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Research Article

Study of Nonleptonic $B_q^* \to D_q V$ and $P_q D^*$ Weak Decays

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Motivated by the powerful capability of measurement for the b hadron decays at LHC and SuperKEKB/Belle-II, the nonleptonic $\overline{B}^* \to D\overline{D}^*$, $D\rho^-$, DK^{*-}, πD^*, and KD^* decays are studied. With the amplitudes calculated with factorization approach and the form factors evaluated with the BSW model, branching fractions and polarization fractions are firstly presented. Numerically, the CKM-favored $\overline{B}_q^* \to D_q D_s^{*-}$ and $D_q\rho^-$ decays have branching fractions $\sim 10^{-8}$, which should be sought for with priority and firstly observed by LHC and Belle-II. The $\overline{B}_q^* \to D_q K^*$ and $D_q\rho$ decays are dominated by the longitudinal polarization states, while the parallel polarization fractions of $\overline{B}_q^* \to D_q\overline{D}^*$ decays are comparable with the longitudinal ones; numerically, $f_\parallel + f_L \simeq 95\%$ and $f_L : f_\parallel \simeq 5 : 4$. Some comparisons between $\overline{B}_q^{*0} \to D_q V$ and their corresponding $\overline{B}_q^0 \to D_q^* V$ decays are performed, and the relation $f_{L,\parallel}(\overline{B}^{*0} \to DV) \simeq f_{L,\parallel}(\overline{B}^0 \to D^* V)$ is found. With the implication of $SU(3)$ flavor symmetry, the ratios R_{du} and R_{ds} are discussed and suggested to be verified experimentally.

1. Introduction

The b physics plays an important role in testing the flavor dynamics of Standard Model (SM), exploring the source of CP violation, searching the indirect hints of new physics, investigating the underling mechanisms of QCD, and so forth and thus attracts much experimental and theoretical attention. With the successful performance of BABAR, Belle, CDF, and D0 in the past years, many $B_{u,d,s}$ meson decays have been well measured. Thanks to the ongoing LHCb experiment [1] at LHC and forthcoming Belle-II experiment [2] at SuperKEKB, experimental analysis of B meson decays is entering a new frontier of precision. By then, besides $B_{u,d,s}$ mesons, the rare decays of some other b-flavored hadrons are hopefully to be observed, which may provide much more extensive space for b physics.

The excited states $B_{u,d,s}^*$ with quantum number of $n^{2s+1}L_J = 1^3S_1$ and $J^P = 1^-$ (n, L, s, J, and P are the quantum numbers of radial, orbital, spin, total angular momenta, and parity, resp.), which will be referred to as B^* in this paper, had been observed by CLEO, Belle, LHCb, and so on [3]. However, except for their masses, there is no more experimental information due to the fact that the production of B^* mesons is mainly through $\Upsilon(5S)$ decays at e^+e^- colliders and the integrated luminosity is not high enough for probing the B^* rare decays. Moreover, B^* decays are dominated by the radiative processes $B^* \to B\gamma$, and the other decay modes are too rare to be measured easily. Fortunately, with annual integrated luminosity ~ 13 ab^{-1} [2] and the cross section of $\Upsilon(5S)$ production in e^+e^- collisions $\sigma(e^+e^- \to \Upsilon(5S)) = (0.301 \pm 0.002 \pm 0.039)$ nb [4], it is expected that about $4 \times 10^9 \Upsilon(5S)$ samples could be produced per year at the forthcoming super-B factory SuperKEKB/Belle-II, which implies that the B^* rare decays with branching fractions $\gtrsim 10^{-9}$ are possible to be observed. Besides, due to the much larger production cross section of pp collisions, experiments at LHC [5, 6] also possibly provide some experimental information for B^* decays.

With the rapid development of experiment, accordingly, the theoretical evaluations for B^* weak decays are urgently needed and worthful. Nonleptonic B^* weak decays allow one to overconstrain parameters obtained from B meson decay, test various models, and improve our understanding on the strong interactions and the mechanism responsible for heavy meson weak decay. The observation of an anomalous production rate of B^* weak decays would be a hint of possible new physics beyond SM. In addition, the B^* weak decay provides one unique opportunity of observing the weak decay of a vector meson, where polarization effects can be used as tests of the underlying structure and dynamics of hadrons. To our knowledge, few previous theoretical works come close to studying B^* weak decays. Compared with the $B^* \to PP$ decays, which are suppressed dynamically by the orbital angular momentum of final states, $B^* \to PV$ decays are expected to have much larger branching fractions and hence are generally much easier to be measured. So, in this paper, we will estimate the observables of nonleptonic two-body $B^* \to PV$ weak decay to offer a ready reference.

Our paper is organized as follows. In Section 2, after a brief review of the effective Hamiltonian and factorization approach, the explicit amplitudes of $B^*_{u,d,s} \to D^{(*)}_{u,d,s}M$ decays are calculated. In Section 3, the numerical results and discussions are presented. Finally, we summarize in Section 4.

2. Theoretical Framework

Within SM, the effective Hamiltonian responsible for nonleptonic B^* weak decay is [7]

$$\mathscr{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q,q'=u,c} \left[V_{qb} V^*_{q'p} \sum_{i=1}^{2} C_i(\mu) O_i(\mu) + V_{qb} V^*_{qp} \cdot \sum_{i=3}^{10} C_i(\mu) O_i(\mu) \right] + \text{h.c.}, \tag{1}$$

where $p = d$ or s, $V_{qb}V^*_{q'p}$ is the product of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, and C_i are Wilson coefficients, which describe the short-distance contributions and are calculated perturbatively; the explicit expressions of local four-quark operators O_i are

$$\begin{aligned}
O_1 &= (\overline{q}_\alpha b_\alpha)_{V-A} (\overline{p}_\beta q'_\beta)_{V-A}, \\
O_2 &= (\overline{q}_\alpha b_\beta)_{V-A} (\overline{p}_\alpha q'_\beta)_{V-A}, \\
O_3 &= (\overline{p}_\alpha b_\alpha)_{V-A} \sum_{p'} (\overline{p}'_\beta p'_\beta)_{V-A}, \\
O_4 &= (\overline{p}_\alpha b_\beta)_{V-A} \sum_{p'} (\overline{p}'_\beta p'_\alpha)_{V-A}, \\
O_5 &= (\overline{p}_\alpha b_\alpha)_{V-A} \sum_{p'} (\overline{p}'_\beta p'_\beta)_{V+A}, \\
O_6 &= (\overline{p}_\alpha b_\beta)_{V-A} \sum_{p'} (\overline{p}'_\beta p'_\alpha)_{V+A}, \\
O_7 &= (\overline{p}_\alpha b_\alpha)_{V-A} \sum_{p'} \frac{3}{2} Q_{p'} (\overline{p}'_\beta p'_\beta)_{V+A}, \\
O_8 &= (\overline{p}_\alpha b_\beta)_{V-A} \sum_{p'} \frac{3}{2} Q_{p'} (\overline{p}'_\beta p'_\alpha)_{V+A}, \\
O_9 &= (\overline{p}_\alpha b_\alpha)_{V-A} \sum_{p'} \frac{3}{2} Q_{p'} (\overline{p}'_\beta p'_\beta)_{V-A}, \\
O_{10} &= (\overline{p}_\alpha b_\beta)_{V-A} \sum_{p'} \frac{3}{2} Q_{p'} (\overline{p}'_\beta p'_\alpha)_{V-A},
\end{aligned} \tag{2}$$

where $(\overline{q}_1 q_2)_{V\pm A} = \overline{q}_1 \gamma_\mu (1 \pm \gamma_5) q_2$, α and β are color indices, $Q_{p'}$ is the electric charge of the quark p' in the unit of $|e|$, and p' denotes the active quark at the scale $\mu \sim \mathcal{O}(m_b)$; *that is*, $p' = u, d, c, s$, and b.

To obtain the decay amplitudes, the remaining and also the most intricate work is how to calculate hadronic matrix elements $\langle PV|O_i|B^*\rangle$. With the factorization approach [8–11] based on the color transparency mechanism [12, 13], in principle, the hadronic matrix element could be factorized as

$$\begin{aligned}
\langle PV | O_i | B^* \rangle &= a \langle P | J_\mu | B^* \rangle \langle V | J^\mu | 0 \rangle \\
&+ b \langle V | J_\mu | B^* \rangle \langle P | J^\mu | 0 \rangle \\
&+ c \langle PV | J_\mu | 0 \rangle \langle 0 | J^\mu | B^* \rangle .
\end{aligned} \tag{3}$$

Due to the unnecessary complexity of hadronic matrix element $\langle V|J_\mu|B^*\rangle$ and power suppression of annihilation contributions, we only consider one simple scenario where pseudoscalar meson picks up the spectator quark in B^* meson; *that is*, $a = 1$, $b = 0$, and $c = 0$ in (3) for the moment. Two current matrix elements can be further parameterized by decay constants and transition form factors:

$$\begin{aligned}
&\langle V(p,\epsilon) | \overline{q}_1 \gamma_\mu q_2 | 0 \rangle = f_V m_V \epsilon^*_\mu, \\
&\langle P(p_P) | \overline{q} \gamma_\mu b | \overline{B}^*(p_{B^*}, \eta) \rangle = \frac{2V(q^2)}{m_{B^*} + m_P} \cdot \varepsilon_{\mu\nu\rho\sigma} \eta^\nu p_P^\rho p_{B^*}^\sigma, \\
&\langle P(p_P) | \overline{q} \gamma^\mu \gamma_5 b | \overline{B}^*(p_{B^*}, \eta) \rangle = i2m_{B^*} A_0(q^2) \frac{\eta \cdot q}{q^2} q^\mu \\
&+ i(m_P + m_{B^*}) A_1(q^2) \left(\eta^\mu - \frac{\eta \cdot q}{q^2} q^\mu \right) + iA_2(q^2) \\
&\cdot \frac{\eta \cdot q}{m_P + m_{B^*}} \left[(p_{B^*} + p_P)^\mu - \frac{(m^2_{B^*} - m^2_P)}{q^2} q^\mu \right],
\end{aligned} \tag{4}$$

where ϵ and η are the polarization vector, f_V is the decay constant of vector meson, V and $A_{0,1,2}$ are transition form factors, $q = p_{B^*} - p_P$, and the sign convention $\epsilon^{0123} = 1$. Even though some improved approaches, such as the QCD factorization [14, 15], the perturbative QCD scheme [16, 17], and the soft-collinear effective theory [18–21], are presented to evaluate higher order QCD corrections and reduce the renormalization scale dependence, the naive factorization (NF) approximation is a useful tool of theoretical estimation. Because there is no available experimental measurement for now, the NF approach is good enough to give a preliminary analysis, and so it is adopted in our evaluation.

With the above definitions, the hadronic matrix elements considered here can be decomposed into three scalar invariant amplitudes $S_{1,2,3}$:

$$\left\langle PV \left| O_i \right| B^* \right\rangle = \epsilon^{*\mu} \eta^{\nu} \left\{ S_1 g_{\mu\nu} + S_2 \frac{\left(p_{B^*} + p_P\right)_\mu p_{V\nu}}{m_{B^*} m_V} + i S_3 \varepsilon_{\mu\nu\rho\sigma} \frac{2 p_{B^*}^{\rho} p_P^{\sigma}}{m_{B^*} m_V} \right\}, \tag{5}$$

where the amplitudes $S_{1,2,3}$ describe the s, d, and p wave contributions, respectively, and are explicitly written as

$$S_1 = -i f_V \left(m_{B^*} + m_P\right) m_V A_1, \tag{6}$$

$$S_2 = -i 2 f_V m_{B^*} m_V^2 \frac{A_2}{m_{B^*} + m_P}, \tag{7}$$

$$S_3 = +i 2 f_V m_{B^*} m_V^2 \frac{V}{m_{B^*} + m_P}. \tag{8}$$

Alternatively, one can choose the helicity amplitudes H^{λ} ($\lambda = 0, +, -$),

$$\begin{aligned} H_{PV}^{0} &= -S_1 x - S_2 \left(x^2 - 1\right), \\ H_{PV}^{\pm} &= -S_1 \pm S_3 \sqrt{x^2 - 1}, \end{aligned} \tag{9}$$

with

$$x \equiv \frac{p_{B^*} \cdot p_V}{m_{B^*} m_V} = \frac{m_{B^*}^2 - m_P^2 + m_V^2}{2 m_{B^*} m_V}. \tag{10}$$

Now, with the formulae given above and the effective coefficients α_i defined as

$$\begin{aligned} \alpha_1 &= C_1 + \frac{C_2}{N_c}, \\ \alpha_2 &= C_2 + \frac{C_1}{N_c}, \\ \alpha_4 &= C_4 + \frac{C_3}{N_c}, \\ \alpha_{4,EW} &= C_{10} + \frac{C_9}{N_c}, \end{aligned} \tag{11}$$

we present the amplitudes of nonleptonic two-body $\overline{B}^*$ decays as follows:

(i) For $\overline{B}_q^* \to D_q \overline{D}^*$ decays (the spectator $q = u$, d, and s),

$$\begin{aligned} &\mathcal{A}^{\lambda}\left(\overline{B}_q^* \longrightarrow D_q D^{*-}\right) \\ &\quad = H_{DD^{*-}}^{\lambda} \left[V_{cb} V_{cd}^* \left(\alpha_1 + \alpha_4 + \alpha_{4,EW}\right) + V_{ub} V_{ud}^* \left(\alpha_4 + \alpha_{4,EW}\right)\right], \\ &\mathcal{A}^{\lambda}\left(\overline{B}_q^* \longrightarrow D_q D_s^{*-}\right) \\ &\quad = H_{DD_s^{*-}}^{\lambda} \left[V_{cb} V_{cd}^* \left(\alpha_1 + \alpha_4 + \alpha_{4,EW}\right) + V_{ub} V_{ud}^* \left(\alpha_4 + \alpha_{4,EW}\right)\right]. \end{aligned} \tag{12}$$

(ii) For $\overline{B}_q^{*0} \to D_q V$ decays (the spectator $q = d$ and s and $V = \rho^-$ and K^{*-}),

$$\begin{aligned} \mathcal{A}^{\lambda}\left(\overline{B}_q^{*0} \longrightarrow D_q \rho^-\right) &= H_{D\rho^-}^{\lambda} V_{cb} V_{cd}^* \alpha_1, \\ \mathcal{A}^{\lambda}\left(\overline{B}_q^{*0} \longrightarrow D_q K^{*-}\right) &= H_{DK^{*-}}^{\lambda} V_{cb} V_{cs}^* \alpha_1. \end{aligned} \tag{13}$$

(iii) For $\overline{B}^* \to \pi D^*$ decays,

$$\begin{aligned} \mathcal{A}^{\lambda}\left(B^{*-} \longrightarrow \pi^- \overline{D}^{*0}\right) &= H_{\pi^- \overline{D}^{*0}}^{\lambda} V_{ub} V_{cd}^* \alpha_2, \\ \sqrt{2}\mathcal{A}^{\lambda}\left(B^{*-} \longrightarrow \pi^0 D^{*-}\right) &= H_{\pi^0 D^{*-}}^{\lambda} V_{ub} V_{cd}^* \alpha_1, \\ \sqrt{2}\mathcal{A}^{\lambda}\left(B^{*-} \longrightarrow \pi^0 D_s^{*-}\right) &= H_{\pi^0 D_s^{*-}}^{\lambda} V_{ub} V_{cs}^* \alpha_1, \\ -\sqrt{2}\mathcal{A}^{\lambda}\left(\overline{B}^{*0} \longrightarrow \pi^0 D^{*0}\right) &= H_{\pi^0 D^{*0}}^{\lambda} V_{cb} V_{ud}^* \alpha_2, \\ -\sqrt{2}\mathcal{A}^{\lambda}\left(\overline{B}^{*0} \longrightarrow \pi^0 \overline{D}^{*0}\right) &= H_{\pi^0 \overline{D}^{*0}}^{\lambda} V_{ub} V_{cd}^* \alpha_2, \\ \mathcal{A}^{\lambda}\left(\overline{B}^{*0} \longrightarrow \pi^+ D^{*-}\right) &= H_{\pi^+ D^{*-}}^{\lambda} V_{ub} V_{cd}^* \alpha_1, \\ \mathcal{A}^{\lambda}\left(\overline{B}^{*0} \longrightarrow \pi^+ D_s^{*-}\right) &= H_{\pi^+ D_s^{*-}}^{\lambda} V_{ub} V_{cs}^* \alpha_1. \end{aligned} \tag{14}$$

(iv) For $\overline{B}^* \to K D^*$ decays,

$$\begin{aligned} \mathcal{A}^{\lambda}\left(B^{*-} \longrightarrow K^- \overline{D}^{*0}\right) &= H_{K^- \overline{D}^{*0}}^{\lambda} V_{ub} V_{cs}^* \alpha_2, \\ \mathcal{A}^{\lambda}\left(\overline{B}^{*0} \longrightarrow \overline{K}^0 \overline{D}^{*0}\right) &= H_{\overline{K}^0 \overline{D}^{*0}}^{\lambda} V_{ub} V_{cs}^* \alpha_2, \\ \mathcal{A}^{\lambda}\left(\overline{B}^{*0} \longrightarrow \overline{K}^0 D^{*0}\right) &= H_{\overline{K}^0 D^{*0}}^{\lambda} V_{cb} V_{us}^* \alpha_2, \\ \mathcal{A}^{\lambda}\left(\overline{B}_s^{*0} \longrightarrow K^+ D^{*-}\right) &= H_{K^+ D^{*-}}^{\lambda} V_{ub} V_{cd}^* \alpha_1, \\ \mathcal{A}^{\lambda}\left(\overline{B}_s^{*0} \longrightarrow K^0 \overline{D}^{*0}\right) &= H_{K^0 \overline{D}^{*0}}^{\lambda} V_{ub} V_{cd}^* \alpha_2, \\ \mathcal{A}^{\lambda}\left(\overline{B}_s^{*0} \longrightarrow K^0 D^{*0}\right) &= H_{K^0 D^{*0}}^{\lambda} V_{cb} V_{ud}^* \alpha_2, \\ \mathcal{A}^{\lambda}\left(\overline{B}_s^{*0} \longrightarrow K^+ D_s^{*-}\right) &= H_{K^+ D_s^{*-}}^{\lambda} V_{ub} V_{cs}^* \alpha_1. \end{aligned} \tag{15}$$

In the rest frame of $\overline{B}^*$ meson, the branching fraction can be written as

$$\mathcal{B}\left(\overline{B}^* \longrightarrow PV\right) = \frac{1}{3}\frac{G_F^2}{2}\frac{1}{8\pi}\frac{p_c}{m_{B^*}^2\Gamma_{\text{tot}}(B^*)}\sum_\lambda\left|\mathcal{A}_\lambda\left(\overline{B}^* \longrightarrow PV\right)\right|^2, \quad (16)$$

where the momentum of final states is

$$p_c = \frac{\sqrt{\left[m_{B^*}^2-(m_P+m_V)^2\right]\left[m_{B^*}^2-(m_P-m_V)^2\right]}}{2m_{B^*}}. \quad (17)$$

The longitudinal, parallel, and perpendicular polarization fractions are defined as

$$f_{L,\parallel,\perp} = \frac{\left|\mathcal{A}_{0,\parallel,\perp}\right|^2}{\left|\mathcal{A}_0\right|^2+\left|\mathcal{A}_\parallel\right|^2+\left|\mathcal{A}_\perp\right|^2}, \quad (18)$$

where $\mathcal{A}_\parallel$ and $\mathcal{A}_\perp$ are parallel and perpendicular amplitudes:

$$\mathcal{A}_{\parallel,\perp} = \frac{1}{\sqrt{2}}\left(\mathcal{A}_- \pm \mathcal{A}_+\right). \quad (19)$$

3. Numerical Results and Discussion

Firstly, we would like to clarify the input parameters used in our numerical evaluations. For the CKM matrix elements, we adopt the Wolfenstein parameterization [22] and choose the four parameters A, λ, ρ, and η as [23]

$$\begin{aligned} A &= 0.810^{+0.018}_{-0.024}, \\ \lambda &= 0.22548^{+0.00068}_{-0.00034}, \\ \overline{\rho} &= 0.1453^{+0.0133}_{-0.0073}, \\ \overline{\eta} &= 0.343^{+0.011}_{-0.012}, \end{aligned} \quad (20)$$

with $\overline{\rho} = \rho(1-\lambda^2/2)$ and $\overline{\eta} = \eta(1-\lambda^2/2)$.

The decay constants of light vector mesons are [24]

$$\begin{aligned} f_\rho &= (216 \pm 3)\ \text{MeV}, \\ f_{K^*} &= (220 \pm 5)\ \text{MeV}. \end{aligned} \quad (21)$$

For the decay constants of $D^*_{(s)}$ mesons, we will take [25]

$$\begin{aligned} f_{D^*} &= (252.2 \pm 22.3 \pm 4)\ \text{MeV}, \\ f_{D_s^*} &= (305.5 \pm 26.8 \pm 5)\ \text{MeV}, \end{aligned} \quad (22)$$

which agree well with the results of the other QCD sum rules [26, 27] and lattice QCD with $N_f = 2$ [28].

Besides the decay constants, the $B^* \rightarrow P$ transition form factors are also essential inputs to estimate branching ratios for nonleptonic $B^* \rightarrow PV$ decay. In this paper, the Bauer-Stech-Wirbel (BSW) model [10] is employed to evaluate the form factors $A_1(0)$, $A_2(0)$, and $V(0)$, which could be written as the overlap integrals of wave functions of mesons [10]:

$$\begin{aligned} V^{B^*\to P}(0) &= \frac{m_b-m_q}{m_{B^*}-m_P}J^{B^*\to P}, \\ A_1^{B^*\to P}(0) &= \frac{m_b+m_q}{m_{B^*}+m_P}J^{B^*\to P}, \\ A_2^{B^*\to P}(0) &= \frac{2m_{B^*}}{m_{B^*}-m_P}A_0^{B^*\to P}(0) - \frac{m_{B^*}+m_P}{m_{B^*}-m_P}A_1^{B^*\to P}(0), \\ A_0^{B^*\to P}(0) &= \int d^2p_\perp \int_0^1 dx\varphi_P(\vec{p}_\perp, x)\,\sigma_z\varphi_V^{1,0}(\vec{p}_\perp, x), \\ J^{B^*\to P} &= \sqrt{2}\int d^2p_\perp \int_0^1 dx\varphi_P(\vec{p}_\perp, x)\,i\sigma_y\varphi_V^{1,-1}(\vec{p}_\perp, x), \end{aligned} \quad (23)$$

where $\vec{p}_\perp$ is the transverse quark momentum, $\sigma_{y,z}$ are the Pauli matrix acting on the spin indices of the decaying quark, and m_q represents the mass of nonspectator quark of pseudoscalar meson. With the meson wave function $\varphi_M(\vec{p}_\perp, x)$ as solution of a relativistic scalar harmonic oscillator potential [10] and $\omega = 0.4$ GeV which determines the average transverse quark momentum through $\langle p_\perp^2\rangle = \omega^2$, we get the numerical results of the transition form factors summarized in Table 1. In our following evaluation, these numbers and 15% of them are used as default inputs and uncertainties, respectively.

Table 1: The numerical results of form factors within BSW model.

Transition	$V(0)$	$A_1(0)$	$A_2(0)$
$B^* \to D$	0.76	0.75	0.62
$B^* \to K$	0.41	0.42	0.35
$B^* \to \pi$	0.35	0.38	0.30
$B_s^* \to D_s$	0.72	0.69	0.59
$B_s^* \to K$	0.30	0.29	0.26

To evaluate the branching fractions, the total decay widths (or lifetimes) $\Gamma_{\text{tot}}(B^*)$ are necessary. However, there is no available experimental or theoretical information for $\Gamma_{\text{tot}}(B^*)$ until now. Because of the fact that the QED radiative processes $B^* \rightarrow B\gamma$ dominate the decays of B^* mesons, we will take the approximation $\Gamma_{\text{tot}}(B^*) \simeq \Gamma(B^* \rightarrow B\gamma)$. The theoretical predictions on $\Gamma(B^* \rightarrow B\gamma)$ have been widely evaluated in various theoretical models, such as relativistic quark model [29, 30], QCD sum rules [31], light cone QCD sum rules [32], light front quark model [33], heavy quark effective theory with vector meson dominance hypothesis

Table 2: The CP-averaged branching fractions of nonleptonic B^* weak decays.

Decay modes	Class	CKM factors	$\mathscr{B}$
$B^{*-} \to D^0 D^{*-}$	T, P, and P_{ew}	λ^3	$(3.9^{+0.2+1.3+0.7}_{-0.2-1.1-0.5}) \times 10^{-10}$
$\overline{B}^{*0} \to D^+ D^{*-}$	T, P, and P_{ew}	λ^3	$(1.2^{+0.1+0.4+0.2}_{-0.1-0.4-0.1}) \times 10^{-9}$
$B^{*-} \to D^0 D_s^{*-}$	T, P, and P_{ew}	λ^2	$(1.1^{+0.1+0.4+0.2}_{-0.1-0.3-0.1}) \times 10^{-8}$
$\overline{B}^{*0} \to D^+ D_s^{*-}$	T, P, and P_{ew}	λ^2	$(3.4^{+0.2+1.1+0.5}_{-0.2-1.0-0.4}) \times 10^{-8}$
$\overline{B}_s^{*0} \to D_s^+ D^{*-}$	T, P, and P_{ew}	λ^3	$(2.3^{+0.1+0.8+0.8}_{-0.1-0.7-0.5}) \times 10^{-9}$
$\overline{B}_s^{*0} \to D_s^+ D_s^{*-}$	T, P, and P_{ew}	λ^2	$(6.4^{+0.3+2.1+2.1}_{-0.4-1.9-1.3}) \times 10^{-8}$
$\overline{B}^{*0} \to D^+ K^{*-}$	T	λ^3	$(7.6^{+0.4+1.9+1.2}_{-0.4-1.7-0.9}) \times 10^{-10}$
$\overline{B}_s^{*0} \to D_s^+ K^{*-}$	T	λ^3	$(1.5^{+0.1+0.4+0.5}_{-0.1-0.3-0.3}) \times 10^{-9}$
$\overline{B}^{*0} \to D^+ \rho^-$	T	λ^2	$(1.3^{+0.1+0.3+0.2}_{-0.1-0.3-0.2}) \times 10^{-8}$
$\overline{B}_s^{*0} \to D_s^+ \rho^-$	T	λ^2	$(2.6^{+0.1+0.6+0.9}_{-0.1-0.6-0.5}) \times 10^{-8}$
$B^{*-} \to \pi^- \overline{D}^{*0}$	C	λ^4	$(3.1^{+0.2+0.8+0.6}_{-0.2-0.6-0.4}) \times 10^{-14}$
$B^{*-} \to \pi^0 D^{*-}$	T	λ^4	$(4.6^{+0.4+1.4+0.9}_{-0.4-1.2-0.6}) \times 10^{-13}$
$\overline{B}^{*0} \to \pi^+ D^{*-}$	T	λ^4	$(2.9^{+0.2+0.9+0.5}_{-0.2-0.8-0.3}) \times 10^{-12}$
$\overline{B}^{*0} \to \pi^0 D^{*0}$	C	λ^2	$(1.2^{+0.1+0.4+0.2}_{-0.1-0.3-0.1}) \times 10^{-10}$
$\overline{B}^{*0} \to \pi^0 \overline{D}^{*0}$	C	λ^4	$(4.9^{+0.3+1.4+0.8}_{-0.3-1.2-0.6}) \times 10^{-14}$
$B^{*-} \to \pi^0 D_s^{*-}$	T	λ^3	$(1.3^{+0.1+0.4+0.2}_{-0.1-0.3-0.2}) \times 10^{-11}$
$\overline{B}^{*0} \to \pi^+ D_s^{*-}$	T	λ^3	$(8.1^{+0.6+2.5+1.3}_{-0.7-2.2-1.0}) \times 10^{-11}$
$B^{*-} \to K^- \overline{D}^{*0}$	C	λ^3	$(7.4^{+0.6+2.1+1.4}_{-0.6-1.9-1.0}) \times 10^{-13}$
$\overline{B}^{*0} \to \overline{K}^0 D^{*0}$	C	λ^3	$(1.7^{+0.1+0.5+0.3}_{-0.1-0.4-0.2}) \times 10^{-11}$
$\overline{B}^{*0} \to \overline{K}^0 \overline{D}^{*0}$	C	λ^3	$(2.3^{+0.2+0.7+0.4}_{-0.2-0.6-0.3}) \times 10^{-12}$
$\overline{B}_s^{*0} \to K^+ D^{*-}$	T	λ^4	$(4.3^{+0.3+1.2+1.4}_{-0.4-1.1-0.9}) \times 10^{-12}$
$\overline{B}_s^{*0} \to K^0 D^{*0}$	C	λ^2	$(3.6^{+0.2+1.0+1.2}_{-0.2-0.9-0.7}) \times 10^{-10}$
$\overline{B}_s^{*0} \to K^0 \overline{D}^{*0}$	C	λ^4	$(1.4^{+0.1+0.4+0.5}_{-0.1-0.3-0.3}) \times 10^{-13}$
$\overline{B}_s^{*0} \to K^+ D_s^{*-}$	T	λ^3	$(1.2^{+0.1+0.3+0.4}_{-0.1-0.3-0.2}) \times 10^{-10}$

[34], or covariant model [35]. In this paper, the most recent results [33, 35]

$$\Gamma\left(B^{*+} \longrightarrow B^+\gamma\right) = \left(468^{+73}_{-75}\right)\ \text{eV}, \tag{24}$$

$$\Gamma\left(B^{*0} \longrightarrow B^0\gamma\right) = (148 \pm 20)\ \text{eV}, \tag{25}$$

$$\Gamma\left(B_s^{*0} \longrightarrow B_s^0\gamma\right) = (68 \pm 17)\ \text{eV}, \tag{26}$$

which agree with the other theoretical results, are approximately treated as Γ_{tot} in our numerical estimate.

With the aforementioned values of input parameters and the theoretical formula, we present theoretical predictions for the observables of $\overline{B}^* \to D\overline{D}^*$, $D\rho$, DK^*, πD^*, and KD^* decays, in which only the (color-suppressed) tree induced decay modes are evaluated due to the fact that the branching fractions of loop induced decays are very small and hard to be measured soon. Our numerical results for the branching fractions and the polarization fractions are summarized in Tables 2 and 3. In Table 2, the first, second, and third theoretical errors are caused by uncertainties of the CKM parameters, hadronic parameters (decay constants and form factors), and total decay widths, respectively. From Tables 2 and 3, the following could be found:

Table 3: The polarization fractions f_L and $f_\parallel$ (in the units of percent).

Decay modes	f_L	$f_\parallel$
$B^{*-} \to D^0 D^{*-}$	54^{+2}_{-2}	40^{+2}_{-2}
$\overline{B}^{*0} \to D^+ D^{*-}$	54^{+2}_{-2}	40^{+2}_{-2}
$B^{*-} \to D^0 D_s^{*-}$	52^{+1}_{-2}	43^{+2}_{-2}
$\overline{B}^{*0} \to D^+ D_s^{*-}$	52^{+1}_{-2}	43^{+2}_{-2}
$\overline{B}_s^{*0} \to D_s^+ D^{*-}$	54^{+2}_{-2}	40^{+2}_{-2}
$\overline{B}_s^{*0} \to D_s^+ D_s^{*-}$	52^{+2}_{-2}	42^{+2}_{-2}
$\overline{B}^{*0} \to D^+ K^{*-}$	85^{+1}_{-1}	13^{+1}_{-1}
$\overline{B}_s^{*0} \to D_s^+ K^{*-}$	85^{+1}_{-1}	13^{+1}_{-1}
$\overline{B}^{*0} \to D^+ \rho^-$	88^{+1}_{-1}	10^{+1}_{-1}
$\overline{B}_s^{*0} \to D_s^+ \rho^-$	88^{+1}_{-1}	10^{+1}_{-1}

(1) The hierarchy of branching fractions is clear. (i) The branching fractions of $\overline{B}^* \to \pi D^*$ and KD^* decays are much smaller than the ones of $\overline{B}^* \to D\overline{D}^*$, $D\rho$, and DK^* decays, which is caused by the fact that the form factors of $\overline{B}^* \to D$ transition are much larger than those of $\overline{B}^* \to \pi$ and $\overline{B}^* \to K$ transitions. (ii) For $\overline{B}^* \to D\overline{D}^*$, $D\rho$, and DK^* decays, the hierarchy is induced by two factors: one is the CKM factor (see the third column of Table 2), and the other is $\Gamma_{\text{tot}}(B^{*\pm}) > \Gamma_{\text{tot}}(B_d^{*0}) > \Gamma_{\text{tot}}(B_s^{*0})$ (see (24), (25), and (26)).

(2) Besides small form factors, the $\overline{B}^* \to \pi D^*$, KD^* decays are either color suppressed or the CKM factors suppressed. So they have very small branching fractions (see Table 2) and are hardly measured soon. Most of the CKM-favored and tree-dominated $\overline{B}^* \to D\overline{D}^*$, $D\rho$, and DK^* decays, enhanced by the relatively large $\overline{B}^* \to D$ transition form factors, have large branching fractions, $\gtrsim 10^{-9}$, and thus could be measured in the near future. In particular, branching ratios for $\overline{B}_q^* \to D_q\overline{D}_s^{*-}$ and $D_q\rho$ decays can reach up to 10^{-8} and hence should be sought for with priority and firstly observed at the high statistics LHC and Belle-II experiments.

The numerical results and above analyses are based on the NF, in which the QCD corrections are not included. Fortunately, for the color-allowed tree amplitude α_1, the NF estimate is stable due to the relatively small QCD corrections [15]. For instance, in $B \to \pi\pi$ and $B \to D^*L$ decays, the results $\alpha_1(\pi\pi) = (1.020)_{LO} + (0.018 + 0.018i)_{NLO}$ [14] and $\alpha_1(D^*L) = (1.025)_{LO} + (0.019 + 0.013i)_{NLO}$ [15] indicate clearly that the $\mathcal{O}(\alpha_s)$ correction is only about 2% and thus trivial numerically. For the color-suppressed decay modes listed in Table 2, even though the NF estimates would suffer significant $\mathcal{O}(\alpha_s)$ correction (about 46% in $B \to \pi\pi$ decays, e.g., [36]), they still escape the experimental scope due to their small branching factions $< 10^{-9}$ and thus will not be discussed further. In the following analyses, we will pay our attention only to the color allowed tree-dominated $\overline{B}^* \to D\overline{D}^*$, $D\rho$, and DK^* decays.

(3) For the $B^{*-} \to D^0 D_{(s)}^{*-}$ and $\overline{B}^{*0} \to D^+ D_{(s)}^{*-}$ decays, the $SU(3)$ flavor symmetry implies the relations

$$\begin{aligned} &\mathcal{A}\left(B^{*-} \longrightarrow D^0 D_s^{*-}\right) \simeq \mathcal{A}\left(\overline{B}^{*0} \longrightarrow D^+ D_s^{*-}\right), \\ &\mathcal{A}\left(B^{*-} \longrightarrow D^0 D^{*-}\right) \simeq \mathcal{A}\left(\overline{B}^{*0} \longrightarrow D^+ D^{*-}\right). \end{aligned} \tag{27}$$

Further considering the theoretical prediction $\Gamma(B^{*+} \to B^+\gamma)/\Gamma(B^{*0} \to B^0\gamma) \approx 3$ (see (24) and (25)) and assumption $\Gamma_{\text{tot}}(B^*) \simeq \Gamma(B^* \to B\gamma)$, one may find the ratios

$$R_{du} \equiv \frac{\mathcal{B}\left(\overline{B}^{*0} \longrightarrow D^+ D_s^{*-}\right)}{\mathcal{B}\left(B^{*-} \longrightarrow D^0 D_s^{*-}\right)} \simeq \frac{\Gamma\left(B^{*+} \longrightarrow B^+\gamma\right)}{\Gamma\left(B^{*0} \longrightarrow B^0\gamma\right)} \overset{\text{theo.}}{\approx} 3, \tag{28}$$

$$R'_{du} \equiv \frac{\mathcal{B}\left(\overline{B}^{*0} \longrightarrow D^+ D^{*-}\right)}{\mathcal{B}\left(B^{*-} \longrightarrow D^0 D^{*-}\right)} \simeq R_{du}, \tag{29}$$

which are satisfied in our numerical evaluations. Experimentally, the first relation (28) is hopefully to be tested soon due to the large branching fractions. For the other potentially detectable $\overline{B}_{d,s}^{*0} \to D\overline{D}^*$, $D\rho$, and DK^* decay modes, with branching fractions $\gtrsim 10^{-9}$, the U-spin symmetry implies relations

$$\begin{aligned} &\mathcal{A}\left(\overline{B}^{*0} \longrightarrow D^+ D^{*-}\right) \simeq \mathcal{A}\left(\overline{B}_s^{*0} \longrightarrow D_s^+ D^{*-}\right), \\ &\mathcal{A}\left(\overline{B}^{*0} \longrightarrow D^+ D_s^{*-}\right) \simeq \mathcal{A}\left(\overline{B}_s^{*0} \longrightarrow D_s^+ D_s^{*-}\right), \\ &\mathcal{A}\left(\overline{B}^{*0} \longrightarrow D^+ K^{*-}\right) \simeq \mathcal{A}\left(\overline{B}_s^{*0} \longrightarrow D_s^+ K^{*-}\right), \\ &\mathcal{A}\left(\overline{B}^{*0} \longrightarrow D^+ \rho^-\right) \simeq \mathcal{A}\left(\overline{B}_s^{*0} \longrightarrow D_s^+ \rho^-\right). \end{aligned} \tag{30}$$

As similar to R_{du}, one also could get the ratio and relation

$$\begin{aligned} R_{ds} &\equiv \frac{\mathcal{B}\left(\overline{B}^{*0} \longrightarrow D^+ D^{*-}, D^+ D_s^{*-}, D^+ K^{*-}, D^+ \rho^-\right)}{\mathcal{B}\left(\overline{B}_s^{*0} \longrightarrow D_s^+ D^{*-}, D_s^+ D_s^{*-}, D_s^+ K^{*-}, D_s^+ \rho^-\right)} \\ &\simeq \frac{\Gamma\left(B_s^{*0} \longrightarrow B_s^0\gamma\right)}{\Gamma\left(B^{*0} \longrightarrow B^0\gamma\right)} \overset{\text{theo.}}{\approx} 2, \end{aligned} \tag{31}$$

which is also satisfied in our numerical evaluation. So, it is obvious that such ratios, R_{du} and R_{ds}, are useful for probing $\tau_{B^{*0}}/\tau_{B^{*\pm}}$ and $\tau_{B^{*0}}/\tau_{B_s^{*0}}$, respectively, and further testing the theoretical predictions of $\Gamma(B^{*+} \to B^+\gamma)/\Gamma(B^{*0} \to B^0\gamma)$ and $\Gamma(B_s^{*0} \to B_s^0\gamma)/\Gamma(B^{*0} \to B^0\gamma)$ in various models, such as the results in [29–35].

(4) Besides branching fraction, the polarization fractions $f_{L,\parallel,\perp}$ are also important observables. For the potentially detectable decay modes with branching fractions $\gtrsim 10^{-9}$, our numerical results of $f_{L,\parallel}$ are summarized in Table 3. For the helicity amplitudes $\mathcal{A}_\lambda$, the formal hierarchy pattern

$$\mathcal{A}_0 : \mathcal{A}_- : \mathcal{A}_+ = 1 : \frac{\Lambda_{\text{QCD}}}{m_b} : \left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)^2 \tag{32}$$

is naively expected. Hence, $\overline{B}^* \to PV$ decays are generally dominated by the longitudinal polarization state and satisfy $f_L \sim 1 - 1/m_{B^*}^2$ [37]. For $\overline{B}^* \to DV$ ($V = K^*, \rho$) decays, in the heavy quark limit, the helicity amplitudes H^λ given by (9) could be simplified as

$$\begin{aligned} H_{PV}^0 &\simeq i f_V \left[\frac{(m_{B^*} - m_D)(m_{B^*} + m_D)^2}{2m_{B^*}} A_1 + \frac{(m_{B^*} + m_D)(m_{B^*} - m_D)^2}{2m_{B^*}} A_2\right], \\ H_{PV}^\pm &\simeq i f_V \left[\frac{(m_{B^*} - m_D)(m_{B^*} + m_D)^2}{2m_{B^*}} A_1 \mp \frac{(m_{B^*} + m_D)(m_{B^*} - m_D)^2}{2m_{B^*}} V\right] \cdot \frac{2m_{B^*} m_V}{m_{B^*}^2 - m_D^2}. \end{aligned} \tag{33}$$

The transversity amplitudes could be gotten easily through (19). Obviously, due to the helicity suppression factor $2m_{B^*} m_V/(m_{B^*}^2 - m_D^2) \sim 2m_V/m_{B^*} \sim \Lambda_{\text{QCD}}/m_b$, the relation of (32) is roughly fulfilled. As a result, the longitudinal polarization fractions of $\overline{B}^* \to DK^*$ and $D\rho$ decays are very large (see Table 3 for numerical results).

It should be noted that the above analyses and (33) are based on the case of $m_V^2 \ll m_{B^*}^2$ and thus possibly no longer satisfied by $\overline{B}^* \to D\overline{D}^*$ decays because of the unnegligible vector mass m_{D^*}. In fact, for the $\overline{B}^* \to D\overline{D}^*$ decays, (9) are simplified as

$$H_{PV}^0 \simeq i f_{D^*} \left[\frac{(m_{B^*} + m_D) m_{B^*}}{2} A_1 + \frac{m_{B^*}}{2(m_{B^*} + m_D)} \left(m_{B^*}^2 - 4m_{D^*}^2\right) A_2\right], \tag{34}$$

$$H_{PV}^\pm \simeq i f_{D^*} \left[\frac{(m_{B^*} + m_D) m_{B^*}}{2} A_1 \mp \frac{m_{B^*}}{2(m_{B^*} + m_D)} m_{B^*} \sqrt{m_{B^*}^2 - 4m_{D^*}^2} V\right] \cdot \frac{2m_{D^*}}{m_{B^*}}, \tag{35}$$

in which, due to $(m_{D^*}^2 - m_D^2) \ll m_{B^*}^2$, the approximation $x = (m_{B^*}^2 - m_D^2 + m_{D^*}^2)/2m_{B^*}m_{D^*} \simeq m_{B^*}/2m_{D^*}$ is used. Because the so-called helicity suppression factor $2m_{D^*}/m_{B^*} \sim 0.8$ is not small, which is different from the case of $\overline{B}^* \to DV$ decays, it could be easily found that the relation of (32) does not follow. Further considering that $H_{PV}^{\pm}$ are dominated by the term of A_1 in (35) due to its large coefficient, the relation $f_L(D\overline{D}^*) \sim f_{\parallel}(D\overline{D}^*) \gg f_{\perp}(D\overline{D}^*)$ could be easily gotten. Above analyses and findings are confirmed by our numerical results in Table 3, which will be tested by future experiments.

(5) As known, there are many interesting phenomena in B meson decays, so it is worthy to explore the possible correlation between B and B^* decays. Taking $\overline{B}^{*0} \to D^+\rho^-$ and $\overline{B}^0 \to D^{*+}\rho^-$ decays as example, we find that the expressions of their helicity amplitudes (the former one has been given by (33)) are similar to each other except for the replacements $\overline{B}^* \leftrightarrow \overline{B}$ and $D \leftrightarrow D^*$ everywhere in (33). As a result, our analyses in item (4) are roughly suitable for $\overline{B}^0 \to D^{*+}\rho^-$ decay, and the relation

$$f_{L,\parallel}\left(\overline{B}^{*0} \longrightarrow D^+\rho^-\right) \simeq f_{L,\parallel}\left(\overline{B}^0 \longrightarrow D^{*+}\rho^-\right) \qquad (36)$$

is generally expected. Interestingly, our prediction $f_L^{NF}(\overline{B}^{*0} \to D^+\rho^-) = (88\pm1)\%$ is consistent with the result $f_L^{WSB}(\overline{B}^0 \to D^{*+}\rho^-) = 87\%$ [38], which is in a good agreement with the experimental data $f_L^{\text{exp.}}(\overline{B}^0 \to D^{*+}\rho^-) = (88.5 \pm 1.6 \pm 1.2)\%$ [39]. The relation equation (36) follows. In addition, the similar correlation as (36) also exists in the other B^* and corresponding B decays.

4. Summary

In this paper, motivated by the experiments of heavy flavor physics at the running LHC and forthcoming SuperKEKB/Belle-II, the nonleptonic $\overline{B}^* \to D\overline{D}^*$, $D\rho$, DK^*, πD^*, and KD^* weak decay modes are evaluated with factorization approach, in which the transition form factors are calculated with the BSW model and the approximation $\Gamma_{\text{tot}}(B^*) \simeq \Gamma(B^* \to B\gamma)$ is used to evaluate the branching fractions. It is found that (i) there are some obvious hierarchies among branching fractions, in which the $\overline{B}_q^* \to D_q\overline{D}_s^{*-}$ and $D_q\rho^-$ decays have large branching fractions $\sim 10^{-8}$ and hence should be sought for with priority at LHC and Belle-II experiments. (ii) With the implication of $SU(3)$ (or U-spin) flavor symmetry, some useful ratios, R_{du} and R_{ds}, are suggested to be verified experimentally. (iii) The $\overline{B}^{*0} \to DK^*$ and $D\rho$ decays are dominated by the longitudinal polarization states; numerically, $f_L \sim [80\%, 90\%]$. While the parallel polarization fractions of $\overline{B}^* \to D\overline{D}^*$ decays are comparable with the longitudinal ones; numerically, $f_L : f_{\parallel} \simeq 5:4$. In addition, comparing with $B \to VV$ decays, the relation $f_{L,\parallel}(\overline{B}^{*0} \to DV) \simeq f_{L,\parallel}(\overline{B}^0 \to DV)$ is generally expected. These results and findings are waiting for confirmation from future LHC and Belle-II experiments.

Conflict of Interests

The authors declare that there is no conflict of interests.

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