

Research Article

Energy Conditions of Built-In Inflation Models in $f(T)$ Gravitational Theories

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We examine the validity of energy conditions of built-in inflation models in $f(T)$ gravitational theories. For this purpose, we formulate the inequalities of energy conditions by assuming the flat and nonflat Friedmann-Robertson-Walker (FRW) universe. We find the feasible constraints on the constants of integration and evaluate their possible ranges graphically for the consistency of these energy conditions for flat, closed, and open universes. We constrain the constants of integration for flat space-time from the inflation epoch while the closed and open universe constants are constrained from late universe.

1. Introduction

Recent observations show that the present expansion of our universe is accelerating [1–12]. Since that the dark energy (DE) problem has become a rich topic in fundamental physics. Nowadays, there are several explanations of the acceleration of the universe. To include the cosmological constant within the framework of general relativity (GR) is among these explanations and is the simplest one, which is known as Λ CDM model. On the other hand, to create accelerating FRW universe one needs to describe it by quintessence/phantom-fluid or another kind of inhomogeneous fluid which satisfies suitable equation of state (EoS). This means that dark fluid has EoS very close to -1 which represents an important point in favor of a cosmological constant-like representation of the DE. However, the actual small value of cosmological constant leads to several problems. The first one is the “cosmological constant problem.” In quantum field theory, the cosmological constant appears as the vacuum energy density, which has to be included in gravity theory, as the vacuum effect [13]. However, the expected value of vacuum energy density results in being of order 120 magnitude larger than the observed value.

Supersymmetry and strings theories were aiming to solve this problem by different ways, but up till now a successful answer seems to be far away [14].

The presence of an early accelerated epoch in the universe scenario or inflation puts a new problem to the standard cosmology, and various suggestions to build an acceptable inflationary model exist (scalar, spinor, (non)abelian vector theory, and so on). Apart from that, the scenarios to describe the early-time and the late-time accelerations are usually very similar and it is natural to anticipate that the same theory lies behind both. Since GR with matter and radiation correctly describes the intermediate (decelerated epoch) expansion of the universe, it is acceptable to anticipate that a different gravitational theory manages the FRW background evolution at high and small energy without the introduction of any other dark components. Warm intermediate scenario of the cosmological inflation in $f(T)$ gravity in the limit of high dissipation is discussed in [15]. In [16] an investigation of the energy conditions (including null, weak, strong, and dominant) in generalized teleparallel gravities including pure $f(T)$, teleparallel gravity with nonminimally coupled scalar field and $f(T)$ with nonminimally coupled scalar field models has been discussed.

The amended gravitational theories represent a generalization of Einstein's gravity, where some combination of curvature invariants replaces or is added into the classical Einstein-Hilbert action formed by the Ricci scalar term R . Thus, in this frame, the early- and the late-time acceleration may be caused by the fact that some dominant terms of gravitational action become essential at high or small curvatures. In addition, some other related problem of cosmological constant could be solved. The study of the full evolutionary cosmic picture, since the Big Bang epoch to its final providence, has become extensive among scientists. To end, many studies have been done on different strategies. These studies are classified into two categories: modifications of the matter sector of Einstein-Hilbert Lagrangian density and the extension of the gravitational framework of GR by adding some terms describing the DE source. The cosmological constant [17], Chaplygin gas matter with its different modified versions [18], and scalar field models like quintessence [19, 20] are some important examples of the first category. The second category is the modified theories of gravity [21]. Some examples of such modified theories are $f(R)$ gravity (the Ricci scalar of the Einstein-Hilbert action functional is replaced by a generic function $f(R)$) [22, 23], Gauss-Bonnet gravity (including Gauss-Bonnet invariant term) [24, 25], $f(T)$ theory (based on the torsion tensor and its corresponding scalar) [26–32], $f(R, T)$ gravity (where T is the trace of the energy-momentum tensor) [33–37], the scalar-tensor theories (based on both scalar and tensor fields) [38–40], and so forth.

The rich class of gravitational modifications is the one involving torsion. It is interesting to mention that teleparallel equivalent of general relativity (TEGR) has been constructed by Einstein by including torsionless Levi-Civita connection instead of curvatureless Weitzenböck connection and the vierbein as the fundamental ingredient for the theory [41–44]. Consequently, the corresponding formulation replaces the Ricci tensor and Ricci scalar by the torsion tensor and torsion scalar, respectively. Further, its modified form has been proposed and discussed by numerous authors like [26, 27, 45, 46].

The energy conditions have been widespread to investigate various issues in GR and cosmology [47–51]. Energy conditions have been explored in different modified theories to constrain the free variables like scalar-tensor theory [52], modified Gauss-Bonnet gravity [53, 54], $f(R)$ gravity [55], $f(T)$ gravity where T is the torsion scalar [56], teleparallel gravity with nonminimally coupled scalar field and $f(T)$ with nonminimally coupled scalar field models [16], $f(R)$ gravity with nonminimal coupling to matter [57, 58], $f(R, L_m)$ gravity [59], $f(R, T)$ gravity, and $f(R, T, R_{\mu\nu}T^{\mu\nu})$ [34–37].

The formulation of this paper is as follows. In Section 2, preliminaries of $f(T)$ gravitational theories are given. In Section 3, the tetrad field that reproduces FRW space-time is given and the three flat, closed, and open models are provided. In Section 4, we give a brief description of the energy conditions in the context of GR and also their extension to modified frameworks of gravity. In Section 5, we analyze the inequalities of the energy condition by studying the three models presented in Section 3. The final section presents the main results.

2. Preliminaries of $f(T)$

In a space-time having absolute parallelism, the tetrad field h_i^μ [60] defines the nonsymmetric affine connection:

$$\Gamma_{\nu\lambda}^\mu \stackrel{\text{def.}}{=} h_i^\mu h_{\nu,\lambda}^i, \quad (1)$$

where $h_{\nu,\lambda}^i = \partial_\lambda h_\nu^i$ (we use the Greek indices μ, ν, \dots for local holonomic space-time coordinates and the Latin indices i, j, \dots to label (co)frame components).

The curvature tensor defined by $\Gamma_{\nu\lambda}^\mu$, given by (1), is identically vanishing. The metric tensor $g_{\mu\nu}$, is defined by

$$g_{\mu\nu} \stackrel{\text{def.}}{=} \eta_{ij} h_\mu^i h_\nu^j, \quad (2)$$

with $\eta_{\mu\nu} = (+1, -1, -1, -1)$ denoting the metric of Minkowski space-time. The torsion components and the contortion are given, respectively, as

$$T_{\nu\lambda}^\mu \stackrel{\text{def.}}{=} \Gamma_{\lambda\nu}^\mu - \Gamma_{\nu\lambda}^\mu = h_i^\mu (\partial_\nu h_\lambda^i - \partial_\lambda h_\nu^i), \quad (3)$$

$$K_\lambda^{\mu\nu} \stackrel{\text{def.}}{=} -\frac{1}{2} (T_\lambda^{\mu\nu} - T_\lambda^{\nu\mu} - T_\lambda^{\mu\nu}),$$

where the contortion equals the difference between Weitzenböck and Levi-Civita connection; that is, $K_{\nu\lambda}^\mu = \Gamma_{\nu\lambda}^\mu - \{\}_{\nu\lambda}^\mu$.

The tensor $S_\mu^{\nu\lambda}$ is defined as

$$S_\mu^{\nu\lambda} \stackrel{\text{def.}}{=} \frac{1}{2} (K_\mu^{\nu\lambda} + \delta_\mu^\nu T_\beta^{\beta\lambda} - \delta_\mu^\lambda T_\beta^{\beta\nu}). \quad (4)$$

The torsion scalar is defined as

$$T \stackrel{\text{def.}}{=} T_{\nu\lambda}^\mu S_\mu^{\nu\lambda}. \quad (5)$$

The action of $f(T)$ theory is given by [61]

$$\mathcal{L}(h_\mu^a) = \frac{1}{16\pi} \int d^4x h f(T), \quad (6)$$

$$\text{where } h = \sqrt{-g} = \det(h_\mu^a).$$

We are going to use the units in which $G = c = 1$. The action of $f(T)$ theory is a function of the tetrad fields h_μ^a . By putting the variation of the function $f(T)$ with respect to the tetrad fields h_μ^a to be vanishing, one can obtain the following equations of motion:

$$S_\mu^{\rho\nu} T_{,\rho} f(T)_{TT} + [h^{-1} h_\mu^a \partial_\rho (h h_\alpha^a S_\alpha^{\rho\nu}) - T_{\lambda\mu}^\alpha S_\alpha^{\nu\lambda}] f(T)_T - \frac{1}{4} \delta_\mu^\nu f(T) = -4\pi \Theta_\mu^\nu, \quad (7)$$

where

$$\begin{aligned} T_{,\rho} &= \frac{\partial T}{\partial x^\rho}, \\ f(T)_T &= \frac{\partial f(T)}{\partial T}, \\ f(T)_{TT} &= \frac{\partial^2 f(T)}{\partial T^2}, \end{aligned} \quad (8)$$

and Θ_μ^ν is the energy-momentum tensor. $f(T)$ gravitational theories have been subject of many concerns and it had been indicated that the Lagrangian and the equations of motion of those theories are not variant under local Lorentz transformations [62]. The reasons why setting back local Lorentz symmetry in $f(T)$ theories cannot upgrade to credible dynamics had been explained, even if one relinquishes teleparallelism [63]. The equations of motion of $f(T)$ theories have been stated to differ from those of $f(R)$ theories [64–74], because they are of second order instead of fourth order. Such property has been believed as an indicator, which shows that the theory might be of much interest than this of GR. Because

$$(h_i^\mu) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{L_1 \sin \theta \cos \phi}{4a(t)} & \frac{L_2 \cos \theta \cos \phi - 4r\sqrt{k} \sin \phi}{4ra(t)} & -\frac{L_2 \sin \phi + 4r\sqrt{k} \cos \theta \cos \phi}{4ra(t) \sin \theta} \\ 0 & \frac{L_1 \sin \theta \sin \phi}{4a(t)} & \frac{L_2 \cos \theta \sin \phi + 4r\sqrt{k} \cos \phi}{4ra(t)} & \frac{L_2 \cos \phi - 4r\sqrt{k} \cos \theta \sin \phi}{4ra(t) \sin \theta} \\ 0 & \frac{L_1 \cos \theta}{4a(t)} & \frac{-L_2 \sin \theta}{4ra(t)} & \frac{\sqrt{k}}{a(t)} \end{pmatrix}, \quad (9)$$

of the nonlocality of these theories, $f(T)$ seems to contain more degrees of freedom.

3. Cosmological Modifications of $f(T)$

Recent cosmic observations show that our universe is expanding with an acceleration. We are going to review some derived solutions of FRW in the frame of $f(T)$. In these cosmological applications the universe is taken as homogeneous and isotropic in space, which directly gives rise to the tetrad given by Robertson [75]. This tetrad field has the same metric as FRW metric and can be written in spherical polar coordinate (t, r, θ, ϕ) as

where $a(t)$ is the *scale factor*, $L_1 = 4 + kr^2$, and $L_2 = 4 - kr^2$. Substituting from vierbein (9) into (5), one can get the field equations (7) as

$$4\pi\rho = \frac{R^2 f + 12\dot{R}^2 f_T}{4R^2}, \quad (10)$$

$$4\pi p = \frac{4k(R^2 f_T + 12\dot{R}^2 f_{TT}) - R^4 f - 4R^2(R\ddot{R} + 2\dot{R}^2) f_T + 48\dot{R}^2(R\ddot{R} - \dot{R}^2) f_{TT}}{4R^4}, \quad (11)$$

with ρ and p being the energy density and pressure of the fluid [76]. Equations (10) and (11) are the modified Friedmann equations in the $f(T)$ gravity in its generalized form. The continuity equation of this model has the form

$$\dot{\rho} + 3H[\rho + p] = 0. \quad (12)$$

The torsion contribution of energy density and pressure, ρ_T, p_T , have the form [76]

$$\rho_T = \frac{1}{8\pi} \left(3H^2 - \frac{f}{2} - 6H^2 f_T + \frac{3k}{R^2} \right). \quad (13)$$

$$p_T = \frac{-1}{8\pi} \left[\frac{k}{R^2} (1 + 2f_T + 24H^2 f_{TT}) + 2\dot{H} + 3H^2 - \frac{f}{2} - 2(\dot{H} + 3H^2) f_T + 24\dot{H}H^2 f_{TT} \right]. \quad (14)$$

In the case of spatially flat FRW universe, $k = 0$, the solution of continuity equation (12) gave the scale factor in the form [76]

$$a(t) = \frac{c_1}{2} e^{(t+c_2)/c_1}. \quad (15)$$

The matter density (10) and pressure (11) according to the scale factor (15) read

$$\rho = \frac{c_3 + c_4 \sqrt{e}}{16\pi} = -p. \quad (16)$$

The torsion density (13) and pressure (14) due to the torsion effect are

$$\rho_T = \frac{6 - (c_3 + c_4 \sqrt{e}) c_1^2}{16\pi c_1^2} = -p_T. \quad (17)$$

In the case of the closed FRW universe, $k = +1$, the solution of the continuity equation gave the scale factor in the form [76]

$$a(t) = -c'_1 \sinh\left(\frac{t+c'_2}{c'_1}\right). \quad (18)$$

The matter density (10) and pressure (11) in this case have the form

$$\rho = \frac{1}{16\pi} \left[c'_3 + c'_4 e^{(1/2)\tanh^2((t+c'_2)/c'_1)} \right] = -p, \quad (19)$$

while the torsion density (13) and pressure (14) read

$$\begin{aligned} \rho_T = \frac{-1}{16\pi} \left[c'_3 + c'_4 e^{(1/2)\tanh^2((t+c'_2)/c'_1)} \right. \\ \left. + \frac{6}{c_1'^2} \left(1 - 2 \coth^2\left(\frac{t+c'_2}{c'_1}\right) \right) \right] = -p_T. \end{aligned} \quad (20)$$

In the case of the open FRW universe, $k = -1$, the solution of the continuity equation (12) gave the scale factor in the form [76]

$$a(t) = c''_1 \cosh\left(\frac{t+c''_2}{c''_1}\right). \quad (21)$$

The matter density (10) and pressure (11) are

$$\rho = \frac{1}{16\pi} \left[c''_3 + c''_4 e^{(\coth^2((t+c''_2)/c''_1))/2} \right] = -p. \quad (22)$$

Also, the torsion density (13) for the open universe reads

$$\begin{aligned} \rho_T = \frac{-1}{16\pi} \left[c''_3 + c''_4 e^{(1/2)\coth^2((t+c''_2)/c''_1)} \right. \\ \left. + \frac{6}{c_1''^2} \left(1 - 2 \tanh^2\left(\frac{t+c''_2}{c''_1}\right) \right) \right], \end{aligned} \quad (23)$$

and the torsion pressure (14) becomes

$$\begin{aligned} p_T = \frac{1}{16\pi} \left[c''_3 + c''_4 e^{(1/2)\coth^2((t+c''_2)/c''_1)} \right. \\ \left. - \frac{2}{c_1''^2} \left(1 + 2 \tanh^2\left(\frac{t+c''_2}{c''_1}\right) \right) \right], \end{aligned} \quad (24)$$

where $c_1, c_2, c_3, c_4, c'_1, c'_2, c'_3, c'_4, c''_1, c''_2, c''_3, c''_4$ are constants of integration. To study the energy conditions of the above models, we are going to give a brief review of the energy conditions.

4. Brief Review of Energy Conditions

Energy conditions are of great interest, in order to understand various cosmological geometries and some general results associated with the strong gravitational fields. In GR, there are four explicit forms of energy conditions, namely,

the strong energy condition (SEC), null energy condition (NEC), dominant energy condition (DEC), and weak energy conditions (WEC) [77, 78]. The SEC and NEC arise from the fundamental characteristic of gravitational force that it is attractive along with a well-known geometrical result describing the dynamics of nearby matter bits known as Raychaudhuri equation. Raychaudhuri equation specifies the temporal evolution of the expansion of scalar θ in terms of some tensorial quantities like Ricci tensor, shear tensor $\sigma^{\mu\nu}$, and rotation $\omega^{\mu\nu}$ for both the time- and light-like curves. In a mathematical form these relations are given by

$$\begin{aligned} \frac{d\theta}{d\tau} &= -\frac{1}{3}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}u^\mu u^\nu, \\ \frac{d\theta}{d\tau} &= -\frac{1}{3}\theta^2 - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}k^\mu k^\nu. \end{aligned} \quad (25)$$

Due to attractive nature of gravity, geodesics become closer to each other satisfying $d\theta/d\tau < 0$ and hence yield converging time- and light-like congruences. We can ignore the quadratic terms by making the assumptions that there are infinitesimal distortions in geodesics (time or null); that is, $\omega_{\mu\nu} = 0$, to simplify the resulting inequalities. Consequently, integration of the simplified Raychaudhuri equations gives $\theta = -\tau R_{\mu\nu}u^\mu u^\nu = -\tau R_{\mu\nu}k^\mu k^\nu$ for time-like and null geodesics, respectively. Using $d\theta/d\tau < 0$, this can also be rearranged to

$$\begin{aligned} R_{\mu\nu}u^\mu u^\nu &\geq 0, \\ R_{\mu\nu}k^\mu k^\nu &\geq 0. \end{aligned} \quad (26)$$

Ricci tensor in terms of energy-momentum tensor and its trace has the form

$$R_{\mu\nu} = \Theta_{\mu\nu} - \frac{\Theta}{2} g_{\mu\nu}. \quad (27)$$

Therefore the inequalities of energy conditions take the following forms:

$$\begin{aligned} R_{\mu\nu}u^\mu u^\nu &= \left(\Theta_{\mu\nu} - \frac{\Theta}{2} g_{\mu\nu} \right) u^\mu u^\nu \geq 0, \\ R_{\mu\nu}k^\mu k^\nu &= \left(\Theta_{\mu\nu} - \frac{\Theta}{2} g_{\mu\nu} \right) k^\mu k^\nu \geq 0. \end{aligned} \quad (28)$$

In the case of perfect fluid matter, the SEC and NEC given by (28) impose the following constraints, $\rho + 3p \geq 0$ and $\rho + p \geq 0$, to be satisfied, while the WEC and DEC require these conditions, $\rho \geq 0$ and $\rho \pm p \geq 0$, respectively, for consistency purposes.

The concept of energy conditions can be extended to the case of modified theories of gravity using Raychaudhuri equation which is pure geometrical relation. Thus, its interesting particulars like focussing on geodesic congruences along with the attractive nature of gravity can be used to formulate these conditions in any modified gravitational framework [79]. In such cases, we take the total matter contents of the universe behaving as perfect fluid and, consequently, the respective conditions can be defined by simply replacing

the energy density and pressure, respectively, by an effective energy density and effective pressure as follows:

$$\begin{aligned}
 \text{NEC: } \rho + p &\geq 0, \\
 \text{SEC: } \rho + p &\geq 0, \quad \rho + 3p \geq 0, \\
 \text{WEC: } \rho &\geq 0, \quad \rho + p \geq 0, \\
 \text{DEC: } \rho &\geq 0, \quad \rho \pm 3p \geq 0.
 \end{aligned} \tag{29}$$

It is also interesting to mention here that the violation of these energy conditions ensures the existence of the ghost instabilities (some interesting feature of modified gravity that supports the cosmic acceleration due to DE).

5. Energy Conditions of Built-In Inflation Models

Now we are going to apply the above procedure of the energy conditions to the derived three models of Section 3. For the flat case the above energy conditions, that is, NEC, WEC and, $\text{DEC}(\rho - 3P)$, are satisfied provided that c_3 and c_4 have positive values; however, SEC and $\text{DEC}(\rho + 3P)$ are not satisfied for these values. For the SEC and DEC to be satisfied for the flat case c_3 and c_4 must have negative values. Same discussion can be applied to energy density and pressure due to torsion effect so that NEC, WEC, and $\text{DEC}(\rho - 3P)$ are satisfied provided that c_3 and c_4 be negative, while SEC and $\text{DEC}(\rho + 3P)$ are satisfied provided that c_3 and c_4 be positive. Let us use some appropriate initial conditions that imposed on the scale factor $a(t)$, Hubble function $H(t)$, energy density $\rho(t)$, and so forth, at an initial moment $t_0 = t_{pl} = 10^{-44}$ s, corresponding to the beginning of the inflationary period to determine the constants of integration appearing in the models presented in Section 3. For this epoch we use $a(t_0) = a_0 = 10^{-13}$, $H(t_0) = H_0 = 10^{10} \text{ s}^{-1}$, and $\rho(t_0) = \rho_0 = 10^{-30} \text{ s}^{-2}$. Using these initial conditions we get for the flat model the value of the constants of integration in the following form: $c_1 = 10^{-10}$, $c_2 = 10^{-20}$, $c_3 = -10^{-10}c_4 + 10^{-22}$, and c_4 is an arbitrary. These values of constants satisfy the energy conditions of the flat model provided that $c_4 \geq 10^{-1}$ in order that NEC, WEC, and $\text{DEC}(\rho - 3P)$ are satisfied while $c_4 < 10^{-1}$ so that the SEC and $\text{DEC}(\rho + 3P)$ energy conditions be satisfied. The same discussion can be used to energy density and pressure due to torsion effect such that NEC, WEC, and $\text{DEC}(\rho - 3P)$ are satisfied whatever the value of c_3 . The SEC and $\text{DEC}(\rho + 3P)$ energy conditions due to torsion effect will not be satisfied using the above constants derived from the initial values.

We cannot use the above initial conditions (inflation epoch) to determine the values of the four constants which appeared in the closed and open models. This means that those two models are not consistent with inflation period. For the closed model the sign of the three constants c'_1 , c'_2 , and c'_3 has no effect on the density and pressure in the late epoch. However, for positive sign of constant c'_4 the NEC, WEC, and $\text{DEC}(\rho - 3P)$ are satisfied as clear from Figures 1 and 2 but for negative values of c'_4 the SEC and $\text{DEC}(\rho + P)$ are satisfied

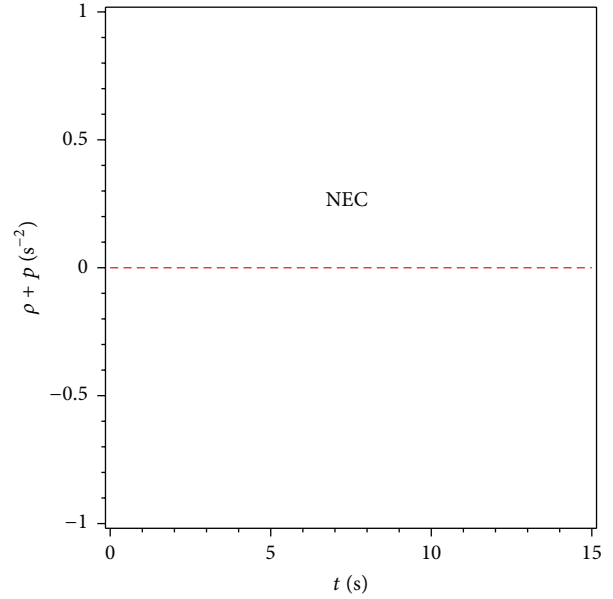


FIGURE 1: The evolution of NEC for closed model. Here the sign of the four constants c'_1 , c'_2 , c'_3 , and c'_4 has no effect on the evolution of NEC.

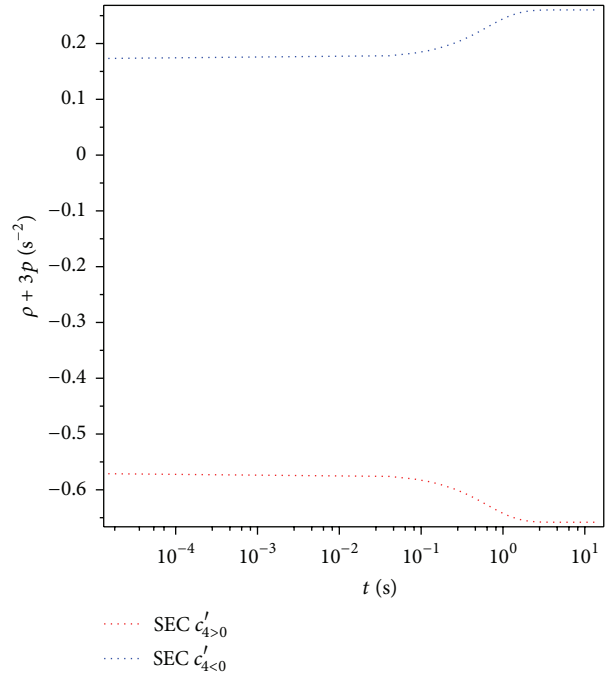


FIGURE 2: The evolution of SEC for closed model. Here the sign of the three constants c'_1 , c'_2 , and c'_3 has no effect on the evolution of SEC; however, c'_4 must have negative value to satisfy the SEC.

as clear from Figures 3 and 4. The same discussion can be applied for open model as clear from Figures 5, 6, 7, and 8 (the unit of the time used in all figures is in second and the unit of the energy density ρ is in second^{-2}).

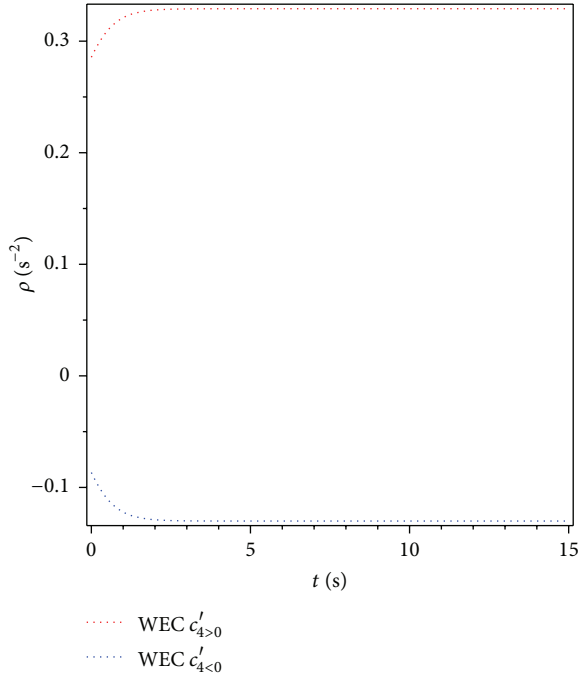


FIGURE 3: The evolution of the WEC for closed model. Here the sign of the three constants c'_1 , c'_2 , and c'_3 has no effect on the evolution of WEC; however, c'_4 must have positive value to satisfy the WEC.

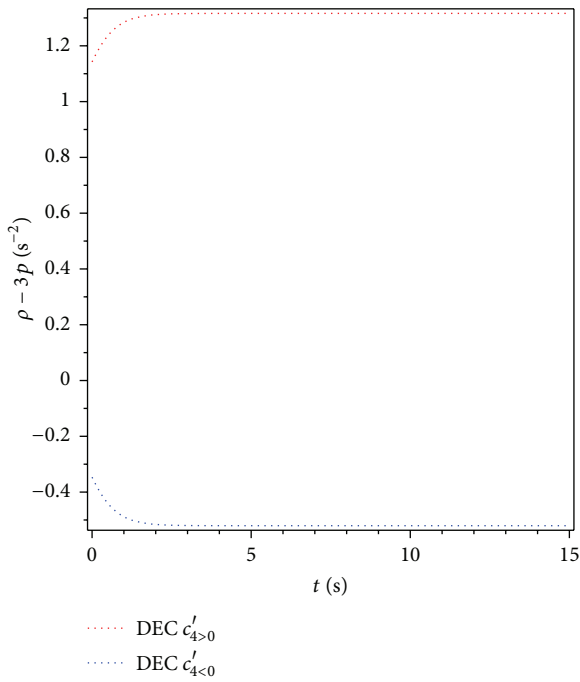


FIGURE 4: The evolution of DEC for closed model. Here the sign of the three constants c'_1 , c'_2 , and c'_3 has no effect on the evolution of DEC; however, c'_4 must have positive value to satisfy the SEC.

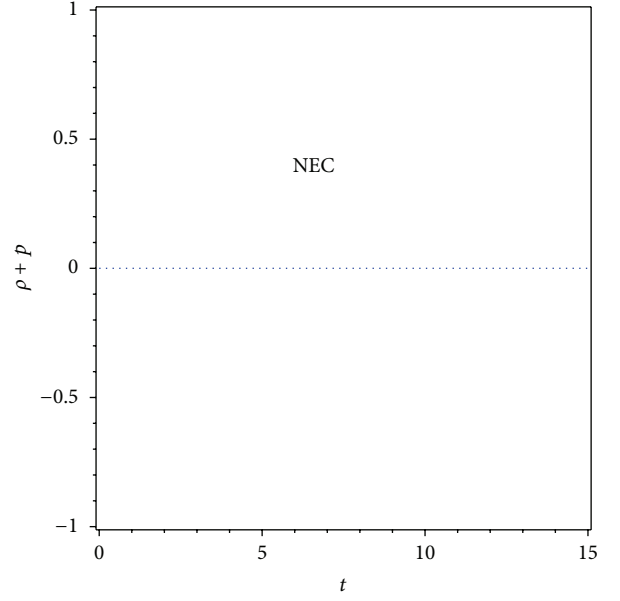


FIGURE 5: The evolution of NEC for open model. Here the sign of the four constants c'_1 , c'_2 , c'_3 , and c'_4 has no effect on the evolution of NEC.

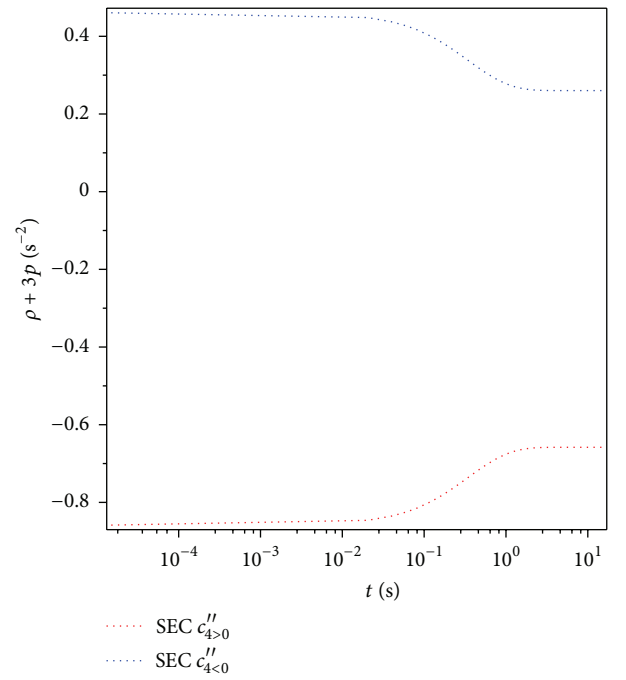


FIGURE 6: The evolution of SEC for open model. Here the sign of the three constants c'_1 , c'_2 , and c'_3 has no effect on the evolution of SEC; however, c'_4 must have negative value to satisfy the SEC.

6. Discussions

Recent observational data suggest that our universe is accelerating. Among the possible explanations for this phenomenon are modifications of gravitational theory [64]. Modification of GR seems to be quite attractive possibility to resolve the

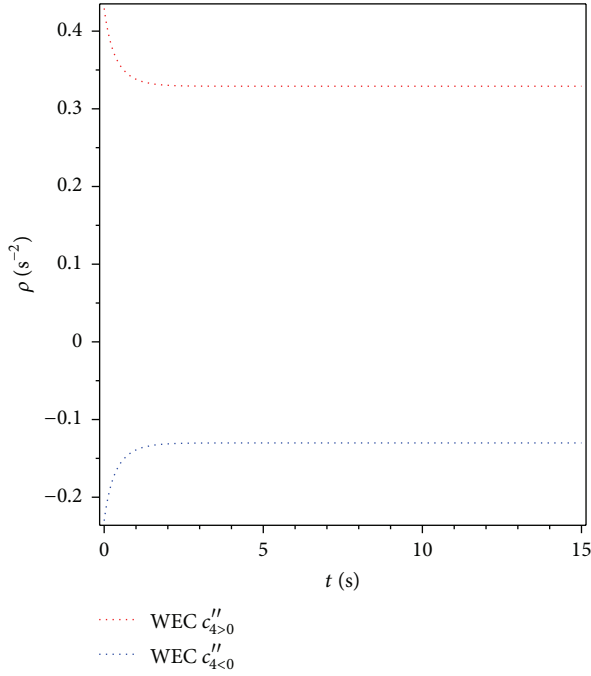


FIGURE 7: The evolution of the WEC for open model. Here the sign of the three constants c'_1 , c'_2 , and c'_3 has no effect on the evolution of WEC; however, c'_4 must have positive value to satisfy the WEC.

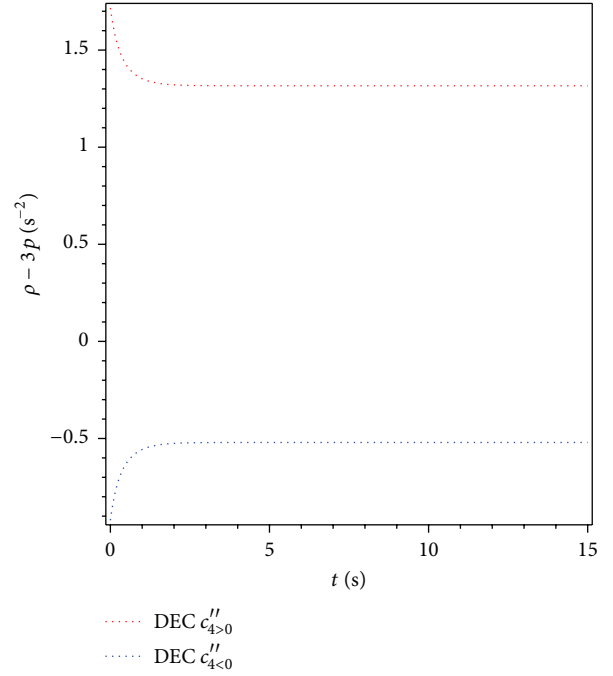


FIGURE 8: The evolution of DEC for open model. Here the sign of the three constants c'_1 , c'_2 , and c'_3 has no effect on the evolution of DEC; however, c'_4 must have positive value to satisfy the SEC.

above-mentioned problem. Most recentlyTEGR has been generalized to $f(T)$ theory, a theory of modified gravity formed in the same spirit as generalizing GR to $f(R)$ gravity [80, 81]. A main merit of $f(T)$ gravity theory is that its gravitational field equation is of second order, the same as GR, while it is of fourth order in metric $f(R)$ gravity. This merit makes the analysis of the cosmological expansion of the universe in $f(T)$ gravity much easier than in $f(R)$ gravity. $f(T)$ gravity has gained significant attention in the literature with promising cosmological implications [56, 82–87]. In such theories, one can develop different cosmological models depending on the choice of function $f(T)$. Different forms of the Lagrangian raise the question of how to constrain such theory on physical grounds.

In this study, we have developed constraints on $f(T)$ gravitational theory by examining the respective energy conditions for flat, open, and closed FRW models [76]. We have analyzed the role of these models parameters graphically in Figures 1–8 by exploring the evolution of NEC, WEC, SEC, and DEC. The obtained results can be summarized as follows:

- (i) The form of the *flat* universe model has an inflationary cosmological model $a(t) \propto e^{Ht}$, $H = \text{const}$. However, the universe compositions have no evolution where the matter density is constant during the expansion which gives a steady state universe.

This model satisfies the energy conditions, NEC, WEC, and $\text{DEC}(\rho - 3P)$, provided that constant $c_3 \geq 10^{-1}$ has positive value. The SEC and $\text{DEC}(\rho + 3P)$ are satisfied for this model when constant $c_3 < 10^{-1}$. Applying the same analysis of energy conditions for

the density and pressure due to torsion effect we show that NEC, WEC, and $\text{DEC}(\rho - 3P)$ are satisfied whatever the value of c_3 . The SEC and $\text{DEC}(\rho + 3P)$ energy conditions due to torsion effect will not be satisfied using the above constants derived from the initial values.

- (ii) In the *closed* universe $k = +1$, we cannot determine the values of the constants using the initial values of the inflation epoch. We show that the energy conditions are satisfied according to the value of constant c'_4 in the late universe. These conditions are shown in Figures 1–4 which show that NEC, WEC, and $\text{DEC}(\rho - 3P)$ are satisfied for positive value of the constant c'_4 while SEC and $\text{DEC}(\rho + 3P)$ are satisfied when constant c'_4 takes a negative value. Same analysis can be applied for the density and pressure due to the effect of torsion and we show that the energy conditions NEC, WEC, and $\text{DEC}(\rho - 3P)$ are satisfied for negative value of constant c'_4 and SEC and $\text{DEC}(\rho + 3P)$ are satisfied for positive value of constant c'_4 .
- (iii) The scale factor of the closed model is given by a hypergeometric function as shown by (18) and the Hubble parameter is not a constant; constant Hubble parameter is a necessary condition for early universe. Therefore, the closed model is not suitable to describe the early universe.
- (iv) The same discussion can be applied for the open universe and the behaviors of energy condition are shown in Figures 5–8.

- (v) Finally as Figures 1 and 5 show $\rho + P = 0$ because all the derived models contain dark energy.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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