi(\alpha^2\xi - 2\lambda^2)} + 6\beta\Omega_m\lambda}$, $-\frac{3}{2} - \frac{1}{2}\sqrt{9 - 12\alpha\sqrt{\xi(\alpha^2\xi - 2\lambda^2)} + 6\beta\Omega_m\lambda}$, $-3\gamma$ |

$3\times 3$ matrix $M$ of the linearized perturbation equations of the autonomous system is shown in the appendix. Therefore, for each critical point of Table 1 we examine the sign of the real part of the eigenvalues of $M$, namely, $r_1$, $r_2$, and $r_3$, to determine the type and stability of the critical point. A fixed point is unstable if $r_1 > 0$, $r_2 > 0$, and $r_3 > 0$. It is saddle if the real part of the eigenvalues has different signs and it is stable for negative real part of eigenvalues. In Table 2 we have calculated the linearized perturbation matrix at each fixed point and corresponding eigenvalues. The stability conditions of each point are presented in Table 3.

We are going now to discuss the corresponding cosmological behavior of each critical point.

*Critical Point A.* Point $A$ represents a dark energy dominated solution that exists for all values of $\alpha$ and $\beta$. For $0 < \xi < 3\alpha^2/4$ it is a stable point (meaning that if the universe reaches this solution, it remains there forever) and thus it can attract the universe at late time. Accelerated expansion occurs for this point because $\omega_{\text{tot}} < -1/3$. Dark energy equation of state at this point is the same as the equation of state of the cosmological constant $\omega_{\text{DE}} = -1$. Point $A$ has the disadvantage that exists for the limiting case $\lambda = 0$ that is for a constant potential. Figure 1 from left to right shows the projections of the phase-space trajectories on the $y - x$, $u - y$, and $x - u$ planes for $\xi = 0.5$ and $\alpha = 1.5$. With these values of the parameters point $A$ is an attractor as it is clear in Figure 1. Also, we can see this stable point in 3-dimensional figure plotted in Figure 4. Also, point $A$ could be a saddle point (meaning that the universe during its evolution can reach this state but does not remain there and evolves to another state) for negative values of the coupling parameter $\xi$.

*Critical Point B.* The critical point $B$ also exists independent of the values of the $\alpha$ and $\beta$ but again for a constant potential. It corresponds to a completely dark energy dominated solution and could be a stable point if $0 < \xi < 3\alpha^2/4$. Note however that since the variable $y$ has the sign of $H$ thus this critical point corresponds to a contracting universe and it is not a physically meaningful solution.

*Critical Point C.* Accelerating dark energy dominated solution $C$ exists for $\xi \geq 2\lambda^2/\alpha^2$ and $\lambda/\alpha > 0$ or $\xi < 0$, $\alpha > 0$ and $\lambda < 0$. This point is an attractor solution if there are the following constraints on the nonminimal coupling parameters:

$$\frac{\lambda^2}{\alpha^2}\left(1 + \sqrt{1 + \frac{\beta^2\Omega_m^2}{4\lambda^2}}\right) < \xi < \frac{\lambda^2}{\alpha^2}\left(1 + \sqrt{1 + \frac{9\left(1 + (2/3)\,\beta\Omega_m\lambda\right)^2}{16\lambda^4}}\right) \tag{22}$$

Table 3: Stability of the critical points of Table 1.

| Label | Stability |
|---|---|
| $A$ | Stable point if $0 < \xi < 3\alpha^2/4$<br>Saddle point if $\xi < 0$ |
| $B$ | Stable point if $0 < \xi < 3\alpha^2/4$<br>Saddle point if $\xi < 0$ |
| $C$ | Stable point (for $\alpha < 0$ and $\beta\lambda > 0$) if<br>$\frac{\lambda^2}{\alpha^2}\left(1+\sqrt{1+\frac{\beta^2\Omega_m^2}{4\lambda^2}}\right) < \xi < \frac{\lambda^2}{\alpha^2}\left(1+\sqrt{1+\frac{9(1+(2/3)\beta\Omega_m\lambda)^2}{16\lambda^4}}\right)$<br>or<br>$\frac{\lambda^2}{\alpha^2}\left(1-\sqrt{1+\frac{9(1+(2/3)\beta\Omega_m\lambda)^2}{16\lambda^4}}\right) < \xi < \frac{\lambda^2}{\alpha^2}\left(1-\sqrt{1+\frac{\beta^2\Omega_m^2}{4\lambda^2}}\right)$<br>Saddle point $\left(\text{for } \alpha > 0, \beta\lambda > 0 \text{ and } \sqrt{\xi(\alpha^2\xi - 2\lambda^2)} > \frac{\beta\Omega_m\lambda}{2\alpha}\right)$ if<br>$\xi < \frac{\lambda^2}{\alpha^2}\left(1-\sqrt{1+\frac{\beta^2\Omega_m^2}{4\lambda^2}}\right)$<br>or<br>$\xi > \frac{\lambda^2}{\alpha^2}\left(1+\sqrt{1+\frac{\beta^2\Omega_m^2}{4\lambda^2}}\right)$<br>Saddle point $\left(\text{for all } \alpha, \beta\lambda > 0 \text{ and } \sqrt{\xi(\alpha^2\xi - 2\lambda^2)} < \left\lvert\frac{\beta\Omega_m\lambda}{2\alpha}\right\rvert\right)$ if<br>$\frac{\lambda^2}{\alpha^2}\left(1-\sqrt{1+\frac{\beta^2\Omega_m^2}{4\lambda^2}}\right) < \xi < 0$<br>and<br>$2\frac{\lambda^2}{\alpha^2} < \xi < \frac{\lambda^2}{\alpha^2}\left(1+\sqrt{1+\frac{\beta^2\Omega_m^2}{4\lambda^2}}\right)$ |
| $D$ | Stable point (for $\alpha > 0$ and $\beta\lambda > 0$) if<br>$\frac{\lambda^2}{\alpha^2}\left(1+\sqrt{1+\frac{\beta^2\Omega_m^2}{4\lambda^2}}\right) < \xi < \frac{\lambda^2}{\alpha^2}\left(1+\sqrt{1+\frac{9(1+(2/3)\beta\Omega_m\lambda)^2}{16\lambda^4}}\right)$<br>or<br>$\frac{\lambda^2}{\alpha^2}\left(1-\sqrt{1+\frac{9(1+(2/3)\beta\Omega_m\lambda)^2}{16\lambda^4}}\right) < \xi < \frac{\lambda^2}{\alpha^2}\left(1-\sqrt{1+\frac{\beta^2\Omega_m^2}{4\lambda^2}}\right)$<br>Saddle point (for $\alpha > 0$ and $\beta\lambda > 0$) if<br>$\frac{\lambda^2}{\alpha^2}\left(1-\sqrt{1+\frac{\beta^2\Omega_m^2}{4\lambda^2}}\right) < \xi < 0$<br>and<br>$2\frac{\lambda^2}{\alpha^2} < \xi < \frac{\lambda^2}{\alpha^2}\left(1+\sqrt{1+\frac{\beta^2\Omega_m^2}{4\lambda^2}}\right)$<br>Saddle point (for $\alpha < 0$ and $\beta\lambda > 0$) if<br>$\xi < \frac{\lambda^2}{\alpha^2}\left(1-\sqrt{1+\frac{\beta^2\Omega_m^2}{4\lambda^2}}\right)$<br>or<br>$\xi > \frac{\lambda^2}{\alpha^2}\left(1+\sqrt{1+\frac{\beta^2\Omega_m^2}{4\lambda^2}}\right)$ |

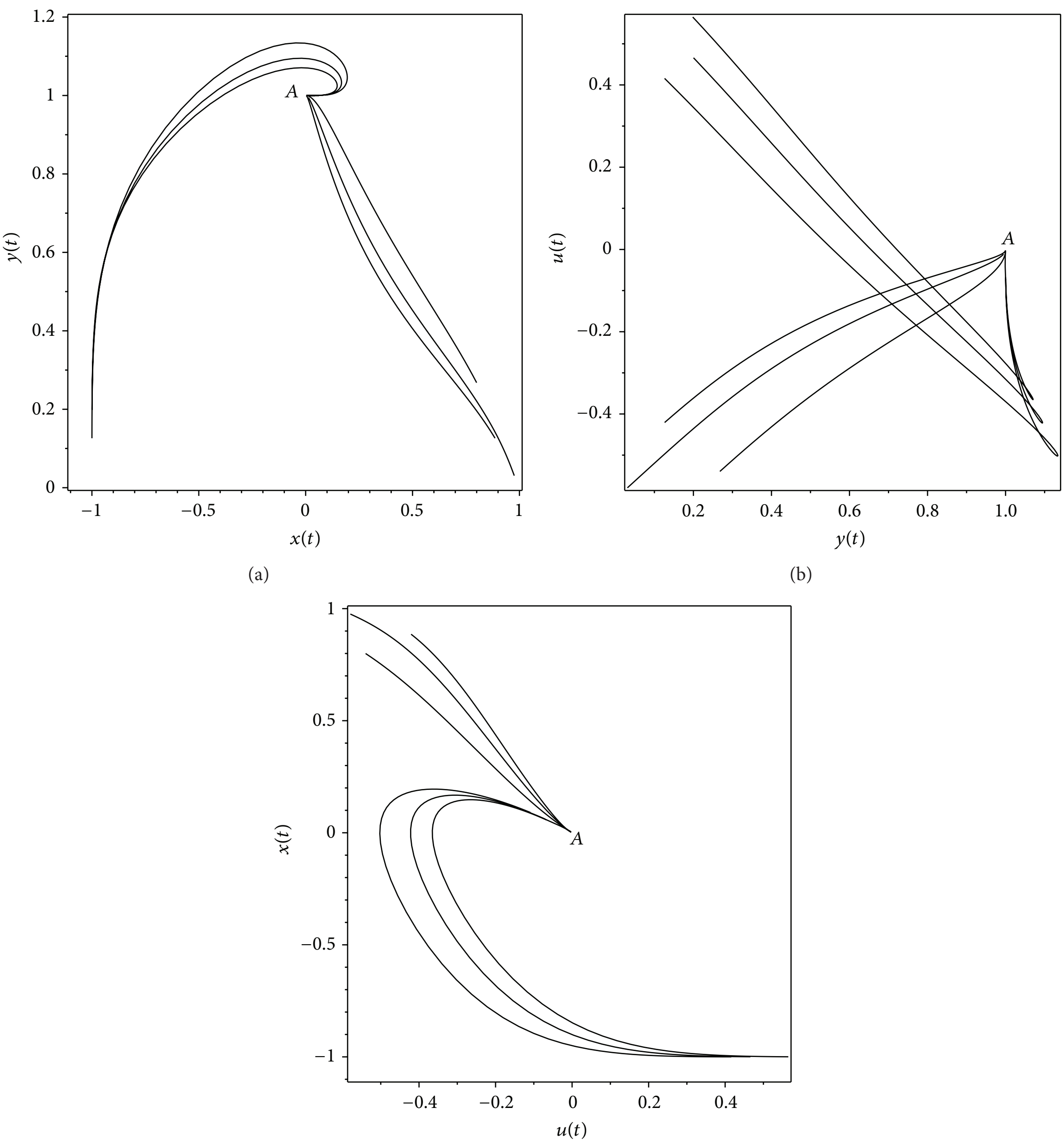



Figure 1: From (a) to (c), the projections of the phase-space trajectories on the $y-x$, $u-y$, and $x-u$ planes with $\xi = 0.5$ and $\alpha = 1.5$. For these values of the parameters, point $A$ is a stable attractor.

or

$$\frac{\lambda^2}{\alpha^2}\left(1-\sqrt{1+\frac{9\left(1+(2/3)\,\beta\Omega_m\lambda\right)^2}{16\lambda^4}}\right) < \xi < \frac{\lambda^2}{\alpha^2}\left(1-\sqrt{1+\frac{\beta^2\Omega_m^2}{4\lambda^2}}\right). \qquad (23)$$

Universe at this point behaves like a cosmological constant with $\omega_{\mathrm{DE}} = -1$. In Figure 2, where the chosen values of the parameters ($\xi = 0.92$ and $\alpha = -1.5$) satisfy existing condition and constraints (22) or (23), the projections of the phase-space trajectories on $y-x$, $u-y$, and $x-u$ are plotted, respectively. Three-dimensional plot is also shown in Figure 4. Point $C$ could be a saddle point if the constraints (22) or (23) are not satisfied as it is mentioned in Table 3.

*Critical Point D*. Similar to the critical point $C$, the fixed point $D$ could be an attractor of the model. In this situation the parameter $\xi$ should be satisfied again in constraint (22) or (23). The only difference is that the point $D$ is stable for

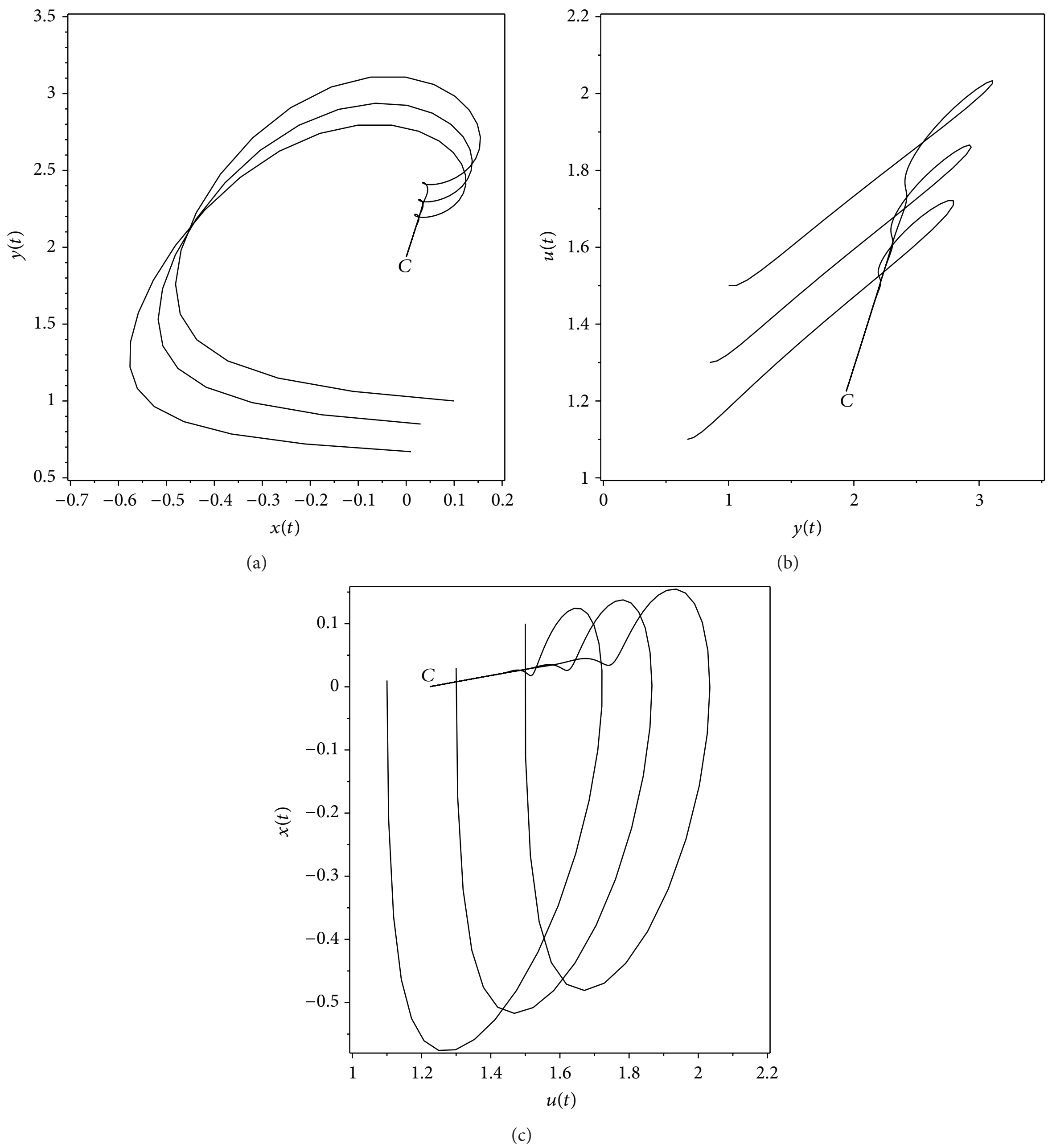



Figure 2: From (a) to (c), the projections of the phase-space trajectories on the $y-x$, $u-y$, and $x-u$ planes with $\xi = 0.92$ and $\alpha = -1.5$. The trajectories are attracted by the point $C$.

positive values of $\alpha$ while $C$ is stable for negative values of $\alpha$. In Figure 3 we have chosen the parameters such that the fixed point $D$ is an attractor point of the model ($\xi = 0.92$ and $\alpha = 1.5$) and Figure 4 shows the corresponding 3-dimensional plot. Conditions needed in order for $D$ to become a saddle point are presented in Table 3.

## 5. Conclusion

Tachyonic teleparallel dark energy is a generalization of the teleparallel dark energy recently proposed by Geng et al. [84]. Such an extension is obtained by replacing a canonical scalar field (quintessence) by a noncanonical scalar field (tachyon). Dynamics of tachyonic teleparallel dark energy has been studied in [90]. In this paper we performed a detailed phase-space analysis of interacting tachyonic teleparallel dark energy where the dark energy sector interacts with the dark matter one. We extracted the critical points of the model and examined their stabilities. We derived the eigenvalues corresponding to each critical point in order to find the stable solutions. We also calculated some of the important cosmological parameters such as the equation of state of dark

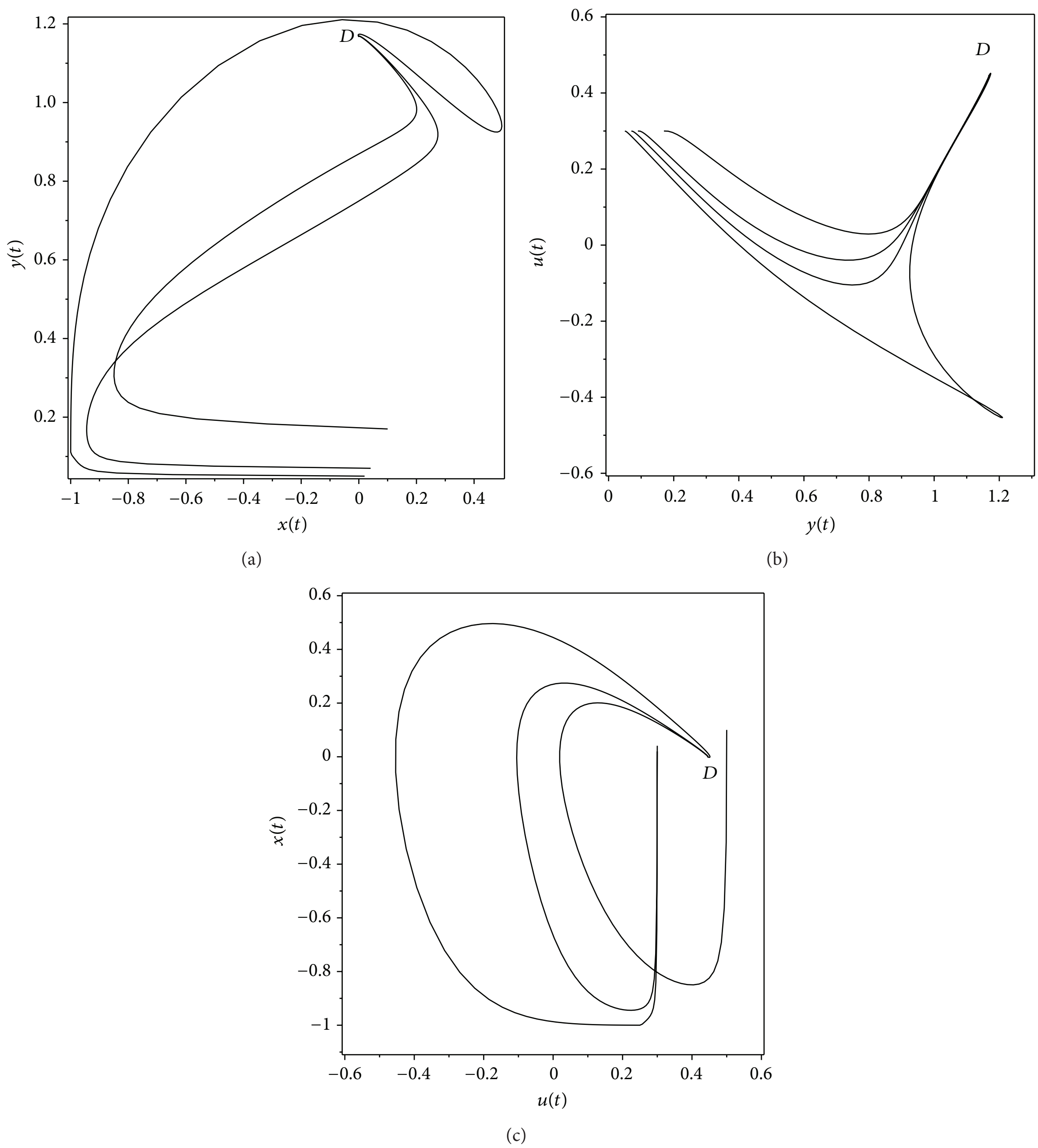
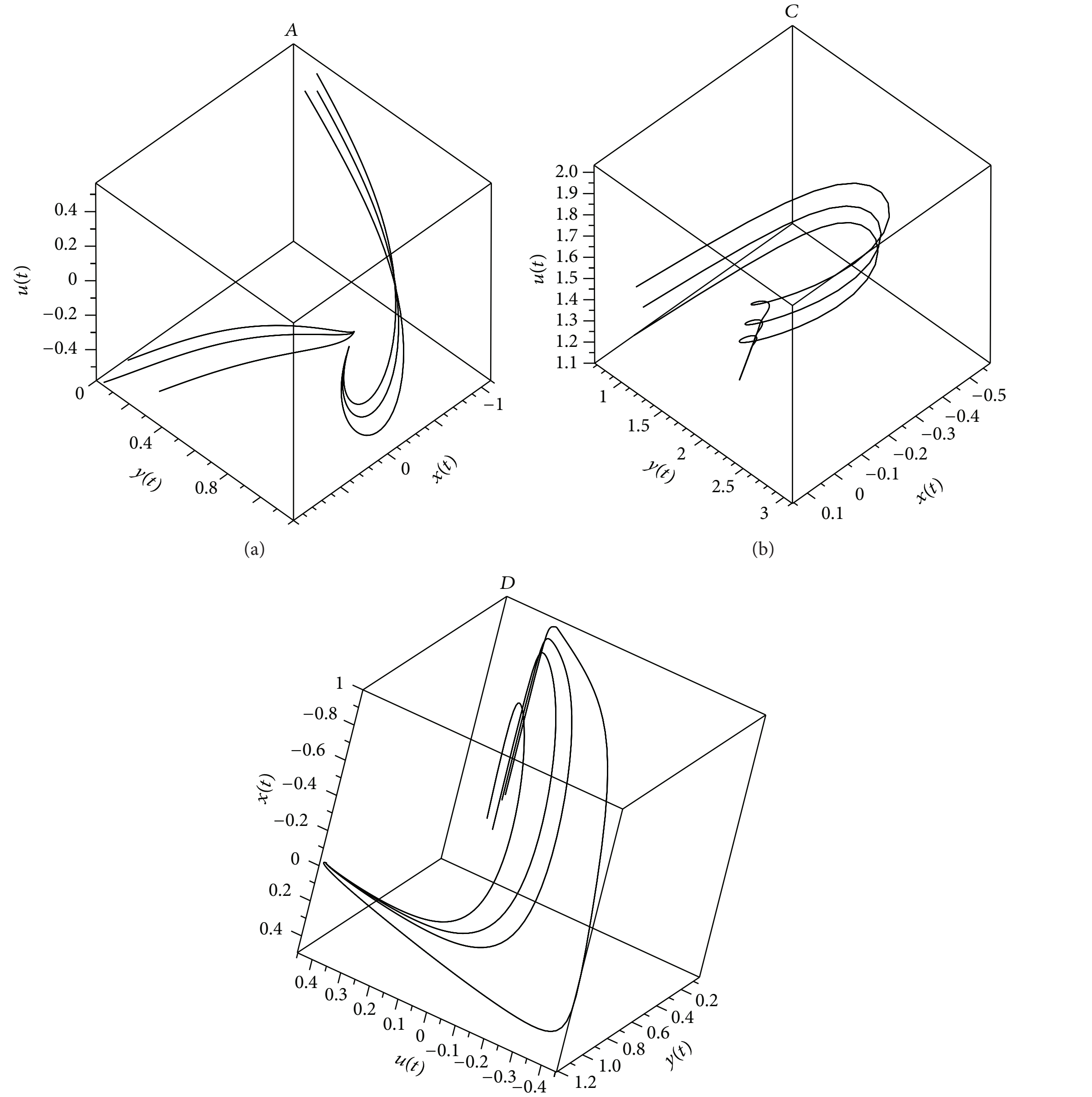



Figure 3: From (a) to (c), the projections of the phase-space trajectories on the $y-x$, $u-y$, and $x-u$ planes with $\xi = 0.92$ and $\alpha = 1.5$. The trajectories are attracted by the point $D$.

energy $\omega_{\mathrm{DE}}$, the total equation of state $\omega_{\mathrm{tot}}$, and deceleration parameter $q$ in order to find whether critical points correspond to an accelerating universe or not. Depending on the nonminimal coupling parameter $\xi$, we found three stable attractors of the model, that is, the point $A$ for a constant potential and points $C$ and $D$ if $\xi$ satisfies (22) or (23). In order to show behavior of the model at the critical points more transparently, we plotted two- and three-dimensional phase-space trajectories of the model in Figures 1–4. These figures show that by choosing parameters of the model suitably, points $A$, $C$, and $D$ are attractors of the model. While all of the stable solutions admit an accelerating universe, they correspond to a complete dark energy domination and thus they are unable to solve coincidence problem. One can consider more complicated interaction terms constructed to solve the coincidence problem [117] or generalize the nonminimal coupling function in order to have a varying parameter $\alpha$ similar to the procedure which was done in




(a) (b) (c)

Figure 4: Three-dimensional phase-space trajectories of the model with stable attractors $A$ (a), $C$ (b), and $D$ (c), respectively.

[88]. Moreover, in order to show that there are no significant equilibrium points lost at infinity one can use the Poincare central projection method to investigate the dynamics at infinity [118]. Such method has been applied for teleparallel dark energy model in [86].

## Appendix

## Stability of the Critical Points

The elements of $3\times 3$ matrix $M$ of the linearized perturbation equations for the real and physically meaningful critical points $(x_c, y_c, u_c)$ of the autonomous system read

$$
\begin{aligned}
M_{11} &= \sqrt{3}\left(-2\lambda x_c y_c - \sqrt{3}\left(1-3x_c^2\right) + 6\alpha\xi u_c x_c \mu_c^{-1} y_c^{-1}\right),\\
M_{12} &= \sqrt{3}\mu_c^{-2}\left(\lambda + 2\alpha\xi u_c \mu_c^{-1} y_c^{-2}\right) + \sqrt{3}\beta y_c^{-2}\Omega_m,\\
M_{13} &= -2\sqrt{3}\alpha\xi\mu_c^{-3} y_c^{-1},\\
M_{21} &= \frac{y_c^2\left(-3\mu_c^3 x_c y_c\left(x_c^2+\gamma-2\right)+4\sqrt{3}\alpha\xi u_c\right)}{2\left(2\xi u_c^2+1\right)} - \frac{\sqrt{3}\lambda y_c^2}{2},
\end{aligned}
$$

$$M_{22} = \frac{y_c\left(9\mu_c y_c\left(x_c^2-\gamma\right)+8\sqrt{3}\alpha\xi x_c u_c\right)}{2\left(2\xi u_c^2+1\right)} - \sqrt{3}\lambda x_c y_c + \frac{3\gamma}{2},$$

$$M_{23} = \frac{2\sqrt{3}\xi y_c^2\left(-\sqrt{3}\mu_c u_c y_c\left(x_c^2-\gamma\right)+2\alpha x_c\right)}{\left(2\xi u_c^2+1\right)^2} - \frac{2\sqrt{3}\alpha\xi x_c y_c^2}{2\xi u_c^2+1},$$

$$M_{31} = \frac{\sqrt{3}\alpha y_c}{2},$$

$$M_{32} = \frac{\sqrt{3}\alpha x_c}{2},$$

$$M_{33} = 0. \tag{A.1}$$

Examining the eigenvalues of the matrix $M$ for each critical point, one determines its stability behavior. We mentioned that although the matrix $M$ has a complicated form, inserting the explicit critical points presented in Table 1 into the elements, it takes a simple form and we can easily calculate its eigenvalues. The corresponding eigenvalues for each critical point are presented in Table 2.

## Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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