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Research Article

Two-Phase Equilibrium Properties in Charged Topological Dilaton AdS Black Holes

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We discuss phase transition of the charged topological dilaton AdS black holes by Maxwell equal area law. The two phases involved in the phase transition could coexist and we depict the coexistence region in P-v diagrams. The two-phase equilibrium curves in P-T diagrams are plotted, the Clapeyron equation for the black hole is derived, and the latent heat of isothermal phase transition is investigated. We also analyze the parameters of the black hole that could have an effect on the two-phase coexistence. The results show that the black holes may go through a small-large phase transition similar to that of a usual nongravity thermodynamic system.

1. Introduction

In recent years, the cosmological constant in n-dimensional AdS and dS spacetime has been regarded as pressure of black hole thermodynamic system with

$$P = -\frac{\Lambda}{8\pi}, \tag{1}$$

and the corresponding conjugate quantity, thermodynamic volume [1–4],

$$V = \left(\frac{\partial M}{\partial P}\right)_{S,Q_i,J_k}. \tag{2}$$

(P-V) critical behaviors in AdS and dS black holes have been extensively studied [5–44]. Using Ehrenfest scheme, [22–29] studied the critical phenomena of a series of black holes in AdS spacetime and proved that the phase transition at critical point is the second-order one, which has also been confirmed in [30–34] by studying thermodynamics and state space geometry of black holes. And a completely simulated gas-liquid system has been put forward [3, 5, 8, 43]. Recently phase transition below critical temperature and phase structure of some black holes have received much attention [45–49].

Although some encouraging results about black hole thermodynamic properties in AdS and dS spacetimes have been achieved and the problems about phase transition of black holes have been extensively discussed, a unified recognition on phase transition of black hole has not been put forward. It is significant to further explore phase equilibrium and phase structure in black holes, which can help to recognize the evolution of black hole. We also expect to provide some relevant information for quantum gravity properties by studying the phase transition of charged topological dilaton AdS black holes.

A scalar field called dilaton appears in the low energy limit of string theory. The presence of the dilaton field has important consequences on the causal structure and the thermodynamic properties of black holes. Much interest has been focused on studies of the dilaton black holes in recent years [50–60]. The isotherms in P-v diagrams of charged topological dilaton AdS black hole in [13] show that there exists thermodynamic unstable region with $\partial P/\partial v > 0$ when temperature is below critical temperature and negative pressure emerges when temperature is below a certain value. This situation also exists in van der Waals-Maxwell gas-liquid system, which has been resolved by Maxwell equal area law. In this paper, using the Maxwell equal area law, we establish a phase transition process for charged topological dilaton AdS black holes, where the issues about thermodynamic unstable states and negative pressure are resolved. By studying the phase transition process, we acquire the two-phase equilibrium properties involving P-T phase diagram, Clapeyron equation, and latent heat of phase change. The results show that the phase transition is the first-order one but the phase transition at critical point belongs to the continuous one though the parameters of the charged topological dilaton black holes have some effects on the two-phase coexistence.

The paper is arranged as follows. The charged topological dilaton AdS black hole as a thermodynamic system is briefly introduced in Section 2. In Section 3, by Maxwell equal area law the phase transition processes at certain temperatures are obtained and the boundary of two-phase equilibrium region is depicted in P-v diagram for a charged topological dilaton AdS black hole. Then some parameters of the black hole are analyzed to find their relation with the two-phase equilibrium. In Section 4, the P-T phase diagrams are plotted; furthermore, Clapeyron equation and latent heat of the isothermal phase change are derived. We make some discussions and conclusions in Section 5. We use the units $G_d = \hbar = k_B = c = 1$ in this paper.

2. Charged Dilaton Black Holes in Anti-de Sitter Space

The Einstein-Maxwell-dilaton action in $(n+1)$-dimensional $(n \geq 3)$ spacetime is [59, 60]

$$S = \frac{1}{16\pi}\int d^{n+1}x\sqrt{-g}\left(R - \frac{4}{n-1}(\nabla\Phi)^2 - U(\Phi) - e^{-4\alpha\Phi/(n-1)}F_{\mu\nu}F^{\mu\nu}\right), \tag{3}$$

where R is the Ricci scalar curvature, Φ is the dilaton field and $U(\Phi)$ is a potential for Φ, α is a constant determining the strength of coupling of the scalar and electromagnetic field, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field tensor, and A_μ is the electromagnetic potential. The dilaton potential is related to the dilaton field and its coupling to the cosmological constant. The field equations

$$\begin{gathered}\nabla^2\Phi = \frac{n-1}{8}\frac{\partial U}{\partial\Phi} - \frac{\alpha}{2}e^{-4\alpha\Phi/(n-1)}F_{\lambda\eta}F^{\lambda\eta},\\ \nabla_\mu\left(e^{-4\alpha\Phi/(n-1)}F^{\mu\nu}\right) = 0.\end{gathered} \tag{4}$$

The topological black hole solutions take the following form [59, 60]:

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2R^2(r)\,d\Omega^2_{k,n-1}, \tag{5}$$

where

$$\begin{aligned} f(r) = &-\frac{k(n-2)\left(\alpha^2+1\right)^2 b^{-2\gamma}r^{2\gamma}}{(\alpha^2-1)(\alpha^2+n-2)} - \frac{m}{r^{(n-1)(1-\gamma)-1}}\\ &+\frac{2q^2\left(\alpha^2+1\right)^2 b^{-2(n-2)\gamma}}{(n-1)(\alpha^2+n-2)}r^{2(n-2)(\gamma-1)}\\ &-\frac{n\left(\alpha^2+1\right)^2 b^{2\gamma}}{l^2(\alpha^2-n)}r^{2(1-\gamma)},\end{aligned} \tag{6}$$

$$\begin{aligned} R(r) &= e^{2\alpha\Phi/(n-1)},\\ \Phi(r) &= \frac{(n-1)\alpha}{2(1+\alpha^2)}\ln\left(\frac{b}{r}\right),\end{aligned} \tag{7}$$

with $\gamma = \alpha^2/(\alpha^2+1)$ and b is an arbitrary constant. The cosmological constant is related to spacetime dimension n by

$$\Lambda = -\frac{n(n-1)}{2l^2}, \tag{8}$$

where l denotes the AdS length scale. In (6), m appears as an integration constant and is related to the ADM (Arnowitt-Deser-Misner) mass of the black hole. According to the definition of mass due to Abbott and Deser [61], the ADM mass of the solution (6) is

$$M = \frac{b^{(n-1)\gamma}(n-1)\,\omega_{n-1}}{16\pi(\alpha^2+1)}m. \tag{9}$$

The electric charge is

$$Q = \frac{q\omega_{n-1}}{4\pi}, \tag{10}$$

where ω_{n-1} represents the volume of constant curvature hypersurface described by $d\Omega^2_{k,n-1}$.

The thermodynamic quantities satisfy the first law of thermodynamics

$$dM = TdS + \phi dQ + VdP. \tag{11}$$

The Hawking temperature and entropy of the topological black hole

$$T = -\frac{\left(\alpha^2+1\right)}{2\pi(n-1)}\left(\frac{k(n-2)(n-1)b^{-2\gamma}}{2(\alpha^2-1)}r_+^{2\gamma-1} + \Lambda b^{2\gamma}r_+^{1-2\gamma} + q^2b^{-2(n-2)\gamma}r_+^{(2n-3)(\gamma-1)-\gamma}\right), \tag{12}$$

$$S = \frac{b^{(n-1)\gamma}\omega_{n-1}r_+^{(n-1)(1-\gamma)}}{4}, \tag{13}$$

where r_+ represents the position of black hole horizon and meets $f(r_+) = 0$. The electric potential

$$\phi = \frac{qb^{(3-n)\gamma}}{r_+^\lambda\lambda}, \tag{14}$$

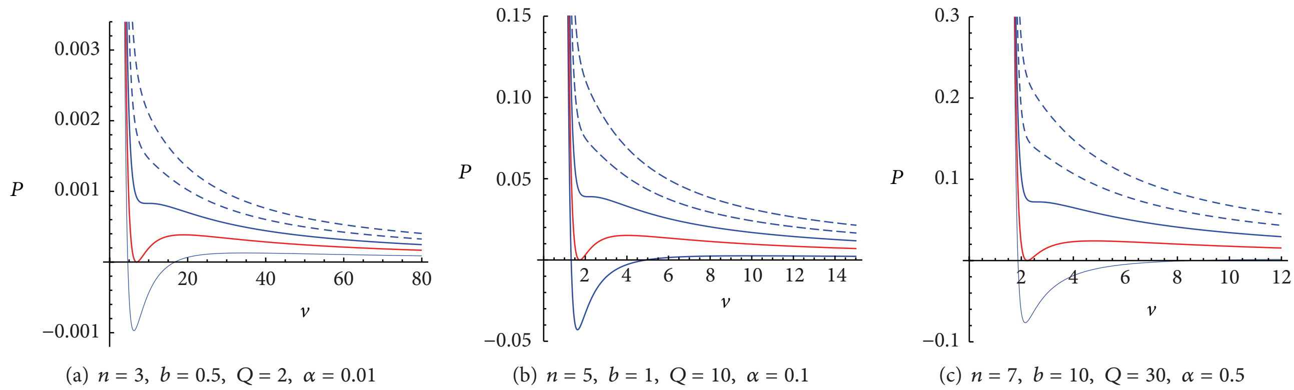

FIGURE 1: Isotherms in P-v diagrams of charged topological dilaton black holes in n-dimensional AdS spacetime.

where $\lambda = (n-3)(1-\gamma)+1$, and the pressure and volume are, respectively,

$$P = \frac{n(n-1)}{16\pi l^2}, \qquad V = -\frac{(\alpha^2+1)\, b^{\gamma(n+1)}\omega_{n-1}}{(\alpha^2-n)} r_+^{n-\gamma(n+1)}. \tag{15}$$

Using (8), (12), and (15) for a fixed charge Q, one can obtain the equation of state $P(v,T)$,

$$\begin{aligned} P &= \frac{T}{v} + \frac{k(n-2)\left(\alpha^2+1\right)^2}{\pi(n-1)\left(\alpha^2-1\right)v^2} \\ &\quad + \frac{Q^2 b^{2(1-n)\gamma} 2\pi}{\omega_{n-1}^2}\left(\frac{v(n-1)}{4\left(\alpha^2+1\right)b^{2\gamma}}\right)^{2(n-1)(\gamma-1)/(1-2\gamma)} \\ &= \frac{T}{v} - \frac{A}{v^2} + \frac{B}{v^{2(n-1)(1-\gamma)/(1-2\gamma)}}, \end{aligned} \tag{16}$$

where specific volume [13]

$$\begin{aligned} v &= \frac{4\left(\alpha^2+1\right)b^{2\gamma}}{(n-1)} r_+^{1-2\gamma}, \\ A &= \frac{k(n-2)\left(\alpha^2+1\right)^2}{\pi(n-1)\left(1-\alpha^2\right)}, \\ B &= \frac{Q^2 b^{2(1-n)\gamma} 2\pi}{\omega_{n-1}^2}\left(\frac{4\left(\alpha^2+1\right)b^{2\gamma}}{(n-1)}\right)^{2(n-1)(1-\gamma)/(1-2\gamma)}. \end{aligned} \tag{17}$$

In Figure 1 we plot the isotherms in P-v diagrams in terms of state equation (16) at different dimension n, charge Q, and parameters b and α. One can see from Figure 1 that there are thermodynamic unstable segments with $\partial P/\partial v > 0$ on the isotherms as temperature $T < T_c$, where T_c is critical temperature. And the negative pressure emerges when temperature is below certain value $\widetilde{T}$. $\widetilde{T}$ and the corresponding specific volume $\widetilde{v}$ can be derived as follows:

$$\begin{aligned} \widetilde{v}^{d-2} &= \frac{B}{A}(d-1), \\ \widetilde{T} &= \frac{A(d-2)}{\widetilde{v}(d-1)}, \\ d &= \frac{2(n-1)(1-\gamma)}{1-2\gamma}. \end{aligned} \tag{18}$$

3. Two-Phase Equilibrium and Maxwell Equal Area Law

The state equation of the charged topological black hole is exhibited by the isotherms in Figure 1, in which the thermodynamic unstable states with $\partial P/\partial v > 0$ may lead to the system automatic expansion or contraction and negative pressure situation has no physical meaning. The cases occur also in van der Waals equation but they have been resolved by Maxwell equal area law.

We extend Maxwell equal area law to $n+1$-dimensional charged topological dilaton AdS black holes to establish a phase transition process for the black hole thermodynamic system. On the isotherm with temperature T_0 ($T_0 < T_c$) in P-v diagram, there exist two points (P_0, v_1) and (P_0, v_2) meeting Maxwell equal area law,

$$P_0\left(v_2-v_1\right) = \int_{v_1}^{v_2} P\,dv, \tag{19}$$

which results in

$$\begin{aligned} P_0\left(v_2-v_1\right) &= T_0 \ln\left(\frac{v_2}{v_1}\right) - A\left(\frac{1}{v_1}-\frac{1}{v_2}\right) \\ &\quad + \frac{B}{d-1}\left(\frac{1}{v_1^{d-1}}-\frac{1}{v_2^{d-1}}\right), \end{aligned} \tag{20}$$

where the two points (P_0, v_1) and (P_0, v_2) are seen as end-points of isothermal phase transition. Considering

$$P_0 = \frac{T_0}{v_1} - \frac{A}{v_1^2} + \frac{B}{v_1^d},$$
$$P_0 = \frac{T_0}{v_2} - \frac{A}{v_2^2} + \frac{B}{v_2^d}, \tag{21}$$

and setting $x = v_1/v_2$, we can get

$$T_0 v_2^{d-1} x^{d-1} = A v_2^{d-2} x^{d-2} (1+x) - B\frac{1-x^d}{1-x}, \tag{22}$$

$$P_0 x^{d-1} v_2^d = A x^{d-2} v_2^{d-2} - B\frac{1-x^{d-1}}{1-x}, \tag{23}$$

$$v_2^{d-2} = \frac{B}{A}\frac{d\left(1-x^{d-1}\right)(1-x)+(d-1)\left(1-x^d\right)\ln x}{x^{d-2}(d-1)(1-x)(2(1-x)+(1+x)\ln x)} = f(x). \tag{24}$$

Substituting (24) into (22) and setting $T_0 = \chi T_c$ $(0 < \chi < 1)$, we obtain

$$\chi T_c x^{d-1} f^{(d-1)/(d-2)}(x) = A f(x) x^{d-2}(1+x) - B\frac{1-x^d}{1-x}. \tag{25}$$

When $x \to 1$, the corresponding state is critical point state. From (24)

$$v_2^{d-2} = v_1^{d-2} = v_c^{d-2} = f(1) = \frac{d(d-1)B}{2A}. \tag{26}$$

Substituting (26) into (22) and (23), the critical temperature and critical pressure are

$$T_c = \frac{2A(d-2)}{(d-1)}\left(\frac{2A}{d(d-1)B}\right)^{1/(d-2)},$$
$$P_c = \frac{A(d-2)}{d}\left(\frac{2A}{d(d-1)B}\right)^{2/(d-2)}. \tag{27}$$

Combining (27) and (25) we can get

$$\chi x^{d-1} f^{(d-1)/(d-2)}(x)\frac{2A(d-2)}{(d-1)}\left(\frac{2A}{d(d-1)B}\right)^{1/(d-2)} = A f(x) x^{d-2}(1+x) - B\frac{1-x^d}{1-x}. \tag{28}$$

For a fixed χ, that is, a fixed T_0, we can get a certain x from (28); then, according to (23) and (24), v_2 and P_0 are obtained. The corresponding v_1 can be obtained from $x = v_1/v_2$. Join the points (v_1, P_0) and (v_2, P_0) on isotherms in P-v diagram, which generate an isobar representing a process of isothermal phase transition or two-phase coexistence situation like that of van der Waals system. Figure 2 shows the isobars on the background of isotherms at different temperature and the boundary of the two-phase equilibrium region by the dot-dashed curve as $n = 5, b = 1, Q = 1, \alpha = 0.01$. The isothermal phase transition process becomes shorter as temperature goes up until it turns into a single point at a certain temperature, which is the critical temperature, and the point corresponds to a critical state of the charged topological dilaton AdS black hole.

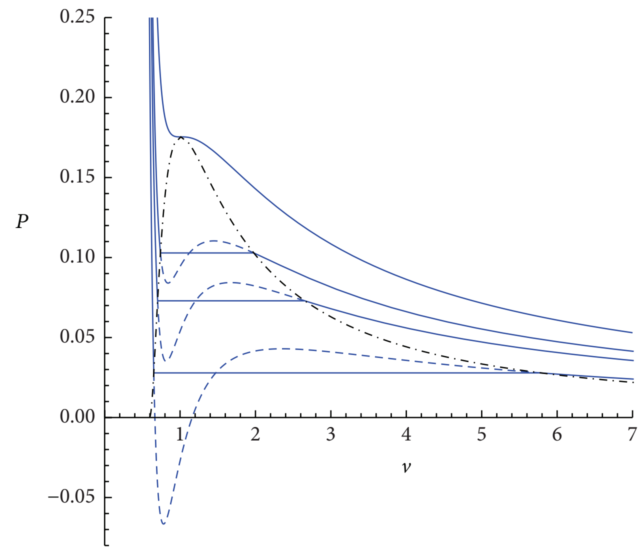

Figure 2: The simulated isothermal phase transition by isobars and the boundary of two-phase coexistence region for the topological dilaton black hole as $n = 5,\ b = 1,\ Q = 1,\ \alpha = 0.01$.

To analyze the effect of parameters α and b on the phase transition processes, we take $\chi = 0.1,\ 0.3,\ 0.5,\ 0.7,\ 0.9$ and calculate the quantities x, v_2, P_0 as $\alpha = 0.1,\ 0.3,\ 0.5$ and $b = 0.2,\ 20,\ 50$, respectively, when $d = 5, Q = 1$. The results are shown in Table 1.

From Table 1, we can see that x is unrelated to b but it is incremental with χ and α. v_2 increases with increasing b but decreases with increasing χ and α. P_0 is incremental with χ and α but decreases with increasing b. So phase transition process becomes shorter with increasing α, and it lengthens as b increases.

4. Two-Phase Coexistent Curves and the Phase Change Latent

Due to lack of knowledge of chemical potential, P-T curves of two-phase equilibrium coexistence for general thermodynamic system are usually obtained by experiment. However, the slope of the curves can be calculated by Clapeyron equation in theory,

$$\frac{dP}{dT} = \frac{L}{T\left(v^\beta - v^\alpha\right)}, \tag{29}$$

where the latent heat of phase change $L = T(s^\beta - s^\alpha)$ and v^α, s^α and v^β, s^β are the molar volumes and molar entropy of phase

Table 1: State quantities at phase transition endpoints with different parameters α and b as $d = 5$, $Q = 1$.

b	χ	$\alpha = 0.1$			$\alpha = 0.3$			$\alpha = 0.5$		
		x	v_2	P_0	x	v_2	P_0	x	v_2	P_0
0.2	0.9	0.531	1.49	0.145	0.546	1.36	0.220	0.577	1.13	0.502
	0.7	0.266	2.62	0.0770	0.279	2.36	0.118	0.308	1.92	0.274
	0.5	0.114	5.70	0.0295	0.121	5.06	0.0458	0.139	4.00	0.109
	0.3	0.0253	24.2	0.00481	0.0279	20.9	0.00771	0.0340	15.6	0.0195
	0.1	$6.32E-5$	$9.28E3$	$4.55E-6$	$8.25E-5$	$6.78E3$	$8.64E-6$	$1.40E-4$	$3.65E3$.	$3.07E-5$
20	0.9	0.531	1.58	0.128	0.546	2.32	0.0753	0.577	4.67	0.0295
	0.7	0.266	2.79	0.068	0.279	4.03	0.0404	0.308	7.91	0.0161
	0.5	0.114	6.06	0.0261	0.121	8.65	0.0157	0.139	16.5	0.00640
	0.3	0.0253	25.7	0.00426	0.0279	35.7	0.00264	0.0340	64.3	0.00115
	0.1	$6.32E-5$	$9.87E3$	$4.02E-6$	$8.25E-5$	$1.16E4$	$2.95E-6$	$1.40E-4$	$1.50E4$	$1.80E-6$
50	0.9	0.531	1.60	0.125	0.546	2.58	0.0608	0.577	6.19	0.0168
	0.7	0.266	2.82	0.0665	0.279	4.49	0.0326	0.308	10.5	0.00915
	0.5	0.114	6.13	0.0254	0.121	9.63	0.0126	0.139	21.9	0.00364
	0.3	0.0253	26.1	0.00415	0.0279	39.7	0.00213	0.0340	85.3	$6.52E-4$
	0.1	$6.32E-5$	$9.99E3$	$3.93E-6$	$8.25E-5$	$1.29E4$	$2.39E-6$	$1.40E-4$	$1.99E4$	$1.03E-6$

α and phase β, respectively. So Clapeyron equation provides a direct experimental verification for some phase transition theories.

Here we investigate the two-phase equilibrium P-T curves and the slope of them for the topological dilaton AdS black holes. Rewrite (22) and (23) as

$$P = y_1(x), \qquad T = y_2(x), \tag{30}$$

where

$$y_1(x) = \frac{\left[Ax^{d-2}f(x) - B\left(1-x^{d-1}\right)/(1-x)\right]}{\left[x^{d-1}f^{d/(d-2)}(x)\right]},$$
$$y_2(x) = \frac{\left[Af(x)x^{d-2}(1+x) - B\left(1-x^{d}\right)/(1-x)\right]}{\left[x^{d-1}f^{(d-1)/(d-2)}(x)\right]}. \tag{31}$$

We plot the P-T curves with $0 < x \le 1$ in Figure 3 when the parameters b, α, and Q take different values, respectively. The curves represent two-phase equilibrium condition for the topological dilaton AdS black holes and the terminal points of the curves represent corresponding critical points.

Figure 3 shows that, for fixed α and Q, both the critical temperature and critical pressure decrease as b increases. Both critical pressure and critical temperature are incremental with increasing α, but two-phase equilibrium pressure decreases with increasing α at certain temperature. The change of two-phase equilibrium curve with parameter Q is similar to that with parameter b. As Q becomes larger the critical pressure and critical temperature become smaller, but at certain temperature the corresponding pressure on P-T curves is larger for larger Q.

From (31), we obtain

$$\frac{dP}{dT} = \frac{y_1'(x)}{y_2'(x)}, \tag{32}$$

where $y'(x) = dy/dx$. Equation (32) represents the slope of two-phase equilibrium P-T curve as function of x.

From (29) and (32) we can get the latent heat of phase change as function of x for $n+1$-dimensional charged topological dilaton AdS black hole,

$$L = T(1-x)\frac{y_1'(x)}{y_2'(x)}f^{1/(d-2)}(x) = (1-x)\frac{y_1'(x)}{y_2'(x)}y_2(x)f^{1/(d-2)}(x). \tag{33}$$

The latent heat of phase change varies with temperature for some usual thermodynamic systems, and the rate of variation

$$\frac{dL}{dT} = C_P^{\beta} - C_P^{\alpha} + \frac{L}{T} - \left[\left(\frac{\partial v^{\beta}}{\partial T}\right)_P - \left(\frac{\partial v^{\alpha}}{\partial T}\right)_P\right]\frac{L}{v^{\beta}-v^{\alpha}}, \tag{34}$$

where C_P^{β} and C_P^{α} are molar heat capacity of phase β and phase α. For $n+1$-dimensional charged topological dilaton AdS black holes, the rate of variation of latent heat of phase transition with temperature can be obtained from (33) and (30):

$$\frac{dL}{dT} = \frac{dL}{dx}\frac{dx}{dT} = \frac{dL}{dx}\frac{1}{y_2'(x)}. \tag{35}$$

Using (33) and (30) we plot L-T curves in Figure 4 as the parameters b, α, and Q take some certain values. From Figure 4 one can see that the effects of T and the parameters α, b, and Q on phase change latent heat L. When T increases, L is not monotonous but increases firstly and then decreases to zero as $T \to T_c$. L decreases with increasing b as other parameters α and Q are fixed. Similarly L decreases with

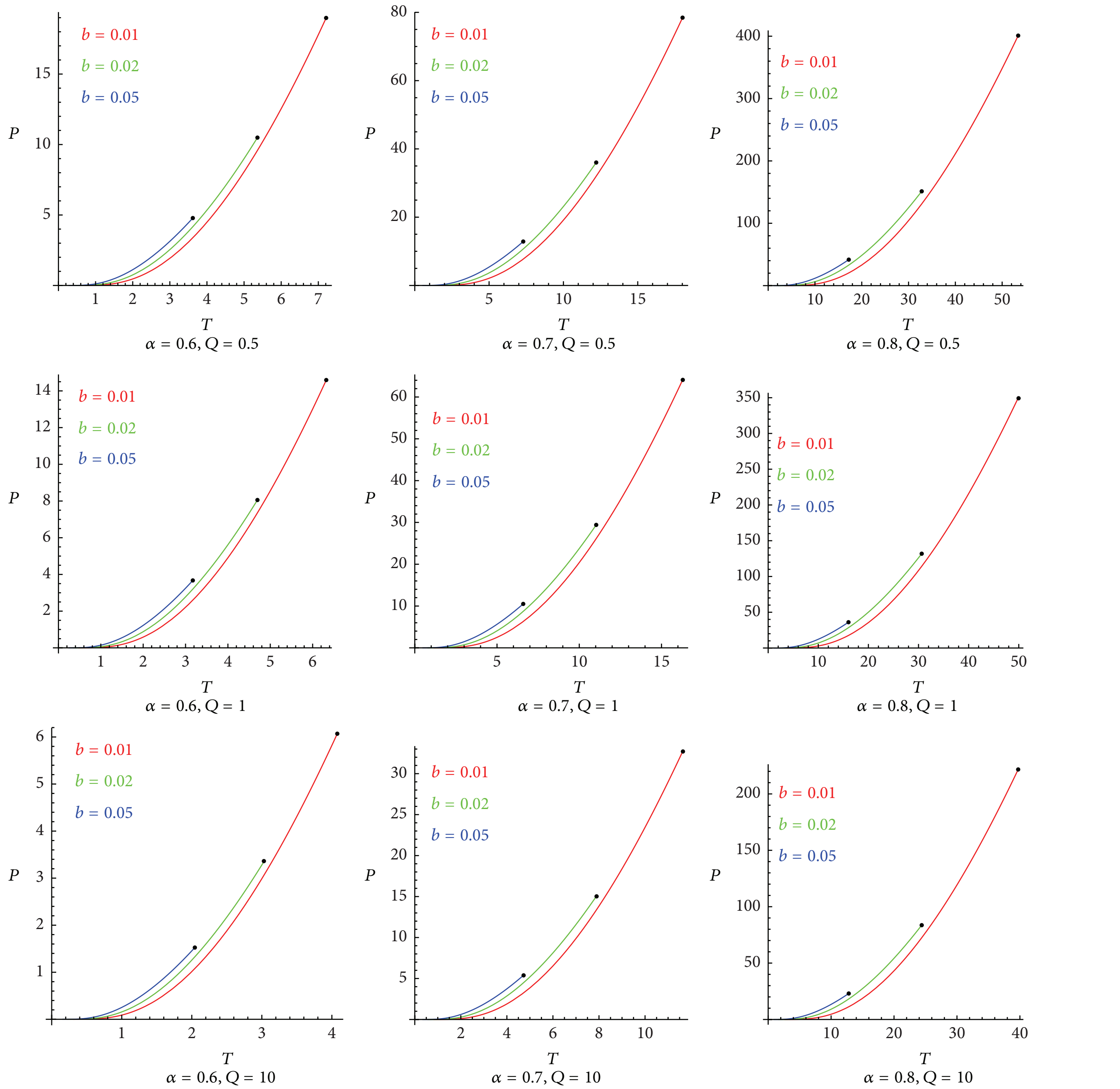

FIGURE 3: Two-phase equilibrium curves in P-T diagrams for the topological dilaton black hole in 5-dimensional AdS spacetime. In each diagram, the longest curves (red) correspond to $b = 0.01$, the curves with medium length (green) meet $b = 0.02$, and the shortest ones (blue) are with $b = 0.05$.

increasing Q for fixed b and α. But L is increment with α for certain b and Q. Among the parameters b, α, and Q, L receives the most effect from b, then α, and lastly Q.

5. Discussions and Conclusions

The charged topological dilaton AdS black hole is regarded as a thermodynamic system, and its state equation has been derived. But when temperature is below critical temperature, thermodynamic unstable situation appears on isotherms, and when temperature reduces to a certain value negative pressure emerges, which can be seen in Figures 1 and 2. However, by Maxwell equal law we established a phase transition process and the problems can be resolved. The phase transition process at a defined temperature happens at a constant pressure, where the system specific volume changes along with the ratio of the two coexistent phases. According to Ehrenfest scheme the phase transition belongs to the first-order one. We draw the isothermal phase transition process

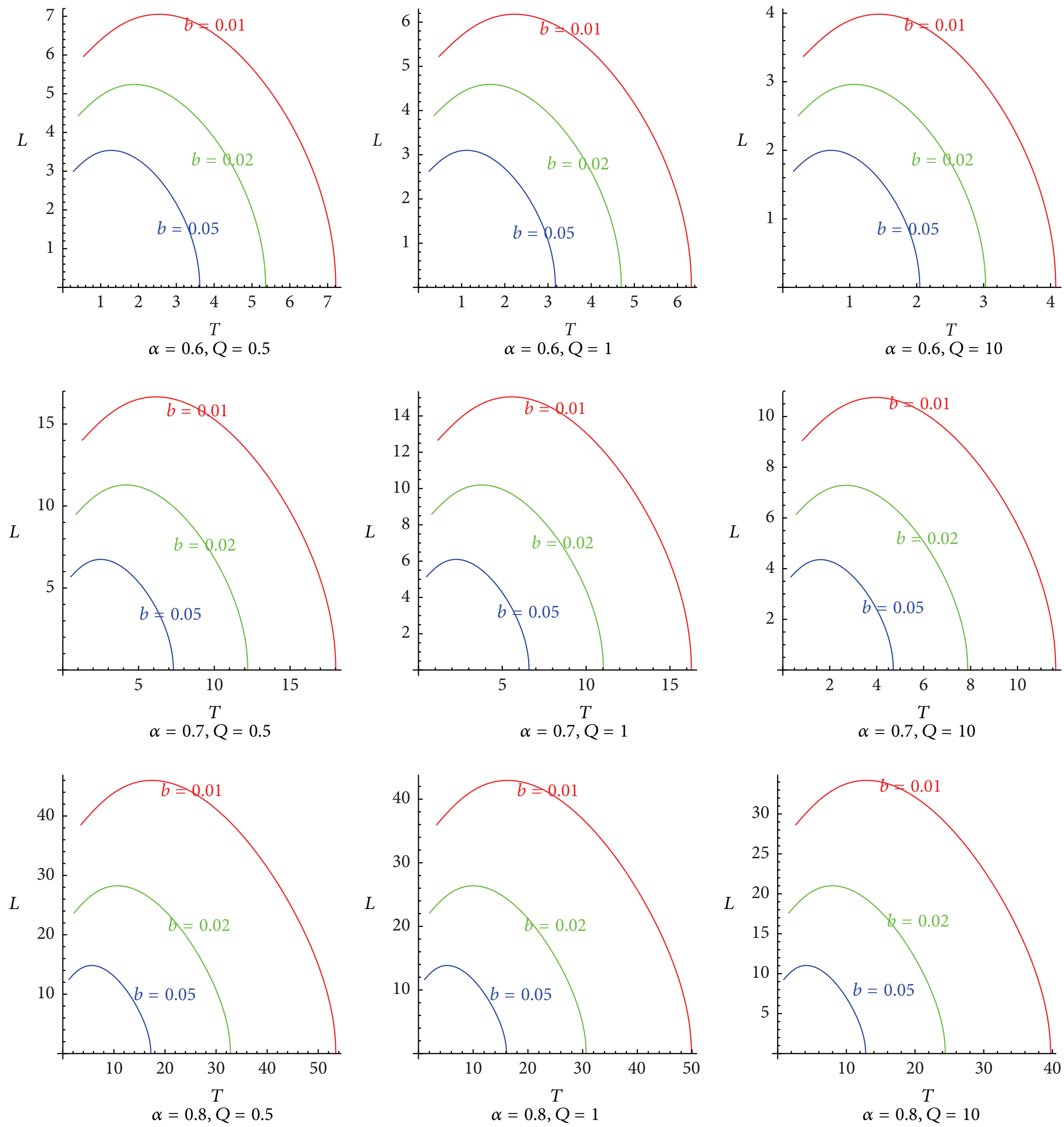

FIGURE 4: L-T curves for the topological dilaton black hole in n-dimensional AdS spacetime as $n = 5$. In each diagram, the highest curves (red) correspond to $b = 0.01$, the middle curves (green) meet $b = 0.02$, and the lowest curves (blue) are with $b = 0.05$.

and depict the boundary of two-phase coexistence region in Figure 2.

Taking black hole as an thermodynamic system, many investigations show that the phase transition of some black holes in AdS spacetime and dS spacetime is similar to that of van der Waals-Maxwell liquid-gas system [3, 5, 13–20, 36–38, 40–44], and phase transition of some other AdS black hole is alike to that of multicomponent superfluid or superconducting system [6, 8–10]. It would make sense if we can seek some observable system, such as van der Waals gas, to back analyze physical nature of black holes by their similar thermodynamic properties. That would help to further understand the thermodynamic quantities, such as entropy, temperature, and heat capacity, of black holes and that is significant for improving self-consistent thermodynamics theory of black holes.

Clapeyron equation of some usual thermodynamic systems agree well with experimental result. In this paper we have plotted the two-phase equilibrium curves in P-T diagrams, derived the slope of the curves, and acquired information on latent heat of phase change by Clapeyron equation, which could create condition for finding some usual thermodynamic systems similar to black holes in thermodynamic properties and provide theoretical basis for experimental research on analogous black holes.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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