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Research Article

Noether Gauge Symmetry of Dirac Field in (2 + 1)-Dimensional Gravity

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We consider a gravitational theory including a Dirac field that is nonminimally coupled to gravity in 2 + 1 dimensions. Noether gauge symmetry approach can be used to fix the form of coupling function $F(\Psi)$ and the potential $V(\Psi)$ of the Dirac field and to obtain a constant of motion for the dynamical equations. In the context of $(2+1)$-dimensional gravity, we investigate cosmological solutions of the field equations using these forms obtained by the existence of Noether gauge symmetry. In this picture, it is shown that, for the nonminimal coupling case, the cosmological solutions indicate both an early-time inflation and late-time acceleration for the universe.

1. Introduction

The general theory of gravitation describes the physical phenomena coming about in a strong gravitational field and the physical behaviour of the universe in a large scale. At the same time, it has a remarkable mathematical and physical perspective. However, as our knowledge about the universe increases by new observational data, some new modified gravitation theories based on the general theory of relativity to understand the reality of the universe are brought out. Moreover, the quantisation of the theory still keeps the most important problem in physics standing. Therefore, the general theory of gravitation is one of the most active research areas.

Given the observational data [1–4], the universe had been accelerated in early time which is called inflation. To investigate the cosmic inflation of the universe or the beginning of expansion, the standard cosmological models were used [5, 6]. Another cosmic acceleration of the universe occurs in the late-time universe and confirms various observational evidences which are the observations of supernovae Type Ia (SNe Ia) [7, 8], cosmic microwave background radiation (CMB) [9, 10], and large-scale structure [11]. To clarify such an accelerated expansion, many authors introduced mysterious cosmic fluid, the so-called dark energy. Several models have been proposed in the literature such as quintessence [12], phantom [13], $F(R)$ [14–16], and $F(T)$ gravity [17, 18]. Recently, to understand the problem $(3+1)$-dimensional spacetime, it has been realized that fermionic or Dirac fields as a gravitational source which cause inflationary period in early universe and dark energy in old universe are being considered [19–23]. In this connection, De Souza and Kremer [24] have analysed a model with Dirac field that is nonminimally coupled to the gravity in the $(3+1)$-dimensional. They have utilized Noether symmetry approach to determine the forms of the potential and the coupling function to show dependence on Dirac fields in the model. Noether symmetry approach firstly introduced by Capozziello and de Ritis [25] to find new cosmological solutions in the $(3+1)$-dimensional gravity has been used to determine the shape of the potential and the coupling function dynamically in the scalar-tensor gravity theory. It is important to note that this approach gives us constants of motion (first integral) for the dynamical equation. Recently, this approach has been extensively studied in various cosmological models, that is, in scalar field cosmology [26], $f(R)$ metric and Palatini theory [27, 28], $f(T)$ theory [29], and teleparallel dark energy model [30]. In all of these studies, the Noether symmetry approach has been considered without a gauge term. If one considers the gauge term, then he may expect extra symmetries which yield new constants of motion, but the term makes the calculations more complicated.

The general theory of gravitation in 3 + 1 dimensions is very difficult because it has complicated calculations. However, in (2 + 1)-dimensional spacetime, it takes much more simple form because Weyl tensor vanishes and Riemann tensor is reduced to Ricci tensor. Moreover, the version in the 2 + 1 dimensions also contains the physical results in the (3 + 1)-dimensional theory [31–34]. Therefore, the (2 + 1)-dimensional spacetime becomes a perfect theoretical laboratory to construct new modified gravitation theories. In particular, if one wants to probe the mathematical and physical properties of the universe in the (2 + 1)-dimensional spacetime by using Dirac spinor fields as a gravitational source, one can simply describe the properties of the universe according to its analogous (3 + 1)-dimensional spacetime because Dirac spinor fields with 4 components in the (3 + 1)-dimensional spacetime are also reduced to the 2-component spinor fields corresponding to one negative and one positive energy states [35]. Furthermore, Dirac spinorial fields can give us useful information about inflation in the early time of the universe because Dirac theory has vacuum which includes Zitterbewegung oscillations between positive and negative energy states. Also, the theory perfectly describes an interaction between Dirac particles and matter. Therefore, Dirac spinorial fields can probe how to expand the universe in late time.

From these points of view, we want to study the Dirac fields as a source of early-time inflation and late-time acceleration in (2 + 1)-dimensional gravity for Friedmann-Robertson-Walker (FRW) background by using Noether gauge symmetry approach. The results to be performed in this study are important in consequence of the (2 + 1)-dimensional gravity and Dirac theory, because they give information about the expansion of the universe in early and late time. Also, it will be the first example carried out, regarding gravity and Dirac fields by using Noether gauge symmetry method. Therefore, the study will satisfy motivations for new studies in the (2 + 1)-dimensional gravity.

This paper is organized as follows. In the following section, we give the field equations of a theory in which the Dirac field is nonminimally coupled to the gravity in the 2 + 1 dimensions. In Section 3, we search the Noether gauge symmetry for the Lagrangian of the theory with the Dirac field. In Section 4, we obtain the solutions of the field equations by using Noether gauge symmetry approach. Finally, in Section 5, we conclude with a summary of the obtained results. Throughout the paper, we use $c = G = \hbar = 1$.

2. The Action and Field Equations

In (2 + 1)-dimensional curved spacetime, the action for a Dirac field which is nonminimally coupled to scalar curvature is given by

$$\begin{aligned}\mathscr{A} = \int d^3x\sqrt{-g}\Big\{&F(\Psi)R \\ &+\frac{\imath}{2}\Big[\overline{\psi}\,\overline{\sigma}^{\mu}(x)\left(\partial_{\mu}-\Omega_{\mu}(x)\right)\psi \\ &-\overline{\psi}\left(\overleftarrow{\partial}_{\mu}+\Omega_{\mu}(x)\right)\overline{\sigma}^{\mu}(x)\psi\Big]-V(\Psi)\Big\},\end{aligned} \tag{1}$$

where $F(\Psi)$ and $V(\Psi)$ are generic functions representing the coupling with gravity and the self-interaction potential of the Dirac field, respectively, and they depend on only functions of the bilinear $\Psi = \overline{\psi}\psi$; g is the determinant of the metric tensor $g_{\mu\nu}$; R is the Ricci scalar; ψ is two-component, particle and antiparticle, Dirac field; $\overline{\psi}$ is adjoint of the ψ and $\overline{\psi} = \psi^{\dagger}\sigma^3$. In this action, $\Omega_{\mu}(x)$ are spin connection and are given as

$$\Omega_{\mu}(x) = \frac{1}{4}g_{\lambda\alpha}\left(e^{i}_{\nu,\mu}e^{\alpha}_{i}-\Gamma^{\alpha}_{\nu\mu}\right)s^{\lambda\nu}(x), \tag{2}$$

where $\Gamma^{\alpha}_{\nu\mu}$ is Christoffel symbol, and $g_{\mu\nu}$ is given in terms of triads, $e^{(i)}_{\mu}(x)$, as follows:

$$g_{\mu\nu}(x) = e^{i}_{\mu}(x)\,e^{j}_{\nu}(x)\,\eta_{ij}, \tag{3}$$

where μ and ν are curved spacetime indices running from 0 to 2. i and j are flat spacetime indices running from 0 to 2 and η_{ij} is the (2 + 1)-dimensional Minkowskian metric with signature $(1, -1, -1)$. The $s^{\lambda\nu}(x)$, spin operators, are given by

$$s^{\lambda\nu}(x) = \frac{1}{2}\left[\overline{\sigma}^{\lambda}(x), \overline{\sigma}^{\nu}(x)\right], \tag{4}$$

where $\overline{\sigma}^{\mu}(x)$ are the spacetime dependent Dirac matrices in the (2 + 1)-dimensional. Thanks to triads, $e^{\mu}_{(i)}(x)$, $\overline{\sigma}^{\mu}(x)$ are related to the flat spacetime Dirac matrices, $\overline{\sigma}^{i}$, as follows:

$$\overline{\sigma}^{\mu}(x) = e^{\mu}_{(i)}(x)\,\overline{\sigma}^{i}, \tag{5}$$

where $\overline{\sigma}^{i}$ are

$$\begin{aligned}\overline{\sigma}^{0} &= \sigma^{3}, \\ \overline{\sigma}^{1} &= i\sigma^{1}, \\ \overline{\sigma}^{2} &= i\sigma^{2}.\end{aligned} \tag{6}$$

σ^1, σ^2, and σ^3 are Pauli matrices [35]. In this representation, the Dirac equation gives important information about the curved spacetime [36]. To analyse the expansion of the universe, we will consider the spatially flat spacetime background in (2 + 1)-dimensional which is analogous to the (3 + 1)-dimensional Friedmann-Robertson-Walker metric as follows:

$$ds^2 = dt^2 - a^2(t)\left[dx^2 + dy^2\right], \tag{7}$$

where $a(t)$ is the scale factor of the universe [33]. The scalar curvature corresponding to the FRW metric (7) takes the form $R = -2(2\ddot{a}/a + \dot{a}^2/a^2)$, where the dot represents differentiation with respect to cosmic time t. Given the background in (7), it is possible to obtain the point-like Lagrangian from action (1) in the following form:

$$L = 2F\dot{a}^2 + 4F'a\dot{a}\dot{\Psi} + \frac{\imath a^2}{2}\left(\overline{\psi}\sigma^3\dot{\psi} - \dot{\overline{\psi}}\sigma^3\psi\right) - a^2V. \tag{8}$$

Here the prime denotes the derivative with respect to the bilinear Ψ. Because of homogeneity and isotropy of the metric, it is assumed that the spinor field only depends on time

t; that is, $\psi = \psi(t)$. The Dirac equations for the spinor field ψ and its adjoint $\overline{\psi}$ are obtained from the point-like Lagrangian (8) such that the Euler-Lagrange equations for ψ and $\overline{\psi}$ are

$$\begin{aligned} &\dot{\psi} + H\psi + iV'\sigma^3\psi + 2iF'\left(2\dot{H} + 3H^2\right)\sigma^3\psi = 0, \\ &\dot{\overline{\psi}} + H\overline{\psi} - iV'\overline{\psi}\sigma^3 - 2iF'\left(2\dot{H} + 3H^2\right)\overline{\psi}\sigma^3 = 0, \end{aligned} \tag{9}$$

where $H = \dot{a}/a$ denotes the Hubble parameter. On the other hand, from the point-like Lagrangian (8) and by considering the Dirac equations, we find the second-order Euler-Lagrange equation for a, that is, the acceleration equation:

$$\frac{\ddot{a}}{a} = -\frac{p_f}{2F}. \tag{10}$$

Finally, we also consider the Hamiltonian constraint equation ($E_L = 0$) associated with Lagrangian (8):

$$E_L = \frac{\partial L}{\partial \dot{a}}\dot{a} + \frac{\partial L}{\partial \dot{\psi}}\dot{\psi} + \dot{\overline{\psi}}\frac{\partial L}{\partial \dot{\overline{\psi}}} - L, \tag{11}$$

which yields the Friedmann equation as follows:

$$H^2 = \frac{\rho_f}{2F}. \tag{12}$$

In (10) and (12), ρ_f and p_f are the effective energy density and pressure of the fermion field, respectively, so that they have the following form:

$$\rho_f = -\left(4F'H\dot{\Psi} + V\right), \tag{13}$$

$$\begin{aligned} p_f = {} & 2F'\left(\ddot{\Psi} + H\dot{\Psi}\right) + 2F''\dot{\Psi}^2 \\ & - \left[2F'\left(2\dot{H} + 3H^2\right) + V'\right]\Psi + V. \end{aligned} \tag{14}$$

In order to solve the field equations, we have to choose a form for the coupling function and for the potential density. To do this, in the following section we will use the Noether gauge symmetry approach.

3. The Noether Gauge Symmetry Approach

Thanks to Pauli matrices, in terms of the components of the spinor field $\psi = (\psi_1, \psi_2)^T$ and its adjoint $\overline{\psi} = ({\psi_1}^\dagger, -{\psi_2}^\dagger)$, Lagrangian (8) can be rewritten as follows:

$$\begin{aligned} L = {} & 2F\dot{a}^2 + 4F'a\dot{a}\sum_{i=1}^{2}\epsilon_i\left(\dot{\psi}_i^\dagger\psi_i + \psi_i^\dagger\dot{\psi}_i\right) \\ & + \frac{ia^2}{2}\left[\sum_{i}^{2}\left(\psi_i^\dagger\dot{\psi}_i - \dot{\psi}_i^\dagger\psi_i\right)\right] - a^2V, \end{aligned} \tag{15}$$

where $\epsilon_i = \left\{\begin{smallmatrix} 1 & \text{for } i=1 \\ -1 & \text{for } i=2 \end{smallmatrix}\right.$.

Noether theorem is useful tool in theoretical physics which states that any differentiable symmetry of an action of a physical system leads to a corresponding conserved quantity [37]. The idea to use Noether symmetry approach without gauge term in generalized theories of gravity studies is not new and was first introduced by Cappoziello et al. to find new cosmological solutions. Another technique is related to the more general symmetries known as Noether gauge symmetries which include nonzero gauge term [38, 39]. Taking into account a gauge term in Noether symmetry equation gives a more general definition of the Noether symmetry.

A vector field $\mathbf{X}$ for the point-like Lagrangian (15) is

$$\mathbf{X} = \tau\frac{\partial}{\partial t} + \alpha\frac{\partial}{\partial a} + \sum_{i=1}^{2}\left(\beta_i\frac{\partial}{\partial\psi_i} + \gamma_i\frac{\partial}{\partial\psi_i^\dagger}\right), \tag{16}$$

where τ, α, β_j, and γ_j depend on t, a, ψ_j, and $\psi_j^\dagger$ and they are determined from the Noether gauge symmetry condition. The first prolongation of $\mathbf{X}$ is given by

$$\mathbf{X}^{[1]} = \mathbf{X} + \alpha_t\frac{\partial}{\partial\dot{a}} + \sum_{i=1}^{2}\left(\beta_{it}\frac{\partial}{\partial\dot{\psi}_i} + \gamma_{it}\frac{\partial}{\partial\dot{\psi}_i^\dagger}\right), \tag{17}$$

in which

$$\begin{aligned} \alpha_t &= D_t\alpha - \dot{a}D_t\tau, \\ \beta_{jt} &= D_t\beta_j - \dot{\psi}_jD_t\tau, \\ \gamma_{jt} &= D_t\gamma_j - \dot{\psi}_j^\dagger D_t\tau. \end{aligned} \tag{18}$$

The vector field $\mathbf{X}$ is a Noether gauge symmetry corresponding to the Lagrangian $L(t, a, \psi_j, \psi_j^\dagger, \dot{a}, \dot{\psi}_j, \dot{\psi}_j^\dagger)$, if the condition

$$\mathbf{X}^{[1]}L + LD_t(\tau) = D_tB \tag{19}$$

holds, where $B(t, a, \psi_j, \psi_j^\dagger, \dot{a}, \dot{\psi}_j, \dot{\psi}_j^\dagger)$ is a gauge function and D_t is the operator of total differentiation with respect to t:

$$D_t = \frac{\partial}{\partial t} + \dot{a}\frac{\partial}{\partial a} + \sum_{i=1}^{2}\left(\dot{\psi}_i\frac{\partial}{\partial\psi_i} + \dot{\psi}_i^\dagger\frac{\partial}{\partial\psi_i^\dagger}\right). \tag{20}$$

The significance of Noether gauge symmetry clearly comes from the fact that if the vector field $\mathbf{X}$ is the Noether gauge symmetry corresponding to the Lagrangian $L(t, a, \psi_j, \psi_j^\dagger, \dot{a}, \dot{\psi}_j, \dot{\psi}_j^\dagger)$, then

$$\begin{aligned} I = {} & \tau L + (\alpha - \tau\dot{a})\frac{\partial L}{\partial\dot{a}} - B \\ & + \sum_{j=1}^{2}\left(\left(\beta_j - \tau\dot{\psi}_j\right)\frac{\partial L}{\partial\dot{\psi}_j} + \left(\gamma_j - \tau\dot{\psi}_j^\dagger\right)\frac{\partial L}{\partial\dot{\psi}_j^\dagger}\right) \end{aligned} \tag{21}$$

is a first integral or a conserved quantity associated with $\mathbf{X}$. Hence the Noether gauge symmetry condition (19) for

Lagrangian (15) leads to the following overdetermined system of differential equations:

$$2F\frac{\partial\alpha}{\partial a}+F'\sum_{i=1}^{2}\epsilon_i\left(\beta_i\psi_i^{\dagger}+\gamma_i\psi_i\right)-F\frac{\partial\tau}{\partial t}+2F'a\sum_{i=1}^{2}\epsilon_i\left(\frac{\partial\beta_i}{\partial a}\psi_i^{\dagger}+\frac{\partial\gamma_i}{\partial a}\psi_i\right)=0, \tag{22}$$

$$F'\frac{\partial\alpha}{\partial\psi_j}=0,\qquad F'\frac{\partial\alpha}{\partial\psi_j^{\dagger}}=0, \tag{23}$$

$$F'\frac{\partial\tau}{\partial\psi_j}=0,\qquad F'\frac{\partial\tau}{\partial\psi_j^{\dagger}}=0,\qquad \frac{\partial\tau}{\partial a}=0, \tag{24}$$

$$\imath\alpha\psi_j+\frac{\imath a}{2}\beta_j-\frac{\imath a}{2}\sum_{i=1}^{2}\left(\frac{\partial\beta_i}{\partial\psi_j^{\dagger}}\psi_i^{\dagger}-\frac{\partial\gamma_i}{\partial\psi_j^{\dagger}}\psi_i\right)-4F'\epsilon_j\psi_j\frac{\partial\alpha}{\partial t}+aV\frac{\partial\tau}{\partial\psi_j^{\dagger}}+\frac{1}{a}\frac{\partial B}{\partial\psi_j^{\dagger}}=0, \tag{25}$$

$$\imath\alpha\psi_j^{\dagger}+\frac{\imath a}{2}\gamma_j+\frac{\imath a}{2}\sum_{i=1}^{2}\left(\frac{\partial\beta_i}{\partial\psi_j}\psi_i^{\dagger}-\frac{\partial\gamma_i}{\partial\psi_j}\psi_i\right)+4F'\epsilon_j\psi_j^{\dagger}\frac{\partial\alpha}{\partial t}-aV\frac{\partial\tau}{\partial\psi_j}-\frac{1}{a}\frac{\partial B}{\partial\psi_j}=0, \tag{26}$$

$$F'\epsilon_j\psi_j^{\dagger}\left(\alpha+a\frac{\partial\alpha}{\partial a}\right)+F\frac{\partial\alpha}{\partial\psi_j}+F'a\left[\epsilon_j\gamma_j-\epsilon_j\psi_j^{\dagger}\frac{\partial\tau}{\partial t}+\sum_{i=1}^{2}\epsilon_i\left(\frac{\partial\beta_i}{\partial\psi_j}\psi_i^{\dagger}+\frac{\partial\gamma_i}{\partial\psi_j}\psi_i\right)\right]+F''\epsilon_j a\psi_j^{\dagger}\sum_{i=1}^{2}\epsilon_i\left(\beta_i\psi_i^{\dagger}+\gamma_i\psi_i\right)=0, \tag{27}$$

$$F'\epsilon_j\psi_j\left(\alpha+a\frac{\partial\alpha}{\partial a}\right)+F\frac{\partial\alpha}{\partial\psi_j^{\dagger}}+F'a\left[\epsilon_j\beta_j-\epsilon_j\psi_j\frac{\partial\tau}{\partial t}+\sum_{i=1}^{2}\epsilon_i\left(\frac{\partial\beta_i}{\partial\psi_j^{\dagger}}\psi_i^{\dagger}+\frac{\partial\gamma_i}{\partial\psi_j^{\dagger}}\psi_i\right)\right]+F''\epsilon_j a\psi_j\sum_{i=1}^{2}\epsilon_i\left(\beta_i\psi_i^{\dagger}+\gamma_i\psi_i\right)=0, \tag{28}$$

$$4F\frac{\partial\alpha}{\partial t}+\frac{\imath a^2}{2}\sum_{i=1}^{2}\left(\frac{\partial\beta_i}{\partial a}\psi_i^{\dagger}-\frac{\partial\gamma_i}{\partial a}\psi_i\right)-a^2V\frac{\partial\tau}{\partial a}-\frac{\partial B}{\partial a}+4F'a\sum_{i=1}^{2}\epsilon_i\left(\frac{\partial\beta_i}{\partial t}\psi_i^{\dagger}+\frac{\partial\gamma_i}{\partial t}\psi_i\right)=0, \tag{29}$$

$$\left(2\alpha+a\frac{\partial\tau}{\partial t}\right)V+\frac{1}{a}\frac{\partial B}{\partial t}+aV'\sum_{i=1}^{2}\epsilon_i\left(\beta_i\psi_i^{\dagger}+\gamma_j\psi_i\right)-\frac{\imath a}{2}\sum_{i=1}^{2}\left(\psi_i^{\dagger}\frac{\partial\beta_i}{\partial t}-\psi_i\frac{\partial\gamma_i}{\partial t}\right)=0. \tag{30}$$

This system given by (22)–(30) is obtained by imposing the fact that the coefficients of $\dot{a}^2$, $\dot{a}$, $\dot{\psi}_j$, $\dot{\psi}_j^{\dagger}$, $\dot{a}\dot{\psi}_j$, $\dot{\psi}_j\dot{\psi}_l$, and so on vanish. Now we will search for a solution of (22)–(30). Equations (23) give us two cases: $F'=0$ or $\partial\alpha/\partial\psi_l=0$ and $\partial\alpha/\partial\psi_l^{\dagger}=0$ ($F'\neq 0$). In this study, we will neglect the case $F'=0$ that corresponds to the fermionic field minimally coupled to gravity. From (24), one can immediately see $\tau=\tau(t)$. From the rest of Noether gauge symmetry equations, the complete solution is obtained as follows:

$$\begin{aligned}\alpha&=-\frac{c_1}{2(n-1)}a,\\ \beta_j&=\frac{c_1}{2(n-1)}\psi_j-\epsilon_j c_2\psi_j,\\ \gamma_j&=\frac{c_1}{2(n-1)}\psi_j^{\dagger}+\epsilon_j c_2\psi_j^{\dagger},\\ \tau&=c_1t+c_3,\\ B&=c_4,\end{aligned} \tag{31}$$

and the coupling and the potential function are power law forms of the function of the bilinear Ψ; that is,

$$F(\Psi)=f_0\Psi^n, \tag{32}$$

$$V(\Psi)=\lambda\Psi^{2-n}, \tag{33}$$

where c_l, λ, f_0, and n ($n\neq 1$) are integration constants.

From the vector field (31), Lagrangian (15) admits three Noether gauge symmetries which are

$$\begin{aligned}\mathbf{X}_1&=\frac{\partial}{\partial t},\\ \mathbf{X}_2&=t\frac{\partial}{\partial t}-\frac{1}{2(n-1)}\left[a\frac{\partial}{\partial a}-\sum_{i=1}^{2}\left(\psi_i\frac{\partial}{\partial\psi_i}+\psi_i^{\dagger}\frac{\partial}{\partial\psi_i^{\dagger}}\right)\right],\\ \mathbf{X}_3&=-\sum_{i=1}^{2}\epsilon_i\left(\psi_i\frac{\partial}{\partial\psi_i}-\psi_i^{\dagger}\frac{\partial}{\partial\psi_i^{\dagger}}\right).\end{aligned} \tag{34}$$

These generators constitute a well-known three-dimensional Lie algebra with commutation relations

$$\begin{aligned}\left[\mathbf{X}_1,\mathbf{X}_2\right]&=\mathbf{X}_1,\\ \left[\mathbf{X}_1,\mathbf{X}_3\right]&=\left[\mathbf{X}_2,\mathbf{X}_3\right]=0.\end{aligned} \tag{35}$$

The three first integrals (or conserved quantities) associated with the Noether gauge symmetries are

$$I_1 = -2F'\dot{a}^2 - 4F'a\dot{a}\dot{\Psi} - a^2V, \tag{36}$$

$$I_2 = tI_1 - \frac{2a}{n-1}\left(F\dot{a} - 2F'\Psi\dot{a} + F'a\dot{\Psi}\right), \tag{37}$$

$$I_3 = \imath a^2\Psi. \tag{38}$$

Here the constant parameter c_4 is assumed to be zero in the gauge function B. We note that the first integral (36) is related to the energy function (11), so that the first integral I_1 vanishes.

4. The Solutions of Field Equations

Since the coupling function F depends on the bilinear function Ψ, from Dirac's equations (9) one gets

$$\dot{\Psi} + 2\frac{\dot{a}}{a}\Psi = 0, \tag{39}$$

which integrates to give

$$\Psi = \frac{\Psi_0}{a^2}, \tag{40}$$

where Ψ_0 is a constant of integration. Inserting (40) into the first integral equation (38), we get $I_3 = \imath\Psi_0$. Considering (40) and the coupling function (32), the first integral (37) can be rewritten as

$$\dot{a} - ka^{2n-1} = 0, \tag{41}$$

where we define $k = (n-1)I_2/2(4n-1)f_0\Psi_0^n$ and $n \neq 1/4$. Now (41) can be used to find out the time dependence of the cosmic scale factor as follows:

$$a(t) = \left[2k(1-n)t + a_0\right]^{1/2(1-n)}, \tag{42}$$

where a_0 is an integration constant. Using solution (42) in the Friedmann equation (12) with (13) and the acceleration equation (10) with (14), we obtain constraint relations between the constants as $\lambda = I_2^2(n-1)^2/2\Psi_0^2 f_0(4n-1)$. Therefore, the solution for the cosmic scale factor that is obtained from the Noether gauge symmetry represents a power law expansion for the universe. It is remarkable from (42) that models with model parameter $n > 1/2$ obey an accelerated power law expansion while for $n < 1/2$ a decelerated expansion occurs. For $n = 1/2$, we have

$$a(t) = -\frac{I_2}{4f_0\sqrt{\Psi_0}}t + a_0. \tag{43}$$

This solution corresponds to the matter dominant universe with the pressure of the Dirac field $p_f = 0$ in the General Relativity in $2+1$ dimensions. Therefore, this solution shows that the Dirac field behaves as a standard pressureless matter field in the $2+1$ dimensions.

For our model, we search whether the fermionic field can provide alternative for dark energy or not. For this purpose we can define the equation of state parameter of the fermionic field by using the energy density (13) and pressure (14) as $\omega_f = p_f/\rho_f$. Considering (32), (33), (40), and (42), we obtain

$$\omega_f = 1 - 2n. \tag{44}$$

According to astrophysical data, the equation of state parameter tends to value -1. For the equation of state parameter less than -1 the dark energy is described by phantom, for $-1 < \omega < -1/3$ the quintessence dark energy is observed, and the case $\omega = -1$ corresponds to the cosmological constant. For our model, if $2/3 < n < 1$, we obtain the quintessence phase; if $n > 1$, we have the phantom phase. In both cases, the universe is both expanding and accelerating. Therefore, the results show that the fermionic field may behave like both quintessence and phantom dark energy field in the late-time universe.

Now, we return to the case $n = 1$ so that the coupling and potential functions have linear forms of Ψ from (32) and (33) as follows:

$$F(\Psi) = f_0\Psi, \qquad V(\Psi) = \lambda\Psi. \tag{45}$$

For this case, we do not need the first integral obtained by Noether gauge symmetry approach because the Friedman equation (12) and acceleration equation (10) can be directly integrated by usual method. Friedmann equation (12) is reduced to

$$\frac{\dot{a}}{a} - \sqrt{\frac{\lambda}{6f_0}} = 0, \tag{46}$$

which has the solution

$$a(t) = a_0 e^{H_0 t}, \quad \text{where } H_0 = \sqrt{\frac{\lambda}{6f_0}} \tag{47}$$

and a_0 is a constant. It is clear that this solution describes an inflationary period, where the cosmic scale factor increases exponentially with the cosmic time. Therefore, one can say that the Dirac field can behave as inflaton field in the $2+1$ dimensions. From (13) and (14), the energy density and the pressure of the Dirac field are given by

$$\rho_f = \frac{\lambda\Psi_0}{3a_0^2}e^{-2H_0 t}, \qquad p_f = -\rho_f, \tag{48}$$

with the equation of state parameter $\omega_f = -1$.

5. Concluding Remarks

The study of cosmological models in the $(2+1)$-dimensional gravitation theories provides mathematical simplicity in understanding the physical models. In the present study, we have considered a theory including the Dirac field which is nonminimally coupled to the gravity in $2+1$ dimension.

Using the Noether gauge symmetry approach, we have determined the explicit forms of the coupling function and potential as power law functions of the bilinear Ψ given by (32) and (33), respectively. The solutions of the field equations for FRW spacetime are presented by using the results obtained from the Noether gauge symmetry approach. It is shown that, in the general case n, the Dirac field plays the role of the dark energy in the late-time universe. For the special case $n = 1/2$, the solution of dynamical equation describes a decelerated universe with a matter dominated behavior in $2 + 1$ dimension. We also consider a model where the coupling function and the potential have linear forms of Ψ (i.e., the case $n = 1$). For such a model, the cosmological solution describes an inflationary period for the early-time universe. Therefore, we may conclude that the Dirac field behaves as an inflaton field.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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